Heat and Mass Transfer (DI05019071)

Complete Lecture Deck

Milav Dabgar

Lecturer, GP Palanpur

July 23, 2026

Course Agenda I

Fourier's Law

- Course: Heat and Mass Transfer (DI05019071)
- Unit 1: Conduction
- Lecture 1: Fourier's Law

Lecture Agenda

- 1. Introduction to Heat Transfer
- 2. Modes of Heat Transfer
- 3. Introduction to Conduction
- 4. Fourier's Law of Heat Conduction
- 5. Assumptions of Fourier's Law
- 6. Concept of Temperature Gradient

What is Heat Transfer?

- Heat Transfer is the science that deals with the rate of exchange of thermal energy between physical systems.
- Driving Force: Temperature Difference.
- Thermodynamics vs. Heat Transfer:
- - Thermodynamics deals with the amount of heat transferred and equilibrium states.
- - Heat Transfer deals with the rate of heat transfer and non-equilibrium phenomena.

Modes of Heat Transfer

- Thermal energy can be transferred in three distinct modes:
- 1. Conduction: Transfer of energy from more energetic particles to adjacent less energetic ones (requires a medium).
- 2. Convection: Transfer of energy between a solid surface and the adjacent fluid that is in motion.
- 3. Radiation: Transfer of energy due to the emission of electromagnetic waves (no medium required).

Focus on Conduction

- Conduction occurs in solids, liquids, and gases.
- In Gases and Liquids: Due to collisions and diffusion of molecules during their random motion.
- In Solids: Due to the combination of lattice vibrations of molecules and energy transport by free electrons.

Fourier's Law of Heat Conduction

- The fundamental law governing heat conduction is Fourier's Law, proposed by Joseph Fourier in 1822.
- Statement: The rate of heat conduction is proportional to the area measured normal to the direction of heat flow, and to the temperature gradient in that direction.

Mathematical Formulation

- Mathematically, Fourier's Law is expressed as:
- $$QnproptoAnfracdTdx$$
- $$Q = -kAnfracdTdx$$
- Where:
- - Q: Rate of heat transfer (W)
- - A: Cross-sectional area (m^2)
- - dT/dx: Temperature gradient (K/m or °C/m)
- - k: Thermal conductivity (W/m·K)

The Negative Sign in Fourier's Law

- Why is there a negative sign in $Q = -kA\frac{dT}{dx}$?
- - Heat always flows in the direction of decreasing temperature (from hot to cold).
- - Therefore, the temperature gradient dT/dx is inherently negative in the direction of positive heat flow (x).
- - The negative sign is inserted to make the heat transfer rate Q a positive quantity.

Assumptions of Fourier's Law

- Fourier's Law is based on several underlying assumptions:
- 1. Conduction of heat takes place under steady-state conditions.
- 2. The heat flow is unidirectional (1D).
- 3. The temperature gradient is constant, leading to a linear temperature profile.
- 4. There is no internal heat generation within the body.
- 5. The bounding surfaces are isothermal.

Temperature Gradient

- Temperature Gradient (dT/dx): The rate of change of temperature with respect to distance.
- For a plane wall of thickness L and surface temperatures T_1 and T_2:
- $$nfracdTdx = nfracT\_2 - T\_1L$$
- Since $T_1 > T_2$, dT/dx is negative.

Applications of Fourier's Law

- Fourier's Law is used to calculate:
- - Heat loss through building walls and roofs.
- - Heat transfer across heat exchanger tubes.
- - Thermal insulation required for pipes and furnaces.
- - Cooling rates of electronic components.

Summary

- Heat transfer is driven by temperature differences.
- Conduction is the transfer of heat through a stationary medium.
- Fourier's Law dictates that $Q = -kA(dT/dx)$.
- The negative sign ensures Q is positive as heat flows down the temperature gradient.

Next Lecture Preview

- Topic: Thermal Conductivity and Thermal Resistance
- Key questions to ponder:
- - What physical properties determine the value of 'k'?
- - How does temperature affect thermal conductivity?
- - Can we analyze heat transfer using electrical circuit concepts?

Thermal conductivity and thermal resistance

- Course: Heat and Mass Transfer (DI05019071)
- Unit 1: Conduction
- Lecture 2: Thermal conductivity and thermal resistance

Lecture Agenda

- 1. Recap of Fourier's Law
- 2. Thermal Conductivity (k) Defined
- 3. Thermal Conductivity of Materials
- 4. Effect of Temperature on k
- 5. The Electrical Analogy
- 6. Concept of Thermal Resistance
- 7. Application to a Plane Wall

What is Thermal Conductivity (k)?

- From Fourier's Law: $k = \frac{Q/A}{-(dT/dx)}$
- Definition: Thermal conductivity is the amount of heat conducted per unit time across unit area and through unit thickness, when a temperature difference of unit degree is maintained.
- It is a thermophysical property of a material.
- High $k \rightarrow$ Good heat conductor.
- Low $k \rightarrow$ Good heat insulator.

Units and Dimensions of k

- Unit of Heat Transfer Rate (Q): Watts (W) or J/s
- Unit of Area (A): m^2
- Unit of Temperature Gradient (dT/dx): K/m or °C/m
- Therefore, Unit of k:
- $$k = nfracWm\hat{}2ncdot(K/m) = W/mncdotK$$
- Dimensional Formula: $[M^1L^1T^{-3}\Theta^{-1}]$

Thermal Conductivity of Different Materials

- Order of Thermal Conductivity:
- Pure Metals ¿ Alloys ¿ Non-metallic Solids ¿ Liquids ¿ Gases
- Examples ($W/m \cdot K$ at room temp):
- - Copper: ˜400
- - Aluminum: ˜237
- - Steel: ˜15 to 50
- - Water: ˜0.6
- - Air: ˜0.026

Why are Metals Good Conductors?

- In solids, heat is conducted by two mechanisms:
- 1. Lattice vibration (k_l)
- 2. Transport by free electrons (k_e)
- Total $k = k_l + k_e$
- In pure metals, free electrons are abundant, making k_e dominant.
- In non-metals, there are no free electrons, so conduction is entirely due to lattice vibration (k_l).

Temperature Dependence of k

- Thermal conductivity is not strictly constant; it varies with temperature.
- Generally modeled as a linear function: $k = k_0(1 + \beta T)$
- Where:
- - k_0: Conductivity at reference temp (e.g., $0^\circ C$)
- - β: Temperature coefficient of thermal conductivity
- For most pure metals, k decreases with increasing T ($\beta < 0$).
- For gases and insulating materials, k increases with increasing T ($\beta > 0$).

The Electrical Analogy

- Heat flow can be modeled similarly to electrical current flow.
- Ohm's Law (Electrical): $I = \frac{\Delta V}{R_e}$
- - I = Current (Flow)
- - ΔV = Voltage Difference (Driving Potential)
- - R_e = Electrical Resistance
- Fourier's Law (Thermal): $Q = \frac{\Delta T}{R_{th}}$
- - Q = Heat Flow Rate (Flow)
- - ΔT = Temperature Difference (Driving Potential)
- - R_{th} = Thermal Resistance

Concept of Thermal Resistance

- Rearranging Fourier's Law for a plane wall of thickness L:
- $$Q = kAnfrac T\_1 - T\_2L$$
- $$Q = nfrac T\_1 - T\_2L/(kA)$$
- Comparing with $Q = \frac{\Delta T}{R_{th}}$, we get:
- $$R\_th = nfracLkA$$
- Thermal Resistance to Conduction (R_{cond}) for a plane wall.

Thermal Circuit for a Plane Wall

- Drawing a thermal circuit helps visualize the problem.
- Nodes represent Temperatures (T_1, T_2).
- Resistors represent Thermal Resistance (R_{th}).
- Current represents Heat Flow (Q).

Significance of Thermal Resistance

- - Simplifies complex heat transfer problems (e.g., composite walls, cylinders).
- - Allows analysis of heat transfer networks in series and parallel.
- - Identifies the 'bottleneck' in heat transfer (the layer with the highest resistance dictates the overall heat flow).
- - Essential for designing insulation systems.

Summary

- Thermal conductivity (k) is a material property defining heat conduction capability.
- Metals have high k due to free electrons; gases have low k.
- The electrical analogy maps Heat Flow (Q) to Current (I) and Temperature Difference (ΔT) to Voltage (ΔV).
- Conduction thermal resistance for a plane wall is $R = L/(kA)$.

Next Lecture Preview

- Topic: Derivation of General Heat Conduction Equation
- Key questions to ponder:
- - How do we model heat transfer in 3-dimensions?
- - What if the material is generating heat internally?
- - How is the general equation reduced to 1D steady state?

Derivation of one-dimensional steady state conduction

- Course: Heat and Mass Transfer (DI05019071)
- Unit 1: Conduction
- Lecture 3: General Heat Conduction Equation

Lecture Agenda

- 1. Need for a General Equation
- 2. The Control Volume and Energy Balance
- 3. Heat Flux Components (x, y, z directions)
- 4. Internal Heat Generation and Storage
- 5. General 3D Conduction Equation
- 6. Simplified Forms (Poisson, Laplace, Fourier)
- 7. Reduction to 1D Steady State Equation

Why a General Equation?

- Fourier's Law $Q = -kA(dT/dx)$ is sufficient for simple 1D steady-state problems.
- However, real problems can involve:
- - Multi-dimensional heat flow (x, y, and z).
- - Unsteady (transient) conditions where temperature changes with time.
- - Internal heat generation (e.g., electrical heaters, chemical reactions, nuclear fuel rods).
- We need a differential equation that governs temperature distribution $T(x, y, z, t)$ in a medium.

The Control Volume

- Consider a small differential element (Control Volume) in a Cartesian coordinate system.
- Dimensions: dx, dy, dz.
- Volume of element: $dV = dx \cdot dy \cdot dz$.
- Heat is conducted into and out of this element across its six faces.

Energy Balance Equation

- Based on the First Law of Thermodynamics (Conservation of Energy):
- (Rate of heat conducted IN)
- + (Rate of heat GENERATED internally)
- = (Rate of heat conducted OUT)
- + (Rate of energy STORED)
-

$$Q\_in + Q\_gen = Q\_out + Q\_stored$$

Heat Flow in X-Direction

- Heat entering left face (at x): $Q_x = -k(dy \cdot dz)\frac{\partial T}{\partial x}$
- Heat leaving right face (at $x + dx$): Using Taylor Series expansion (neglecting higher order terms):
- $$Q\_x + dx = Q\_x + nfracnpartialQ\_xnpartialxdx$$
- Net heat accumulated in x-direction:
- $$Q\_x - Q\_x + dx = -nfracnpartialQ\_xnpartialxdx = nfracnpartial$$

Heat Flow in Y and Z Directions

- Similarly, net heat accumulated in y and z directions:
- Y-direction:
-

$$Q\_y - Q\_y + dy = nfrac npartial npartial y nleft(knfrac npartial T npa$$

- Z-direction:
-

$$Q\_z - Q\_z + dz = nfrac npartial npartial z nleft(knfrac npartial T npa$$

Heat Generation and Energy Storage

- Internal Heat Generation (Q_{gen}):
- Let q_g be heat generated per unit volume (W/m^3).
-

$$Q\_gen = q\_g(dxncdotdyncdotdz)$$

- Energy Storage (Q_{stored}):
- Rate of change of internal energy.
-

$$Q\_stored = mncdotC\_pncdotnfracnpartialTnpartialt = (nrhoncdo$$

- where ρ is density, C_p is specific heat, t is time.

General 3D Conduction Equation

- Substitute all terms back into the energy balance equation and divide by volume ($dx \cdot dy \cdot dz$):
- $$nfrac npartial l npartial x nleft(knfrac npartial l T npartial x nright)+nfrac$$
- If thermal conductivity (k) is constant (isotropic material):
- $$nfrac npartial l\hat{}2 T npartial x\hat{}2 + nfrac npartial l\hat{}2 T npartial y\hat{}2 + nfrac np$$

Thermal Diffusivity (α)

- Thermal Diffusivity, $\alpha = \frac{k}{\rho C_p}$
- - Units: m^2/s
- - Physical Meaning: Ratio of heat conducted (k) to heat stored (ρC_p).
- - High α: Heat propagates rapidly into the medium (e.g., metals).
- - Low α: Heat is mostly absorbed by the material, slow propagation (e.g., wood, insulation).
- Equation becomes: $\nabla^2 T + \frac{q_g}{k} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$

Simplified Forms of the Equation

- 1. Steady-state ($\partial T/\partial t = 0$): Poisson's Equation
- $$nnablâ2T + nfracq\_gk = 0$$
- 2. No heat generation ($q_g = 0$): Fourier's Equation (Transient)
- $$nnablâ2T = nfrac1nalphanfracnpartialTnpartialt$$
- 3. Steady-state & No generation: Laplace Equation
- $$nnablâ2T = 0$$

Reduction to 1D Steady State Conduction

- For 1-Dimensional (x-direction only), Steady-State, No internal heat generation:
- - 1D: $\partial^2 T/\partial y^2 = 0$, $\partial^2 T/\partial z^2 = 0$
- - Steady-state: $\partial T/\partial t = 0$
- - No generation: $q_g = 0$
- The general equation reduces to:
-

$$nfracd\hat{}2T\,dx\hat{}2 = 0$$

- This is the governing equation for a simple plane wall.

Next Lecture Preview

- Topic: Heat Conduction through a Plane Wall and Composite Systems
- Key questions to ponder:
- - How do we apply the 1D steady state equation to real walls?
- - What happens when a wall is made of multiple layers of different materials?
- - How does the electrical analogy make solving composite walls easy?

Heat Conduction through Plane and Composite Walls

- Course: Heat and Mass Transfer (DI05019071)
- Unit 1: Conduction
- Lecture 4: Heat Conduction through Plane and Composite Walls

Lecture Agenda

- 1. Temperature Profile in a Plane Wall
- 2. Thermal Resistance Review
- 3. Concept of Composite Walls
- 4. Thermal Circuit for Composite Walls
- 5. Convection at Boundaries
- 6. Overall Heat Transfer Coefficient (U)
- 7. Thermal Contact Resistance

Conduction in a Plane Wall

- Governing Equation: $\frac{d^2T}{dx^2} = 0$
- Integrating once: $\frac{dT}{dx} = C_1$
- Integrating twice: $T(x) = C_1x + C_2$
- Boundary Conditions:
- 1. At $x = 0$, $T = T_1$
- 2. At $x = L$, $T = T_2$

Temperature Profile

- Applying boundary conditions to $T(x) = C_1 x + C_2$:
- - From BC 1: $C_2 = T_1$
- - From BC 2: $T_2 = C_1 L + T_1 \Rightarrow C_1 = \frac{T_2 - T_1}{L}$
- Resulting Temperature Profile:
- $$T(x) = T\_1 - (T\_1 - T\_2)nfracxL$$
- Conclusion: The temperature distribution in a plane wall at steady state is linear.

Heat Rate through Plane Wall

- Using Fourier's Law: $Q = -kA\frac{dT}{dx}$
- Since $\frac{dT}{dx} = \frac{T_2 - T_1}{L}$, then:
- $$Q = -kAnfrac T\_2 - T\_1 L = kAnfrac T\_1 - T\_2 L$$
- In terms of Thermal Resistance ($R_{th} = L/kA$):
- $$Q = nfrac T\_1 - T\_2 R\_th$$

Composite Walls

- Most real-world structures are composite (multiple layers in series).
- Example: A building wall consisting of brick, insulation, and plasterboard.
- Key Principle: At steady state, the rate of heat transfer (Q) is the SAME through all layers in series.
-

$$Q = Q\_A = Q\_B = Q\_C$$

Thermal Circuit for Composite Walls

- For a 3-layer wall (A, B, C) with interface temperatures T_1, T_2, T_3, T_4:
- Total Resistance $R_{total} = R_A + R_B + R_C$
- $$R\_total = nfracL\_Ak\_AA + nfracL\_Bk\_BA + nfracL\_Ck\_CA$$
- $$Q = nfracT\_1 - T\_4R\_total$$

Adding Convection Boundaries

- Surfaces are usually exposed to fluids (air, water) at temperatures $T_{\infty 1}$ and $T_{\infty 2}$.
- Convection Heat Transfer (Newton's Law of Cooling):
- $$Q = hA(T\_surface - T\_fluid)$$
- Convection Resistance:
- $$R\_conv = nfrac1hA$$
- Where h is the convective heat transfer coefficient $(W/m^2 \cdot K)$.

Comprehensive Thermal Circuit

- Wall with fluid on both sides:
- $R_{total} = R_{conv,in} + R_{wall} + R_{conv,out}$
- $$R\_total = nfrac1h\_inA + nfracLkA + nfrac1h\_outA$$
- $$Q = nfracT\_ninfty, in - T\_ninfty, outR\_total$$

Overall Heat Transfer Coefficient (U)

- It is convenient to express heat transfer as:
-

$$Q = UAnDeltaT\_overall$$

- Where U is the Overall Heat Transfer Coefficient ($W/m^2 \cdot K$).
- Comparing with $Q = \Delta T / R_{total}$, we get:
-

$$UA = nfrac1R\_totalnRightarrowU = nfrac1AncdotR\_total$$

- For a plane wall: $U = \frac{1}{\frac{1}{h_{in}} + \frac{L}{k} + \frac{1}{h_{out}}}$

Thermal Contact Resistance

- When two solid surfaces are pressed together, they only touch at discrete high spots due to microscopic roughness.
- The gaps are filled with air (a poor conductor).
- This creates a resistance to heat flow at the interface, called Thermal Contact Resistance ($R_{t,c}$).
- Results in a sudden temperature drop ($\Delta T_{interface}$) across the junction.

Summary

- Temperature profile in a 1D steady-state plane wall is linear.
- Composite walls are analyzed using series thermal resistance networks.
- Convection at boundaries adds a resistance of $1/(hA)$.
- The Overall Heat Transfer Coefficient (U) represents the total thermal conductance.
- Contact resistance accounts for imperfect mating of surfaces.

Next Lecture Preview

- Topic: Critical radius of insulation for a cylinder
- Key questions to ponder:
- - Does adding insulation always reduce heat loss?
- - How is conduction through a cylinder different from a plane wall?
- - What happens to the surface area as you add layers to a pipe?

Critical radius of insulation for a cylinder

- Course: Heat and Mass Transfer (DI05019071)
- Unit 1: Conduction
- Lecture 5: Critical radius of insulation for a cylinder

Lecture Agenda

- 1. Conduction in Radial Systems (Cylinders)
- 2. Thermal Resistance of a Cylinder
- 3. Adding Insulation: Plane Wall vs. Cylinder
- 4. The Competing Effects in a Cylinder
- 5. Definition of Critical Radius
- 6. Mathematical Derivation of r_c
- 7. Practical Applications

Conduction through a Hollow Cylinder

- Consider a hollow cylinder (pipe) of length L, inner radius r_1, outer radius r_2.
- Heat flows radially outwards.
- Fourier's Law in cylindrical coordinates (1D radial):
- $$Q = -kAnfracdTdr$$
- Important: The area A is not constant! It varies with radius r.
- Surface area at radius r: $A = 2\pi rL$

Thermal Resistance of a Cylinder

- Substitute $A = 2\pi rL$ into Fourier's Law:
- $$Q = -k(2npirL)nfracdT\,dr$$
- Separate variables and integrate from r_1 to r_2 and T_1 to T_2:
- $$Qnint\_r\_1\hat{}r\_2nfracdrr = -2npikLnint\_T\_1\hat{}T\_2dT$$
- $$Q = nfrac2npikL(T\_1 - T\_2)nln(r\_2/r\_1)$$
- Thermal Resistance: $R_{cyl} = \frac{\ln(r_2/r_1)}{2\pi kL}$

Adding Insulation: Plane Wall vs Cylinder

- Plane Wall:
- Adding insulation increases thickness (L).
- Conduction resistance (L/kA) increases.
- Outer area (A) stays constant, so convection resistance ($1/hA$) is constant.
- Result: Total resistance ALWAYS increases. Heat loss ALWAYS decreases.
- Cylinder:
- Adding insulation increases outer radius (r_2).

The Competing Effects in a Cylinder

- When you add insulation to a pipe (increasing outer radius r_0):
- 1. Conduction Resistance ($R_{cond} = \frac{\ln(r_0/r_i)}{2\pi kL}$) INCREASES.
- - Tends to decrease heat transfer.
- 2. Convection Resistance ($R_{conv} = \frac{1}{h(2\pi r_0 L)}$) DECREASES (because outer surface area $A = 2\pi r_0 L$ increases).
- - Tends to increase heat transfer.
- Which effect dominates?

Total Thermal Resistance

- Consider a pipe covered with insulation of outer radius r.
- Total Resistance from pipe surface to ambient air:
- $$R\_total = R\_insulation + R\_convection$$
- $$R\_total = nfracnln(r/r\_i)2npikL + nfrac1h(2npirL)$$
- Where k is insulation conductivity and h is outside convection coefficient.
- Heat transfer $Q = \frac{\Delta T}{R_{total}}$.

Finding the Critical Radius

- To find the radius where heat transfer is maximum, we minimize total resistance.
- Differentiate R_{total} with respect to r and equate to zero:
- $$nfracdR\_totaldr = nfrac12npikLr - nfrac1h2npiLr\hat{}2 = 0$$
- $$nfrac1kr = nfrac1hr\hat{}2$$
- Solving for r, we get the Critical Radius (r_c):
- $$r\_c = nfrackh$$

Understanding the Critical Radius Graph

- Plot of Heat Transfer (Q) vs. Outer Radius (r):
- - The curve starts at bare pipe radius r_i.
- - It rises to a maximum peak at $r = r_c$.
- - It then continuously decreases for $r > r_c$.
- Behavior:
- If $r_i < r_c$: Adding insulation initially INCREASES heat loss up to r_c.
- If $r_i \geq r_c$: Adding insulation DECREASES heat loss immediately.

Physical Significance

- Why does Q increase initially if $r_i < r_c$?
- Because for small radii, the increase in surface area (decreasing convection resistance) overwhelms the added thermal resistance of the insulation.
- Beyond r_c, the added conduction resistance becomes dominant, and heat loss drops.
- Maximum heat loss occurs exactly at $r = r_c$.

Application: Electrical Cables

- Goal: Maximize heat dissipation (prevent overheating).
- Electrical wires are very thin (radius r_i is very small, usually $r_i < r_c$).
- Adding plastic/rubber insulation (k is small, but h for natural convection is also small, making $r_c = k/h$ larger than the wire).
- Result: The plastic coating on wires actually HELPS cool the wire while providing electrical safety!

