Heat and Mass Transfer (DI05019071)

Complete Lecture Deck

Milav Dabgar

Lecturer, GP Palanpur

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Critical Radius of Insulation

- Course: Heat and Mass Transfer (DI05019071)
- Unit 1: Conduction
- Lecture 9: Critical radius of insulation for a cylinder

Lecture Agenda

- 1. The Paradox of Adding Insulation
- 2. Competing Effects: Conduction vs. Convection
- 3. Thermal Resistance Network
- 4. Deriving the Critical Radius
- 5. Physical Significance
- 6. Application: Cables vs. Steam Pipes

The Paradox of Adding Insulation

- For a plane wall: Adding insulation ALWAYS increases thermal resistance and decreases heat transfer.
- For a cylinder: Adding insulation adds conduction resistance, BUT it also increases the outer surface area.
- Larger outer area means lower convection resistance at the surface.
- Question: Which effect dominates?

Thermal Resistance Network

- Consider a pipe of radius r1, coated with insulation up to outer radius r.
- Surrounding fluid has convection coefficient h.
- $R\_total = R\_cond\_insulation + R\_conv\_outer$
- $R\_total = [\ln(r/r1) \ / \ (2\pi Lk)] + [1 \ / \ (h^*2\pi rL)]$

Total Resistance vs. Radius

- Let's plot R_total as a function of the outer insulation radius (r).
- Initially, R_total decreases because the drop in convection resistance is faster than the rise in conduction resistance.
- It reaches a minimum, then begins to increase.
- Minimum resistance = Maximum heat transfer!

Deriving Critical Radius

- To find the minimum, take the derivative of R_total with respect to r and set it to zero.
- $d(R\_total)/dr = [1 \ / \ (2\pi Lkr)] - [1 \ / \ (h*2\pi Lr^2)] = 0$
- $1/(kr) = 1/(hr^2)$
- $r = k \ / \ h$

The Critical Radius (rc)

- Critical radius of insulation: rc = k / h
- Where k is the thermal conductivity of the INSULATION.
- h is the convection coefficient of the SURROUNDING FLUID.
- For a sphere, rc = 2k / h (derived similarly).

Physical Significance of rc

- If r_outer ¡ rc: Adding insulation INCREASES heat transfer (acts as a cooling fin).
- If r_outer = rc: Heat transfer is MAXIMUM.
- If r_outer ¿ rc: Adding insulation DECREASES heat transfer (acts as an insulator).

Application: Electrical Cables

- Electrical wires are very small (radius ¡¡ rc).
- The plastic/rubber coating (insulation) often has an outer radius smaller than rc.
- Result: The 'insulation' actually helps cool the wire and prevents it from melting due to electrical resistance heating.
- It provides electrical insulation but thermal enhancement!

Application: Steam Pipes

- Steam pipes are large (radius ¿ rc).
- Adding ANY amount of insulation to a steam pipe will immediately increase the thermal resistance.
- Result: For large pipes, insulation always acts to conserve heat.
- We don't need to worry about rc for large industrial pipes.

Example Problem

- A 2 mm diameter wire is coated with 1 mm plastic (k = 0.15 W/mK).
- Surrounding air h = 15 W/m^2K.
- Calculate the critical radius and determine if the plastic increases or decreases heat dissipation.

Example Problem Solution

- rc = k / h = 0.15 / 15 = 0.01 m = 10 mm
- Current outer radius of the wire + plastic = 1 mm + 1 mm = 2 mm.
- Since current radius (2 mm) ¡ rc (10 mm), the plastic coating INCREASES the heat dissipation.
- The wire runs cooler with the plastic than if it were bare!

Next Lecture Preview

- Topic: Overall heat transfer coefficient (U)
- How do we combine conduction and convection into one easy-to-use term?
- We will introduce the concept of the Overall Heat Transfer Coefficient.