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Contents

CHAPTER 1

Analog Modulation Techniques

1.1 Amplitude Modulation

Methodology:

Amplitude Modulation Basics Amplitude Modulation (AM) is a process where the amplitude of the carrier signal is varied in accordance with the instantaneous amplitude of the modulating signal.

1. Modulation Index (m):

- $m = \frac{V_m}{V_c}$ where V_m is the peak amplitude of the modulating signal and V_c is the peak amplitude of the carrier signal.
- Also $m = \frac{V_{max} - V_{min}}{V_{max} + V_{min}}$

2. Bandwidth (BW):

- $BW = 2f_m$ where f_m is the maximum frequency of the modulating signal.
- Upper Sideband Frequency (USB): $f_{USB} = f_c + f_m$
- Lower Sideband Frequency (LSB): $f_{LSB} = f_c - f_m$

3. Power Relations:

- Carrier Power: $P_c = \frac{V_c^2}{2R}$
- Total Power: $P_t = P_c \left(1 + \frac{m^2}{2}\right)$
- Sideband Power: $P_{SB} = P_{USB} + P_{LSB} = P_c \frac{m^2}{2}$

4. Current Relations:

- Total Current: $I_t = I_c\sqrt{1 + \frac{m^2}{2}}$

Worked Example:

Calculating Modulation Index and Bandwidth An audio signal of $15\sin(2\pi \times 1500t)$ amplitude modulates a carrier $60\sin(2\pi \times 100000t)$. Calculate the modulation index and bandwidth.

Step 1: Identify the parameters from the given equations.

- Modulating signal: $v_m(t) = 15\sin(2\pi \times 1500t)$
- Peak modulating voltage $V_m = 15$ V
- Modulating frequency $f_m = 1500$ Hz $= 1.5$ kHz
- Carrier signal: $v_c(t) = 60\sin(2\pi \times 100000t)$
- Peak carrier voltage $V_c = 60$ V

- Carrier frequency $f_c = 100000$ Hz $= 100$ kHz

Step 2: Calculate the modulation index (m).

$$m = \frac{V_m}{V_c} = \frac{15}{60} = 0.25$$

Step 3: Calculate the Bandwidth (BW).

$$BW = 2f_m = 2 \times 1500 = 3000 \text{ Hz} = 3 \text{ kHz}$$

Worked Example:

Power Calculation in AM A 400 W carrier is modulated to a depth of 75 percent. Calculate the total power in the modulated wave.

Step 1: Identify the given values.

- Carrier power $P_c = 400$ W
- Modulation index $m = 0.75$

Step 2: Apply the total power formula.

$$P_t = P_c\left(1 + \frac{m^2}{2}\right)$$
$$P_t = 400\left(1 + \frac{(0.75)^2}{2}\right)$$
$$P_t = 400\left(1 + \frac{0.5625}{2}\right)$$
$$P_t = 400(1 + 0.28125)$$
$$P_t = 400 \times 1.28125 = 512.5 \text{ W}$$

Worked Example:

Calculating Index from Voltage Extremes An AM wave has a maximum voltage of 15 V and a minimum voltage of 5 V. Calculate the modulation index and the carrier amplitude.

Step 1: Identify given maximum and minimum voltages.

- $V_{max} = 15$ V
- $V_{min} = 5$ V

Step 2: Calculate modulation index (m).

$$m = \frac{V_{max} - V_{min}}{V_{max} + V_{min}} = \frac{15 - 5}{15 + 5} = \frac{10}{20} = 0.5$$

Step 3: Calculate carrier amplitude (V_c). Since $V_{max} = V_c + V_m$ and $V_{min} = V_c - V_m$ adding them gives:

$$V_{max} + V_{min} = 2V_c$$
$$V_c = \frac{V_{max} + V_{min}}{2}$$
$$V_c = \frac{15 + 5}{2} = 10 \text{ V}$$

1.2 Frequency Modulation

Methodology:

Frequency Modulation Computations In Frequency Modulation (FM) the frequency of the carrier is varied in accordance with the instantaneous amplitude of the modulating signal.

1. Frequency Deviation (δ):

- $\delta = k_f V_m$ where k_f is the frequency sensitivity of the modulator (Hz/V) and V_m is the peak modulating voltage.

2. Modulation Index (m_f):

- $m_f = \frac{\delta}{f_m}$

3. Bandwidth (BW) using Carson Rule:

- $BW \approx 2(\delta + f_m)$ or $BW = 2f_m(m_f + 1)$

4. Total Power:

- The total power in an FM wave remains constant and is equal to the unmodulated carrier power.
- $P_t = \frac{V_c^2}{2R}$

Worked Example:

FM Modulation Index and Bandwidth In an FM system the modulating frequency is 2 kHz and the modulating voltage is 3 V. The frequency deviation is 15 kHz. Determine the modulation index and bandwidth.

Step 1: Identify given parameters.

- Modulating frequency $f_m = 2$ kHz
- Peak modulating voltage $V_m = 3$ V
- Frequency deviation $\delta = 15$ kHz

Step 2: Calculate modulation index (m_f).

$$m_f = \frac{\delta}{f_m} = \frac{15 \text{ kHz}}{2 \text{ kHz}} = 7.5$$

Step 3: Calculate Bandwidth using Carson rule.

$$BW = 2(\delta + f_m) = 2(15 \text{ kHz} + 2 \text{ kHz}) = 2(17 \text{ kHz}) = 34 \text{ kHz}$$

Worked Example:

Effect of Modulating Voltage and Frequency An FM transmission has a frequency deviation of 20 kHz when the modulating audio signal is 5 V at 1 kHz. Calculate the new frequency deviation and modulation index if the modulating signal is changed to 8 V at 2 kHz.

Step 1: Determine the frequency sensitivity (k_f).

- Initial frequency deviation $\delta_1 = 20$ kHz
- Initial modulating voltage $V_{m1} = 5$ V

$$k_f = \frac{\delta_1}{V_{m1}} = \frac{20 \text{ kHz}}{5 \text{ V}} = 4 \text{ kHz/V}$$

Step 2: Calculate new frequency deviation (δ_2).

- New modulating voltage $V_{m2} = 8$ V

$$\delta_2 = k_f V_{m2} = 4 \text{ kHz/V} \times 8 \text{ V} = 32 \text{ kHz}$$

Step 3: Calculate new modulation index (m_{f2}).