Application: Steam Pipes

- Goal: Minimize heat loss (save energy).
- Steam pipes are large (radius r_i is large, usually $r_i > r_c$).
- Since the bare pipe is already larger than the critical radius, ANY insulation added will immediately decrease heat loss.
- Result: Insulating large pipes is always effective for conserving heat.

Summary & Next Lecture

- Thermal resistance of a cylinder is logarithmic: $R = \ln(r_2/r_1)/(2\pi kL)$.
- Critical radius of insulation for a cylinder is $r_c = k/h$.
- If $r < r_c$, adding insulation increases heat transfer.
- Next time: We will wrap up Unit 1 with Overall Heat Transfer Coefficient analysis for cylindrical systems and look at extended surfaces (Fins).

Heat Conduction through Plain Wall

- Course: Heat and Mass Transfer (DI05019071)
- Unit 1: Conduction
- Lecture 6: Heat Conduction through Plain Wall

Lecture Agenda

- 1. 1D Steady State Conduction
- 2. The Plane Wall Model
- 3. Applying Fourier's Law
- 4. Temperature Distribution
- 5. Thermal Resistance Concept
- 6. Electrical Analogy

1D Steady State Conduction

- Assumptions for our model:
- - Steady State: Temperature does not change with time.
- - One-Dimensional: Heat flows in only one direction (x).
- - No Internal Heat Generation.
- - Constant Thermal Conductivity (k).

The Plane Wall Model

- Consider a plane wall of thickness L.
- Area A is perpendicular to the direction of heat transfer.
- Temperatures at surfaces are T1 (at x=0) and T2 (at x=L).
- Assume T1 ¿ T2, so heat flows in the positive x direction.

Applying Fourier's Law

- Fourier's Law: qx = -k * A * (dT/dx)
- Since conditions are steady, the heat transfer rate (qx) is constant throughout the wall.
- Separating variables: qx * dx = -k * A * dT

Temperature Distribution (Derivation)

- Integrate from x=0 to x=L, and T=T1 to T=T2:
- qx * (0 to L) dx = -k * A * (T1 to T2) dT
- qx * L = -k * A * (T2 - T1)
- qx = k * A * (T1 - T2) / L

Temperature Profile Characteristics

- The temperature distribution T(x) can be found by integrating to an arbitrary x:
- T(x) = T1 - (T1 - T2) * (x / L)
- The profile is linear.
- Slope dT/dx is constant.

Thermal Resistance Concept

- Rearranging the heat rate equation:
- qx = (T1 - T2) / (L / kA)
- Let R_th = L / (k * A)
- R_th is the thermal resistance to conduction.
- Units: °C/W or K/W

Electrical Analogy

- Ohm's Law: I = ΔV / R_elec
- Thermal Equivalent: qx = ΔT / R_th
- Current (I) ¡-¿ Heat Rate (qx)
- Voltage drop (ΔV) ¡-¿ Temperature drop (ΔT)
- Electrical Resistance (R_elec) ¡-¿ Thermal Resistance (R_th)

Example Problem Introduction

- Problem: A glass window ($k = 0.78$ W/m·K) is 2 m by 1.5 m and 5 mm thick.
- Temperatures at the inner and outer surfaces are 15°C and -5°C.
- Find the rate of heat loss through the window.

Example Problem Solution

- Area A = 2 * 1.5 = 3 m^2
- Thickness L = 0.005 m
- ΔT = 15 - (-5) = 20 K
- R_th = L / (k*A) = 0.005 / (0.78 * 3) = 0.00214 K/W
- q = ΔT / R_th = 20 / 0.00214 = 9346 W = 9.35 kW

Summary

- 1D steady-state conduction in a plane wall yields a linear temperature profile.
- Heat transfer rate is proportional to ΔT and area, inversely proportional to thickness.
- Thermal resistance ($R_{th} = L/kA$) allows us to use an electrical analogy for heat transfer.

Next Lecture Preview

- Topic: Heat Conduction through Composite Walls
- What happens when a wall is made of multiple layers of different materials?
- We will use the electrical analogy to build series thermal circuits.

Heat Conduction through Composite Walls

- Course: Heat and Mass Transfer (DI05019071)
- Unit 1: Conduction
- Lecture 7: Heat Conduction through Composite Walls

Lecture Agenda

- 1. Recap of Thermal Resistance
- 2. Introduction to Composite Walls
- 3. Series Thermal Circuit
- 4. Overall Heat Transfer Rate Equation
- 5. Temperature Drop Across Layers
- 6. Thermal Contact Resistance

Introduction to Composite Walls

- Most practical walls are not made of a single material.
- Examples: house walls (brick, insulation, wood), furnace walls (firebrick, insulating brick, steel).
- Heat must flow sequentially through each layer.
- This is equivalent to electrical resistors in series.

Series Thermal Circuit

- Consider a wall with 3 layers: A, B, and C.
- Thicknesses: LA, LB, LC. Conductivities: kA, kB, kC.
- The same heat (qx) flows through all layers.
- Total Resistance R_total = R_A + R_B + R_C
- R_total = (LA/kAA) + (LB/kBA) + (LC/kCA)

Overall Heat Transfer Rate

- Using the overall temperature difference:
- qx = (T_inner - T_outer) / R_total
- qx = (T1 - T4) / [(LA/kAA) + (LB/kBA) + (LC/kCA)]
- This assumes perfect thermal contact between layers.

Temperature Drop Across Layers

- Once qx is known, intermediate temperatures can be found.
- qx = (T1 - T2) / R_A =¿ T2 = T1 - qx * R_A
- qx = (T2 - T3) / R_B =¿ T3 = T2 - qx * R_B
- Larger resistance = larger temperature drop.

Thermal Contact Resistance

- In reality, surfaces are rough.
- When two solid surfaces are pressed together, they touch only at high spots.
- Gaps are filled with air, which is a poor conductor.
- This creates an additional resistance at the interface: Contact Resistance (R_t,c).

Accounting for Contact Resistance

- Contact resistance R_t,c = 1 / (hc * A)
- where hc is the contact conductance.
- It is simply added to the series circuit.
- R_total = R_A + R_t,c + R_B

Reducing Contact Resistance

- How to minimize it:
- 1. Increase joint pressure.
- 2. Decrease surface roughness (polish).
- 3. Use thermal grease or conducting paste to displace air in gaps.
- 4. Insert a soft metallic foil (e.g., tin, silver).

Example Problem Introduction

- A furnace wall consists of 200 mm firebrick (k=1.2 W/mK) and 50 mm insulating brick (k=0.15 W/mK).
- Inner temperature is 1000°C, outer is 40°C.
- Calculate the heat loss per unit area.

Example Problem Solution

- R_firebrick = 0.2 / 1.2 = 0.167 (K m^2/W)
- R_insulation = 0.05 / 0.15 = 0.333 (K m^2/W)
- R_total = 0.167 + 0.333 = 0.500 (K m^2/W)
- q/A = (1000 - 40) / 0.500 = 1920 W/m^2

Summary

- Composite walls are modeled as series thermal circuits.
- Total resistance is the sum of individual layer resistances.
- Intermediate temperatures depend on the resistance distribution.
- Contact resistance must be considered for rough interfaces.

Next Lecture Preview

- Topic: Heat Conduction through Cylinders
- How does heat transfer change when the area is not constant?
- We'll explore radial heat conduction in pipes and tubes.

Heat Conduction through Cylinders

- Course: Heat and Mass Transfer (DI05019071)
- Unit 1: Conduction
- Lecture 8: Heat Conduction through Cylinders

Lecture Agenda

- 1. 1D Conduction in a Cylinder
- 2. Fourier's Law in Radial Direction
- 3. Temperature Distribution (Derivation)
- 4. Logarithmic Temperature Profile
- 5. Thermal Resistance of a Cylinder
- 6. Composite Cylindrical Walls

1D Conduction in a Cylinder

- Many applications involve cylindrical geometries: pipes, tubes, wires.
- If a pipe is long and temperatures depend only on radius (r), it's 1D radial conduction.
- Unlike a plane wall, the heat transfer area $A = 2\pi rL$ changes with radius r.

Fourier's Law in Radial Direction

- Fourier's Law: qr = -k * A * (dT/dr)
- Substitute A = 2πrL:
- qr = -k * (2πrL) * (dT/dr)
- At steady state, the heat transfer rate qr is constant.

Temperature Distribution (Derivation)

- Separate variables and integrate from r1 to r2:
- qr * (1/r) dr = -k * 2πL * dT
- qr * ln(r2/r1) = k * 2πL * (T1 - T2)
- qr = 2πLk * (T1 - T2) / ln(r2/r1)

Logarithmic Temperature Profile

- Solving for T(r) gives:
- T(r) = T1 - (T1 - T2) * [ln(r/r1) / ln(r2/r1)]
- The temperature profile is logarithmic, not linear.
- The slope dT/dr decreases as r increases because area increases.

Thermal Resistance of a Cylinder

- Rearranging the heat rate equation to qr = ΔT / R_th:
- R_th_cyl = ln(r2/r1) / (2πLk)
- This is the radial thermal resistance.
- It depends on the ratio of radii, not just the thickness.

Composite Cylindrical Walls

- Example: A pipe covered with a layer of insulation.
- Inner pipe (r1 to r2, k_pipe) and insulation (r2 to r3, k_ins).
- Just like plane walls, these resistances are in series.
- Total Resistance R_total = R_pipe + R_insulation

Series Circuit for Cylinders

- R_total = [ln(r2/r1) / 2πLk_pipe] + [ln(r3/r2) / 2πLk_ins]
- qr = (T_inner - T_outer) / R_total
- Often, the pipe resistance is very small compared to the insulation and can be neglected for metals.

Example Problem Introduction

- A steel pipe (k=50 W/mK) with inner radius 50 mm, outer 60 mm.
- Covered with 40 mm of insulation (k=0.05 W/mK).
- Inner surface is 200°C, outer surface is 30°C.
- Calculate heat loss per meter of pipe length (L=1m).

Example Problem Solution

- r1=0.05m, r2=0.06m, r3=0.10m.
- R_pipe = ln(0.06/0.05) / (2π*1*50) = 0.00058 K/W
- R_ins = ln(0.10/0.06) / (2π*1*0.05) = 1.625 K/W
- R_total = 1.6256 K/W
- q/L = (200 - 30) / 1.6256 = 104.6 W/m

Summary

- Radial conduction has a non-constant area.
- The temperature profile in a cylinder is logarithmic.
- Cylindrical thermal resistance is ln(r2/r1) / (2πLk).
- Composite cylinders can be analyzed using series thermal circuits.

Next Lecture Preview

- Topic: Critical Radius of Insulation
- Does adding insulation to a pipe ALWAYS decrease heat loss?
- We'll explore a counter-intuitive phenomenon regarding cylinders and convection.

Critical Radius of Insulation

- Course: Heat and Mass Transfer (DI05019071)
- Unit 1: Conduction
- Lecture 9: Critical radius of insulation for a cylinder

Lecture Agenda

- 1. The Paradox of Adding Insulation
- 2. Competing Effects: Conduction vs. Convection
- 3. Thermal Resistance Network
- 4. Deriving the Critical Radius
- 5. Physical Significance
- 6. Application: Cables vs. Steam Pipes

The Paradox of Adding Insulation

- For a plane wall: Adding insulation ALWAYS increases thermal resistance and decreases heat transfer.
- For a cylinder: Adding insulation adds conduction resistance, BUT it also increases the outer surface area.
- Larger outer area means lower convection resistance at the surface.
- Question: Which effect dominates?

Thermal Resistance Network

- Consider a pipe of radius r1, coated with insulation up to outer radius r.
- Surrounding fluid has convection coefficient h.
- R_total = R_cond_insulation + R_conv_outer
- R_total = [ln(r/r1) / (2πLk)] + [1 / (h*2πrL)]

Total Resistance vs. Radius

- Let's plot R_total as a function of the outer insulation radius (r).
- Initially, R_total decreases because the drop in convection resistance is faster than the rise in conduction resistance.
- It reaches a minimum, then begins to increase.
- Minimum resistance = Maximum heat transfer!

Deriving Critical Radius

- To find the minimum, take the derivative of R_total with respect to r and set it to zero.
- d(R_total)/dr = [1 / (2πLkr)] - [1 / (h*$2\pi Lr^2$)] = 0
- $1/(kr) = 1/(hr^2)$
- r = k / h

The Critical Radius (rc)

- Critical radius of insulation: $rc = k / h$
- Where k is the thermal conductivity of the INSULATION.
- h is the convection coefficient of the SURROUNDING FLUID.
- For a sphere, $rc = 2k / h$ (derived similarly).

Physical Significance of rc

- If r_outer ¡ rc: Adding insulation INCREASES heat transfer (acts as a cooling fin).
- If r_outer = rc: Heat transfer is MAXIMUM.
- If r_outer ¿ rc: Adding insulation DECREASES heat transfer (acts as an insulator).

Application: Electrical Cables

- Electrical wires are very small (radius ¡¡ rc).
- The plastic/rubber coating (insulation) often has an outer radius smaller than rc.
- Result: The 'insulation' actually helps cool the wire and prevents it from melting due to electrical resistance heating.
- It provides electrical insulation but thermal enhancement!

Application: Steam Pipes

- Steam pipes are large (radius ¿ rc).
- Adding ANY amount of insulation to a steam pipe will immediately increase the thermal resistance.
- Result: For large pipes, insulation always acts to conserve heat.
- We don't need to worry about rc for large industrial pipes.

Example Problem

- A 2 mm diameter wire is coated with 1 mm plastic (k = 0.15 W/mK).
- Surrounding air h = 15 W/m^2K.
- Calculate the critical radius and determine if the plastic increases or decreases heat dissipation.

Example Problem Solution

- rc = k / h = 0.15 / 15 = 0.01 m = 10 mm
- Current outer radius of the wire + plastic = 1 mm + 1 mm = 2 mm.
- Since current radius (2 mm) ¡ rc (10 mm), the plastic coating INCREASES the heat dissipation.
- The wire runs cooler with the plastic than if it were bare!

Next Lecture Preview

- Topic: Overall heat transfer coefficient (U)
- How do we combine conduction and convection into one easy-to-use term?
- We will introduce the concept of the Overall Heat Transfer Coefficient.

Overall Heat Transfer Coefficient

- Course: Heat and Mass Transfer (DI05019071)
- Unit 1: Conduction
- Lecture 10: Overall heat transfer coefficient

Lecture Agenda

- 1. Introduction to Combined Modes
- 2. Defining the Overall Heat Transfer Coefficient (U)
- 3. U for a Plane Wall
- 4. U for a Cylindrical Surface
- 5. Inner vs. Outer Area Basis for U
- 6. Fouling Factors

Combined Modes of Heat Transfer

- Most engineering problems involve multiple modes of heat transfer.
- Example: Hot fluid inside a pipe, cold air outside.
- Heat transfers by: Convection (inside) -¿ Conduction (through pipe wall) -¿ Convection (outside).
- We need a simple way to express this total process.

Defining the Overall Coefficient (U)

- We define the overall heat transfer coefficient, U, such that:
- q = U * A * ΔT_total
- Where ΔT_total is the temperature difference between the two bulk fluids.
- Comparing this to q = ΔT_total / R_total, we see that:
- U * A = 1 / R_total

U for a Plane Wall

- Consider a wall of thickness L, conductivity k, with fluids h_in and h_out.
- R_total = (1/h_in*A) + (L/k*A) + (1/h_out*A)
- Since area A is constant: R_total = (1/A) * [1/h_in + L/k + 1/h_out]
- Therefore: 1/U = 1/h_in + L/k + 1/h_out

U for a Cylindrical Surface

- For a pipe, the inner area (A_in) is not equal to the outer area (A_out).
- R_total = 1/(h_in*A_in) + ln(r_out/r_in)/(2πLk) + 1/(h_out*A_out)
- Because area varies, U depends on which area we use as a reference.
- q = U_in * A_in * ΔT = U_out * A_out * ΔT

Inner vs. Outer Area Basis

- Based on inner area (A_in = 2πr_in*L):
- 1/U_in = 1/h_in + [r_in * ln(r_out/r_in)]/k + (r_in/r_out)*(1/h_out)
- Based on outer area (A_out = 2πr_out*L):
- 1/U_out = (r_out/r_in)*(1/h_in) + [r_out * ln(r_out/r_in)]/k + 1/h_out
- Note: U_in * A_in = U_out * A_out

Fouling Factor Concept

- Over time, surfaces accumulate deposits (rust, scale, algae).
- These deposits act as additional thermal resistance.
- We account for this using a 'fouling factor' (Rf).
- Rf has units of $m^2 \cdot K/W$.

Including Fouling in U

- We simply add the fouling resistances to our total resistance.
- $1/U = 1/h\_in + Rf\_in + L/k + Rf\_out + 1/h\_out$
- Fouling always decreases the overall heat transfer coefficient.
- Designers must overestimate U requirements to account for future fouling.

Typical Values of U (W/m^2K)

- Water to Water: 850 - 1700
- Water to Oil: 110 - 350
- Steam condenser (Water in tubes): 1000 - 6000
- Air to Water: 10 - 50
- Notice that systems involving gases (air) have much lower U values.

Example Problem

- A steel pipe (k=50) has r_in = 20 mm, r_out = 24 mm.
- Inside water h_in = 1000 W/m^2K. Outside air h_out = 15 W/m^2K.
- Calculate U based on the outside area (U_out).

Summary

- The overall heat transfer coefficient (U) combines conduction and convection.
- For cylinders, U must be referenced to a specific area (usually A_in or A_out).
- Fouling factors account for the thermal resistance of scale/dirt build-up.
- U simplifies the calculation of heat transfer in complex systems.

Next Lecture Preview

- Topic: Heat transfer from an extended surface (Fins)
- How can we increase heat transfer when convection is poor?
- We will explore the use of fins to increase surface area.

Heat Transfer from an Extended Surface

- Course: Heat and Mass Transfer (DI05019071)
- Unit 1: Conduction
- Lecture 11: Extended Surfaces (Fins) and their Types

Lecture Agenda

- 1. Limitations of Natural Convection
- 2. Concept of Extended Surfaces
- 3. Why Use Fins?
- 4. Practical Applications
- 5. Classification and Types of Fins

Limitations of Standard Convection

- Newton's Law of Cooling: Q = h A (Ts - T∞)
- To increase heat transfer rate (Q), we can:
- 1. Increase the temperature difference (Ts - T∞) -¿ Often limited by process.
- 2. Increase the convection coefficient (h) -¿ Requires forced convection, adding cost (fans, pumps).
- 3. Increase the surface area (A) -¿ Most practical approach.

Concept of Extended Surfaces

- An extended surface (fin) is a solid that experiences heat transfer by conduction within its boundaries, while heat is transferred by convection (and/or radiation) from its boundaries to the surroundings.
- It effectively increases the surface area 'A' in contact with the fluid.
- Primarily used when the convective heat transfer coefficient (h) is low, particularly for gases.

Why Use Fins?

- Enhance heat dissipation without needing active cooling mechanisms (like pumps).
- Cost-effective and reliable (no moving parts).
- Ideal for air cooling, where the heat transfer coefficient is inherently low compared to liquid cooling.
- Compact design for electronics and automotive applications.

Practical Applications of Fins

- Electronics: Heat sinks for CPUs, GPUs, and power transistors.
- Automotive: Radiators, engine blocks of air-cooled motorcycles.
- HVAC Systems: Condenser and evaporator coils in ACs and refrigerators.
- Power Generation: Cooling towers and transformer casings.

Types of Fins: Overview

- Fins come in various shapes depending on the application, manufacturing constraints, and fluid flow dynamics.
- 1. Straight Fins (Longitudinal)
- 2. Annular Fins (Radial)
- 3. Pin Fins (Spines)

Straight Fins (Longitudinal)

- Attached to a plane wall.
- Can have a uniform cross-section (rectangular) or non-uniform cross-section (triangular, parabolic).
- Rectangular: Easy to manufacture.
- Triangular: Lighter weight, uses less material for the same performance.

Annular Fins (Radial)

- Circumferentially attached to a cylinder.
- Cross-sectional area increases from base to tip.
- Commonly used in pipes transferring hot fluids, or on motorcycle engine cylinders.

Pin Fins (Spines)

- Protrude from a surface like a pin or peg.
- Can be cylindrical, conical, or square in profile.
- Promote turbulence in the fluid, enhancing the convection coefficient (h).
- Often used in compact heat exchangers and electronic heat sinks.

Summary

- Fins enhance heat transfer by increasing the effective surface area.
- They are most beneficial when convection coefficients are low (e.g., in air).
- Various shapes (straight, annular, pin) are used based on thermal and physical requirements.

Next Lecture Preview

- Topic: Heat Flow through a Rectangular Fin
- We will derive the governing differential equation for a simple straight fin.
- Be prepared with fundamental calculus and Fourier's law of conduction.

Heat Flow Through a Rectangular Fin

- Course: Heat and Mass Transfer (DI05019071)
- Unit 1: Conduction
- Lecture 12: Deriving the Fin Equation

Lecture Agenda

- 1. Standard Assumptions for Fin Analysis
- 2. The Rectangular Fin Geometry
- 3. Energy Balance on a Differential Element
- 4. Derivation of the Governing Equation
- 5. The Fin Parameter (m)

Assumptions in Fin Analysis

- 1. Steady-state heat conduction.
- 2. One-dimensional heat flow (temperature varies only along the length 'x').
- 3. Constant thermal conductivity (k) of the fin material.
- 4. Uniform convection coefficient (h) over the entire fin surface.
- 5. No internal heat generation.
- 6. Negligible radiation (or combined into an effective 'h').

Geometry of a Rectangular Fin

- Consider a straight rectangular fin:
- Length = L, Width = w, Thickness = t.
- Cross-sectional Area (Ac) = w * t.
- Perimeter (P) = 2 * (w + t).
- Base temperature = Tb, Ambient temperature = T∞.

Energy Balance Formulation

- Take a differential element of thickness 'dx' at distance 'x'.
- Energy In (Conduction): Qx
- Energy Out (Conduction): Qx+dx
- Energy Out (Convection): Qconv
- Steady State Balance: Qx = Qx+dx + Qconv

Applying Fourier's and Newton's Laws

- From Fourier's Law:
- Qx = -k * Ac * (dT/dx)
- Qx+dx = Qx + (d/dx)(Qx) * dx
- From Newton's Law of Cooling:
- Qconv = h * dAs * (T - T∞)
- Where surface area dAs = P * dx (Perimeter * thickness).

