Heat and Mass Transfer (DI05019071)

Complete Lecture Deck

Milav Dabgar

Lecturer, GP Palanpur

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- Course: Heat and Mass Transfer (DI05019071)
- Unit 1: Conduction
- Lecture 4: Heat Conduction through Plane and Composite Walls

Lecture Agenda

- 1. Temperature Profile in a Plane Wall
- 2. Thermal Resistance Review
- 3. Concept of Composite Walls
- 4. Thermal Circuit for Composite Walls
- 5. Convection at Boundaries
- 6. Overall Heat Transfer Coefficient (U)
- 7. Thermal Contact Resistance

Conduction in a Plane Wall

- Governing Equation: $\frac{d^2T}{dx^2} = 0$
- Integrating once: $\frac{dT}{dx} = C_1$
- Integrating twice: $T(x) = C_1 x + C_2$
- Boundary Conditions:
- 1. At $x = 0$, $T = T_1$
- 2. At $x = L$, $T = T_2$

Temperature Profile

- Applying boundary conditions to $T(x) = C_1 x + C_2$:
- - From BC 1: $C_2 = T_1$
- - From BC 2: $T_2 = C_1 L + T_1 \Rightarrow C_1 = \frac{T_2 - T_1}{L}$
- Resulting Temperature Profile:
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$$T(x) = T\_1 - (T\_1 - T\_2) nfracxL$$
- Conclusion: The temperature distribution in a plane wall at steady state is linear.

- Using Fourier's Law: $Q = -kA\frac{dT}{dx}$
- Since $\frac{dT}{dx} = \frac{T_2 - T_1}{L}$, then:
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$$Q = -kAnfrac T\_2 - T\_1 L = kAnfrac T\_1 - T\_2 L$$
- In terms of Thermal Resistance ($R_{th} = L/kA$):
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$$Q = nfrac T\_1 - T\_2 R\_th$$

Composite Walls

- Most real-world structures are composite (multiple layers in series).
- Example: A building wall consisting of brick, insulation, and plasterboard.
- Key Principle: At steady state, the rate of heat transfer (Q) is the SAME through all layers in series.
- $$Q = Q\_A = Q\_B = Q\_C$$

Thermal Circuit for Composite Walls

- For a 3-layer wall (A, B, C) with interface temperatures T_1, T_2, T_3, T_4:
- Total Resistance $R_{total} = R_A + R_B + R_C$
- $$R\_total = nfrac L\_Ak\_AA + nfrac L\_Bk\_BA + nfrac L\_Ck\_CA$$
- $$Q = nfrac T\_1 - T\_4R\_total$$

Adding Convection Boundaries

- Surfaces are usually exposed to fluids (air, water) at temperatures $T_{\infty 1}$ and $T_{\infty 2}$.
- Convection Heat Transfer (Newton's Law of Cooling):
- $$Q = hA(T\_surface - T\_fluid)$$
- Convection Resistance:
- $$R\_conv = nfrac1hA$$
- Where h is the convective heat transfer coefficient $(W/m^2 \cdot K)$.

Comprehensive Thermal Circuit

- Wall with fluid on both sides:
- $R_{total} = R_{conv,in} + R_{wall} + R_{conv,out}$
- $$R\_total = nfrac1h\_inA + nfracLkA + nfrac1h\_outA$$
- $$Q = nfracT\_ninfty, in - T\_ninfty, outR\_total$$

Overall Heat Transfer Coefficient (U)

- It is convenient to express heat transfer as:
- $$Q = UAnDeltaT\_overall$$
- Where U is the Overall Heat Transfer Coefficient $(W/m^2 \cdot K)$.
- Comparing with $Q = \Delta T/R_{total}$, we get:
- $$UA = nfrac1R\_totalnRightarrowU = nfrac1AncdotR\_total$$
- For a plane wall: $U = \frac{1}{\frac{1}{h_{in}} + \frac{L}{k} + \frac{1}{h_{out}}}$

Thermal Contact Resistance

- When two solid surfaces are pressed together, they only touch at discrete high spots due to microscopic roughness.
- The gaps are filled with air (a poor conductor).
- This creates a resistance to heat flow at the interface, called Thermal Contact Resistance ($R_{t,c}$).
- Results in a sudden temperature drop ($\Delta T_{interface}$) across the junction.

Summary

- Temperature profile in a 1D steady-state plane wall is linear.
- Composite walls are analyzed using series thermal resistance networks.
- Convection at boundaries adds a resistance of $1/(hA)$.
- The Overall Heat Transfer Coefficient (U) represents the total thermal conductance.
- Contact resistance accounts for imperfect mating of surfaces.

Next Lecture Preview

- Topic: Critical radius of insulation for a cylinder
- Key questions to ponder:
- - Does adding insulation always reduce heat loss?
- - How is conduction through a cylinder different from a plane wall?
- - What happens to the surface area as you add layers to a pipe?