- New modulating frequency $f_{m2} = 2$ kHz

$$m_{f2} = \frac{\delta_2}{f_{m2}} = \frac{32 \text{ kHz}}{2 \text{ kHz}} = 16$$

1.3 Phase Modulation

Methodology:

Phase Modulation Formulas In Phase Modulation (PM) the phase of the carrier is varied in accordance with the instantaneous amplitude of the modulating signal.

1. Phase Deviation ($\Delta\theta$):

- $\Delta\theta = k_p V_m$ where k_p is the phase sensitivity (radians/V).

2. Modulation Index (m_p):

- $m_p = \Delta\theta$ (in radians)

3. Frequency Deviation in PM (δ):

- $\delta = f_m \Delta\theta = f_m m_p$

4. Bandwidth (BW) using Carson Rule:

- $BW \approx 2(\delta + f_m) = 2f_m(m_p + 1)$

Worked Example:

Phase Modulation Index and Deviation A carrier signal is phase-modulated by a 4 kHz modulating signal. The maximum phase deviation is 2.5 radians. Determine the modulation index and the equivalent frequency deviation.

Step 1: Identify given parameters.

- Modulating frequency $f_m = 4$ kHz
- Phase deviation $\Delta\theta = 2.5$ rad

Step 2: Determine modulation index (m_p).

$$m_p = \Delta\theta = 2.5 \text{ rad}$$

Step 3: Calculate equivalent frequency deviation (δ).

$$\delta = m_p f_m = 2.5 \times 4 \text{ kHz} = 10 \text{ kHz}$$

1.4 Comparison and Multi-tone Modulation

Methodology:

Multi-tone Amplitude Modulation When a carrier is modulated by multiple frequencies simultaneously the total modulation index and power calculations adjust as follows.

1. Total Modulation Index (m_t):

- $m_t = \sqrt{m_1^2 + m_2^2 + \cdots + m_n^2}$

2. Total Power (P_t):

- $P_t = P_c \left(1 + \frac{m_t^2}{2}\right)$

3. Total Current (I_t):

- $I_t = I_c \sqrt{1 + \frac{m_t^2}{2}}$

Worked Example:

Multi-tone AM Power Calculation A transmitter radiates 9 kW with the carrier unmodulated and 10.125 kW when the carrier is sinusoidally modulated. If another sine wave corresponding to 40 percent modulation is transmitted simultaneously calculate the total radiated power.

Step 1: Find the first modulation index (m_1).

- Carrier power $P_c = 9$ kW
- Total power with first signal $P_{t1} = 10.125$ kW

$$P_{t1} = P_c \left(1 + \frac{m_1^2}{2}\right)$$
$$10.125 = 9 \left(1 + \frac{m_1^2}{2}\right)$$
$$1.125 = 1 + \frac{m_1^2}{2}$$
$$0.125 = \frac{m_1^2}{2}$$
$$m_1^2 = 0.25 \implies m_1 = 0.5$$

Step 2: Find the total modulation index (m_t).

- Second modulation index $m_2 = 0.4$

$$m_t = \sqrt{m_1^2 + m_2^2}$$
$$m_t = \sqrt{(0.5)^2 + (0.4)^2}$$
$$m_t = \sqrt{0.25 + 0.16}$$
$$m_t = \sqrt{0.41} \approx 0.6403$$

Step 3: Calculate the new total radiated power.

$$P_{t\_new} = P_c \left(1 + \frac{m_t^2}{2}\right)$$
$$P_{t\_new} = 9 \left(1 + \frac{0.41}{2}\right)$$
$$P_{t\_new} = 9(1 + 0.205)$$
$$P_{t\_new} = 9 \times 1.205 = 10.845 \text{ kW}$$

CHAPTER 2

Analog Receivers

In this chapter, we focus on the fundamental computational problems related to analog radio receivers, specifically the superheterodyne receiver. Key numerical topics include calculating local oscillator frequency, image frequency, image rejection ratio, and tuning capacitor ranges.

2.1 Local Oscillator and Image Frequency Calculations

Methodology:

Local Oscillator and Image Frequencies In a superheterodyne receiver, the incoming radio frequency (RF) signal f_s is mixed with a local oscillator (LO) frequency f_{LO} to produce a fixed intermediate frequency (IF) f_{IF}.

1. Local Oscillator Frequency (f_{LO}): The local oscillator can operate either above or below the signal frequency.

- **High-side injection** (most common): $f_{LO} = f_s + f_{IF}$
- **Low-side injection**: $f_{LO} = f_s - f_{IF}$

Unless specified otherwise, high-side injection is typically assumed for AM broadcast receivers.

2. Image Frequency (f_{si}): The image frequency is an unwanted signal frequency that can also mix with the local oscillator to produce the intermediate frequency. It is located at twice the intermediate frequency away from the desired signal frequency.

- For high-side injection: $f_{si} = f_s + 2f_{IF}$
- For low-side injection: $f_{si} = f_s - 2f_{IF}$

Worked Example:

Calculating Local Oscillator and Image Frequencies An AM superheterodyne receiver is tuned to a signal frequency of 1000 kHz and uses an intermediate frequency (IF) of 455 kHz. Assuming high-side injection is used determine the local oscillator frequency and the image frequency.

Solution:

Step 1: Identify given parameters.

- Signal frequency, $f_s = 1000$ kHz
- Intermediate frequency, $f_{IF} = 455$ kHz

Step 2: Calculate the Local Oscillator Frequency. Using the formula for high-side injection:

$$f_{LO} = f_s + f_{IF}$$

$$f_{LO} = 1000 \text{ kHz} + 455 \text{ kHz} = 1455 \text{ kHz}$$

Step 3: Calculate the Image Frequency. Using the formula for image frequency:

$$f_{si} = f_s + 2f_{IF}$$

$$f_{si} = 1000 \text{ kHz} + 2(455 \text{ kHz})$$

$$f_{si} = 1000 \text{ kHz} + 910 \text{ kHz} = 1910 \text{ kHz}$$

2.2 Image Rejection Ratio (IRR)

Methodology:

Image Rejection Ratio Calculation The Image Rejection Ratio (IRR) is a measure of a receiver's ability to reject the unwanted image frequency signal. It depends on the Quality factor (Q) of the tuned RF circuit.

1. Calculate the parameter ρ:

$$\rho = \left(\frac{f_{si}}{f_s}\right) - \left(\frac{f_s}{f_{si}}\right)$$

where f_s is the signal frequency and f_{si} is the image frequency.

2. Calculate the IRR (α):

$$IRR = \sqrt{1 + (Q \cdot \rho)^2}$$

where Q is the quality factor of the tuned circuit.