Deriving the Differential Equation

- Substitute into balance: $Qx = [Qx + (d/dx)(Qx) * dx] + h*P*dx*(T - T\infty)$
- Simplify: $-(d/dx)(Qx) * dx = h*P*dx*(T - T\infty)$
- Divide by dx: $-(d/dx)(-k*Ac * dT/dx) = h*P*(T - T\infty)$
- $k * Ac * (d^2T/dx^2) = h*P*(T - T\infty)$

The General Governing Equation

- Rearranging the terms:
- d^2T/dx^2 - (h*P / k*Ac) * (T - T∞) = 0
- To simplify, we introduce the excess temperature θ = T - T∞.
- Since T∞ is constant, $d\theta/dx = dT/dx$.
- Equation becomes: $d^2\theta/dx^2$ - (h*P / k*Ac) * θ = 0

The Fin Parameter (m)

- Let m^2 = h*P / k*Ac
- The parameter 'm' has units of m^1.
- The equation simplifies to:
- $d^2\theta/dx^2$ - $m^2\theta = 0$
- This is a second-order, linear, homogeneous ordinary differential equation.

General Solution of the Equation

- The general solution for $d^2\theta/dx^2$ - $m^2\theta = 0$ is:
- $\theta(x) = C1 * e\hat{}(mx) + C2 * e\hat{}(-mx)$
- Where C1 and C2 are constants of integration.
- To find C1 and C2, we need two boundary conditions.

Summary

- Assumed 1D steady-state heat conduction.
- Derived energy balance on a small dx element.
- Obtained the governing equation: $d^2\theta/dx^2$ - $m^2\theta = 0$.
- The fin parameter m relies on geometry and material properties.

Next Lecture Preview

- Topic: Analysis of an Infinitely Long Fin
- We will apply specific boundary conditions to solve for C1 and C2.
- We'll evaluate the heat transfer rate for a very long fin.

Infinitely Long Fin Analysis

- Course: Heat and Mass Transfer (DI05019071)
- Unit 1: Conduction
- Lecture 13: The Infinitely Long Fin

Lecture Agenda

- 1. Recap of the General Solution
- 2. Defining Boundary Conditions for an Infinite Fin
- 3. Solving for Constants C1 and C2
- 4. Temperature Distribution Profile
- 5. Heat Transfer Rate (Q_fin)

The General Solution Recap

- Governing Equation: $d^2\theta/dx^2$ - $m^2\theta = 0$
- General Solution: $\theta(x) = C1 * e\hat{}(mx) + C2 * e\hat{}(-mx)$
- Where: $\theta(x) = T(x)$ - $T\infty$
- And $m = (hP/kAc)$

Boundary Condition 1: The Fin Base

- At the base of the fin (attached to the wall):
- x = 0
- Temperature is equal to the base temperature: T(0) = Tb
- In terms of excess temperature:
- $\theta(0)$ = Tb - T∞ = θb
- Therefore: θb = C1*e^0 + C2*e^0 =¿ θb = C1 + C2

Boundary Condition 2: The Infinite Tip

- For a very long fin, as x approaches infinity ($x \to \infty$):
- The fin eventually cools down to the ambient fluid temperature.
- $T(\infty) = T\infty$
- In terms of excess temperature:
- $\theta(\infty) = T\infty$ - $T\infty = 0$

Evaluating Constant C1

- Apply Boundary Condition 2 to the general solution:
- $\theta(\infty)$ = C1 * e^(m*∞) + C2 * e^(-m*∞) = 0
- Since e^(-m*∞) $\rightarrow$ 0, the second term vanishes.
- We are left with: C1 * e^(∞) = 0
- Since e^(∞) is infinity, for the equation to hold true, C1 must be 0.
- Result: C1 = 0

Evaluating Constant C2

- Substitute C1 $= 0$ into Boundary Condition 1:
- θb $=$ C1 $+$ C2
- θb $= 0 +$ C2
- Result: C2 $= \theta$b
- Now substitute C1 and C2 back into the general solution.

Temperature Distribution Equation

- $\theta(x) = \theta b$ * e^(-mx)
- Or in terms of absolute temperatures:
- (T(x) - T∞) / (Tb - T∞) = e^(-mx)
- This indicates an exponential decay of temperature along the length of the fin.

Heat Transfer Rate (Q_fin)

- How much heat does the fin transfer?
- Method 1: Integrate convection over the entire surface.
- Method 2: Heat entering the base by conduction must equal total heat convected.
- Use Fourier's Law at x = 0:
- Q_fin = -k * Ac * (dT/dx) at x=0

Calculating Q_fin

- We know: $\theta(x) = \theta b$ * e^(-mx)
- Differentiate: $d\theta/dx$ = -m * θb * e^(-mx)
- At x = 0: $(dT/dx)_{x=0}$ = -m * θb
- Substitute into Fourier's Law:
- Q_fin = -k * Ac * (-m * θb) = k * Ac * m * θb

Summary

- For an infinite fin, tip temperature T(∞) = T∞.
- Temperature profile is an exponential decay: θ(x) = θb * e^(-mx).
- Heat transfer rate: Q_fin = (hPkAc) * (Tb - T∞).
- This is the theoretical maximum heat a fin of this cross-section can transfer.

Next Lecture Preview

- Topic: Efficiency and Effectiveness of Fins
- How do real, finite fins perform compared to our infinite ideal?
- We will define the metrics to evaluate fin performance.

Efficiency and Effectiveness of Fins

- Course: Heat and Mass Transfer (DI05019071)
- Unit 1: Conduction
- Lecture 14: Fin Performance Metrics

Lecture Agenda

- 1. The Need for Performance Metrics
- 2. Fin Efficiency (_fin)
- 3. Formula for Fin Efficiency
- 4. Fin Effectiveness (ε_fin)
- 5. When are Fins Justified?
- 6. Relationship between Efficiency and Effectiveness

Why Measure Performance?

- Adding a fin increases surface area (good for convection).
- BUT adding a fin also adds conduction resistance.
- Temperature drops along the fin, so the tip dissipates less heat than the base.
- We need metrics to ensure the area gain outweighs the conduction penalty.

Fin Efficiency (_fin) - Concept

- Fin Efficiency compares actual heat transfer to an ideal case.
- Ideal Case: The entire fin is at the base temperature (Tb).
- If k is infinite, there is no temperature drop, and maximum heat is dissipated.
- _fin = (Actual Heat Transfer from Fin) / (Ideal Heat Transfer if entire fin was at Tb)

Fin Efficiency Formula

- _fin = Q_fin / Q_max
- Q_max = h * A_fin * (Tb - T∞)
- For a long fin: Q_fin = (hPkAc) * θb
- Substitute and simplify:
- _fin = 1 / (m * Lc) [For infinitely long fin, though usually applied to insulated tip fin: tanh(mL)/mL]

Graphical Representation of Efficiency

- Efficiency depends heavily on the parameter mL.
- As length (L) increases, _fin decreases.
- As thermal conductivity (k) increases, m decreases, and _fin increases.
- Fins should be made of high 'k' materials (Aluminum, Copper).

Fin Effectiveness (ε_fin) - Concept

- Effectiveness compares the heat transfer WITH the fin to the heat transfer WITHOUT the fin.
- ε_fin = (Heat transfer with fin) / (Heat transfer from base area without fin)
- ε_fin = Q_fin / (h * Ac * (Tb - T∞))

Evaluating Effectiveness

- For an infinitely long fin:
- Q_fin = (hPkAc) * θb
- ε_fin = [(hPkAc) * θb] / [h * Ac * θb]
- ε_fin = (k*P / h*Ac)

When is a Fin Justified?

- Rule of Thumb: ε_fin must be ¿ 2 to justify the cost and weight of adding fins.
- If ε_fin ¡ 1, the fin is acting as an insulator!
- To maximize effectiveness:
- 1. High thermal conductivity (k)
- 2. High perimeter-to-area ratio (P/Ac) -¿ Thin fins
- 3. Low convection coefficient (h) -¿ Gases

Relationship between _fin and ε_fin

- ε_fin = Q_fin / (h * Ac * θb)
- _fin = Q_fin / (h * A_fin * θb)
- Dividing the two equations gives:
- ε_fin = (A_fin / Ac) * _fin
- Effectiveness is the Efficiency scaled by the area ratio.

Summary

- Efficiency (): How well the fin utilizes its material (ideal = uniform Tb).
- Effectiveness (ε): How much the fin improves total heat transfer.
- Fins are justified when ε ¿ 2.
- Best used in applications with low convection coefficients (like air cooling).

Next Lecture Preview

- Topic: Numerical Examples on Extended Surfaces
- We will solve practical problems involving rectangular fins.
- Bring your calculators!

Numerical Examples on Extended Surfaces

- Course: Heat and Mass Transfer (DI05019071)
- Unit 1: Conduction
- Lecture 15: Problem Solving

Lecture Agenda

- 1. Problem Solving Strategy
- 2. Numerical 1: Infinitely Long Fin Application
- 3. Numerical 2: Evaluating Fin Efficiency
- 4. Numerical 3: Justifying Fin Addition (Effectiveness)

Problem Solving Strategy

- Step 1: Identify geometry (Rectangular, Pin) and extract L, w, t, D.
- Step 2: Calculate Area (Ac) and Perimeter (P).
- Step 3: Calculate the fin parameter 'm' = (hP/kAc).
- Step 4: Determine the appropriate boundary condition (Infinite, Insulated tip, etc.).
- Step 5: Apply the correct formula for Q_fin or _fin.

Numerical 1: Problem Statement

- A long copper rod (k = 398 W/mK) of 5 mm diameter extends from a surface at 120°C.
- The ambient air is at 25°C with a convection coefficient of 20 W/m^2K.
- Assuming the rod is infinitely long, determine the heat transfer rate from the rod.

Numerical 1: Step 1 & 2 (Geometry)

- Diameter D = 0.005 m
- Perimeter P = π * D = π * 0.005 = 0.0157 m
- Cross-sectional Area Ac = $(\pi/4)$ * D^2 = $(\pi/4)$ * $(0.005)^2$ = 1.963×10 m^2
- Base excess temp θb = Tb - T∞ = 120 - 25 = 95°C

Numerical 1: Step 3 & 5 (m & Q_fin)

- Calculate m = (hP / kAc)
- m = ((20 * 0.0157) / (398 * 1.963×10)) = (0.314 / 0.00781) ≈ 6.34 m^1
- For infinite fin: Q_fin = (hPkAc) * θb
- Alternatively: Q_fin = k * Ac * m * θb
- Q_fin = 398 * 1.963×10 * 6.34 * 95 ≈ 4.70 W

Numerical 2: Problem Statement

- A rectangular aluminum fin (k = 200 W/mK) is 3 mm thick and 7.5 cm long.
- It is attached to a wall at 300°C. Ambient air is 50°C with h = 10 W/m^2K.
- Determine the fin efficiency assuming an insulated tip.

Numerical 2: Geometry and 'm'

- Assume width w = 1 m. Thickness t = 0.003 m, L = 0.075 m.
- P = 2*(w+t) $\approx$ 2w = 2 m (since t ¡¡ w)
- Ac = w*t = 1 * 0.003 = 0.003 m^2
- m = (hP / kAc) = ((10 * 2) / (200 * 0.003)) = (20 / 0.6) = 33.33 $\approx$ 5.77 m^{1}
- mL = 5.77 * 0.075 = 0.433

Numerical 2: Calculating Efficiency

- Efficiency formula for insulated tip: _fin = tanh(mL) / mL
- _fin = tanh(0.433) / 0.433
- tanh(0.433) $\approx$ 0.407
- _fin = 0.407 / 0.433 $\approx$ 0.94 or 94%

Numerical 3: Effectiveness Check

- For the same fin in Numerical 2, what is the effectiveness?
- ε_fin = (A_fin / Ac) * _fin
- A_fin = P * L = 2 * 0.075 = 0.15 m^2
- Ac = 0.003 m^2
- ε_fin = (0.15 / 0.003) * 0.94 = 50 * 0.94 = 47
- Since 47 ¿¿ 2, the use of this fin is highly justified.

Summary

- Systematically break down problems: Geometry -¿ m parameter -¿ formula.
- Be careful with units, especially mm vs. m.
- For thin rectangular fins, Perimeter P $\approx$ 2w.
- Efficiency and Effectiveness ε are easily linked by the area ratio.

Next Lecture Preview

- Topic: Unit 2 - Convection
- We are wrapping up Conduction.
- Next, we move to the fluid dynamics of heat transfer: Newton's Law of Cooling in depth.

Newton's law of cooling

- Course: Heat and Mass Transfer (DI05019071)
- Unit 2: Convection
- Lecture 16: Newton's Law of Cooling

Lecture Agenda

- 1. Introduction to Convection
- 2. Mechanism of Convective Heat Transfer
- 3. Types of Convection
- 4. Newton's Law of Cooling
- 5. Convection Heat Transfer Coefficient
- 6. Factors Affecting the Heat Transfer Coefficient

Introduction to Convection

- Convection is the mode of heat transfer between a solid surface and the adjacent liquid or gas that is in motion.
- It involves the combined effects of:
- 1. Conduction (at the surface)
- 2. Fluid motion (macroscopic movement of fluid parcels)
- Without fluid motion, heat transfer between a solid surface and an adjacent fluid is purely by conduction.

Mechanism of Convection

- Near the solid surface, fluid velocity is zero (no-slip condition).
- Heat is first transferred to this stationary fluid layer by conduction.
- This heat is then carried away by the macroscopic motion of the fluid away from the surface.
- Therefore, convection is essentially conduction combined with fluid advection.

Types of Convection

- Convection is broadly classified based on how the fluid motion is initiated:
- 1. Forced Convection: Fluid is forced to flow over the surface by external means (pump, fan, wind).
- 2. Free (Natural) Convection: Fluid motion is caused by buoyancy forces due to density differences induced by temperature gradients.

Newton's Law of Cooling

- Governs the rate of convection heat transfer.
- Statement: The rate of heat transfer is directly proportional to the surface area and the temperature difference between the surface and the fluid.
- Equation: q = h * A * (T_s - T_\infty)
- Where:
- - q: Rate of heat transfer (W)
- - h: Convection heat transfer coefficient (W/m^2K)
- - A: Surface area (m^2)

Understanding the Equation

- Equation details: q = h * A * (T_s - T_\infty)
- T_s: Temperature of the surface.
- T_\infty: Temperature of the fluid sufficiently far from the surface (free-stream temperature).
- The direction of heat transfer depends on which temperature is higher. If T_s ¿ T_\infty, heat flows from the surface to the fluid.

Convection Heat Transfer Coefficient (h)

- It is not a property of the fluid.
- It is an experimentally determined parameter whose value depends on all the variables influencing convection.
- Units: Watts per square meter Kelvin ($W/m^2 \cdot K$) or $W/m^2 \cdot {}^\circ C$.
- It characterizes how effectively heat is transferred by convection.

Factors Affecting 'h'

- The convection heat transfer coefficient depends heavily on:
- 1. Fluid properties (dynamic viscosity, thermal conductivity, density, specific heat).
- 2. Surface geometry (flat plate, cylinder, sphere).
- 3. Surface roughness.
- 4. Fluid velocity (flow type: laminar vs. turbulent).

Typical Values of 'h'

- Free Convection (Gases): 2 - 25 $W/m^2 \cdot K$
- Free Convection (Liquids): 50 - 1,000 $W/m^2 \cdot K$
- Forced Convection (Gases): 25 - 250 $W/m^2 \cdot K$
- Forced Convection (Liquids): 50 - 20,000 $W/m^2 \cdot K$
- Boiling and Condensation: 2,500 - 100,000 $W/m^2 \cdot K$

Comparison: Conduction vs. Convection

- Conduction:
- - Mechanism: Molecular vibration/electron drift.
- - Medium: Solid (mostly), stationary fluid.
- - Law: Fourier's Law (q = -kA(dT/dx)).
- Convection:
- - Mechanism: Macroscopic fluid motion + conduction.
- - Medium: Moving fluid.
- - Law: Newton's Law of Cooling (q = hA(ΔT)).

Summary

- Convection combines conduction with fluid motion.
- Newton's Law of Cooling is q = h * A * (T_s - T_\infty).
- The convection heat transfer coefficient 'h' is an empirical parameter, not a material property.
- 'h' depends on fluid properties, flow conditions, and surface geometry.

Next Lecture Preview

- Topic: Concept of forced and free convection
- Key questions:
- - How do buoyancy forces create fluid motion in free convection?
- - When does forced convection transition from laminar to turbulent flow?

Concept of forced and free convection

- Course: Heat and Mass Transfer (DI05019071)
- Unit 2: Convection
- Lecture 17: Forced and Free Convection

Lecture Agenda

- 1. Recap of Convection Basics
- 2. What is Forced Convection?
- 3. Mechanisms and Examples of Forced Convection
- 4. What is Free (Natural) Convection?
- 5. Mechanisms of Free Convection (Buoyancy)
- 6. Forced vs. Free Convection Comparison
- 7. Mixed Convection

Recap of Convection

- Convection involves heat transfer by macroscopic fluid motion.
- Governed by Newton's Law of Cooling: q = hA(T_s - T_\infty)
- The driving force for heat transfer is the temperature difference.
- But what drives the fluid motion? This question leads us to our two types.

Forced Convection

- Definition: Fluid motion is artificially induced by an external agent.
- External agents include fans, blowers, pumps, or wind.
- The fluid is forced to flow over a surface or through a tube, regardless of the temperature differences present.

Mechanisms of Forced Convection

- High fluid velocities are achievable.
- Results in thinner boundary layers (we will discuss these later).
- Consequently, forced convection yields much higher heat transfer coefficients (h) compared to free convection.
- Flow can be easily controlled to achieve desired cooling/heating rates.

Examples of Forced Convection

- 1. Air conditioning and heating systems (fans blowing air).
- 2. Automobile radiators (water pump circulating coolant).
- 3. Cooling of electronic components (CPU cooling fans).
- 4. Flow of fluids through heat exchangers in chemical plants.

Free (Natural) Convection

- Definition: Fluid motion is induced by natural means, without any external agent.
- The motion is driven entirely by density differences within the fluid.
- These density differences are created by temperature gradients in the fluid itself.

The Mechanism: Buoyancy Forces

- 1. A hot surface heats the adjacent fluid.
- 2. The fluid expands, its density decreases.
- 3. The lighter, warmer fluid experiences an upward buoyancy force and rises.
- 4. Cooler, denser fluid moves in from the sides to replace it.
- This continuous circulation is natural convection.

Examples of Free Convection

- 1. Cooling of a hot cup of coffee in a still room.
- 2. Heat dissipation from standard home radiators (despite the name, they operate mostly via free convection and radiation).
- 3. Cooling of power transmission lines on a calm day.
- 4. Oceanic and atmospheric circulation patterns.

Forced vs. Free Convection

- — Feature — Forced Convection — Free Convection —
- ——————
- — Cause of Flow — External agent (fan/pump) — Buoyancy forces (density diff) —
- — Fluid Velocity — Generally high — Generally low —
- — Heat Transfer Coeff (h) — High — Low —
- — Control — Easily controllable — Difficult to control —

Mixed Convection

- In reality, free convection is always present as long as there is a temperature difference and gravity.
- Mixed convection occurs when both forced and free convection effects are significant.
- Example: A slow fan blowing air over a very hot surface.
- Buoyancy forces can either assist or oppose the forced flow.

Summary

- Forced convection uses external power (fans, pumps) to move fluid.
- Free convection relies on buoyancy forces generated by temperature-induced density changes.
- Forced convection generally achieves much higher heat transfer rates than free convection.
- Mixed convection happens when both mechanisms are of comparable strength.

Next Lecture Preview

- Topic: Overall heat transfer coefficient - conduction and convection
- Key questions:
- - How do we calculate heat transfer when a wall has convection on both sides?
- - What is the overall heat transfer coefficient 'U'?

Overall Heat Transfer Coefficient

- Course: Heat and Mass Transfer (DI05019071)
- Unit 2: Convection
- Lecture 18: Overall Heat Transfer Coefficient

Lecture Agenda

- 1. Real-world Heat Transfer Scenarios
- 2. The Thermal Resistance Concept Revisited
- 3. Defining Convection Resistance
- 4. Heat Transfer through a Plane Wall (Convection on both sides)
- 5. The Overall Heat Transfer Coefficient (U)
- 6. U for Cylindrical Geometries (Pipes)
- 7. Fouling and Thermal Contact Resistance

Real-world Scenarios

- In most engineering problems, heat transfer involves multiple modes in series.
- Example 1: Heat lost from a room through a wall to the cold outside air. (Inner convection -¿ Wall conduction -¿ Outer convection).
- Example 2: Heat exchanger tube. Hot fluid inside, cold fluid outside. (Inner convection -¿ Tube wall conduction -¿ Outer convection).

Thermal Resistance Revisited

- Recall from Unit 1: Heat transfer can be modeled analogously to electrical current.
- Current (I) -¿ Heat transfer rate (q)
- Voltage Difference (ΔV) -¿ Temperature Difference (ΔT)
- Electrical Resistance (R_e) -¿ Thermal Resistance (R_t)
- $q = \Delta T$ / R_total

Convection Resistance

- Newton's Law of Cooling: q = h * A * (T_s - T_\infty)
- Rearranging into the resistance format (q = ΔT / R):
- q = (T_s - T_\infty) / (1 / (h*A))
- Therefore, Convection Thermal Resistance is:
- R_conv = 1 / (h * A)

Heat Transfer through a Plane Wall

- Consider a wall of thickness 'L', thermal conductivity 'k', area 'A'.
- Hot fluid (T_h, h_1) on one side, cold fluid (T_c, h_2) on the other.
- Thermal circuit consists of three resistances in series:
- 1. R_conv1 = 1 / (h_1 * A)
- 2. R_cond = L / (k * A)
- 3. R_conv2 = 1 / (h_2 * A)

The Overall Heat Transfer Coefficient (U)

- To simplify calculations, we define the Overall Heat Transfer Coefficient (U).
- Equation: q = U * A * ΔT_overall
- Where ΔT_overall = T_hot_fluid - T_cold_fluid.
- Comparing with q = ΔT / R_total, we get:
- U * A = 1 / R_total =¿ U = 1 / (R_total * A)

U for a Plane Wall

- For our plane wall with convection on both sides:
- R_total = (1 / h_1*A) + (L / k*A) + (1 / h_2*A)
- Since U = 1 / (R_total * A), we can factor out 'A':
- U = 1 / [(1/h_1) + (L/k) + (1/h_2)]
- Units for U are $W/m^2 \cdot K$ (same as h).

