Heat and Mass Transfer (DI05019071)

Complete Lecture Deck

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Steady State Diffusion Through a Plain Membrane

- Course: Heat and Mass Transfer (DI05019071)
- Unit 5: Mass Transfer
- Lecture 44: Steady State Diffusion Through a Plain Membrane

Lecture Agenda

- 1. Problem Definition and Assumptions
- 2. The Governing Equation
- 3. Deriving the Concentration Profile
- 4. Mass Transfer Rate Calculation
- 5. The Concept of Mass Transfer Resistance
- 6. Practical Applications

Problem Definition & Assumptions

- **Scenario**: A species A (e.g., hydrogen gas) diffuses through a plain solid membrane (species B, e.g., a steel plate) of thickness L.
- **Assumptions**:
- 1. One-dimensional mass transfer (x-direction).
- 2. Steady-state conditions (∂C_A/∂t = 0).
- 3. No chemical reactions (Generation rate R_A = 0).
- 4. The solid membrane is stationary (bulk velocity = 0).
- 5. Constant diffusion coefficient D_AB.

The Governing Equation

- Starting from the general mass diffusion equation:
- $\partial C\_A / \partial t = D\_AB * (\partial\hat{}2\ C\_A / \partial x\hat{}2) + R\_A$
- Applying assumptions (Steady state, no reaction):
- $0 = D\_AB * (d\hat{}2\ C\_A / dx\hat{}2)$
- Therefore:
- $d\hat{}2\ C\_A / dx\hat{}2 = 0$

Concentration Profile Derivation

- Equation: d^2 C_A / dx^2 = 0
- Integrating once: dC_A / dx = C1
- Integrating twice: C_A(x) = C1 * x + C2
- **Boundary Conditions**:
- At x = 0, C_A = C_A1
- At x = L, C_A = C_A2
- Resulting Profile: C_A(x) = C_A1 - (C_A1 - C_A2) * (x / L)

Rate of Mass Transfer

- Now apply Fick's Law to find the molar transfer rate (N_A * Area):
- N_A = - D_AB * (dC_A / dx)
- From our profile, dC_A / dx = - (C_A1 - C_A2) / L
- Therefore, Molar Flux: N_A = D_AB * (C_A1 - C_A2) / L
- Total Molar Rate (W_A) in kmol/s for Area A:
- W_A = N_A * A = [D_AB * A * (C_A1 - C_A2)] / L

Mass Transfer Resistance Concept

- Rearranging the rate equation to fit the Ohm's law form (Rate = Driving Force / Resistance):
- $W_A = (C_{A1} - C_{A2}) / R_{mass}$
- Where the Mass Transfer Resistance for a plain membrane is:
- $R_{mass} = L / (D_{AB} * A)$
- (Analogy: Thermal resistance of a plane wall $R_{th} = L / (k * A)$)

Mass Transfer in Composite Membranes

- The resistance concept allows easy analysis of composite (multi-layer) membranes.
- For membranes in series, mass transfer resistances simply add up:
- R_total = R_m1 + R_m2 + ... = L1/(D_1*A) + L2/(D_2*A) + ...
- W_A = (C_A,in - C_A,out) / R_total

Practical Application: Gas Diffusion in Solids

- A common scenario is a gas diffusing through a solid container wall (e.g., Helium escaping a balloon, Hydrogen diffusing through steel).
- The concentration of gas at the solid surface (C_{A1}) is determined by **solubility**.
- Henry's Law (or similar relations) relates the partial pressure of the gas outside to the concentration just inside the solid surface: $C_{A1} = S * p_{A1}$ (where S is solubility).

Example Problem Concept

- **Problem**: Hydrogen gas is maintained at 3 bar and 1 bar on opposite sides of a plastic membrane 2 mm thick. Determine mass transfer rate.
- **Steps**:
- 1. Convert pressures to surface concentrations (C_{A1}, C_{A2}) using given solubility.
- 2. Calculate mass transfer resistance: $R_{mass} = L / (D_{AB} * A)$
- 3. Calculate Rate: $W_A = (C_{A1} - C_{A2}) / R_{mass}$

Summary

- 1D steady-state diffusion through a plain membrane yields a linear concentration profile.
- The mass transfer rate is analogous to heat conduction through a plane wall.
- Mass Transfer Resistance is defined as $R\_mass = L / (D\_AB * A)$.
- For gas diffusion into solids, boundary concentrations are determined by solubility and partial pressures.

Next Lecture Preview

- Topic: Steady State Equimolar Counter Diffusion & Convective Mass Transfer
- Key questions to ponder:
- - What happens if two gases are diffusing into each other in opposite directions at the same rate?
- - How do we analyze mass transfer when the bulk fluid is moving (convection)?
- - What is the mass transfer coefficient?