3. Express IRR in Decibels (dB):

$$IRR_{dB} = 20 \log_{10}(IRR)$$

Worked Example:

Calculating Image Rejection Ratio A superheterodyne receiver has an IF of 455 kHz and is tuned to 1200 kHz. The RF stage has a quality factor (Q) of 80. Calculate the image rejection ratio in dB.

Solution:

Step 1: Identify given parameters.

- $f_s = 1200$ kHz
- $f_{IF} = 455$ kHz
- $Q = 80$

Step 2: Calculate the Image Frequency (f_{si}). Assuming high-side injection:

$$f_{si} = f_s + 2f_{IF} = 1200 + 2(455) = 1200 + 910 = 2110 \text{ kHz}$$

Step 3: Calculate the parameter ρ.

$$\rho = \frac{f_{si}}{f_s} - \frac{f_s}{f_{si}}$$

$$\rho = \frac{2110}{1200} - \frac{1200}{2110}$$

$$\rho \approx 1.7583 - 0.5687 = 1.1896$$

Step 4: Calculate the Image Rejection Ratio (IRR).

$$IRR = \sqrt{1 + (Q \cdot \rho)^2}$$

$$IRR = \sqrt{1 + (80 \cdot 1.1896)^2}$$

$$IRR = \sqrt{1 + (95.168)^2} = \sqrt{1 + 9056.95} \approx \sqrt{9057.95} \approx 95.17$$

Step 5: Convert IRR to decibels (dB).

$$IRR_{dB} = 20 \log_{10}(95.17)$$

$$IRR_{dB} \approx 20 \times 1.9785 \approx 39.57 \text{ dB}$$

2.3 Tuning Capacitor Range Calculations

Methodology:

Tuning Capacitor Range The resonant frequency of an LC tuned circuit is given by:

$$f = \frac{1}{2\pi\sqrt{LC}}$$

From this, the capacitance is inversely proportional to the square of the frequency:

$$C \propto \frac{1}{f^2}$$

Capacitance Ratio: For a receiver tuning over a frequency range from f_{min} to f_{max}, the required tuning capacitor range is given by the ratio of maximum to minimum capacitance:

$$\frac{C_{max}}{C_{min}} = \left(\frac{f_{max}}{f_{min}}\right)^2$$

This concept applies to both the RF tuning section (tuning f_s) and the local oscillator section (tuning f_{LO}).

Worked Example:

RF and Local Oscillator Tuning Ranges An AM radio receiver needs to tune across the broadcast band from 540 kHz to 1600 kHz. The receiver uses an IF of 455 kHz. Determine the required capacitance tuning ratio for the RF section and the local oscillator section.

Solution:

Step 1: Calculate the RF Section Capacitance Ratio. The RF section must tune the signal frequency f_s from $f_{min} = 540$ kHz to $f_{max} = 1600$ kHz.

$$\text{Ratio}_{RF} = \frac{C_{max}}{C_{min}} = \left(\frac{1600}{540}\right)^2$$

$$\text{Ratio}_{RF} = (2.963)^2 \approx 8.78$$

The RF tuning capacitor needs a maximum-to-minimum capacitance ratio of approximately $8.78 : 1$.

Step 2: Determine Local Oscillator Frequencies. Assuming high-side injection, $f_{LO} = f_s + f_{IF}$.

- Minimum LO frequency: $f_{LO,min} = f_{min} + f_{IF} = 540 + 455 = 995$ kHz
- Maximum LO frequency: $f_{LO,max} = f_{max} + f_{IF} = 1600 + 455 = 2055$ kHz

Step 3: Calculate the LO Section Capacitance Ratio. The local oscillator must tune from 995 kHz to 2055 kHz.

$$\text{Ratio}_{LO} = \frac{C_{max,LO}}{C_{min,LO}} = \left(\frac{2055}{995}\right)^2$$

$$\text{Ratio}_{LO} = (2.065)^2 \approx 4.26$$

The local oscillator tuning capacitor needs a capacitance ratio of approximately $4.26 : 1$.

CHAPTER 3

Pulse Modulation Techniques and Sampling Theory

3.1 Sampling Theory

Methodology:

Nyquist Rate and Nyquist Interval Calculations The sampling theorem states that a bandlimited signal $x(t)$ with maximum frequency f_m can be exactly reconstructed from its samples if the sampling frequency f_s satisfies specific conditions:

1. **Nyquist Rate** (f_N): The minimum sampling frequency required to avoid aliasing.

$$f_N = 2f_m$$

   where f_m is the maximum frequency component in the message signal.

2. **Nyquist Interval** (T_N): The maximum time interval between samples.

$$T_N = \frac{1}{f_N} = \frac{1}{2f_m}$$

3. **Sampling Frequency** (f_s): To avoid aliasing completely without distortion, the general condition is $f_s \geq 2f_m$.

Worked Example:

Calculate Nyquist Rate and Interval for a Given Signal Find the Nyquist rate and Nyquist interval for the continuous-time signal $x(t) = 5\cos(1000\pi t) + 2\sin(2000\pi t)$.
Solution:

1. Identify the frequencies of individual components in the signal. The general form for a component is $A\cos(2\pi ft)$ or $A\sin(2\pi ft)$.
   - For $5\cos(1000\pi t)$: $2\pi f_1 = 1000\pi \implies f_1 = 500$ Hz
   - For $2\sin(2000\pi t)$: $2\pi f_2 = 2000\pi \implies f_2 = 1000$ Hz

2. Find the maximum frequency f_m present in the signal.

$$f_m = \max(f_1, f_2) = \max(500, 1000) = 1000 \text{ Hz}$$

3. Calculate the Nyquist Rate f_N.

$$f_N = 2f_m = 2 \times 1000 = 2000 \text{ Hz}$$

4. Calculate the Nyquist Interval T_N.

$$T_N = \frac{1}{f_N} = \frac{1}{2000} = 0.5 \text{ ms}$$

The Nyquist rate is 2000 Hz and the Nyquist interval is 0.5 ms.

Worked Example:

Calculate Nyquist Rate for Product of Signals Find the Nyquist rate for the signal $y(t) = 10\cos(400\pi t) \cdot \cos(600\pi t)$.