U for Cylindrical Geometries

- For pipes, the inner area ($A_i = 2\pi r_i L$) is different from the outer area ($A_o = 2\pi r_o L$).
- Because A is not constant, U depends on which area it is based on:
- $U_i * A_i = U_o * A_o = 1 / R_{total}$
- $U_i = 1 / [(1/h_i) + (A_i * \ln(r_o/r_i)/(2\pi kL)) + (A_i/(A_o * h_o))]$

Fouling Factor (R_f)

- Over time, surfaces accumulate deposits (scale, rust, algae).
- These deposits act as additional thermal insulation, reducing heat transfer.
- This added resistance is called the Fouling Factor (R_f).
- R_total = R_conv1 + R_f1 + R_cond + R_f2 + R_conv2

Impact of Fouling on U

- Fouling always increases the total thermal resistance.
- Therefore, fouling always decreases the Overall Heat Transfer Coefficient (U).
- Equipment must be designed with extra capacity (a clean U value vs a dirty U value) to allow for fouling over its operational life.

Summary

- Convection thermal resistance is R_conv = 1 / (hA).
- The Overall Heat Transfer Coefficient (U) combines all series resistances: U*A = 1/R_total.
- For pipes, U must be specified relative to the inner or outer area.
- Fouling adds thermal resistance and degrades performance over time.

Next Lecture Preview

- Topic: Dimensionless numbers and their physical significance
- Key questions:
- - How do we simplify complex convection problems?
- - What are the Reynolds, Prandtl, and Nusselt numbers?

Dimensionless Numbers in Convection

- Course: Heat and Mass Transfer (DI05019071)
- Unit 2: Convection
- Lecture 19: Dimensionless Numbers (Re, Pr, Nu)

Lecture Agenda

- 1. The Challenge of Finding 'h'
- 2. What is Dimensional Analysis?
- 3. Reynold's Number (Re): Definition & Significance
- 4. Prandtl Number (Pr): Definition & Significance
- 5. Nusselt Number (Nu): Definition & Significance
- 6. Empirical Correlations (Nu = f(Re, Pr))

The Challenge of Finding 'h'

- As we learned, 'h' depends on many variables:
- Fluid velocity (V), density (ρ), viscosity (μ), specific heat (Cp), thermal conductivity (k), and characteristic length (L).
- Mathematically: h = f(V, ρ, μ, Cp, k, L)
- Performing experiments by varying one parameter at a time while holding the others constant is practically impossible and very expensive.

Dimensional Analysis

- Dimensional analysis groups these many variables into a few dimensionless groups.
- Benefits:
- 1. Drastically reduces the number of variables (e.g., from 7 variables to 3 dimensionless groups).
- 2. Makes experimental data generalizable (scale models can be used to predict real-world behavior).
- Buckingham Pi Theorem is often used to derive these groups.

Reynold's Number (Re)

- Definition: Re = (ρ * V * L) / μ
- Where:
- - ρ = fluid density (kg/m^3)
- - V = fluid velocity (m/s)
- - L = characteristic length (m)
- - μ = dynamic viscosity (kg/m·s)

Significance of Reynold's Number

- Physical Significance: Re is the ratio of Inertia forces to Viscous forces.
- Re = (Inertia Forces) / (Viscous Forces)
- - High Re (Inertia dominates): Turbulent flow, chaotic mixing.
- - Low Re (Viscosity dominates): Laminar flow, smooth layers.
- Re is used to determine if a flow is laminar or turbulent.

Prandtl Number (Pr)

- Definition: Pr = (μ * Cp) / k = ν / α
- Where:
- - μ = dynamic viscosity, Cp = specific heat, k = thermal conductivity.
- - ν = kinematic viscosity (μ/ρ) = momentum diffusivity.
- - α = thermal diffusivity (k/(ρ*Cp)).

Significance of Prandtl Number

- Physical Significance: Pr is the ratio of Molecular Momentum Diffusivity to Thermal Diffusivity.
- $\text{Pr} = \nu \ / \ \alpha$
- It describes the relative thickness of the velocity boundary layer compared to the thermal boundary layer.
- - Liquid metals: Pr ¡¡ 1 (heat diffuses faster than momentum).
- - Gases: Pr $\approx$ 1 (similar boundary layers).
- - Oils: Pr ¿¿ 1 (momentum diffuses faster than heat).

Nusselt Number (Nu)

- Definition: Nu = (h * L) / k_fluid
- Where:
- - h = convection heat transfer coefficient.
- - L = characteristic length.
- - k_fluid = thermal conductivity of the fluid.

Significance of Nusselt Number

- Physical Significance: Nu is the ratio of Convection heat transfer to Conduction heat transfer in the fluid layer.
- Nu = (q_convection) / (q_conduction_if_fluid_was_stationary)
- Nu = 1 means heat transfer is purely by conduction.
- Higher Nu means more effective convection.
- Nu is the parameter we want to solve for to find 'h'.

Empirical Correlations

- Through dimensional analysis, we find that Nu is a function of Re and Pr for forced convection.
- General form: Nu = C * Reˆm * Prˆn
- Engineers perform experiments to find the constants C, m, and n for different geometries (flat plate, pipe, sphere) and flow types (laminar, turbulent).
- Workflow: Calculate Re and Pr -¿ Use correlation to find Nu -¿ Calculate h from Nu.

Summary

- Dimensional analysis simplifies convection by grouping variables.
- Re (Inertia/Viscous) determines flow regime (laminar vs turbulent).
- Pr (Momentum diff/Thermal diff) is a fluid property relating boundary layers.
- Nu (Convection/Conduction) is the dimensionless heat transfer coefficient.
- Convection problems are solved using Nu = f(Re, Pr) correlations.

Next Lecture Preview

- Topic: Boundary layer definition and characteristics
- Key questions:
- - What happens to the fluid exactly at the solid surface?
- - How do the velocity and thermal boundary layers develop?

Boundary Layers in Convection

- Course: Heat and Mass Transfer (DI05019071)
- Unit 2: Convection
- Lecture 20: Boundary Layer Definition and Thickness

Lecture Agenda

- 1. Concept of a Boundary Layer
- 2. The Velocity Boundary Layer
- 3. Velocity Boundary Layer Thickness ()
- 4. The Thermal Boundary Layer
- 5. Thermal Boundary Layer Thickness (_t)
- 6. Relationship Between Boundary Layers (Role of Pr)

Concept of a Boundary Layer

- When a fluid flows over a solid surface, the fluid particles adjacent to the surface stick to it.
- This is called the 'no-slip condition' (Velocity = 0 at the wall).
- These stationary particles slow down the adjacent layer of fluid, which slows down the next layer, and so on.
- This region of retarded flow near the surface is the Boundary Layer.

The Velocity Boundary Layer

- It is the region of the flow where the effects of viscous shearing forces caused by fluid friction are felt.
- Inside the boundary layer: Velocity gradients are large (du/dy is significant).
- Outside the boundary layer (free stream): Velocity is uniform, and viscous effects are negligible.
- The boundary layer grows in thickness as you move further down the plate.

Velocity Boundary Layer Thickness ()

- Definition: The distance 'y' from the surface at which the local fluid velocity 'u' reaches 99% of the free-stream velocity 'U_∞'.
- Mathematical definition: y = where u = 0.99 U_∞
- It defines the edge of the boundary layer.
- increases with distance 'x' from the leading edge.

Laminar vs. Turbulent Boundary Layers

- The boundary layer starts as laminar (smooth, ordered flow).
- At a critical distance (determined by critical Reynold's number), the flow becomes unstable and transitions to turbulent.
- Turbulent boundary layers are thicker, have intense mixing, and result in much higher friction and heat transfer rates.

The Thermal Boundary Layer

- Similar to velocity, if the fluid and surface are at different temperatures, a thermal boundary layer develops.
- Fluid particles at the surface achieve thermal equilibrium with the surface ($T = T\_s$).
- These particles exchange heat with adjacent fluid layers.
- It is the region where temperature gradients are present in the flow.

Thermal Boundary Layer Thickness (_t)

- Definition: The distance 'y' from the surface at which the temperature difference (T - T_s) equals 99% of the maximum temperature difference (T_∞ - T_s).
- Mathematical definition: y = _t where (T - T_s)/(T_∞ - T_s) = 0.99
- Like the velocity boundary layer, _t grows along the length of the surface.

Significance of Thermal Boundary Layer

- The convection heat transfer coefficient 'h' is inversely proportional to the thermal boundary layer thickness (_t).
- Fourier's law at the wall: q = -k_fluid * (dT/dy)_at_wall
- Newton's law: q = h * (T_s - T_∞)
- Thinner boundary layer -¿ Steeper temperature gradient -¿ Higher 'h'.

Relationship Between and _t

- Which boundary layer is thicker: velocity () or thermal (_t)?
- It depends entirely on the fluid's Prandtl Number (Pr $= \nu$ / α).
- Pr $\approx$ 1 (Gases): $\approx$ _t. Boundary layers grow at similar rates.
- Pr ¡¡ 1 (Liquid Metals): _t ¿¿ . Heat diffuses much faster than momentum.
- Pr ¿¿ 1 (Oils): ¿¿ _t. Momentum diffuses much faster than heat.

Boundary Layer Equations (Overview)

- For laminar flow over a flat plate (Blasius solution):
- Velocity BL thickness: $\approx$ 5.0 * x / (Re_x)
- Thermal BL thickness: _t = / (Prˆ(1/3))
- Notice how boundary layer thickness is inversely proportional to the square root of Reynolds number.

Summary

- Boundary layers are regions near the surface where velocity and temperature gradients are significant.
- Thickness is defined at the 99% free-stream value.
- Thinner thermal boundary layers result in higher heat transfer coefficients.
- Prandtl number dictates the relative thickness of velocity and thermal boundary layers.

Next Lecture Preview

- Topic: Boiling regimes and general aspects of condensation heat transfer
- Key questions:
- - What happens to heat transfer when a fluid boils?
- - What are the different regimes of boiling?
- - Film vs. Dropwise condensation: which is better?

Boiling and Condensation

- Course: Heat and Mass Transfer (DI05019071)
- Unit 2: Convection
- Lecture 21: Boiling Regimes and Condensation

Lecture Agenda

- 1. Convection with Phase Change
- 2. Pool Boiling vs. Flow Boiling
- 3. The Boiling Curve (Nukiyama Curve)
- 4. Natural Convection Boiling Regime
- 5. Nucleate Boiling Regime
- 6. Transition and Film Boiling Regimes
- 7. Critical Heat Flux (Burnout Point)
- 8. General Aspects of Condensation
- 9. Filmwise vs. Dropwise Condensation

Convection with Phase Change

- Boiling and condensation are forms of convection associated with a change in phase (liquid to gas, or gas to liquid).
- Latent heat of vaporization plays a massive role.
- Heat transfer coefficients (h) for phase change are typically orders of magnitude higher than single-phase forced convection.
- Examples: Power plant boilers, steam condensers, refrigeration systems.

Pool Boiling vs. Flow Boiling

- Pool Boiling: Boiling occurs in a quiescent (still) fluid. Fluid motion is due entirely to natural convection currents and bubble motion. (e.g., boiling water in a pan).
- Flow Boiling: Fluid is forced to move over a heated surface by external means (pump) while boiling occurs. (e.g., water flowing through a reactor core).
- We will focus primarily on Pool Boiling to understand the regimes.

The Boiling Curve

- Plots Heat Flux (q") on the y-axis vs. Excess Temperature (ΔT_excess = T_surface - T_saturation) on the x-axis.
- Log-log scale is usually used.
- Discovered by Nukiyama in 1934.
- Identifies four distinct boiling regimes as surface temperature increases.

1. Natural Convection Boiling

- Occurs at low excess temperatures (ΔT_excess ¡ 5°C for water).
- Fluid near the surface is superheated slightly.
- Heat is transferred by free convection to the liquid surface where evaporation occurs.
- No bubbles form on the heating surface yet.

2. Nucleate Boiling

- Occurs at 5°C ¡ ΔT_excess ¡ 30°C.
- Bubbles begin to form at nucleation sites (scratches, pits) on the surface.
- Bubbles detach, rise, and cause intense fluid mixing.
- Extremely high heat transfer rates. This is the most desirable operating regime for industrial boilers.

Critical Heat Flux (Burnout Point)

- The peak of the nucleate boiling curve (around $\Delta T\_excess \approx 30^{\circ}C$ for water).
- Also known as Critical Heat Flux (CHF) or Departure from Nucleate Boiling (DNB).
- At this point, bubble formation is so rapid that bubbles start to merge before detaching.
- Maximum possible heat transfer rate in nucleate boiling.

3 & 4. Transition & Film Boiling

- Transition Boiling (30°C ¡ ΔT ¡ 120°C): Bubbles merge to form an unstable vapor film. Heat flux actually drops as T_surface increases because vapor is a poor conductor.
- Film Boiling (ΔT ¿ 120°C): A stable, continuous vapor film completely covers the surface. (Leidenfrost effect).
- Radiation becomes a significant mode of heat transfer across the vapor film.

General Aspects of Condensation

- Occurs when vapor comes into contact with a surface whose temperature is below the saturation temperature of the vapor.
- Latent heat is released to the surface.
- Two main types of condensation on solid surfaces:
- 1. Filmwise Condensation
- 2. Dropwise Condensation

Filmwise vs. Dropwise Condensation

- Filmwise: The liquid wets the surface completely, forming a continuous liquid film. The film acts as a thermal resistance.
- Dropwise: The liquid does NOT wet the surface. Vapor condenses in discrete drops which roll off, leaving bare surface exposed.
- Dropwise condensation achieves heat transfer rates up to 10 times higher than filmwise condensation.

Summary

- Boiling and condensation offer very high heat transfer rates due to phase change.
- The boiling curve maps regimes: Natural convection, Nucleate (desired), Transition, and Film (dangerous).
- Critical Heat Flux is a dangerous limit where a vapor blanket forms.
- Dropwise condensation is much more efficient than filmwise condensation.

Unit 3 Preview

- Unit 3: Radiation
- Key questions:
- - How does heat travel through a vacuum like space?
- - What is electromagnetic thermal radiation?
- - What makes a body 'black', 'white', or 'grey' in heat transfer?

Radiation Heat Transfer

- Course: Heat and Mass Transfer (DI05019071)
- Unit 3: Radiation
- Lecture 22: Absorptivity, Reflectivity, and Transmissivity

Lecture Agenda

- 1. Introduction to Thermal Radiation
- 2. Electromagnetic Spectrum
- 3. Interaction of Radiation with Matter
- 4. Definitions: Absorptivity, Reflectivity, Transmissivity
- 5. Conservation of Energy Relationship
- 6. Surface Characteristics

What is Thermal Radiation?

- Radiation is energy emitted by matter in the form of electromagnetic waves (or photons).
- Does NOT require an intervening medium (can occur in a vacuum).
- All matter at a temperature above absolute zero (0 K) emits thermal radiation.
- Travels at the speed of light.

The Electromagnetic Spectrum

- Thermal radiation spans a specific portion of the EM spectrum.
- Wavelength range: ~0.1 to 100 micrometers (μm).
- Includes a portion of ultraviolet (UV), all visible light, and infrared (IR).
- Heat transfer is mostly concerned with the infrared region.

Interaction of Radiation with Matter

- When thermal radiation strikes a surface, three things can happen:
- 1. It is absorbed by the material (increases internal energy).
- 2. It is reflected away from the surface.
- 3. It is transmitted through the material.
- Total incident radiation is called Irradiation (G).

Absorptivity (α)

- Absorptivity (α): The fraction of incident radiation absorbed by the surface.
- Mathematical definition: α = G_abs / G
- Where G_abs is absorbed radiation and G is total incident irradiation.
- Range: $0 \leq \alpha \leq 1$
- Depends on the nature of the surface, temperature, and wavelength of incident radiation.

Reflectivity (ρ)

- Reflectivity (ρ): The fraction of incident radiation reflected by the surface.
- Mathematical definition: ρ = G_ref / G
- Where G_ref is reflected radiation.
- Range: $0 \leq \rho \leq 1$
- Types of reflection: Specular (mirror-like) and Diffuse (scattered in all directions).

Transmissivity (τ)

- Transmissivity (τ): The fraction of incident radiation transmitted through the material.
- Mathematical definition: τ = G_tr / G
- Where G_tr is transmitted radiation.
- Range: $0 \leq \tau \leq 1$
- Solid opaque materials generally have $\tau = 0$.

Conservation of Energy

- By the principle of conservation of energy:
- Total Incident Radiation = Absorbed + Reflected + Transmitted
- G = G_abs + G_ref + G_tr
- Dividing by G gives:
- $1 = \alpha + \rho + \tau$

Special Cases: Opaque Surfaces

- For opaque materials (e.g., metals, wood, brick):
- No radiation is transmitted: $\tau = 0$
- The conservation equation simplifies to:
- $\alpha + \rho = 1$
- Therefore, a highly reflective opaque surface must have low absorptivity, and vice versa.

Special Cases: Transparent Gases

- For highly transparent media (e.g., dry air, pure gases):
- Very little radiation is absorbed or reflected.
- $\alpha \approx 0$, $\rho \approx 0$
- Transmissivity approaches 1: $\tau \approx 1$
- Note: Some gases (like CO2 and water vapor) absorb specific wavelengths.

Summary

- Thermal radiation requires no medium and travels as EM waves.
- Irradiation (G) interacts via absorption, reflection, and transmission.
- Properties: α (absorptivity), ρ (reflectivity), τ (transmissivity).
- Conservation law: $\alpha + \rho + \tau = 1$.

Next Lecture Preview

- Topic: Black, white, and grey bodies
- Key questions to ponder:
- - What makes a surface a 'perfect' emitter and absorber?
- - How do real surfaces differ from ideal theoretical surfaces?

Ideal and Real Surfaces in Radiation

- Course: Heat and Mass Transfer (DI05019071)
- Unit 3: Radiation
- Lecture 23: Black, White, and Grey Bodies

Lecture Agenda

- 1. The Black Body Concept
- 2. Characteristics of a Black Body
- 3. Experimental Realization of a Black Body
- 4. The White Body Concept
- 5. The Opaque Body
- 6. The Grey Body Approximation
- 7. Real Surfaces vs. Grey Bodies

What is a Black Body?

- A black body is an idealized physical body that absorbs all incident electromagnetic radiation.
- Key Property: $\alpha = 1$
- Since it absorbs everything, it reflects and transmits nothing: $\rho = 0$, $\tau = 0$.
- It is a perfect absorber, regardless of wavelength or angle of incidence.

Characteristics of a Black Body

- 1. Perfect Absorber: Absorbs all incident radiation.
- 2. Perfect Emitter: For a prescribed temperature and wavelength, no surface can emit more energy than a black body.
- 3. Diffuse Emitter: Radiation emitted by a black body is independent of direction.
- Serves as a standard against which the radiative properties of real surfaces are compared.

Is a Black Body Actually 'Black'?

- The term 'black body' is a thermodynamic concept.
- Visual appearance depends on its temperature.
- At room temperature, it appears black (absorbs all visible light).
- At high temperatures, it can glow red, yellow, or white (e.g., the Sun, a filament).
- Snow and ice absorb almost all infrared radiation ($\alpha \approx 0.95$), acting nearly as black bodies in the IR spectrum!

Experimental Realization of a Black Body

- A large cavity with a very small opening acts as a near-perfect black body.
- Radiation entering the hole undergoes multiple internal reflections.
- With each reflection, a fraction is absorbed.
- Almost none of the incident radiation escapes back out the hole.
- The hole acts as a black body surface of area equal to the hole's area.

What is a White Body?

- A white body is an idealized surface that reflects all incident radiation.
- Key Property: $\rho = 1$
- Therefore: $\alpha = 0$, $\tau = 0$.
- It absorbs no energy and transmits no energy.
- Like the black body, it is a theoretical extreme.

Opaque Bodies

- As discussed previously, an opaque body transmits no radiation.
- Key Property: $\tau = 0$
- Conservation equation: $\alpha + \rho = 1$
- Most solids we analyze in engineering (metals, walls, machinery) are modeled as opaque bodies.

Real Surfaces vs. Idealizations

- Real surfaces emit and absorb less radiation than a black body.
- Their properties (α, ρ, τ) depend strongly on:
- - The wavelength (λ) of the radiation.
- - The direction (angle) of the radiation.
- This makes analyzing real surfaces mathematically complex.

The Grey Body Concept

- To simplify calculations, we use the Grey Body approximation.
- A Grey Body is defined as a surface whose properties (α, ρ) are independent of wavelength and direction.
- It absorbs a constant fraction of incident radiation across all wavelengths.
- 0 ¡ α_grey ¡ 1 (Constant)

Why Assume a Grey Body?

- Allows use of total (average) properties instead of spectral (wavelength-dependent) properties.
- Most engineering applications involve relatively narrow temperature ranges where the grey body approximation is highly accurate.
- Makes analytical solutions to radiation networks possible.

Summary

- Black Body: $\alpha = 1$ (Perfect absorber and emitter).
- White Body: $\rho = 1$ (Perfect reflector).
- Opaque Body: $\tau = 0$ ($\alpha + \rho = 1$).
- Grey Body: Properties are independent of wavelength (constant α).

Next Lecture Preview

- Topic: Emissive power and emissivity
- Key questions to ponder:
- - How do we quantify the energy emitted by a black body versus a grey body?
- - What is emissivity and how does it relate to absorptivity?

Emissive Power and Emissivity

- Course: Heat and Mass Transfer (DI05019071)
- Unit 3: Radiation
- Lecture 24: Emissive Power and Emissivity

Lecture Agenda

- 1. Definition of Emissive Power
- 2. Spectral vs. Total Emissive Power
- 3. Emissive Power of a Black Body
- 4. Definition of Emissivity
- 5. Emissivity of Real vs. Grey Bodies
- 6. Factors Affecting Emissivity

What is Emissive Power (E)?

- Emissive Power (E) is the radiant energy emitted by a surface per unit time per unit area.
- Units: Watts per square meter (W/m^2).
- It depends on:
- - Surface temperature (T)
- - Surface characteristics (material, roughness)
- It represents the total heat flux leaving a surface due to its own temperature.

Spectral Emissive Power (E_λ)

- Radiation is emitted across a range of wavelengths.
- Spectral Emissive Power (E_λ) is the rate of energy emission per unit area per unit wavelength interval at a specific wavelength λ.
- Units: $W/(m^2 \cdot \mu m)$.
- It shows how emitted energy is distributed across the electromagnetic spectrum.

Total Emissive Power (E)

- Total Emissive Power is the total radiation emitted over ALL wavelengths.
- Mathematically, it is the integral of spectral emissive power from λ=0 to λ=∞.
- $E = \int E_\lambda \, d\lambda$
- This is the total area under the spectral emissive power curve.

