Heat and Mass Transfer (DI05019071)

Complete Lecture Deck

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LMTD for Parallel and Counter Flow Exchangers

- Course: Heat and Mass Transfer (DI05019071)
- Unit 4: Heat Exchanger
- Lecture 32: LMTD for Parallel and Counter Flow Exchangers

Lecture Agenda

- 1. Introduction to Mean Temperature Difference
- 2. Temperature Distribution in Parallel Flow
- 3. Derivation of LMTD for Parallel Flow
- 4. Temperature Distribution in Counter Flow
- 5. Derivation of LMTD for Counter Flow
- 6. Comparison of Flow Types

Introduction to LMTD

- In a heat exchanger, temperatures of both fluids change as they flow.
- The temperature difference ΔT between hot and cold fluids varies along the length.
- Newton's Law of Cooling: $q = U A \Delta T\_mean$
- We need an appropriate average ΔT, which is the Logarithmic Mean Temperature Difference (LMTD).

Why Not Arithmetic Mean?

- Arithmetic Mean Temperature Difference (AMTD) = $(\Delta T1 + \Delta T2) / 2$
- AMTD is only accurate if the temperature profiles are linear.
- In reality, heat transfer rate decreases as ΔT decreases, leading to exponential temperature profiles.
- Therefore, AMTD overestimates the actual mean temperature difference.

Temperature Distribution in Parallel Flow

- Both hot and cold fluids enter at the same end and flow in the same direction.
- Hot fluid temperature (Th) drops; cold fluid temperature (Tc) rises.
- ΔT is maximum at the inlet (ΔT1) and minimum at the outlet (ΔT2).
- $\Delta T1 = Th,in - Tc,in$
- $\Delta T2 = Th,out - Tc,out$

Derivation of LMTD for Parallel Flow (Part 1)

- Consider a small area element dA.
- Heat transfer: $dq = U\ dA\ (Th - Tc) = U\ dA\ \Delta T$
- Energy balance for hot fluid: $dq = -\ m\_h\ cp\_h\ dTh = -\ Ch\ dTh$
- Energy balance for cold fluid: $dq = m\_c\ cp\_c\ dTc = Cc\ dTc$
- $d(\Delta T) = dTh - dTc = -dq(1/Ch + 1/Cc)$

Derivation of LMTD for Parallel Flow (Part 2)

- Substitute $dq = U\, dA\, \Delta T$ into the equation:
- $d(\Delta T) = - U\, dA\, \Delta T\, (1/Ch + 1/Cc)$
- Rearranging: $d(\Delta T)/\Delta T = - U\, (1/Ch + 1/Cc)\, dA$
- Integrating from inlet (1) to outlet (2):
- $\ln(\Delta T2\ /\ \Delta T1) = - U\, A\, (1/Ch + 1/Cc)$

Final LMTD Formula for Parallel Flow

- From overall energy balance: $1/Ch = (Th,in - Th,out)/q$ and $1/Cc = (Tc,out - Tc,in)/q$
- Substitute these back:
- $\ln(\Delta T2 / \Delta T1) = - U A / q * (\Delta T1 - \Delta T2)$
- Rearranging for q: $q = U A [(\Delta T1 - \Delta T2) / \ln(\Delta T1 / \Delta T2)]$
- Thus, $LMTD = (\Delta T1 - \Delta T2) / \ln(\Delta T1 / \Delta T2) = (\Delta T2 - \Delta T1) / \ln(\Delta T2 / \Delta T1)$

Temperature Distribution in Counter Flow

- Fluids enter at opposite ends and flow in opposite directions.
- Inlet 1: Hot fluid in, Cold fluid out. $\Delta T1 = Th,in - Tc,out$
- Inlet 2: Hot fluid out, Cold fluid in. $\Delta T2 = Th,out - Tc,in$
- Temperature difference ΔT is more uniform along the length.

Derivation of LMTD for Counter Flow

- Energy balances are similar, but dTc is negative along positive x.
- dq = - Ch dTh = - Cc dTc (since cold fluid flows opposite)
- $d(\Delta T)$ = dTh - dTc = -dq(1/Ch - 1/Cc)
- Integrating gives: $\ln(\Delta T2 / \Delta T1)$ = - U A (1/Ch - 1/Cc)

Final LMTD Formula for Counter Flow

- Substituting the capacity rates as before yields the exact same mathematical form:
- LMTD = $(\Delta T1 - \Delta T2) / \ln(\Delta T1 / \Delta T2)$
- IMPORTANT: The definitions of $\Delta T1$ and $\Delta T2$ are different from parallel flow!
- Parallel: $\Delta T1 = T_{h,in} - T_{c,in}$; $\Delta T2 = T_{h,out} - T_{c,out}$
- Counter: $\Delta T1 = T_{h,in} - T_{c,out}$; $\Delta T2 = T_{h,out} - T_{c,in}$

Comparison: Parallel vs Counter Flow

- For the same inlet and outlet temperatures, LMTD_counter ¿ LMTD_parallel.
- Higher LMTD means a smaller required Area (A) for the same heat transfer (q).
- Counter flow is more efficient and requires a smaller heat exchanger.
- Counter flow allows Tc,out ¿ Th,out, which is impossible in parallel flow.

Summary & Next Lecture

- LMTD is the appropriate average temperature difference for heat exchangers.
- The formula is LMTD $= (\Delta T1 - \Delta T2) / \ln(\Delta T1 / \Delta T2)$.
- ΔT definitions depend heavily on the flow arrangement.
- Counter flow is thermally more efficient than parallel flow.
- Next Lecture: LMTD for Condensers and Evaporators, and Cross-flow arrangements.