Solution:

1. Express the product of trigonometric functions as a sum of individual frequency components. Use the trigonometric identity $\cos A \cos B = \frac{1}{2}[\cos(A+B) + \cos(A-B)]$.

$$y(t) = 10\left(\frac{\cos(400\pi t + 600\pi t) + \cos(400\pi t - 600\pi t)}{2}\right)$$

$$y(t) = 5[\cos(1000\pi t) + \cos(-200\pi t)]$$

   Since the cosine function is even $(\cos(-\theta) = \cos(\theta))$:

$$y(t) = 5\cos(1000\pi t) + 5\cos(200\pi t)$$

2. Identify the frequencies of the resulting components.
   - Component 1: $2\pi f_1 = 1000\pi \implies f_1 = 500$ Hz
   - Component 2: $2\pi f_2 = 200\pi \implies f_2 = 100$ Hz
3. Find the maximum frequency f_m.

$$f_m = \max(500, 100) = 500 \text{ Hz}$$

4. Calculate the Nyquist Rate.

$$f_N = 2f_m = 2 \times 500 = 1000 \text{ Hz}$$

3.2 Pulse Code Modulation

Methodology:

PCM Parameter Calculations In Pulse Code Modulation (PCM) systems converting an analog signal to a digital signal:

1. **Number of Quantization Levels (L):** For an n-bit encoder.

$$L = 2^n$$

2. **Step Size (Δ):** The voltage difference between adjacent quantization levels.

$$\Delta = \frac{V_{max} - V_{min}}{L} = \frac{V_{p-p}}{L} = \frac{V_{p-p}}{2^n}$$

3. **Maximum Quantization Error (Q_e):** The maximum deviation introduced by quantization.

$$(Q_e)_{max} = \pm\frac{\Delta}{2}$$

4. **Bit Rate (R_b):** The total rate at which bits are transmitted per second.

$$R_b = n \cdot f_s$$

   where n is the number of bits per sample and f_s is the sampling frequency.

5. **Transmission Bandwidth (BW):** The minimum baseband bandwidth required to transmit the PCM signal.
$$BW \geq \frac{R_b}{2} \implies BW_{min} = \frac{n \cdot f_s}{2}$$

Worked Example:

Calculate PCM Parameters for an Audio Signal An audio signal with a bandwidth of 4 kHz is sampled at 25% above the Nyquist rate. The signal is quantized into 256 levels. Calculate: (1) Sampling frequency (2) Number of bits per sample (3) Bit rate and (4) Minimum transmission bandwidth.
Solution:

1. **Calculate Sampling Frequency (f_s):** Given the maximum frequency $f_m = 4$ kHz. The Nyquist rate is $f_N = 2f_m = 2 \times 4$ kHz $= 8$ kHz. The actual sampling rate is 25% above the Nyquist rate:
$$f_s = f_N + 0.25 f_N = 1.25 \times 8 \text{ kHz} = 10 \text{ kHz}$$
2. **Calculate Number of bits per sample (n):** Given the number of quantization levels $L = 256$.
$$L = 2^n \implies 256 = 2^n \implies n = 8 \text{ bits/sample}$$
3. **Calculate Bit Rate (R_b):**
$$R_b = n \cdot f_s = 8 \text{ bits/sample} \times 10 \text{ kHz} = 80 \text{ kbps}$$
4. **Calculate Minimum Transmission Bandwidth (BW):**
$$BW_{min} = \frac{R_b}{2} = \frac{80 \text{ kbps}}{2} = 40 \text{ kHz}$$

Worked Example:

Calculate Step Size and Quantization Error A sinusoidal signal ranging from -5 V to $+5$ V is to be quantized using an 8-bit uniform quantizer. Calculate the step size and the maximum quantization error.
Solution:

1. Identify the given values: Maximum voltage $V_{max} = 5$ V, minimum voltage $V_{min} = -5$ V, $n = 8$ bits.
2. Calculate the peak-to-peak voltage (V_{p-p}):
$$V_{p-p} = V_{max} - V_{min} = 5 - (-5) = 10 \text{ V}$$
3. Calculate the number of quantization levels (L):
$$L = 2^n = 2^8 = 256$$
4. Calculate the step size (Δ):
$$\Delta = \frac{V_{p-p}}{L} = \frac{10}{256} = 0.0390625 \text{ V} \approx 39.06 \text{ mV}$$
5. Calculate the maximum quantization error (Q_e):
$$(Q_e)_{max} = \frac{\Delta}{2} = \frac{39.06 \text{ mV}}{2} = 19.53 \text{ mV}$$

Methodology:

Signal to Quantization Noise Ratio The Signal to Quantization Noise Ratio (SQNR) measures the quality of the quantized signal relative to the quantization noise.

1. **For a full-load sinusoidal signal:** The SQNR expressed in decibels (dB) for an n-bit uniform PCM system is approximated by the formula:

$$\text{SQNR (dB)} \approx 1.76 + 6.02n$$

2. **Rule of Thumb:** Each additional bit in the quantizer increases the SQNR by approximately 6 dB.

Worked Example:

Calculate Required Bits for a Target SQNR A PCM system is required to maintain a minimum Signal to Quantization Noise Ratio of 40 dB for a sinusoidal input. Determine the minimum number of bits per sample required.

Solution:

1. Start with the SQNR formula for a sinusoidal signal:

$$\text{SQNR (dB)} = 1.76 + 6.02n$$

2. Substitute the required minimum SQNR value:

$$40 = 1.76 + 6.02n$$

3. Solve for the number of bits n:

$$6.02n = 40 - 1.76$$
$$6.02n = 38.24$$
$$n = \frac{38.24}{6.02} \approx 6.35$$

4. Since the number of bits must be an integer, round up to the next higher integer to ensure the system meets or exceeds the minimum SQNR:

$$n = 7 \text{ bits/sample}$$

The minimum number of bits required is 7.

3.3 Delta Modulation

Methodology:

Delta Modulation Parameter Calculations Delta Modulation (DM) is a highly simplified 1-bit PCM system where the step size Δ is fixed.

1. **Bit Rate (R_b):** Since only $n = 1$ bit is transmitted per sample, the bit rate is equal to the sampling frequency.

$$R_b = f_s$$

2. **Slope Overload Condition:** This distortion occurs when the analog signal changes faster than the modulator can track. To avoid slope overload, the step size Δ must satisfy:

$$\frac{\Delta}{T_s} \geq \left|\frac{dm(t)}{dt}\right|_{max} \implies \Delta \cdot f_s \geq \left|\frac{dm(t)}{dt}\right|_{max}$$

3. **For a Sinusoidal Signal:** If the message signal is $m(t) = A_m \sin(2\pi f_m t)$, the maximum rate of change (slope) is $2\pi f_m A_m$. The condition to avoid slope overload becomes:

$$\Delta \cdot f_s \geq 2\pi f_m A_m \implies A_m \leq \frac{\Delta \cdot f_s}{2\pi f_m}$$

Worked Example:

Calculate Step Size to Avoid Slope Overload A sinusoidal signal $m(t) = 2\sin(2\pi \times 1000t)$ is applied to a Delta Modulator operating at a sampling frequency of 50 kHz. Determine the minimum step size required to avoid slope overload distortion.