Black Body Emissive Power (E_b)

- Recall: A black body is a perfect emitter.
- For a given temperature T, no surface can emit more energy than a black body.
- E_b is the maximum possible emissive power at temperature T.
- Real surfaces always emit less: E ¡ E_b (at the same T).

Introducing Emissivity (ε)

- Emissivity (ε) is the ratio of the emissive power of a real surface to the emissive power of a black body at the same temperature.
- ε = E / E_b
- Where:
- E = Emissive power of the real surface
- E_b = Emissive power of a black body at same T

Properties of Emissivity

- Emissivity is a dimensionless number.
- Range: $0 \leq \varepsilon \leq 1$
- For a perfect Black Body: $\varepsilon = 1$ (since E = E_b)
- For a White Body: $\varepsilon = 0$ (emits nothing, reflects everything)
- For real surfaces: ε is typically between 0.05 and 0.95.

Spectral vs. Total Emissivity

- Spectral Emissivity (ε_λ): The emissivity at a specific wavelength.
- $\varepsilon\_\lambda = \mathrm{E}\_\lambda$ / $\mathrm{E}\_\mathrm{b}\lambda$
- Total Emissivity (ε): The average emissivity over all wavelengths.
- Real surfaces have emissivities that change with wavelength. This complicates calculations.

The Grey Body Approximation (Again)

- For a Grey Body, spectral emissivity is constant across all wavelengths.
- $\varepsilon\_\lambda = \varepsilon =$ constant
- This means the total emissivity is equal to the spectral emissivity.
- The emissive power curve of a grey body is simply a scaled-down version of the black body curve: E_grey $= \varepsilon$ * E_b.

Factors Affecting Emissivity

- Emissivity of a surface depends on:
- 1. Material type (metals vs non-metals)
- 2. Surface condition (smooth vs rough, polished vs oxidized)
- 3. Temperature (emissivity generally changes slightly with T)
- Example: Polished copper $\varepsilon \approx 0.03$. Heavily oxidized copper $\varepsilon \approx 0.78$.

Summary

- Emissive Power (E): Radiant energy emitted per unit area (W/m^2).
- Total E is the integral of Spectral E_λ over all wavelengths.
- Emissivity (ε = E / E_b): Ratio of real emission to black body emission.
- Grey body assumes constant ε across all wavelengths.

Next Lecture Preview

- Topic: Laws of Radiation
- Key questions to ponder:
- - How exactly do we calculate E_b for a black body? (Stefan-Boltzmann Law)
- - What is the mathematical relationship between emissivity and absorptivity? (Kirchhoff's Law)

Fundamental Laws of Radiation

- Course: Heat and Mass Transfer (DI05019071)
- Unit 3: Radiation
- Lecture 25: Laws of Radiation

Lecture Agenda

- 1. Planck's Law of Distribution
- 2. Wien's Displacement Law
- 3. Stefan-Boltzmann Law
- 4. Kirchhoff's Law
- 5. Lambert's Cosine Law

1. Planck's Distribution Law

- Formulated by Max Planck (1900).
- Describes the spectral emissive power of a black body (E_bλ) as a function of wavelength (λ) and temperature (T).
- Equation:
- E_bλ = C_1 / [λ^5 * (exp(C_2 / λT) - 1)]
- Where C_1 and C_2 are radiation constants.

Planck's Law: Spectral Distribution

- Plotting Planck's law shows bell-shaped curves for different temperatures.
- Key observations:
- - Emitted radiation varies continuously with wavelength.
- - At any wavelength, magnitude increases with increasing temperature.
- - The peak of the curve shifts to shorter wavelengths at higher temperatures.

2. Wien's Displacement Law

- Derived from Planck's Law by finding the maximum of the curve.
- States that the wavelength at which maximum emission occurs (λ_max) is inversely proportional to absolute temperature (T).
- Equation: λ_max * T = 2898 μm·K
- This explains why hotter objects change color (red to yellow to blue-white).

3. Stefan-Boltzmann Law

- Provides the TOTAL emissive power of a black body.
- Obtained by integrating Planck's Law from λ=0 to ∞.
- Equation for Black Body: E_b = σ * T^4
- σ = Stefan-Boltzmann constant = 5.67 $\times$ 10 W/(m^2·K)
- T must be in Kelvin!

Stefan-Boltzmann for Real Surfaces

- Real surfaces emit less than a black body.
- We incorporate Emissivity (ε) into the Stefan-Boltzmann law.
- Equation for Real Surface: E $= \varepsilon$ * σ * T^4
- For a Grey Body, ε is a constant independent of wavelength.

4. Kirchhoff's Law

- Relates emission and absorption properties.
- States that at thermal equilibrium, the emissivity of a surface equals its absorptivity.
- Equation: $\varepsilon = \alpha$
- Conditions: Strict equilibrium is required, but it is broadly applied to grey bodies as an engineering approximation.

Implications of Kirchhoff's Law

- Since $\varepsilon = \alpha$, and for opaque bodies $\alpha + \rho = 1$:
- We can substitute: $\varepsilon + \rho = 1$
- Therefore, Emissivity = 1 - Reflectivity ($\varepsilon = 1 - \rho$)
- Highly reflective surfaces (like aluminum foil) have very low emissivity.

5. Lambert's Cosine Law

- Describes directional characteristics of diffuse emission.
- States that the radiation intensity emitted by a diffuse surface is directly proportional to the cosine of the angle between the observer's line of sight and the surface normal.
- Equation: I_θ = I_n * cos(θ)
- Where I_n is intensity in the normal direction.

Lambert's Law: Diffuse Emitters

- A surface that obeys Lambert's Cosine Law is called a Lambertian surface or perfectly diffuse surface.
- Interestingly, while intensity varies with $\cos(\theta)$, the apparent area viewed by the observer also varies by $\cos(\theta)$.
- Result: A Lambertian surface appears equally bright from any viewing angle! (e.g., the Sun, matte paper).

Summary of Laws

- Planck: E_bλ vs λ distribution curve.
- Wien: Peak wavelength λ_max $\propto$ 1/T.
- Stefan-Boltzmann: Total E_b $= \sigma$T.
- Kirchhoff: Emissivity ε = Absorptivity α.
- Lambert: Intensity I_θ = I_n cos(θ).

Next Lecture Preview

- Topic: Concept of Shape Factor and Radiation Shields
- Key questions to ponder:
- - When two surfaces exchange heat, how much of surface 1's radiation actually hits surface 2?
- - How can we use reflective materials to drastically reduce radiation heat transfer?

Shape Factors and Radiation Shields

- Course: Heat and Mass Transfer (DI05019071)
- Unit 3: Radiation
- Lecture 26: Shape Factors, Radiation Shields, and Numerical Analysis

Lecture Agenda

- 1. Concept of Shape Factor (View Factor)
- 2. Shape Factor Algebra (Reciprocity & Summation)
- 3. Concept of Radiation Shields
- 4. Heat Transfer Reduction by Shields
- 5. Numerical Examples & Problem Solving

What is a Shape Factor?

- Also known as View Factor or Configuration Factor (F_ij).
- Definition: The fraction of radiation leaving surface 'i' that directly strikes surface 'j'.
- Purely a geometric property.
- Depends on the sizes, shapes, orientation, and distance between the two surfaces.
- $0 \leq F\_ij \leq 1$

Self-Viewing Surfaces

- Can a surface radiate to itself? Yes, depending on its geometry.
- Flat or Convex surface: F_ii = 0 (cannot see itself).
- Concave surface: F_ii ¿ 0 (can see itself).
- A portion of radiation leaving a concave cavity strikes another part of the same cavity.

Shape Factor Algebra: Reciprocity Theorem

- Relates the shape factor F_ij to F_ji.
- Equation: A_i * F_ij = A_j * F_ji
- Where A_i and A_j are the surface areas.
- Significance: If you know F_12, you can easily calculate F_21 without complex integration.

Shape Factor Algebra: Enclosure Theorem

- Also known as the Summation Rule.
- For an enclosure of N surfaces, all radiation leaving one surface must strike some surface within the enclosure.
- Equation: Σ F_ij $= 1$ (summed from j=1 to N)
- Example for a 3-surface enclosure (leaving surface 1):
- F_11 + F_12 + F_13 $= 1$

Radiation Heat Exchange (Grey Surfaces)

- For two infinite parallel grey surfaces, the net heat transfer is:
- q_12 = (σ * (T_1^4 - T_2^4)) / [(1/ε_1) + (1/ε_2) - 1]
- This formula uses thermal resistance concepts.
- - Surface resistances: (1-ε)/εA
- - Space resistance: 1/A*F_12

Concept of Radiation Shields

- Radiation Shield: A highly reflective (low emissivity) material placed between two surfaces to reduce radiation heat transfer.
- They do not generate or remove heat; they add resistance to the radiation network.
- Commonly used in cryogenics, spacecraft (multi-layer insulation), and furnaces.

Effectiveness of a Radiation Shield

- Inserting 'n' shields with the same emissivity as the main plates reduces heat transfer by a factor of (n + 1).
- q_with_n_shields = q_without_shields / (n + 1)
- Example: One shield cuts heat transfer exactly in half (if ε_shield = ε_plates).
- To maximize effectiveness, shield emissivity should be as close to zero as possible.

Numerical Example 1: Shape Factor

- Problem: A long concentric cylinder arrangement. Inner cylinder (1) radius r1, outer cylinder (2) radius r2.
- Find F_11, F_12, F_21, and F_22.
- Solution steps:
- 1. Inner cylinder is convex: F_11 = 0.
- 2. Summation rule for inner: F_11 + F_12 = 1 =¿ F_12 = 1.
- 3. Reciprocity: A1*F12 = A2*F21 =¿ F_21 = (A1/A2)*1 = r1/r2.
- 4. Summation for outer: F_21 + F_22 = 1 =¿ F_22 = 1 - (r1/r2).

Numerical Example 2: Heat Transfer

- Problem: Two large parallel plates at T1=800K (ε1=0.8) and T2=400K (ε2=0.6). Find heat flux (q/A).
- Formula: q/A = σ*(T1^4 - T2^4) / [(1/ε1) + (1/ε2) - 1]
- Calculation:
- σ*(800^4 - 400^4) = 5.67e-8 * (4096e8 - 256e8) = 21772.8 W/m^2
- Denominator: (1/0.8) + (1/0.6) - 1 = 1.25 + 1.667 - 1 = 1.917
- Result: q/A = 21772.8 / 1.917 = 11,357 W/m^2.

Summary of Unit 3: Radiation

- Radiation requires no medium; it's governed by temperature and surface properties (α, ρ, τ, ε).
- Black bodies are ideal references; Grey bodies approximate real surfaces.
- Stefan-Boltzmann (E=σT) is the core governing law.
- Shape factors (geometry) and Radiation shields (resistances) allow us to solve real-world system networks.

Looking Ahead

- Next Unit: Unit 4 - Heat Exchangers
- We will combine conduction, convection, and (rarely) radiation to analyze industrial equipment designed to transfer heat between two fluids.

Classification of Heat Exchangers

- Course: Heat and Mass Transfer (DI05019071)
- Unit 4: Heat Exchanger
- Lecture 27: Classification

Lecture Agenda

- 1. What is a Heat Exchanger?
- 2. Applications
- 3. Classification by Flow Arrangement
- 4. Classification by Construction
- 5. Regenerators vs. Recuperators

What is a Heat Exchanger?

- A heat exchanger is a device that facilitates the exchange of heat between two fluids that are at different temperatures.
- The fluids may be separated by a solid wall to prevent mixing or they may be in direct contact.
- Primary goal: Heating or cooling of a fluid, or changing its state (condensation or evaporation).

Applications

- 1. Power plants (Boilers, Condensers, Cooling towers)
- 2. HVAC systems (Heating, Ventilation, and Air Conditioning)
- 3. Automotive industry (Radiators, Oil coolers)
- 4. Chemical and food processing industries
- 5. Refrigeration systems (Evaporators, Condensers)

Classification by Flow Arrangement

- Heat exchangers can be classified based on the relative direction of fluid motion:
- 1. Parallel Flow: Both fluids move in the same direction.
- 2. Counter Flow: Fluids move in opposite directions.
- 3. Cross Flow: Fluids move perpendicular to each other.

Parallel Flow Heat Exchangers

- Hot and cold fluids enter at the same end, flow in the same direction, and leave at the same end.
- Large temperature difference at the inlet, which rapidly decreases along the length.
- Outlet temperature of cold fluid can never exceed the outlet temperature of the hot fluid.

Counter Flow Heat Exchangers

- Hot and cold fluids enter at opposite ends and flow in opposite directions.
- More uniform temperature difference along the length.
- Highly efficient: Cold fluid outlet temperature can exceed the hot fluid outlet temperature.

Cross Flow Heat Exchangers

- Fluids move perpendicular to each other.
- Commonly used when one of the fluids is a gas (like air).
- Sub-classified into Mixed and Unmixed flow depending on whether the fluid is allowed to mix in the transverse direction.

Classification by Construction

- Based on how they are built, heat exchangers fall into several categories:
- 1. Tubular (e.g., Shell and Tube, Double pipe)
- 2. Plate (e.g., Plate and frame)
- 3. Extended Surface (e.g., Finned tube)
- 4. Regenerative vs Recuperative

Shell and Tube Heat Exchangers

- Consists of a large shell with a bundle of tubes inside.
- One fluid flows inside the tubes, the other flows outside over the tubes (in the shell).
- Baffles are used in the shell to direct flow and increase turbulence and heat transfer.
- Very common in industrial processes due to robustness.

Compact Heat Exchangers

- Designed to realize a large heat transfer surface area per unit volume.
- Typically area density greater than 700 m^2/m^3.
- Often use closely spaced fins or plates.
- Ideal for applications where weight and volume are restricted (e.g., aerospace, automotive).

Summary

- Heat exchangers transfer thermal energy between fluids.
- Flow arrangements include parallel, counter, and cross flow.
- Counter flow is generally the most thermally efficient.
- Construction types include shell and tube, plate, and compact designs depending on application needs.

Next Lecture Preview

- Topic: Heat Exchanger Analysis
- - Overall energy balance
- - Heat transfer rate equations
- - Assumptions used in thermal analysis of heat exchangers

Heat Exchanger Analysis

- Course: Heat and Mass Transfer (DI05019071)
- Unit 4: Heat Exchanger
- Lecture 28: Heat Exchanger Analysis

Lecture Agenda

- 1. Goals of Heat Exchanger Analysis
- 2. Overall Energy Balance
- 3. Heat Transfer Rate Equations
- 4. Key Assumptions in Analysis
- 5. Heat Capacity Rates
- 6. Need for Mean Temperature Difference

Goals of Heat Exchanger Analysis

- Analysis typically involves two types of problems:
- 1. Performance evaluation: Determining the heat transfer rate and fluid outlet temperatures for an existing heat exchanger.
- 2. Design problem: Determining the required surface area to achieve specific outlet temperatures given flow rates and inlet temperatures.

Overall Energy Balance

- Based on the First Law of Thermodynamics (Conservation of Energy).
- In a steady-flow device, the rate of heat transfer from the hot fluid equals the rate of heat transfer to the cold fluid.
- Q_hot_rejected = Q_cold_absorbed (assuming no heat loss to surroundings).

Heat Transfer Rate Equations

- For the hot fluid: Q = m_h * c_ph * (T_hi - T_ho)
- For the cold fluid: Q = m_c * c_pc * (T_co - T_ci)
- Where: m = mass flow rate, c_p = specific heat, T_i = inlet temp, T_o = outlet temp.

Key Assumptions in Analysis

- 1. Steady-flow operating conditions.
- 2. Kinetic and potential energy changes are negligible.
- 3. Specific heats of fluids remain constant.
- 4. Axial heat conduction along the tubes is negligible.
- 5. The outer surface of the heat exchanger is perfectly insulated (no heat loss).

Temperature Profiles

- Temperature of fluids changes continuously as they flow through the exchanger.
- The driving force for heat transfer is the temperature difference (T_h - T_c) at any location.
- This local difference varies along the length of the exchanger.

Heat Capacity Rates

- A useful parameter is the heat capacity rate, C.
- Defined as the product of mass flow rate and specific heat.
- C_h = m_h * c_ph (for hot fluid)
- C_c = m_c * c_pc (for cold fluid)
- Equations become: Q = C_h * (T_hi - T_ho) = C_c * (T_co - T_ci)

Phase Change Considerations

- If a fluid undergoes a phase change (like boiling or condensation), its temperature remains constant.
- In this case, the heat transfer rate is Q = m * h_fg
- Where h_fg is the latent heat of vaporization or condensation.
- The heat capacity rate C approaches infinity during phase change.

Newton's Law of Cooling in HE

- Locally, heat transfer rate is expressed as: dQ = U * dA * (T_h - T_c)
- Where U is the overall heat transfer coefficient.
- dA is a differential surface area.
- T_h and T_c are local temperatures.

Need for Mean Temperature Difference

- Integrating dQ = U * dA * delta_T gives Q = U * A * delta_T_mean
- Since delta_T varies along the length, we cannot use a simple arithmetic average.
- We require a properly weighted average temperature difference to calculate total Q accurately.

Summary

- Energy balances allow us to relate heat transfer to fluid temperature changes.
- Q = C_h * delta_T_h = C_c * delta_T_c
- Assumptions like steady-state and zero heat loss simplify analysis.
- A mean temperature difference is required to use Q = U * A * delta_T_m.

Next Lecture Preview

- Topic: Log Mean Temperature Difference (LMTD)
- - Derivation of LMTD for parallel flow
- - Derivation of LMTD for counter flow
- - Application to condensers and evaporators

Log Mean Temperature Difference (LMTD)

- Course: Heat and Mass Transfer (DI05019071)
- Unit 4: Heat Exchanger
- Lecture 29: LMTD for Parallel, Counter Flow, Condenser & Evaporator

Lecture Agenda

- 1. The Concept of LMTD
- 2. Derivation of LMTD for Parallel Flow
- 3. Derivation of LMTD for Counter Flow
- 4. Comparing Parallel vs. Counter Flow
- 5. LMTD for Condensers
- 6. LMTD for Evaporators

The Concept of LMTD

- From Newton's Law of Cooling, total heat transfer Q = U * A * delta_T_m.
- delta_T_m must account for the non-linear temperature profiles of the fluids.
- By integrating the local heat transfer equations along the length, we obtain a logarithmic average.
- This is called the Log Mean Temperature Difference (LMTD).

Derivation Setup: Parallel Flow

- Consider a differential area dA in a parallel flow exchanger.
- Hot fluid temperature changes by dT_h (negative).
- Cold fluid temperature changes by dT_c (positive).
- Local temperature difference delta_T = T_h - T_c.

LMTD Parallel Flow Equation

- Integrating d(delta_T) / delta_T from inlet to outlet yields:
- LMTD = (delta_T_1 - delta_T_2) / ln(delta_T_1 / delta_T_2)
- Where for Parallel Flow:
- delta_T_1 = T_hi - T_ci (Inlet temperature difference)
- delta_T_2 = T_ho - T_co (Outlet temperature difference)

Derivation Setup: Counter Flow

- For counter flow, fluids move in opposite directions.
- Hot fluid enters at one end, cold fluid enters at the other.
- The derivation steps are similar, but the temperature boundary conditions change.

LMTD Counter Flow Equation

- The LMTD formula remains the same:
- LMTD = (delta_T_1 - delta_T_2) / ln(delta_T_1 / delta_T_2)
- However, the definitions change for Counter Flow:
- delta_T_1 = T_hi - T_co (Hot inlet minus Cold outlet)
- delta_T_2 = T_ho - T_ci (Hot outlet minus Cold inlet)

Comparing Parallel vs. Counter Flow

- For the same inlet and outlet temperatures, LMTD_counter ¿ LMTD_parallel.
- Since Q = U * A * LMTD, a larger LMTD means a smaller required surface area (A) for the same Q and U.
- Therefore, counter flow heat exchangers are more compact and economical.

Condensers in Heat Exchangers

- In a condenser, the hot fluid (vapor) condenses at a constant temperature (T_h = constant).
- The cold fluid absorbs heat and its temperature rises.
- Since T_h is constant, parallel and counter flow configurations yield the same temperature profile.

Evaporators in Heat Exchangers

- In an evaporator, the cold fluid (liquid) boils at a constant temperature (T_c = constant).
- The hot fluid rejects heat and its temperature drops.
- Similar to condensers, the flow direction (parallel vs counter) does not affect the LMTD.

LMTD for Phase Change

- For condensers and evaporators, use the standard LMTD formula.
- delta_T_1 and delta_T_2 are simply the temperature differences at the two ends.
- LMTD_parallel = LMTD_counter for any phase change exchanger where one temperature is constant.

Summary

- LMTD is the exact mean temperature difference for heat exchanger analysis.
- Counter flow LMTD is greater than parallel flow LMTD for the same conditions.
- Phase change devices have identical LMTD regardless of flow arrangement.

Next Lecture Preview

- Topic: Overall Heat Transfer Coefficient
- - Thermal resistance network in heat exchangers
- - Definition of U based on inner and outer areas
- - Fouling factors and their impact on performance

Overall Heat Transfer Coefficient

- Course: Heat and Mass Transfer (DI05019071)
- Unit 4: Heat Exchanger
- Lecture 30: Overall Heat Transfer Coefficient & Fouling

Lecture Agenda

- 1. Heat Transfer Process in a Tube
- 2. Thermal Resistance Network
- 3. Overall Heat Transfer Coefficient (U)
- 4. U based on Inside vs. Outside Area
- 5. Introduction to the Fouling Factor
- 6. Design Considerations for Fouling

Heat Transfer Process in a Tube

- Heat transfers from hot fluid to cold fluid through three mechanisms:
- 1. Convection from hot fluid to the inner wall.
- 2. Conduction through the solid tube wall.
- 3. Convection from the outer wall to the cold fluid.

Thermal Resistance Network

- Total thermal resistance R_total = R_conv,i + R_wall + R_conv,o
- R_conv,i = 1 / (h_i * A_i) (Inner convection resistance)
- R_wall = ln(r_o / r_i) / (2 * pi * k * L) (Cylindrical wall conduction resistance)
- R_conv,o = 1 / (h_o * A_o) (Outer convection resistance)

Overall Heat Transfer Coefficient (U)

- Q = (T_h - T_c) / R_total = U * A * (T_h - T_c)
- Therefore, U * A = 1 / R_total
- U is the overall heat transfer coefficient.
- It represents the total ability of the system to transfer heat.