Solution:

1. Identify the given parameters from the message signal $m(t)$: Peak amplitude $A_m = 2$ V Message frequency $f_m = 1000$ Hz Sampling frequency $f_s = 50$ kHz $= 50,000$ Hz

2. Calculate the maximum slope of the message signal:

$$\left|\frac{dm(t)}{dt}\right|_{max} = 2\pi f_m A_m = 2\pi(1000)(2) = 4000\pi \text{ V/s}$$

3. Set up the inequality to avoid slope overload:

$$\Delta \cdot f_s \geq \left|\frac{dm(t)}{dt}\right|_{max}$$

4. Substitute the calculated values and solve for the minimum step size Δ:

$$\Delta \cdot 50000 \geq 4000\pi$$

$$\Delta \geq \frac{4000\pi}{50000} = \frac{4\pi}{50} = \frac{12.566}{50} \approx 0.251 \text{ V}$$

The minimum step size required to avoid slope overload is 0.251 V or 251 mV.

Worked Example:

Determine Maximum Amplitude for Delta Modulator A Delta Modulation system is designed to operate at 3 times the Nyquist rate for a signal with a maximum frequency of 3 kHz. The step size is fixed at 250 mV. Find the maximum amplitude of a 3 kHz sinusoidal input for which slope overload does not occur.

Solution:

1. Calculate the Nyquist rate: Given $f_m = 3$ kHz, the Nyquist rate is $f_N = 2 \times 3$ kHz $= 6$ kHz.

2. Calculate the sampling frequency f_s: The system operates at 3 times the Nyquist rate.

$$f_s = 3 \times f_N = 3 \times 6 \text{ kHz} = 18 \text{ kHz} = 18000 \text{ Hz}$$

3. Identify the step size Δ and signal frequency f_m: $\Delta = 250$ mV $= 0.25$ V $f_m = 3$ kHz $= 3000$ Hz

4. Use the condition for avoiding slope overload to solve for the maximum amplitude $A_{m(max)}$:

$$A_{m(max)} = \frac{\Delta \cdot f_s}{2\pi f_m}$$

5. Substitute the values into the rearranged formula:

$$A_{m(max)} = \frac{0.25 \times 18000}{2\pi \times 3000} = \frac{4500}{6000\pi} = \frac{4.5}{6\pi} = \frac{0.75}{\pi} \approx 0.2387 \text{ V}$$

The maximum amplitude of the 3 kHz input signal without causing slope overload is approximately 0.239 V.

CHAPTER 4

Waveform Coding and Multiplexing

4.1 Pulse Code Modulation

Methodology:

PCM Bit Rate and Bandwidth Pulse Code Modulation (PCM) converts an analog signal into a digital sequence. The essential computational parameters are determined as follows:

1. **Sampling Rate (f_s):** According to the Nyquist theorem, the minimum sampling rate must be twice the maximum message frequency (f_m).
$$f_s \geq 2f_m$$
2. **Number of Quantization Levels (L):** If n bits are used per sample, the number of levels is:
$$L = 2^n$$
3. **Bit Rate (R_b):** The transmission bit rate is the product of the number of bits per sample and the sampling frequency:
$$R_b = nf_s$$
4. **Minimum Transmission Bandwidth (BW):** The minimum baseband bandwidth required for PCM transmission is half of the bit rate:
$$BW_{min} = \frac{R_b}{2}$$

Worked Example:

Bit Rate and Bandwidth of Voice Signal An analog voice signal with a maximum frequency of 4 kHz is to be transmitted using PCM. The signal is sampled at 20% above the Nyquist rate and quantized into 256 levels. Calculate the sampling frequency, number of bits per sample, transmission bit rate, and minimum bandwidth required.

Solution:

1. **Identify given parameters:** Message frequency $f_m = 4$ kHz
Number of levels $L = 256$
2. **Calculate the Nyquist rate:**
$$f_{nyq} = 2f_m = 2 \times 4000 = 8000 \text{ Hz}$$
3. **Calculate actual sampling frequency (f_s):** The signal is sampled at 20% above the Nyquist rate:
$$f_s = f_{nyq} + 0.2f_{nyq} = 1.2 \times 8000 = 9600 \text{ Hz}$$
4. **Calculate number of bits per sample (n):**
$$L = 2^n \implies 256 = 2^n \implies n = 8 \text{ bits/sample}$$

5. **Calculate transmission bit rate (R_b):**

$$R_b = n \times f_s = 8 \times 9600 = 76800 \text{ bps} = 76.8 \text{ kbps}$$

6. **Calculate minimum transmission bandwidth (BW_{min}):**

$$BW_{min} = \frac{R_b}{2} = \frac{76800}{2} = 38400 \text{ Hz} = 38.4 \text{ kHz}$$

4.2 Quantization Error and SQNR

Methodology:

Signal to Quantization Noise Ratio When an analog signal is quantized, an error is introduced called quantization noise. The Signal-to-Quantization Noise Ratio (SQNR) measures the quality of the quantization process.

1. **Step Size (Δ):** For a signal with peak-to-peak amplitude $V_{pp} = V_{max} - V_{min}$ and L quantization levels:

$$\Delta = \frac{V_{pp}}{L}$$

2. **Quantization Noise Power (N_q):** Assuming a uniform distribution of quantization error:

$$N_q = \frac{\Delta^2}{12}$$

3. **Signal Power (P_s):** For a sinusoidal signal of peak amplitude A_m:

$$P_s = \frac{A_m^2}{2}$$

4. **SQNR in Decibels:** The ratio in dB is given by:

$$SQNR_{dB} = 10 \log_{10} \left(\frac{P_s}{N_q} \right)$$

For a full-scale sinusoidal test signal, a simplified formula dependent only on the number of bits n is:

$$SQNR_{dB} \approx 1.76 + 6.02n$$

Worked Example:

Calculating Step Size and SQNR A sinusoidal signal with a peak amplitude of 5 V is quantized by a 10-bit uniform PCM system. Calculate the step size, quantization noise power, and the SQNR in decibels.
Solution:

1. **Identify given parameters:** Peak amplitude $A_m = 5$ V
Number of bits $n = 10$