U based on Inside vs. Outside Area

- Since the inner area (A_i) and outer area (A_o) of a tube are different, U depends on which area is used.
- U_i * A_i = U_o * A_o = 1 / R_total
- 1 / (U_i * A_i) = 1/(h_i*A_i) + R_wall + 1/(h_o*A_o)
- When specifying U, one must specify if it is U_i or U_o.

Simplifying U for Thin-Walled Tubes

- If the tube wall is very thin and highly conductive, R_wall approaches 0.
- Also, A_i is approximately equal to A_o.
- The equation simplifies to: $1/U = 1/h\_i + 1/h\_o$
- U is dominated by the smaller convection coefficient (the larger resistance).

The Fouling Factor (Rf)

- Over time, surfaces accumulate deposits (scale, rust, algae).
- These deposits act as additional insulation, reducing heat transfer.
- The effect of these deposits is represented by a fouling factor, R_f.
- R_f has units of m^2 K / W.

Impact of Fouling on Heat Transfer

- Fouling adds additional thermal resistances to our network.
- New equation for U based on outer area:
- 1 / U_o = (A_o/A_i)(1/h_i) + (A_o/A_i)R_f,i + A_o*R_wall + R_f,o + 1/h_o
- As R_f increases, U decreases, and Q drops.

Typical Values of Fouling Factors

- Distilled water: R_f = 0.0001 m^2 K/W
- City water: R_f = 0.0002 m^2 K/W
- River water: R_f = 0.001 m^2 K/W
- Fuel oil: R_f = 0.0009 m^2 K/W
- Notice that dirtier fluids have much higher fouling factors.

Design Considerations for Fouling

- Engineers must oversize heat exchangers to account for future fouling.
- A heat exchanger designed with a clean U value will fail to meet requirements after a few months.
- Periodic cleaning schedules must be planned to restore U to design levels.

Summary

- U incorporates all convective and conductive resistances.
- U must be referenced to a specific area (inside or outside).
- Fouling adds thermal resistance and degrades performance over time.
- Heat exchangers must be oversized to compensate for fouling.

Next Lecture Preview

- Topic: Effectiveness of Heat Exchanger (NTU Method)
- - What to do when outlet temperatures are unknown
- - Definition of heat exchanger effectiveness
- - The Number of Transfer Units (NTU)

Effectiveness of Heat Exchanger (NTU Method)

- Course: Heat and Mass Transfer (DI05019071)
- Unit 4: Heat Exchanger
- Lecture 31: Effectiveness - NTU Method

Lecture Agenda

- 1. Limitations of the LMTD Method
- 2. The NTU Method Overview
- 3. Maximum Possible Heat Transfer (Q_max)
- 4. Heat Exchanger Effectiveness (Epsilon)
- 5. Number of Transfer Units (NTU)
- 6. Epsilon-NTU Relations

Limitations of the LMTD Method

- LMTD is easy to use if all inlet and outlet temperatures are known (Design problems).
- If only the inlet temperatures are known, outlet temperatures must be guessed and iterated.
- Performance evaluation problems (finding Q for a given exchanger) are tedious with LMTD.
- Need a method that avoids iteration.

The Effectiveness-NTU Method

- Developed by Kays and London in 1955.
- Predicts the outlet temperatures and heat transfer rate without iteration.
- Based on a dimensionless parameter called heat exchanger effectiveness (epsilon).
- Highly preferred for performance evaluation of existing heat exchangers.

Determining C_min and C_max

- Recall heat capacity rates: C_h = m_h * c_ph and C_c = m_c * c_pc.
- Compare C_h and C_c.
- The smaller value is designated as C_min.
- The larger value is designated as C_max.
- The fluid with C_min will experience the largest temperature change.

Maximum Possible Heat Transfer (Q_max)

- What is the maximum heat transfer theoretically possible?
- It occurs in a very long counter-flow heat exchanger.
- The fluid with C_min would reach the inlet temperature of the other fluid.
- Q_max = C_min * (T_h,in - T_c,in)

Heat Exchanger Effectiveness

- Effectiveness (epsilon) = Actual heat transfer rate / Maximum possible heat transfer rate
- epsilon = Q / Q_max
- Since Q = C_c(T_c,out - T_c,in) = C_h(T_h,in - T_h,out)
- epsilon is a dimensionless number between 0 and 1.

Number of Transfer Units (NTU)

- NTU is a dimensionless parameter widely used for heat exchanger analysis.
- NTU = (U * A) / C_min
- It is a measure of the heat transfer surface area (A).
- A larger NTU means a physically larger heat exchanger.

Capacity Ratio (c)

- Another dimensionless parameter is the capacity ratio, c.
- c = C_min / C_max
- c ranges from 0 to 1.
- For phase change (boiling or condensation), C_max approaches infinity, so c = 0.

Effectiveness Relations: Parallel Flow

- Effectiveness can be expressed as a function of NTU and c: epsilon = f(NTU, c)
- For Parallel Flow:
- epsilon = [1 - exp(-NTU(1+c))] / (1+c)
- Charts are available to look up epsilon based on NTU and c.

Effectiveness Relations: Counter Flow

- For Counter Flow (when c ¡ 1):
- epsilon = [1 - exp(-NTU(1-c))] / [1 - c*exp(-NTU(1-c))]
- If c = 1 (C_min = C_max): epsilon = NTU / (1 + NTU)
- Counter flow charts show higher effectiveness for the same NTU.

Summary

- The NTU method eliminates iteration when outlet temperatures are unknown.
- Effectiveness (epsilon) compares actual heat transfer to theoretical maximum.
- NTU = UA / C_min is a dimensionless measure of size.
- Effectiveness charts map epsilon, NTU, and c.

Next Lecture Preview

- Topic: Advanced Heat Exchanger Concepts
- - Concept of heat pipe
- - Compact heat exchanger design
- - Specialized applications

LMTD for Parallel and Counter Flow Exchangers

- Course: Heat and Mass Transfer (DI05019071)
- Unit 4: Heat Exchanger
- Lecture 32: LMTD for Parallel and Counter Flow Exchangers

Lecture Agenda

- 1. Introduction to Mean Temperature Difference
- 2. Temperature Distribution in Parallel Flow
- 3. Derivation of LMTD for Parallel Flow
- 4. Temperature Distribution in Counter Flow
- 5. Derivation of LMTD for Counter Flow
- 6. Comparison of Flow Types

Introduction to LMTD

- In a heat exchanger, temperatures of both fluids change as they flow.
- The temperature difference ΔT between hot and cold fluids varies along the length.
- Newton's Law of Cooling: $q = U\ A\ \Delta T\_mean$
- We need an appropriate average ΔT, which is the Logarithmic Mean Temperature Difference (LMTD).

Why Not Arithmetic Mean?

- Arithmetic Mean Temperature Difference (AMTD) = (ΔT1 + ΔT2) / 2
- AMTD is only accurate if the temperature profiles are linear.
- In reality, heat transfer rate decreases as ΔT decreases, leading to exponential temperature profiles.
- Therefore, AMTD overestimates the actual mean temperature difference.

Temperature Distribution in Parallel Flow

- Both hot and cold fluids enter at the same end and flow in the same direction.
- Hot fluid temperature (Th) drops; cold fluid temperature (Tc) rises.
- ΔT is maximum at the inlet ($\Delta T1$) and minimum at the outlet ($\Delta T2$).
- $\Delta T1 =$ Th,in - Tc,in
- $\Delta T2 =$ Th,out - Tc,out

Derivation of LMTD for Parallel Flow (Part 1)

- Consider a small area element dA.
- Heat transfer: dq = U dA (Th - Tc) = U dA ΔT
- Energy balance for hot fluid: dq = - m_h cp_h dTh = - Ch dTh
- Energy balance for cold fluid: dq = m_c cp_c dTc = Cc dTc
- d(ΔT) = dTh - dTc = -dq(1/Ch + 1/Cc)

Derivation of LMTD for Parallel Flow (Part 2)

- Substitute $dq = U\ dA\ \Delta T$ into the equation:
- $d(\Delta T) = -\ U\ dA\ \Delta T\ (1/Ch + 1/Cc)$
- Rearranging: $d(\Delta T)/\Delta T = -\ U\ (1/Ch + 1/Cc)\ dA$
- Integrating from inlet (1) to outlet (2):
- $\ln(\Delta T2\ /\ \Delta T1) = -\ U\ A\ (1/Ch + 1/Cc)$

Final LMTD Formula for Parallel Flow

- From overall energy balance: $1/Ch = (Th,in - Th,out)/q$ and $1/Cc = (Tc,out - Tc,in)/q$
- Substitute these back:
- $\ln(\Delta T2 / \Delta T1) = - U A / q * (\Delta T1 - \Delta T2)$
- Rearranging for q: $q = U A [(\Delta T1 - \Delta T2) / \ln(\Delta T1 / \Delta T2)]$
- Thus, $LMTD = (\Delta T1 - \Delta T2) / \ln(\Delta T1 / \Delta T2) = (\Delta T2 - \Delta T1) / \ln(\Delta T2 / \Delta T1)$

Temperature Distribution in Counter Flow

- Fluids enter at opposite ends and flow in opposite directions.
- Inlet 1: Hot fluid in, Cold fluid out. $\Delta T1 =$ Th,in - Tc,out
- Inlet 2: Hot fluid out, Cold fluid in. $\Delta T2 =$ Th,out - Tc,in
- Temperature difference ΔT is more uniform along the length.

Derivation of LMTD for Counter Flow

- Energy balances are similar, but dTc is negative along positive x.
- dq = - Ch dTh = - Cc dTc (since cold fluid flows opposite)
- d(ΔT) = dTh - dTc = -dq(1/Ch - 1/Cc)
- Integrating gives: ln(ΔT2 / ΔT1) = - U A (1/Ch - 1/Cc)

Final LMTD Formula for Counter Flow

- Substituting the capacity rates as before yields the exact same mathematical form:
- LMTD = (ΔT1 - ΔT2) / ln(ΔT1 / ΔT2)
- IMPORTANT: The definitions of ΔT1 and ΔT2 are different from parallel flow!
- Parallel: ΔT1 = Th,in - Tc,in ; ΔT2 = Th,out - Tc,out
- Counter: ΔT1 = Th,in - Tc,out ; ΔT2 = Th,out - Tc,in

Comparison: Parallel vs Counter Flow

- For the same inlet and outlet temperatures, LMTD_counter ¿ LMTD_parallel.
- Higher LMTD means a smaller required Area (A) for the same heat transfer (q).
- Counter flow is more efficient and requires a smaller heat exchanger.
- Counter flow allows Tc,out ¿ Th,out, which is impossible in parallel flow.

Summary & Next Lecture

- LMTD is the appropriate average temperature difference for heat exchangers.
- The formula is LMTD = (ΔT1 - ΔT2) / ln(ΔT1 / ΔT2).
- ΔT definitions depend heavily on the flow arrangement.
- Counter flow is thermally more efficient than parallel flow.
- Next Lecture: LMTD for Condensers and Evaporators, and Cross-flow arrangements.

LMTD for Condensers and Evaporators

- Course: Heat and Mass Transfer (DI05019071)
- Unit 4: Heat Exchanger
- Lecture 33: Condensers, Evaporators, and Cross-flow

Lecture Agenda

- 1. Concept of Phase Change in Exchangers
- 2. Temperature Profiles in Condensers
- 3. Temperature Profiles in Evaporators
- 4. LMTD for Phase Change
- 5. Cross-flow and Multipass Exchangers
- 6. The Correction Factor (F)

Phase Change in Heat Exchangers

- In many industrial processes, one fluid undergoes a phase change (boiling or condensation).
- During phase change at a constant pressure, the temperature of the fluid remains constant.
- Specific heat capacity (cp) is effectively infinite during phase change.
- Therefore, heat capacity rate ($C = m\ cp$) approaches infinity.

Temperature Distribution in Condensers

- Hot fluid (vapor) condenses into liquid at a constant temperature (Th).
- Cold fluid temperature (Tc) increases as it absorbs the latent heat.
- $T_{h,in} = T_{h,out} = T_h$ (constant line on the graph).
- T_c rises from $T_{c,in}$ to $T_{c,out}$.

Temperature Distribution in Evaporators

- Cold fluid (liquid) boils into vapor at a constant temperature (Tc).
- Hot fluid temperature (Th) decreases as it provides the latent heat.
- $T_{c,in} = T_{c,out} = T_c$ (constant line on the graph).
- Th drops from $T_{h,in}$ to $T_{h,out}$.

LMTD for Condensers and Evaporators

- Because one temperature is constant, $\Delta T1$ and $\Delta T2$ are the same regardless of flow direction.
- Parallel flow and counter flow arrangements give the exact same LMTD.
- Condenser: $\Delta T1 = Th - Tc,in$; $\Delta T2 = Th - Tc,out$
- Evaporator: $\Delta T1 = Th,in - Tc$; $\Delta T2 = Th,out - Tc$
- $LMTD = (\Delta T1 - \Delta T2) / \ln(\Delta T1 / \Delta T2)$

Beyond Parallel and Counter Flow

- Pure parallel or counter flow is often impractical for large surface areas.
- Industry uses Cross-flow (fluids flow perpendicular to each other).
- Industry also uses Multipass shell-and-tube exchangers (e.g., 1 shell pass, 2 tube passes).
- The standard LMTD formula assumes pure parallel or counter flow.

The Correction Factor (F)

- For cross-flow and multipass exchangers, we modify the LMTD equation.
- q = U A F LMTD_cf
- Where LMTD_cf is the LMTD calculated assuming pure counter flow.
- F is a correction factor (0 ¡ F $\leq$ 1).
- If F ¡ 0.75, the heat exchanger design is generally considered uneconomical.

How to Find F

- F is found using published charts (Bowman, Mueller, and Nagle charts).
- The charts depend on two dimensionless temperature ratios: P and R.
- P (Capacity ratio) = (t2 - t1) / (T1 - t1)
- R (Temperature ratio) = (T1 - T2) / (t2 - t1)
- (Where T = shell side temps, t = tube side temps)

Correction Factor for Phase Change

- What happens to the Correction Factor (F) in a condenser or evaporator?
- Since one fluid temperature is constant, R = 0 or R = infinity.
- Looking at the F-charts, for these values, F = 1.0 exactly.
- Conclusion: For phase change, q = U A LMTD (no correction needed for cross/multipass).

Steps for LMTD Method

- 1. Calculate heat duty (q) if temperatures and mass flows are known.
- 2. Determine unknown inlet/outlet temperatures using energy balance.
- 3. Calculate LMTD for counter flow arrangement.
- 4. Find P and R, then read F from the appropriate chart (if cross/multipass).
- 5. Calculate Area A = q / (U F LMTD_cf).

Limitations of the LMTD Method

- LMTD is great for sizing problems (finding Area A when temps are known).
- LMTD is difficult for performance problems (finding outlet temps when A is known).
- If outlet temps are unknown, LMTD requires tedious trial-and-error iterations.
- To solve this, we will introduce the Effectiveness-NTU method in upcoming lectures.

Summary & Next Lecture

- Flow direction doesn't matter for condensers and evaporators.
- Cross-flow and multipass designs require a correction factor F.
- $F = 1$ for any phase change process.
- LMTD method requires iteration if outlet temperatures are unknown.
- Next Lecture: Overall Heat Transfer Coefficient and Fouling Factors.

Overall Heat Transfer Coefficient

- Course: Heat and Mass Transfer (DI05019071)
- Unit 4: Heat Exchanger
- Lecture 34: Overall Heat Transfer Coefficient and Fouling

Lecture Agenda

- 1. Recap of Heat Transfer Modes
- 2. The Thermal Resistance Network
- 3. Defining the Overall Heat Transfer Coefficient (U)
- 4. U based on Inner and Outer Areas
- 5. The Problem of Fouling
- 6. Fouling Factors (Rf) and their effect on U

The Heat Transfer Path

- In a tubular heat exchanger, heat transfers from the hot fluid to the cold fluid via 3 steps:
- 1. Convection from hot fluid to the inner pipe wall.
- 2. Conduction through the pipe wall.
- 3. Convection from the outer pipe wall to the cold fluid.
- q = ΔT_total / R_total

Thermal Resistance Network

- R_total = R_conv,inner + R_cond,wall + R_conv,outer
- R_conv,inner = 1 / (hi Ai)
- R_cond,wall = ln(ro/ri) / (2π k L) (For a cylindrical pipe)
- R_conv,outer = 1 / (ho Ao)
- R_total = 1 / (hi Ai) + ln(ro/ri) / (2π k L) + 1 / (ho Ao)

Defining U (Clean Surfaces)

- We define U such that: q = U A ΔT = ΔT / R_total
- Therefore, U A = 1 / R_total
- Since Ai Ao for a pipe, we can define U based on either area.
- Ui Ai = Uo Ao = 1 / R_total
- Ui = 1 / [Ai/Ai*hi + Ai*ln(ro/ri)/(2π k L) + Ai/(Ao*ho)]

Approximations for Thin Tubes

- For thin-walled tubes (like in most compact heat exchangers), Ai $\approx$ Ao $\approx$ A.
- The wall resistance is often very small compared to convection.
- If R_cond,wall $\approx$ 0, then:
- 1 / U $\approx$ 1 / hi + 1 / ho
- U $\approx$ (hi * ho) / (hi + ho)

The Problem of Fouling

- Over time, heat exchanger surfaces accumulate deposits.
- Scaling (minerals), rust, biological growth (algae), or suspended solids.
- This deposit acts as an additional layer of insulation.
- This phenomenon is called 'fouling' and it severely reduces U.

The Fouling Factor (Rf)

- We account for this degradation using a Fouling Factor, Rf.
- Rf has units of $m^2 \cdot K/W$ or $(hr \cdot ft^2 \cdot {}^\circ F)/Btu$.
- It represents the thermal resistance of the fouling layer.
- There can be fouling on the inside (Rfi) and the outside (Rfo).

U with Fouled Surfaces

- The total thermal resistance now has 5 terms:
- R_total = 1/(hi Ai) + Rfi/Ai + R_wall + Rfo/Ao + 1/(ho Ao)
- 1 / (Uo Ao) = 1/(hi Ai) + Rfi/Ai + ln(ro/ri)/(2πkL) + Rfo/Ao + 1/(ho Ao)
- Fouling always decreases the Overall Heat Transfer Coefficient.

Typical Values of Fouling Factors

- Distilled water: $R_f \approx 0.0001\ m^2 \cdot K/W$
- City water: $R_f \approx 0.0002\ m^2 \cdot K/W$
- River water: $R_f \approx 0.0005\ m^2 \cdot K/W$
- Fuel oil: $R_f \approx 0.0009\ m^2 \cdot K/W$
- Untreated cooling tower water requires frequent cleaning.

Controlling Heat Transfer

- The overall U is dominated by the smallest convection coefficient (hi or ho).
- If hi is water (large) and ho is air (very small), U will be close to ho.
- To improve a heat exchanger, focus on improving the side with the lowest 'h'.
- This is why fins are added to the air side of car radiators!

Example Application

- Given: hi = 1000 W/m^2K, ho = 100 W/m^2K (thin tube).
- Clean U $\approx$ (1000*100) / (1100) = 90.9 W/m^2K.
- If a fouling factor of 0.002 m^2K/W develops on the inside:
- 1/U_dirty = 1/90.9 + 0.002 = 0.011 + 0.002 = 0.013
- U_dirty = 76.9 W/m^2K (a 15% drop in performance).

Summary & Next Lecture

- U combines convection and conduction into a single term.
- Uo and Ui differ due to differing surface areas.
- Fouling introduces additional thermal resistance, lowering U.
- Designers must use fouled U-values to ensure long-term operation.
- Next Lecture: The Effectiveness-NTU Method.

Effectiveness of Heat Exchanger and NTU Method

- Course: Heat and Mass Transfer (DI05019071)
- Unit 4: Heat Exchanger
- Lecture 35: Effectiveness-NTU Method (Part 1)

Lecture Agenda

- 1. Why not use LMTD?
- 2. Heat Capacity Rate (C)
- 3. Maximum Possible Heat Transfer (q_max)
- 4. Heat Exchanger Effectiveness (ε)
- 5. The Number of Transfer Units (NTU)
- 6. Capacity Ratio (c)

The Limitation of LMTD

- LMTD is excellent for 'Sizing' problems:
- - You know inlet and outlet temps, you find LMTD, then find Area.
- LMTD is terrible for 'Performance' problems:
- - You have a specific heat exchanger (Area known).
- - You know inlet temps, but NOT outlet temps.
- - Finding outlet temps requires guessing them, calculating LMTD, and iterating until things match.

Heat Capacity Rate (C)

- First, we define the Heat Capacity Rate, C.
- C = mass flow rate (m) * specific heat (cp)
- Ch = m_h * cp_h (Hot fluid capacity rate)
- Cc = m_c * cp_c (Cold fluid capacity rate)
- Units: W/K or J/s·K

Identifying C_min and C_max

- Calculate both Ch and Cc.
- The smaller value is designated as C_min.
- The larger value is designated as C_max.
- The fluid with C_min will experience the LARGER temperature change (ΔT) in the heat exchanger.
- Since q = C * ΔT, if C is small, ΔT must be large for the same q.

Maximum Possible Heat Transfer (q_max)

- What is the theoretical maximum heat transfer possible?
- It happens in an infinitely long counter-flow heat exchanger.
- The fluid with C_min will reach the inlet temperature of the other fluid.
- q_max = C_min * (Th,in - Tc,in)
- The max temperature difference is (Th,in - Tc,in).

Defining Effectiveness (ε)

- Effectiveness (ε) is the ratio of actual heat transfer to maximum possible heat transfer.
- ε = q_actual / q_max
- Effectiveness is dimensionless and $0 \leq \varepsilon \leq 1$.
- q_actual = ε * C_min * (Th,in - Tc,in)

Calculating Actual Heat Transfer

- Once q_actual is found via effectiveness, outlet temps are easy:
- q_actual = Ch * (Th,in - Th,out)
- Th,out = Th,in - (q_actual / Ch)
- q_actual = Cc * (Tc,out - Tc,in)
- Tc,out = Tc,in + (q_actual / Cc)

How to find Effectiveness?

- Effectiveness depends on the geometry of the heat exchanger and two dimensionless groups.
- ε = function(NTU, c, flow arrangement)
- We need to define NTU and c to use the formulas and charts.

Number of Transfer Units (NTU)

- NTU is a dimensionless parameter indicating the 'size' or heat transfer capability.
- NTU = (U * A) / C_min
- Higher NTU means a larger heat exchanger (more Area or better U).
- Practically, for NTU ¿ 3, the size of the heat exchanger grows massively for very small gains in effectiveness.