2. **Calculate number of quantization levels (L):**

$$L = 2^n = 2^{10} = 1024$$

3. **Calculate step size (Δ):** Peak-to-peak voltage $V_{pp} = 2 \times A_m = 10$ V

$$\Delta = \frac{V_{pp}}{L} = \frac{10}{1024} \approx 0.00976 \text{ V}$$

4. **Calculate quantization noise power (N_q):**

$$N_q = \frac{\Delta^2}{12} = \frac{(0.00976)^2}{12} \approx \frac{0.0000953}{12} \approx 7.94 \times 10^{-6} \text{ W}$$

5. **Calculate signal power (P_s):**

$$P_s = \frac{A_m^2}{2} = \frac{5^2}{2} = 12.5 \text{ W}$$

6. **Calculate SQNR in dB using direct ratio:**

$$SQNR_{dB} = 10\log_{10}\left(\frac{12.5}{7.94 \times 10^{-6}}\right) = 10\log_{10}(1574307.3) \approx 61.97 \text{ dB}$$

7. *Alternatively, using the approximation formula:*

$$SQNR_{dB} \approx 1.76 + 6.02(10) = 61.96 \text{ dB}$$

4.3 Delta Modulation

Methodology:

Delta Modulation Step Size for Slope Overload Delta Modulation (DM) uses a single bit per sample to indicate whether the signal has increased or decreased. The core computational problem is avoiding slope overload distortion.

1. **Bit Rate (R_b):** Since there is 1 bit per sample, bit rate equals the sampling frequency:

$$R_b = f_s$$

2. **Maximum Signal Slope:** For a sinusoidal signal $m(t) = A_m \sin(2\pi f_m t)$, the maximum rate of change (slope) is:

$$\left|\frac{dm(t)}{dt}\right|_{max} = 2\pi f_m A_m$$

3. **Maximum Modulator Slope:** The maximum rate at which the DM staircase approximation can change is:

$$\text{Modulator Slope} = \Delta \cdot f_s$$

Where Δ is the step size.

4. **Condition to Avoid Slope Overload:** The modulator slope must be greater than or equal to the maximum signal slope:

$$\Delta \cdot f_s \geq 2\pi f_m A_m$$

5. **Minimum Step Size (Δ_{min}):**

$$\Delta_{min} = \frac{2\pi f_m A_m}{f_s}$$

Worked Example:

Preventing Slope Overload in Delta Modulation A sinusoidal message signal $m(t) = 3\sin(2\pi \times 10^3 t)$ is applied to a Delta Modulator operating with a sampling frequency of 32 kHz. Determine the minimum step size required to prevent slope overload distortion. What is the resulting bit rate?

Solution:

1. **Identify signal parameters:** From $m(t)$, peak amplitude $A_m = 3$ V

Message frequency $f_m = 1000$ Hz
Sampling frequency $f_s = 32000$ Hz

2. **Calculate maximum signal slope:**

$$\text{Max Slope} = 2\pi f_m A_m = 2\pi(1000)(3) = 6000\pi \approx 18849.5 \text{ V/s}$$

3. **Set up the condition for no slope overload:**

$$\Delta \cdot f_s \geq 2\pi f_m A_m$$

4. **Solve for minimum step size (Δ_{min}):**

$$\Delta \geq \frac{18849.5}{32000}$$

$$\Delta_{min} = 0.589 \text{ V}$$

5. **Determine the bit rate (R_b):** Because DM uses 1 bit per sample:

$$R_b = 1 \times f_s = 32000 \text{ bps} = 32 \text{ kbps}$$

4.4 Multiplexing

Methodology:

Time Division Multiplexing Parameters Time Division Multiplexing (TDM) interleaves multiple digital signals in time.

1. **Frame Rate (f_{frame}):** If all channels are sampled at the same frequency f_s, the frame rate equals the sampling frequency:
$$f_{frame} = f_s$$

2. **Number of Bits per Frame (N_{frame}):** If there are K channels, each outputting n bits per sample, and a extra framing/synchronization bits are added per frame:

$$N_{frame} = (K \times n) + a$$

3. **Total TDM Bit Rate (R_{TDM}):** The overall transmission rate is the bits per frame multiplied by the frame rate:
$$R_{TDM} = N_{frame} \times f_{frame}$$

Worked Example:

TDM Bit Rate Calculation Four voice channels, each with a bandwidth of 3 kHz, are multiplexed using TDM. Each channel is sampled at 8 kHz and encoded using 8-bit PCM. One synchronization bit is added at the end of each frame. Calculate the number of bits per frame, the frame rate, and the total transmission bit rate.
Solution:

1. **Identify parameters:** Number of channels $K = 4$
Bits per sample $n = 8$
Sync bits per frame $a = 1$
Sampling frequency $f_s = 8000$ Hz

2. **Calculate frame rate (f_{frame}):** One frame contains one sample from each channel, generated at the

sampling rate:

$$f_{frame} = f_s = 8000 \text{ frames/second}$$

3. **Calculate number of bits per frame (N_{frame}):**

$$N_{frame} = (K \times n) + a$$

$$N_{frame} = (4 \times 8) + 1 = 32 + 1 = 33 \text{ bits/frame}$$

4. **Calculate total TDM bit rate (R_{TDM}):**

$$R_{TDM} = N_{frame} \times f_{frame}$$

$$R_{TDM} = 33 \times 8000 = 264000 \text{ bps} = 264 \text{ kbps}$$

Methodology:

Frequency Division Multiplexing Bandwidth Frequency Division Multiplexing (FDM) shifts the frequency of multiple analog baseband signals so they can be transmitted over a single channel without overlapping. Guard bands are inserted between adjacent channels to prevent cross-talk.

1. **Individual Channel Bandwidth (BW_i):** Determined by the modulation scheme used to shift the signal (e.g., for AM, $BW_i = 2f_m$; for SSB, $BW_i = f_m$).
2. **Number of Guard Bands:** For K multiplexed channels, there are typically $K - 1$ guard bands required between them.
3. **Total FDM Bandwidth (BW_{total}):** The sum of all individual channel bandwidths plus the sum of all guard band bandwidths (BW_g):

$$BW_{total} = \sum_{i=1}^{K} BW_i + (K - 1) \times BW_g$$

Worked Example:

FDM Total Bandwidth Three distinct baseband signals with bandwidths of 10 kHz, 20 kHz, and 30 kHz are multiplexed using FDM. Single Sideband (SSB) modulation is used, so the modulated bandwidth equals the baseband bandwidth. A guard band of 5 kHz is inserted between adjacent multiplexed channels. Find the total bandwidth required for the FDM transmission.

Solution:

1. **Identify channel bandwidths:** $BW_1 = 10$ kHz, $BW_2 = 20$ kHz, $BW_3 = 30$ kHz.
2. **Identify number of channels and guard bands:** Number of channels $K = 3$.
   Number of guard bands $= K - 1 = 3 - 1 = 2$.
3. **Calculate total signal bandwidth:**

$$\text{Signal BW} = BW_1 + BW_2 + BW_3 = 10 + 20 + 30 = 60 \text{ kHz}$$

4. **Calculate total guard band bandwidth:**

$$\text{Guard BW} = 2 \times BW_g = 2 \times 5 = 10 \text{ kHz}$$

5. **Calculate total FDM bandwidth:**

$$BW_{total} = \text{Signal BW} + \text{Guard BW} = 60 + 10 = 70 \text{ kHz}$$

CHAPTER 5

Antenna and Wave Propagation

Antennas act as transducers between guided electromagnetic waves and free-space waves. Wave propagation studies how these electromagnetic waves travel from the transmitting antenna to the receiving antenna through various layers of the Earth's atmosphere. In this chapter, we focus on the fundamental computational aspects of antenna parameters and wave propagation mechanisms such as Line-of-Sight distance and Maximum Usable Frequency (MUF).

5.1 Fundamental Antenna Parameters

The performance of an antenna is quantified using several key parameters including Directivity, Gain, Radiation Efficiency and Effective Aperture.

Methodology:

Calculating Antenna Gain and Efficiency The efficiency and gain of an antenna relate to its physical losses and its directivity.

1. **Radiation Efficiency (η):** The ratio of power radiated (P_{rad}) to the total input power (P_{in}). It depends on radiation resistance (R_r) and loss resistance (R_l).

$$\eta = \frac{P_{rad}}{P_{in}} = \frac{R_r}{R_r + R_l}$$

2. **Directivity (D):** The ratio of radiation intensity in a given direction to that of an isotropic source.

3. **Gain (G):** Gain incorporates the antenna's efficiency.

$$G = \eta \times D$$

4. **Gain in Decibels (dB):** Gain is frequently expressed in logarithmic scale.

$$G_{dB} = 10 \log_{10}(G)$$

Worked Example:

Antenna Efficiency and Gain Calculation An antenna has a radiation resistance of $72\,\Omega$ and a loss resistance of $8\,\Omega$. If its directivity is 20, calculate its radiation efficiency and gain in decibels.

Step 1: Identify given parameters.

Radiation resistance $(R_r) = 72\,\Omega$

Loss resistance $(R_l) = 8\,\Omega$

Directivity $(D) = 20$

Step 2: Calculate radiation efficiency (η).

$$\eta = \frac{R_r}{R_r + R_l} = \frac{72}{72 + 8} = \frac{72}{80} = 0.90 \text{ (or } 90\%)$$

Step 3: Calculate Gain (G).

$$G = \eta \times D = 0.90 \times 20 = 18$$

Step 4: Convert Gain to decibels (G_{dB}).

$$G_{dB} = 10 \log_{10}(18) \approx 10 \times 1.255 = 12.55\,\text{dB}$$

Final Answer: The radiation efficiency is 90% and the gain is 12.55 dB.

Methodology:

Calculating Effective Aperture The effective aperture (A_e) measures how effectively an antenna captures electromagnetic power from a passing wave.

1. **Determine wavelength (λ):** If frequency (f) is given, find $\lambda = \frac{c}{f}$, where $c = 3 \times 10^8\,\text{m/s}$.
2. **Apply the Aperture formula:**

$$A_e = \frac{\lambda^2}{4\pi} \times G$$

where G is the absolute (linear) power gain, not the dB value.

Worked Example:

Effective Area of a Parabolic Reflector Calculate the effective area of an antenna operating at 3 GHz with a gain of 1000.

Step 1: Calculate wavelength (λ).

$$\lambda = \frac{c}{f} = \frac{3 \times 10^8}{3 \times 10^9} = 0.1\,\text{m}$$

Step 2: Apply the effective aperture formula.
Given $G = 1000$.

$$A_e = \frac{\lambda^2}{4\pi} \times G = \frac{(0.1)^2}{4\pi} \times 1000 = \frac{0.01}{12.566} \times 1000 = \frac{10}{12.566} \approx 0.796\,\text{m}^2$$

Final Answer: The effective aperture of the antenna is approximately $0.796\,\text{m}^2$.

5.2 EIRP and Friis Transmission

When transmitting power, it is essential to determine the equivalent power density at the receiver and the actual received power in free space.

Methodology:

EIRP and Power Density

1. **Equivalent Isotropic Radiated Power EIRP:** The total power that an isotropic antenna would have to radiate to produce the same peak power density as the actual antenna.

$$EIRP = P_t \times G_t$$

Where P_t is the transmitted power and G_t is the absolute gain of the transmitting antenna.

2. **Power Density P_D:** The power per unit area at a distance d from the transmitting antenna.

$$P_D = \frac{EIRP}{4\pi d^2} = \frac{P_t G_t}{4\pi d^2}$$

Worked Example:

Power Density Calculation A directional antenna with a gain of 50 radiates a power of 20 W. Calculate the Equivalent Isotropic Radiated Power and the power density at a distance of 5 km from the antenna.

Step 1: Calculate EIRP.

$P_t = 20\,\text{W}$ and $G_t = 50$.

$$EIRP = P_t \times G_t = 20 \times 50 = 1000\,\text{W}$$

Step 2: Calculate Power Density.

Distance $d = 5\,\text{km} = 5000\,\text{m}$.

$$P_D = \frac{EIRP}{4\pi d^2} = \frac{1000}{4\pi(5000)^2} = \frac{1000}{4\pi(25 \times 10^6)} = \frac{1000}{3.1416 \times 10^8}$$

$$P_D = \frac{1000}{314159265} \approx 3.18 \times 10^{-6}\,\text{W/m}^2 = 3.18\,\mu\text{W/m}^2$$

Final Answer: The EIRP is 1000 W and the power density at 5 km is $3.18\,\mu\text{W/m}^2$.

Methodology:

Using the Friis Equation The Friis Equation calculates the power received (P_r) by a receiving antenna in free-space line-of-sight propagation:

1. **Ensure consistent units:** Gain must be in linear scale (convert from dB using $G = 10^{G_{dB}/10}$). Transmitted power (P_t) and distance (d) must be in standard units (Watts and meters).

2. **Calculate wavelength (λ):** using $\lambda = c/f$.