Capacity Ratio (c)

- Capacity ratio 'c' is the ratio of the minimum to maximum heat capacity rates.
- c = C_min / C_max
- Since C_min $\leq$ C_max, the ratio c must be: $0 \leq c \leq 1$.
- If c = 0, one fluid is undergoing phase change (C_max = infinity).
- If c = 1, both fluids have the same capacity rate (balanced flow).

Effectiveness Formulas Preview

- Parallel Flow: ε = [1 - exp(-NTU*(1+c))] / (1+c)
- Counter Flow (c¡1): ε = [1 - exp(-NTU*(1-c))] / [1 - c*exp(-NTU*(1-c))]
- Counter Flow (c=1): ε = NTU / (1 + NTU)
- These are standard formulas provided in data books.

Summary & Next Lecture

- LMTD requires iteration for performance problems.
- q_max occurs when the C_min fluid reaches the opposite inlet temperature.
- Effectiveness ε = q_actual / q_max.
- ε depends on NTU (UA/Cmin) and c (Cmin/Cmax).
- Next Lecture: Using Effectiveness charts and solving numerical problems.

Effectiveness of Heat Exchanger: Applications

- Course: Heat and Mass Transfer (DI05019071)
- Unit 4: Heat Exchanger
- Lecture 36: Applying the NTU Method

Lecture Agenda

- 1. Reading Effectiveness-NTU Charts
- 2. Special Case: Phase Change ($c = 0$)
- 3. Special Case: Balanced Flow ($c = 1$)
- 4. Step-by-Step: Performance Problems
- 5. Step-by-Step: Design/Sizing Problems
- 6. Unit 4 Summary

Effectiveness-NTU Charts

- Instead of complex formulas, engineers often use charts.
- X-axis: NTU (Number of Transfer Units)
- Y-axis: Effectiveness (ε, from 0 to 1 or 0 to 100%)
- Curves: Different values of capacity ratio 'c' (from 0 to 1).

Chart Observations

- For a given NTU and given flow type, effectiveness increases as 'c' decreases.
- Maximum effectiveness always occurs at c = 0.
- For NTU ¿ 3, the curves flatten out.
- Increasing NTU (making the exchanger larger/more expensive) yields diminishing returns in performance.

Special Case: Phase Change (c = 0)

- For condensers and evaporators, C_max = infinity, so c = 0.
- Looking at the formulas or charts, the flow arrangement (parallel vs counter) disappears from the math.
- ε = 1 - exp(-NTU) (for all flow arrangements!)
- This perfectly matches our conclusion from the LMTD method.

Special Case: Balanced Flow (c = 1)

- If C_min = C_max, then c = 1.
- Example: Gas-to-gas heat recovery with equal flow rates.
- Parallel Flow: ε = 0.5 * [1 - exp(-2*NTU)] (Max ε = 50%)
- Counter Flow: ε = NTU / (1 + NTU) (Max ε approaches 100%)
- This proves counter flow is vastly superior for balanced flows.

Solving Performance Problems

- Goal: Find outlet temps when Area (A) is given.
- 1. Calculate C_h, C_c. Determine C_min, C_max, and c.
- 2. Calculate NTU = UA / C_min.
- 3. Find ε using formulas or charts.
- 4. Calculate q_max = C_min * (Th,in - Tc,in).
- 5. Calculate actual q = ε * q_max.
- 6. Calculate Th,out and Tc,out using q.

Solving Design (Sizing) Problems

- Goal: Find Area (A) when inlet and outlet temps are known.
- 1. Calculate C_h, C_c. Determine C_min, C_max, and c.
- 2. Calculate actual q, and q_max.
- 3. Calculate required ε = q / q_max.
- 4. Find required NTU using charts (or inverted formulas).
- 5. Calculate Area = (NTU * C_min) / U.

Example Problem: Performance

- A counter-flow exchanger has U = 200 W/m^{2}K, A = 10 m^2.
- Hot water: 2 kg/s, cp=4180 J/kgK, inlet = 90°C.
- Cold water: 1 kg/s, cp=4180 J/kgK, inlet = 20°C.
- Step 1: Ch = 8360 W/K. Cc = 4180 W/K. C_min = Cc = 4180.
- Step 2: c = 4180/8360 = 0.5.

Example Problem: Solution

- Step 3: NTU = UA/C_min = (200*10)/4180 = 0.478.
- Step 4: Use chart or formula for Counter flow (NTU=0.478, c=0.5) -¿ $\varepsilon \approx 0.35$
- Step 5: q_max = 4180 * (90 - 20) = 292,600 W
- Step 6: q_actual = 0.35 * 292,600 = 102,410 W
- Step 7: Tc,out = 20 + 102,410/4180 = 44.5°C

Choosing Between Methods

- When to use LMTD:
- - Sizing a new heat exchanger (finding Area).
- - All inlet and outlet temperatures are known or easily found.
- When to use Effectiveness-NTU:
- - Evaluating an existing heat exchanger.
- - Finding outlet temperatures.
- - Comparing different heat exchanger designs (ε is a great metric).

Unit 4 Summary

- Unit 4 covered the analysis of heat exchangers.
- We learned basic classifications (parallel, counter, cross flow).
- We defined the Overall Heat Transfer Coefficient (U) and fouling.
- We derived and applied the LMTD method.
- We defined Effectiveness, NTU, and applied the NTU method.

Next Unit Preview

- Unit 5: Mass Transfer
- We will move away from moving heat, and look at moving mass.
- Topics will include Fick's Law of Diffusion, concentration gradients, and modes of mass transfer.
- Prepare by reviewing concentration definitions (molarity, mass fraction).

Concept of Heat Pipe

- Course: Heat and Mass Transfer (DI05019071)
- Unit 4: Heat exchanger
- Lecture 37: Concept of Heat Pipe

Lecture Agenda

- 1. Introduction to Heat Pipes
- 2. Working Principle
- 3. Components of a Heat Pipe
- 4. Working Fluid Selection
- 5. Capillary Action and Wick Structures
- 6. Advantages and Limitations
- 7. Applications

What is a Heat Pipe?

- A heat pipe is a highly efficient heat-transfer device that combines the principles of both thermal conductivity and phase transition.
- It can transfer heat between two solid interfaces efficiently.
- It has an effective thermal conductivity much higher than solid conductors like copper or aluminum.
- Operates passively without any moving parts or external power supply.

Working Principle

- Heat is applied to the evaporator section, causing the working fluid to vaporize.
- The vapor pressure drives the vapor through the adiabatic section to the condenser.
- At the condenser, heat is removed, and the vapor condenses back into a liquid.
- The liquid returns to the evaporator via capillary action in the wick structure.

Components of a Heat Pipe

- 1. Container: Provides structural integrity and isolates the working fluid from the outside environment.
- 2. Working Fluid: Absorbs and releases latent heat. Chosen based on operating temperature.
- 3. Wick Structure: Provides the capillary pumping pressure to return the liquid to the evaporator.

Working Fluid Selection

- Selection depends heavily on the operating temperature range.
- Cryogenic: Helium, Nitrogen (0-200 K)
- Room Temperature: Water, Ammonia, Methanol (200-500 K)
- Liquid Metals: Sodium, Potassium, Lithium (500-1500 K)
- Properties desired: High latent heat, high surface tension, high thermal conductivity, and good thermal stability.

Wick Structures and Capillary Action

- The wick is responsible for pumping the condensed liquid back to the evaporator against gravity or friction.
- Types of Wicks:
- - Sintered Metal Powder: Excellent capillary force, can work against gravity.
- - Grooved Tube: Low capillary force but low flow resistance; best for horizontal use.
- - Screen Mesh: Balance between capillary force and permeability.

Heat Pipe Limitations

- Capillary Limit: Wick cannot pump liquid back fast enough (dry out).
- Sonic Limit: Vapor velocity reaches the speed of sound, choking the flow.
- Entrainment Limit: High vapor velocity strips liquid droplets from the wick.
- Boiling Limit: Film boiling occurs in the evaporator, blocking liquid flow.

Advantages of Heat Pipes

- Very high effective thermal conductivity.
- Passive operation (no moving parts, zero maintenance).
- Isothermal operation over long distances.
- Lightweight and compact compared to solid metal conductors.
- Flexible geometries (can be bent or flattened).

Applications: Electronics Cooling

- Laptops and PCs: Moving heat from CPU/GPU to the cooling fan at the edge.
- Smartphones: Ultra-thin vapor chambers (flattened heat pipes) for processor cooling.
- Power Electronics: Cooling high-power transistors and LEDs.

Applications: Aerospace & Industrial

- Aerospace: Thermal control of satellites, moving heat from sun-facing side to shaded radiators.
- HVAC: Energy recovery ventilators (air-to-air heat exchangers).
- Industrial: Waste heat recovery systems, nuclear reactors.

Summary

- Heat pipes transfer heat efficiently using evaporation and condensation.
- They consist of a sealed container, working fluid, and wick structure.
- They operate passively with very high effective thermal conductivity.
- Widely used in electronics, aerospace, and energy recovery.

Next Lecture Preview

- Topic: Compact Heat Exchangers
- Key questions to ponder:
- - What makes a heat exchanger 'compact'?
- - Where are compact heat exchangers used, and why are they necessary?

Compact Heat Exchangers

- Course: Heat and Mass Transfer (DI05019071)
- Unit 4: Heat exchanger
- Lecture 38: Compact Heat Exchangers

Lecture Agenda

- 1. Introduction and Definition
- 2. Why Use Compact Heat Exchangers?
- 3. Classification and Types
- 4. Plate-Fin Heat Exchangers
- 5. Tube-and-Fin Heat Exchangers
- 6. Pressure Drop Considerations
- 7. Applications

Definition of a Compact Heat Exchanger

- A compact heat exchanger is defined by a high area density (ratio of heat transfer surface area to its volume, denoted by β).
- General criteria:
- β = Area / Volume ¿ 700 m^2/m^3 (for gas-to-fluid).
- β ¿ 400 m^2/m^3 (for liquid or phase-change applications).
- They incorporate extended surfaces (fins) to achieve this high density.

Why Use Compact Heat Exchangers?

- Gas flows have low convection heat transfer coefficients (h).
- To achieve a desired heat transfer rate ($Q = h * A * \Delta T$), a large area (A) is required.
- Space and weight are often severely constrained (e.g., aerospace, automotive).
- Compact designs provide massive area within a small footprint, compensating for the low 'h' of gases.

Classification and Types

- Compact heat exchangers are typically cross-flow types and are classified mainly by their construction:
- 1. Plate-Fin Heat Exchangers (PFHE)
- 2. Tube-and-Fin Heat Exchangers
- 3. Printed Circuit Heat Exchangers (PCHE)
- 4. Regenerators (Rotary matrix)

Plate-Fin Heat Exchangers (PFHE)

- Constructed from alternating layers of corrugated fins and flat separator plates.
- Fluids flow through the passages created by the fins.
- Allows for multiple fluid streams (3 or more) in a single unit.
- Primarily made of brazed aluminum for lightweight and good conductivity.

Types of Fins in PFHE

- Fin geometry dictates heat transfer enhancement and flow friction.
- 1. Plain Fins: Straight channels, low friction, lower heat transfer.
- 2. Wavy Fins: Induce swirling, better heat transfer.
- 3. Offset Strip / Serrated Fins: Break boundary layers, very high heat transfer, high pressure drop.
- 4. Louvered Fins: Slices in the fin direct flow, very common in radiators.

Tube-and-Fin Heat Exchangers

- Consist of round or flat tubes with continuous fins attached to the outside.
- Typically used when one fluid is a liquid (in the tube) and the other is a gas (across the fins).
- Flat tubes offer less aerodynamic drag on the gas side compared to round tubes.

Pressure Drop and Pumping Power

- Trade-off: Increasing surface density and boundary layer interruption increases heat transfer but also increases flow friction.
- Higher pressure drop means a larger fan or pump is required.
- Pumping power costs can exceed the material costs of the exchanger over its lifetime.
- Optimization is required to balance Colburn j-factor (heat transfer) and Friction factor (f).

Applications: Automotive & HVAC

- Automotive: Radiators, charge air coolers (intercoolers), oil coolers, AC condensers.
- HVAC: Air conditioning evaporator and condenser coils, heat recovery wheels.
- Electronics: CPU/GPU heatsinks (fin arrays).

Applications: Aerospace & Cryogenics

- Aerospace: Environmental control systems, engine oil coolers.
- Cryogenics: Air separation plants, liquefaction of natural gas (LNG).
- Cryogenic PFHEs can achieve temperature approaches of less than 2°C between streams.

Summary

- Compact heat exchangers have area densities ¿ 700 m^2/m^3.
- They are essential for gas-flow applications where 'h' is low.
- Plate-fin and tube-fin are the most common configurations.
- Design requires a careful balance between heat transfer performance and pressure drop.

Next Lecture Preview

- Topic: Numerical Examples on Heat Exchangers (Part 1)
- Key questions to ponder:
- - How do we apply the LMTD method to solve real-world problems?
- - What are the step-by-step procedures for parallel and counter-flow designs?

Numerical examples on Heat Exchangers (Part 1)

- Course: Heat and Mass Transfer (DI05019071)
- Unit 4: Heat exchanger
- Lecture 39: Numerical examples (LMTD Method)

Lecture Agenda

- 1. Problem Solving Strategy
- 2. Recap of Key Formulas (Energy Balance & LMTD)
- 3. Example 1: Parallel Flow Heat Exchanger
- 4. Example 2: Counter Flow Heat Exchanger
- 5. Comparison and Discussion

Problem Solving Strategy

- Step 1: Identify the flow arrangement (Parallel, Counter, Cross, etc.).
- Step 2: Note down given values (mass flow rates, specific heats, inlet/outlet temperatures, U).
- Step 3: Perform an overall energy balance to find any missing temperatures: Q = mh*Cph*ΔTh = mc*Cpc*ΔTc.
- Step 4: Calculate temperature differences at both ends (ΔT1, ΔT2).
- Step 5: Calculate LMTD (ΔTm).
- Step 6: Use Q = U * A * ΔTm to find the unknown (Area, U, or Q).

Recap of Key Formulas

- Energy Balance: Q = m_h * C_ph * (T_h,in - T_h,out) = m_c * C_pc * (T_c,out - T_c,in)
- Heat Transfer Rate: Q = U * A * ΔT_lm
- LMTD Formula: ΔT_lm = (ΔT1 - ΔT2) / ln(ΔT1 / ΔT2)
- For Parallel Flow:
- - ΔT1 = T_h,in - T_c,in
- - ΔT2 = T_h,out - T_c,out
- For Counter Flow:
- - ΔT1 = T_h,in - T_c,out
- - ΔT2 = T_h,out - T_c,in

Example 1: Problem Statement (Parallel Flow)

- Water (Cp = 4.18 kJ/kg.K) enters a parallel flow heat exchanger at 15°C at a rate of 0.5 kg/s.
- It is heated by hot oil (Cp = 2.1 kJ/kg.K) entering at 100°C at a rate of 0.8 kg/s.
- The overall heat transfer coefficient is 350 $W/m^2.K$.
- The outlet temperature of the oil is 60°C.
- Find: (a) The rate of heat transfer, (b) Outlet temperature of water, (c) The surface area of the heat exchanger.

Example 1: Solution Step 1 & 2

- Step 1: Calculate Heat Transfer Rate (Q)
- Q = m_oil * Cp_oil * (T_oil,in - T_oil,out)
- Q = 0.8 kg/s * 2.1 kJ/kg.K * (100 - 60) K
- Q = 0.8 * 2.1 * 40 = 67.2 kW (or 67,200 W)
-
- Step 2: Find Outlet Temperature of Water (T_c,out)
- Q = m_water * Cp_water * (T_c,out - T_c,in)
- 67.2 = 0.5 * 4.18 * (T_c,out - 15)
- T_c,out - 15 = 67.2 / 2.09 = 32.15°C
- T_c,out = 47.15°C

Example 1: Solution Step 3 & 4

- Step 3: Calculate $\Delta T1$ and $\Delta T2$ for Parallel Flow
- $\Delta T1 = T\_h,in - T\_c,in = 100 - 15 = 85°C$
- $\Delta T2 = T\_h,out - T\_c,out = 60 - 47.15 = 12.85°C$
-
- Step 4: Calculate LMTD
- $\Delta T\_lm = (85 - 12.85) / \ln(85 / 12.85)$
- $\Delta T\_lm = 72.15 / \ln(6.615)$
- $\Delta T\_lm = 72.15 / 1.889 = 38.19°C$

Example 1: Final Calculation

- Step 5: Calculate Surface Area (A)
- Formula: $Q = U * A * \Delta T\_lm$
- Rearranging for A: $A = Q / (U * \Delta T\_lm)$
- Substitute values (ensure Q is in Watts):
- $A = 67{,}200\ W / (350\ W/m^2.K * 38.19\ K)$
- $A = 67{,}200 / 13366.5$
- Area (A) $= 5.03\ m^2$

Example 2: Problem Statement (Counter Flow)

- Let's use the same fluids, temperatures, and flow rates, but arrange the exchanger in Counter Flow.
- Oil: Inlet = 100°C, Outlet = 60°C. Q is still 67.2 kW.
- Water: Inlet = 15°C, Outlet = 47.15°C.
- U = 350 $W/m^2.K$
- Find: The surface area of the heat exchanger in counter-flow arrangement.
- Compare this area to the parallel flow case.

Example 2: LMTD Calculation

- Calculate $\Delta T1$ and $\Delta T2$ for Counter Flow:
- $\Delta T1 = T\_h,in - T\_c,out$
- $\Delta T1 = 100 - 47.15 = 52.85°C$
-
- $\Delta T2 = T\_h,out - T\_c,in$
- $\Delta T2 = 60 - 15 = 45°C$
-
- Calculate LMTD:
- $\Delta T\_lm = (52.85 - 45) / \ln(52.85 / 45)$
- $\Delta T\_lm = 7.85 / \ln(1.174)$
- $\Delta T\_lm = 7.85 / 0.1608 = 48.81°C$

Example 2: Final Area & Comparison

- Calculate Area:
- $A = Q / (U * \Delta T\_lm)$
- $A = 67{,}200 / (350 * 48.81)$
- $A = 67{,}200 / 17083.5 = 3.93\ m^2$
-
- Comparison:
- Parallel Flow Area $= 5.03\ m^2$
- Counter Flow Area $= 3.93\ m^2$
- Counter-flow requires ~22% less surface area for the exact same duty.

Summary

- The LMTD method is straightforward when inlet and outlet temperatures are known or easily found via energy balance.
- Care must be taken to define $\Delta T1$ and $\Delta T2$ correctly based on the flow arrangement.
- Counter-flow configurations always yield a higher LMTD than parallel-flow for the same temperatures, resulting in a smaller required area.

Next Lecture Preview

- Topic: Numerical Examples on Heat Exchangers (Part 2)
- Key topics:
- - What happens when we don't know the outlet temperatures?
- - Introduction to the Effectiveness-NTU method for problem-solving.
- - Solving cross-flow and shell-and-tube problems.

Numerical examples on Heat Exchangers (Part 2)

- Course: Heat and Mass Transfer (DI05019071)
- Unit 4: Heat exchanger
- Lecture 40: Numerical examples (NTU Method & Cross-Flow)

Lecture Agenda

- 1. When to use LMTD vs. NTU Method
- 2. Recap of Effectiveness-NTU Formulas
- 3. Example 3: Performance Prediction (NTU Method)
- 4. Recap of Multipass Correction Factor (F)
- 5. Example 4: Shell and Tube Exchanger with 'F' Factor

LMTD vs. Effectiveness-NTU Method

- LMTD Method is best for SIZING problems:
- - You know inlet and outlet temperatures.
- - You need to find the required Surface Area (A).
-
- Effectiveness-NTU Method is best for RATING/PERFORMANCE problems:
- - You know the Surface Area (A) and inlet temperatures.
- - You need to predict outlet temperatures and Heat Transfer Rate (Q).

Recap: Effectiveness-NTU Formulas

- Heat Capacity Rates: C_h = m_h * Cp_h, C_c = m_c * Cp_c
- Identify C_min (the smaller of C_h and C_c) and C_max.
- Capacity Ratio: c = C_min / C_max
- Maximum possible heat transfer: Q_max = C_min * (T_h,in - T_c,in)
- Number of Transfer Units (NTU) = (U * A) / C_min
- Effectiveness (ε) = Q_actual / Q_max
- ε is found from charts or formulas based on NTU, c, and flow arrangement.

Example 3: Problem Statement (NTU Method)

- A counter-flow heat exchanger has a surface area of 10 m^2 and U = 200 $W/m^2.K$.
- Hot fluid (Cp = 2.0 kJ/kg.K) enters at 120°C at 1.0 kg/s.
- Cold fluid (Cp = 4.18 kJ/kg.K) enters at 20°C at 0.5 kg/s.
- Find: The rate of heat transfer and the outlet temperatures.

Example 3: Capacity Rates and NTU

- Step 1: Calculate Capacity Rates
- C_h = 1.0 kg/s * 2000 J/kg.K = 2000 W/K
- C_c = 0.5 kg/s * 4180 J/kg.K = 2090 W/K
- Therefore, C_min = C_h = 2000 W/K.
- Capacity ratio (c) = C_min / C_max = 2000 / 2090 = 0.957
-
- Step 2: Calculate NTU
- NTU = (U * A) / C_min = (200 * 10) / 2000 = 2000 / 2000 = 1.0

Example 3: Effectiveness and Q_actual

- Step 3: Find Effectiveness (ε) for Counter Flow
- Formula for Counter Flow: ε = [1 - exp(-NTU*(1-c))] / [1 - c*exp(-NTU*(1-c))]
- ε = [1 - exp(-1*(1-0.957))] / [1 - 0.957*exp(-1*(1-0.957))]
- ε = [1 - exp(-0.043)] / [1 - 0.957*exp(-0.043)]
- ε = [1 - 0.9579] / [1 - 0.957(0.9579)] = 0.0421 / (1 - 0.9167) = 0.505 (or 50.5%)
-
- Step 4: Calculate Q_max and Q_actual
- Q_max = C_min * (T_h,in - T_c,in) = 2000 * (120 - 20) = 200,000 W = 200 kW
- Q_actual = ε * Q_max = 0.505 * 200 kW = 101 kW

Example 3: Outlet Temperatures

- Step 5: Calculate Outlet Temperatures
- From Q = C_h * (T_h,in - T_h,out)
- 101,000 = 2000 * (120 - T_h,out)
- 50.5 = 120 - T_h,out =¿ T_h,out = 69.5°C
-
- From Q = C_c * (T_c,out - T_c,in)
- 101,000 = 2090 * (T_c,out - 20)
- 48.33 = T_c,out - 20 =¿ T_c,out = 68.33°C

Recap: Multipass Correction Factor (F)

- For Cross-Flow and Multipass Shell-and-Tube exchangers, the flow is neither pure parallel nor pure counter-flow.
- We calculate LMTD assuming pure counter-flow, then apply a correction factor (F).
- Q = U * A * F * ΔT_lm,CF
- F is obtained from charts based on parameters P and R:
- - P = (t2 - t1) / (T1 - t1) (Effectiveness parameter)
- - R = (T1 - T2) / (t2 - t1) (Capacity ratio parameter)

Example 4: Problem Statement (Shell & Tube)

- A 1-shell pass, 2-tube pass heat exchanger is used to heat water in tubes from 20°C to 70°C.
- Hot oil in the shell cools from 140°C to 90°C.
- Total heat transfer rate Q = 150 kW.
- U = 400 $W/m^2.K$.
- Find the required surface area.

Example 4: Solution

- Step 1: Calculate LMTD for Counter Flow
- $\Delta T1 = 140 - 70 = 70°C$, $\Delta T2 = 90 - 20 = 70°C$
- Since $\Delta T1 = \Delta T2$, LMTD $= 70°C$.
-
- Step 2: Find P and R parameters
- Let tube fluid be lower case (t), shell fluid be upper case (T).
- $P = (70 - 20) / (140 - 20) = 50 / 120 = 0.417$
- $R = (140 - 90) / (70 - 20) = 50 / 50 = 1.0$
-
- Step 3: From chart for 1-2 exchanger, at P=0.417, R=1.0, F ≈ 0.85

Example 4: Final Area Calculation

- Step 4: Calculate Area
- Formula: $Q = U * A * F * \Delta T\_lm$
- $150{,}000\ W = 400\ W/m^2.K * A * 0.85 * 70\ K$
- $150{,}000 = A * (400 * 0.85 * 70)$
- $150{,}000 = A * 23{,}800$
- $Area = 150{,}000 / 23{,}800 = 6.30\ m^2$

Summary & Unit 4 Conclusion

- NTU Method is essential when outlet temperatures are unknown (rating/performance problems).
- Correction factor (F) is mandatory for multipass and cross-flow designs when using LMTD.
- This concludes Unit 4: Heat Exchangers. You should now be able to analyze and size various heat transfer equipment.

Modes of Mass Transfer

- Course: Heat and Mass Transfer (DI05019071)
- Unit 5: Mass Transfer
- Lecture 41: Modes of Mass Transfer

Lecture Agenda

- 1. Introduction to Mass Transfer
- 2. Driving Force for Mass Transfer
- 3. Everyday Examples
- 4. Molecular Diffusion
- 5. Convective Mass Transfer
- 6. Analogy between Heat and Mass Transfer

What is Mass Transfer?

- Mass transfer is the net movement of mass from one location, usually meaning a stream, phase, fraction or component, to another.
- It occurs in mixtures containing two or more chemical species.
- Mass transfer occurs whenever there is a difference in the concentration of a species in a mixture.

The Driving Force for Mass Transfer

- The driving force for heat transfer is a temperature gradient.
- The driving force for mass transfer is a **concentration gradient**.
- Species in a mixture will naturally move from a region of higher concentration to a region of lower concentration.
- The system tends to reach a state of equilibrium where concentration is uniform.

Everyday Examples of Mass Transfer

- Dissolving sugar in a cup of coffee.
- Drying of wet clothes (water evaporating into the air).
- Spreading of a perfume scent in a room.
- Oxygen dissolving into blood in the lungs.
- Sweating to cool the human body.

Modes of Mass Transfer: Overview

- Similar to heat transfer, mass transfer can occur in different modes:
- 1. **Molecular Diffusion**: Mass transfer by random molecular motion in a stagnant fluid or solid.
- 2. **Convective Mass Transfer**: Mass transfer between a boundary surface and a moving fluid or between two immiscible moving fluids.

Molecular Diffusion

- Analogous to heat conduction.
- Occurs in stationary fluids (gases and liquids) or solid mediums.
- Transport occurs due to the random thermal motion of molecules.
- Governed by Fick's Law of Diffusion.

Convective Mass Transfer

- Analogous to heat convection.
- Involves the transport of mass between a boundary surface (solid or liquid) and a moving fluid.
- Relies on the bulk motion of the fluid.
- Can be classified into:
- - Free (natural) convective mass transfer
- - Forced convective mass transfer

Analogy between Heat and Mass Transfer

- Many mass transfer concepts can be mapped directly to heat transfer:
- **Heat Transfer** — **Mass Transfer**
- Temperature Gradient (dT/dx) — Concentration Gradient (dC/dx)
- Heat Conduction — Molecular Diffusion
- Heat Convection — Convective Mass Transfer
- Fourier's Law — Fick's Law
- Thermal Conductivity (k) — Diffusion Coefficient (D)

Key Differences

- Despite the strong analogy, there are key differences:
- 1. No mass transfer equivalent to thermal radiation.
- 2. Mass transfer strictly requires a mixture; heat transfer can occur in a pure substance.
- 3. High rates of mass transfer can induce a velocity at the fluid-solid boundary, altering the flow field (unlike heat transfer where the boundary is impermeable).

Summary

- Mass transfer is driven by a concentration gradient.
- The two main modes are molecular diffusion (analogous to conduction) and convective mass transfer (analogous to convection).
- Molecular diffusion relies on random microscopic motion, whereas convective mass transfer relies on bulk fluid flow.
- The heat and mass transfer analogy provides a strong foundation for understanding mass transfer equations.

Next Lecture Preview

- Topic: Concentrations, Velocities, and Fluxes
- Key questions to ponder:
- - How do we mathematically define the concentration of a mixture?
- - What is the difference between mass concentration and molar concentration?
- - How do we define the average velocity of a mixture where different species are moving at different speeds?

Concentrations, Velocities and Fluxes

- Course: Heat and Mass Transfer (DI05019071)
- Unit 5: Mass Transfer
- Lecture 42: Concentrations, Velocities and Fluxes

Lecture Agenda

- 1. Mixture Composition
- 2. Mass Concentration (Density)
- 3. Molar Concentration
- 4. Mass and Mole Fractions
- 5. Mixture Velocities
- 6. Mass and Molar Fluxes

Mixture Composition: An Overview

- To describe mass transfer mathematically, we must quantify the composition of a mixture.
- A mixture consists of two or more chemical species (e.g., Species A, Species B, Species C, ...).
- The concentration of a species can be expressed in two primary ways:
- 1. On a mass basis (Mass Concentration)
- 2. On a molar basis (Molar Concentration)

Mass Concentration (Partial Density)

- Mass concentration (ρ_A) of species A is the mass of species A per unit volume of the mixture.
- ρ_A = m_A / V (Unit: kg/m^3)
- The total mass density (ρ) of the mixture is the sum of the mass concentrations of all species:
- $\rho = \rho\_A + \rho\_B + ... + \rho\_n = \Sigma(\rho\_i)$

Molar Concentration

- Molar concentration (C_A) of species A is the number of moles of species A per unit volume.
- C_A = n_A / V (Unit: kmol/m^3 or mol/L)
- The total molar concentration (C) of the mixture is the sum of the molar concentrations of all species:
- C = C_A + C_B + ... + C_n = Σ(C_i)
- Relation between mass and molar concentration: ρ_A = C_A * M_A (where M_A is molecular weight).

Mass and Mole Fractions

- Mass Fraction (m_A): The ratio of the mass concentration of species A to the total density.
- m_A = ρ_A / ρ (Note: Σ(m_i) = 1)
- Mole Fraction (x_A or y_A): The ratio of the molar concentration of species A to the total molar concentration.
- x_A = C_A / C (Note: Σ(x_i) = 1)
- Note: x_A is typically used for liquids/solids, y_A for gases.

Mixture Velocity

- In a moving mixture, each species may have its own absolute velocity (v_i).
- The overall velocity of the mixture can be defined in two ways:
- 1. Mass-average velocity (v)
- 2. Molar-average velocity (V)
- This is important because mass transfer involves movement relative to the bulk fluid motion.

Mass and Molar Average Velocities

- **Mass-Average Velocity (v)**:
- v = Σ(ρ_i * v_i) / ρ
- It is the velocity based on the momentum of the mixture.
- **Molar-Average Velocity (V)**:
- V = Σ(C_i * v_i) / C
- It is the velocity based on the molar flow of the mixture.

Concept of Diffusion Velocity

- The velocity of a species relative to the bulk mixture velocity is its **diffusion velocity**.
- Mass diffusion velocity of species A = (v_A - v)
- Molar diffusion velocity of species A = (v_A - V)
- Diffusion velocity tells us how fast a species is moving purely due to concentration gradients, independent of the bulk flow.

Mass and Molar Fluxes

- **Flux** is the amount of species transferred per unit area per unit time.
- **Absolute Mass Flux (n_A)** = ρ_A * v_A (kg/m^2·s)
- **Mass Diffusion Flux (j_A)** = ρ_A * (v_A - v)
- **Absolute Molar Flux (N_A)** = C_A * v_A (kmol/m^2·s)
- **Molar Diffusion Flux (J_A)** = C_A * (v_A - V)
- Relationship: n_A = j_A + ρ_A * v

Summary

- Concentration can be expressed by mass (ρ_A) or by moles (C_A).
- Mass fractions (m_A) and mole fractions (x_A) sum to 1.
- A mixture has a mass-average velocity and a molar-average velocity.
- Fluxes can be defined based on absolute velocity or diffusion velocity.
- Total flux = Diffusion flux + Convective (Bulk) flux.

Next Lecture Preview

- Topic: Fick's Law of Diffusion
- Key questions to ponder:
- - How is diffusion flux mathematically related to the concentration gradient?
- - What is the physical significance of the diffusion coefficient (diffusivity)?
- - How does Fick's Law compare to Fourier's Law of heat conduction?

Fick's Law of Diffusion

- Course: Heat and Mass Transfer (DI05019071)
- Unit 5: Mass Transfer
- Lecture 43: Fick's Law

Lecture Agenda

- 1. Introduction to Fick's First Law
- 2. The Mathematical Equation
- 3. Diffusion Coefficient (Diffusivity)
- 4. Characteristics of Diffusivity in different phases
- 5. Fick's Law: Mass vs. Molar Basis
- 6. General Mass Diffusion Equation

Introduction to Fick's First Law

- Proposed by Adolf Fick in 1855.
- It states that the mass diffusion flux of a constituent in a mixture is proportional to the concentration gradient.
- Diffusion always occurs in the direction of decreasing concentration.
- It perfectly mirrors Fourier's Law of heat conduction.

Fick's First Law (Mass Basis)

- The equation for mass diffusion flux (j_A) in the x-direction:
- j_A = - D_AB * (d(ρ_A) / dx)
- Where:
- - j_A = Mass diffusion flux of species A (kg/m^2·s)
- - D_AB = Diffusion coefficient (or mass diffusivity) of species A in species B (m^2/s)
- - d(ρ_A) / dx = Concentration gradient of species A (kg/m^4)

Diffusion Coefficient (Diffusivity)

- Denoted as D_AB (diffusion of A into B).
- Unit: m^2/s (Same unit as thermal diffusivity, α, and kinematic viscosity, ν).
- It is a property of the specific binary mixture (A and B), the temperature, and the pressure.
- A higher D_AB means mass diffuses more rapidly.

Characteristics of Diffusivity

- **In Gases**: Diffusion is rapid. D_AB is relatively high (approx. 10^-5 m^2/s). D_AB increases with temperature and decreases with pressure.
- **In Liquids**: Diffusion is much slower due to close molecular packing (approx. 10^-9 m^2/s).
- **In Solids**: Diffusion is extremely slow (approx. 10^-14 m^2/s). Depends heavily on the solid structure (crystalline, amorphous).

Fick's Law (Molar Basis)

- Chemical engineers often prefer the molar basis.
- Molar diffusion flux (J_A):
- J_A = - D_AB * (dC_A / dx)
- Where:
- - J_A = Molar diffusion flux (kmol/m^2·s)
- - C_A = Molar concentration (kmol/m^3)
- - dC_A / dx = Molar concentration gradient (kmol/m^4)

Fick's Law using Mass/Mole Fractions

- If total mixture density (ρ) is constant:
- j_A = - ρ * D_AB * (dm_A / dx)
- If total molar concentration (C) is constant (typical for ideal gas mixtures at constant T and P):
- J_A = - C * D_AB * (dx_A / dx)
- This form is highly useful in derivation and problem solving.

Total Absolute Flux Equation

- Recall from last lecture: Total Absolute Flux = Diffusion Flux + Bulk Flux
- n_A = j_A + ρ_A * v
- Substituting Fick's Law for j_A:
- n_A = - D_AB * (dρ_A / dx) + ρ_A * v
- Similarly for Molar Flux:
- N_A = - D_AB * (dC_A / dx) + x_A * (N_A + N_B)

General Mass Diffusion Equation

- Analogous to the general heat conduction equation (Heat Equation).
- Derived from species conservation over a control volume.
- Governing Equation (1D, stationary medium, no bulk flow):
- ∂C_A / ∂t = D_AB * (∂^2 C_A / ∂x^2) + R_A
- (Where R_A is the rate of generation of species A due to chemical reactions).

Summary

- Fick's First Law relates diffusion flux linearly to the concentration gradient.
- The negative sign indicates mass moves from high to low concentration.
- Diffusivity (D_AB) depends on the mixture, temperature, and pressure, and is highest in gases.
- The total absolute mass flux includes both Fickian diffusion and bulk fluid motion.

Next Lecture Preview

- Topic: Steady state diffusion through a plain membrane
- Key questions to ponder:
- - How do we apply Fick's law to a solid membrane?
- - What happens to the concentration profile across a membrane in steady state?
- - How can we use the concept of 'mass transfer resistance' to solve problems quickly?

Steady State Diffusion Through a Plain Membrane

- Course: Heat and Mass Transfer (DI05019071)
- Unit 5: Mass Transfer
- Lecture 44: Steady State Diffusion Through a Plain Membrane

Lecture Agenda

- 1. Problem Definition and Assumptions
- 2. The Governing Equation
- 3. Deriving the Concentration Profile
- 4. Mass Transfer Rate Calculation
- 5. The Concept of Mass Transfer Resistance
- 6. Practical Applications

Problem Definition & Assumptions

- **Scenario**: A species A (e.g., hydrogen gas) diffuses through a plain solid membrane (species B, e.g., a steel plate) of thickness L.
- **Assumptions**:
- 1. One-dimensional mass transfer (x-direction).
- 2. Steady-state conditions (∂C_A/∂t = 0).
- 3. No chemical reactions (Generation rate R_A = 0).
- 4. The solid membrane is stationary (bulk velocity = 0).
- 5. Constant diffusion coefficient D_AB.

The Governing Equation

- Starting from the general mass diffusion equation:
- ∂C_A / ∂t = D_AB * (∂^2 C_A / ∂x^2) + R_A
- Applying assumptions (Steady state, no reaction):
- 0 = D_AB * (d^2 C_A / dx^2)
- Therefore:
- d^2 C_A / dx^2 = 0

Concentration Profile Derivation

- Equation: d^2 C_A / dx^2 = 0
- Integrating once: dC_A / dx = C1
- Integrating twice: C_A(x) = C1 * x + C2
- **Boundary Conditions**:
- At x = 0, C_A = C_A1
- At x = L, C_A = C_A2
- Resulting Profile: C_A(x) = C_A1 - (C_A1 - C_A2) * (x / L)

Rate of Mass Transfer

- Now apply Fick's Law to find the molar transfer rate (N_A * Area):
- N_A = - D_AB * (dC_A / dx)
- From our profile, dC_A / dx = - (C_A1 - C_A2) / L
- Therefore, Molar Flux: N_A = D_AB * (C_A1 - C_A2) / L
- Total Molar Rate (W_A) in kmol/s for Area A:
- W_A = N_A * A = [D_AB * A * (C_A1 - C_A2)] / L

Mass Transfer Resistance Concept

- Rearranging the rate equation to fit the Ohm's law form (Rate = Driving Force / Resistance):
- W_A = (C_A1 - C_A2) / R_mass
- Where the Mass Transfer Resistance for a plain membrane is:
- R_mass = L / (D_AB * A)
- (Analogy: Thermal resistance of a plane wall R_th = L / (k * A))

Mass Transfer in Composite Membranes

- The resistance concept allows easy analysis of composite (multi-layer) membranes.
- For membranes in series, mass transfer resistances simply add up:
- R_total = R_m1 + R_m2 + ... = L1/(D_1*A) + L2/(D_2*A) + ...
- W_A = (C_A,in - C_A,out) / R_total

Practical Application: Gas Diffusion in Solids

- A common scenario is a gas diffusing through a solid container wall (e.g., Helium escaping a balloon, Hydrogen diffusing through steel).
- The concentration of gas at the solid surface (C_A1) is determined by **solubility**.
- Henry's Law (or similar relations) relates the partial pressure of the gas outside to the concentration just inside the solid surface: C_A1 = S * p_A1 (where S is solubility).

Example Problem Concept

- **Problem**: Hydrogen gas is maintained at 3 bar and 1 bar on opposite sides of a plastic membrane 2 mm thick. Determine mass transfer rate.
- **Steps**:
- 1. Convert pressures to surface concentrations (C_A1, C_A2) using given solubility.
- 2. Calculate mass transfer resistance: R_mass = L / (D_AB * A)
- 3. Calculate Rate: W_A = (C_A1 - C_A2) / R_mass

Summary

- 1D steady-state diffusion through a plain membrane yields a linear concentration profile.
- The mass transfer rate is analogous to heat conduction through a plane wall.
- Mass Transfer Resistance is defined as R_mass = L / (D_AB * A).
- For gas diffusion into solids, boundary concentrations are determined by solubility and partial pressures.

Next Lecture Preview

- Topic: Steady State Equimolar Counter Diffusion & Convective Mass Transfer
- Key questions to ponder:
- - What happens if two gases are diffusing into each other in opposite directions at the same rate?
- - How do we analyze mass transfer when the bulk fluid is moving (convection)?
- - What is the mass transfer coefficient?

Equimolar Counter Diffusion & Convective Mass Transfer

- Course: Heat and Mass Transfer (DI05019071)
- Unit 5: Mass Transfer
- Lecture 45: Equimolar Counter Diffusion & Convective Mass Transfer

Lecture Agenda

- 1. What is Equimolar Counter Diffusion?
- 2. Derivation of Flux for Equimolar Counter Diffusion
- 3. Concentration Profile
- 4. Introduction to Convective Mass Transfer (Topic 5.6)
- 5. The Mass Transfer Coefficient
- 6. Dimensionless Numbers (Sh, Sc, Le)
- 7. Course Wrap-up

Equimolar Counter Diffusion: Concept

- Occurs in binary gas mixtures (A and B) where the molar fluxes of A and B are equal in magnitude but opposite in direction.
- Condition: N_A = - N_B
- Therefore, total molar flux N = N_A + N_B = 0.
- Physical example: Distillation processes, or two large tanks containing different ideal gases connected by a tube at constant pressure and temperature.

Governing Equation (Equimolar)

- Recall the total absolute molar flux equation:
- N_A = - D_AB * (dC_A / dx) + x_A * (N_A + N_B)
- Since N_A = -N_B, the bulk convective term (N_A + N_B) is zero.
- Thus, N_A = - D_AB * (dC_A / dx)
- Conclusion: In equimolar counter diffusion, the total absolute flux is exactly equal to the diffusion flux. The bulk mixture is strictly stationary (molar-average velocity V = 0).

Flux and Concentration Profile (Equimolar)

- Since N_A is constant (steady state), integrating Fick's law over length L gives:
- N_A = D_AB * (C_A1 - C_A2) / L
- For ideal gases, C_A = p_A / (Ru * T). Substituting this:
- N_A = D_AB * (p_A1 - p_A2) / (Ru * T * L)
- The concentration (and partial pressure) profile is linear across the diffusion path.

Convective Mass Transfer (Topic 5.6)

- When mass transfer occurs between a surface and a moving fluid, it involves both diffusion and bulk fluid motion.
- Analogous to convective heat transfer (Newton's Law of Cooling).
- The flow regime (laminar or turbulent) heavily influences the mass transfer rate.
- A concentration boundary layer develops over the surface.

The Mass Transfer Coefficient (h_m)

- Convective mass transfer rate is governed by an equation analogous to Newton's Law of Cooling.
- Rate equation: N_A = h_m * (C_A,surface - C_A,infinity)
- Where:
- - N_A = Molar convective flux (kmol/m^2·s)
- - h_m = Convective mass transfer coefficient (m/s)
- - C_A,surface = Concentration at the surface
- - C_A,infinity = Free stream concentration

Finding h_m: Dimensionless Numbers

- Like 'h' in heat transfer, 'h_m' is difficult to calculate analytically. We rely on empirical correlations using dimensionless numbers.
- In heat transfer, we used Nusselt (Nu) = f(Reynolds, Prandtl).
- In mass transfer, we use Sherwood (Sh) = f(Reynolds, Schmidt).

Sherwood and Schmidt Numbers

- **Sherwood Number (Sh)**: Ratio of convective mass transfer to mass diffusion.
- Sh = (h_m * L_c) / D_AB [Analogue to Nusselt Number]
- **Schmidt Number (Sc)**: Ratio of momentum diffusivity to mass diffusivity.
- Sc = ν / D_AB = μ / (ρ * D_AB) [Analogue to Prandtl Number]
- Typical Correlation: Sh = C * Re^m * Sc^n

The Lewis Number (Le)

- **Lewis Number (Le)**: Links heat and mass transfer. It is the ratio of thermal diffusivity to mass diffusivity.
- Le $= \alpha$ / D_AB $=$ Sc / Pr
- Significance:
- - If Le $= 1$, thermal and concentration boundary layers grow at the exact same rate.
- - For many gas mixtures (like water vapor in air), Le is approximately 1, simplifying simultaneous heat and mass transfer analysis.

Summary of Mass Transfer

- Equimolar counter diffusion occurs when two species diffuse in opposite directions at equal molar rates (N_A = -N_B).
- Convective mass transfer is analogous to convective heat transfer.
- The mass transfer coefficient (h_m) is determined using the Sherwood and Schmidt dimensionless numbers.
- The heat and mass transfer analogy is a powerful engineering tool.

Course Conclusion

- This concludes the Heat and Mass Transfer course (DI05019071).
- We have covered:
- - Conduction (Fourier's Law)
- - Convection (Newton's Law of Cooling)
- - Radiation (Stefan-Boltzmann Law)
- - Heat Exchangers
- - Mass Transfer (Fick's Law)
- Best of luck with your final examinations!