3. **Apply Friis Transmission Formula:**

$$P_r = P_t \times G_t \times G_r \times \left(\frac{\lambda}{4\pi d}\right)^2$$

Where: P_r = Received Power
P_t = Transmitted Power
G_t = Gain of transmitting antenna
G_r = Gain of receiving antenna
d = Separation distance

Worked Example:

Received Power in Free Space A transmitter operates at 300 MHz and radiates 50 W of power. The transmitting antenna has a gain of 10 dB and the receiving antenna has a gain of 15 dB. If the receiver is located 10 km away in free space, determine the received power.

Step 1: Convert gains to linear values.

$$G_t = 10^{10/10} = 10^1 = 10$$

$$G_r = 10^{15/10} = 10^{1.5} \approx 31.62$$

Step 2: Calculate wavelength (λ).

$$\lambda = \frac{3 \times 10^8}{300 \times 10^6} = 1\,\text{m}$$

Step 3: Apply the Friis Transmission Formula.

Given $P_t = 50\,\text{W}$, $d = 10,000\,\text{m}$.

$$P_r = 50 \times 10 \times 31.62 \times \left(\frac{1}{4\pi(10000)}\right)^2$$

$$P_r = 15810 \times \left(\frac{1}{125663.7}\right)^2 \approx 15810 \times (7.957 \times 10^{-6})^2$$

$$P_r \approx 15810 \times (6.33 \times 10^{-11}) \approx 1.00 \times 10^{-6}\,\text{W} = 1\,\mu\text{W}$$

Final Answer: The received power is approximately $1\,\mu$W.

5.3 Line of Sight Propagation

In space wave propagation, radio waves travel in a straight line from transmitting antenna to receiving antenna. Due to the Earth's curvature, the maximum transmission distance (Line of Sight) is limited by the heights of both antennas.

Methodology:

Calculating Radio Horizon and Line of Sight Distance Radio waves undergo slight refraction in the atmosphere. The "effective earth radius" model modifies the physical horizon distance formula.

1. **Radio Horizon of a single antenna:** The distance to the radio horizon d from an antenna of height h (in meters) is given by:

$$d \approx 4.12\sqrt{h} \quad \text{(Result in km)}$$

2. **Maximum Line of Sight Distance (d_{max}):** If the transmitting antenna has height h_t and receiving antenna has height h_r (both in meters), the maximum distance for communication is:

$$d_{max} = d_t + d_r = 4.12\sqrt{h_t} + 4.12\sqrt{h_r}$$

Where d_{max} is measured in kilometers.

Worked Example:

Maximum Range of Space Wave Communication A transmitting television antenna has a height of 144 m and the receiving antenna has a height of 25 m. Calculate the maximum Line-of-Sight distance between them.

Step 1: Identify given values.
$h_t = 144\,\text{m}$
$h_r = 25\,\text{m}$
Step 2: Calculate radio horizon for the transmitter (d_t).

$$d_t = 4.12\sqrt{h_t} = 4.12\sqrt{144} = 4.12 \times 12 = 49.44\,\text{km}$$

Step 3: Calculate radio horizon for the receiver (d_r).

$$d_r = 4.12\sqrt{h_r} = 4.12\sqrt{25} = 4.12 \times 5 = 20.60\,\text{km}$$

Step 4: Find maximum Line-of-Sight distance (d_{max}).

$$d_{max} = d_t + d_r = 49.44 + 20.60 = 70.04\,\text{km}$$

Final Answer: The maximum Line-of-Sight distance is 70.04 km.

5.4 Sky Wave Propagation

Sky wave propagation relies on the reflection (refraction) of radio waves from the Earth's ionosphere. It is primarily used for high-frequency (HF) communication.

Methodology:

Critical Frequency MUF and Skip Distance

1. **Critical Frequency (f_c):** The highest frequency that is returned to Earth when sent vertically upward. It depends on the maximum electron density (N_{max}, in electrons per cubic meter).

$$f_c = 9\sqrt{N_{max}}$$

2. **Maximum Usable Frequency (MUF):** The highest frequency that can be used for communication between two specific points. It is determined by the secant law, where θ_i is the angle of incidence at the ionosphere.

$$f_{MUF} = f_c \sec\theta_i = \frac{f_c}{\cos\theta_i}$$

3. **Skip Distance (D_{skip}):** The minimum distance from the transmitter at which a sky wave of given frequency $f_{MUF} > f_c$ is returned to Earth. Given the virtual height of the ionospheric layer h:

$$D_{skip} = 2h\sqrt{\left(\frac{f_{MUF}}{f_c}\right)^2 - 1}$$

Worked Example:

Critical Frequency from Electron Density Calculate the critical frequency for an ionospheric layer if the maximum electron density is 1.44×10^{12} electrons/m^3.

Step 1: Identify given parameter.

$N_{max} = 1.44 \times 10^{12}$

Step 2: Apply the critical frequency formula.

$$f_c = 9\sqrt{N_{max}}$$

$$f_c = 9\sqrt{1.44 \times 10^{12}} = 9 \times (1.2 \times 10^6)$$

$$f_c = 10.8 \times 10^6\,\text{Hz} = 10.8\,\text{MHz}$$

Final Answer: The critical frequency is 10.8 MHz.

Worked Example:

Maximum Usable Frequency and Skip Distance A high-frequency radio link is established via the ionosphere. The critical frequency of the layer is 5 MHz and the virtual height of the layer is 200 km. If the angle of incidence of the radio wave is 60°, calculate the Maximum Usable Frequency (MUF) and the Skip Distance.

Step 1: Calculate the MUF. Given $f_c = 5\,\text{MHz}$ and $\theta_i = 60°$.

$$f_{MUF} = \frac{f_c}{\cos\theta_i} = \frac{5\,\text{MHz}}{\cos(60°)} = \frac{5}{0.5} = 10\,\text{MHz}$$

Step 2: Calculate the Skip Distance (D_{skip}). Given virtual height $h = 200\,\text{km}$.

$$D_{skip} = 2h\sqrt{\left(\frac{f_{MUF}}{f_c}\right)^2 - 1}$$

$$D_{skip} = 2(200)\sqrt{\left(\frac{10}{5}\right)^2 - 1} = 400\sqrt{2^2 - 1} = 400\sqrt{4-1} = 400\sqrt{3}$$

$$D_{skip} \approx 400 \times 1.732 = 692.8\,\text{km}$$

Final Answer: The Maximum Usable Frequency is 10 MHz and the skip distance is 692.8 km.