Lecture 5.45

Simple Numerical on Brayton Cycle

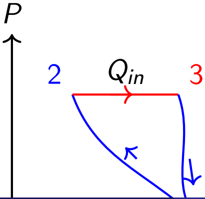
Agenda

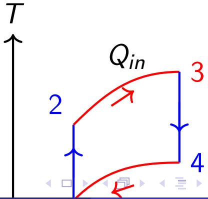
- Review of the Ideal Brayton Cycle
- Key Formulas and Thermodynamic Relations
- Numerical Problem 1: Ideal Brayton Cycle
- Step-by-step Solution to Problem 1
- Numerical Problem 2: Actual Cycle with Component Efficiencies
- Step-by-step Solution to Problem 2
- Summary and Key Takeaways

Review of the Ideal Brayton Cycle

Processes

- **Process 1-2:** Isentropic compression in the compressor ($s_1 = s_2$). Work is done on the gas.
- **Process 2-3:** Constant-pressure heat addition in the combustion chamber ($P_2 = P_3$). Heat Q_{in} is supplied.
- **Process 3-4:** Isentropic expansion in the turbine ($s_3 = s_4$). Work is extracted from the gas.
- **Process 4-1:** Constant-pressure heat rejection in the exhaust ($P_4 = P_1$). Heat Q_{out} is rejected.





Key Formulas and Thermodynamic Relations

- Pressure Ratio $(r_p) = P_2/P_1 = P_3/P_4$
- Isentropic Temperature Relation:
  $T_2/T_1 = (P_2/P_1)^{(k-1)/k} = r_p^{(k-1)/k}$
- Compressor Work $(W_c) = c_p(T_2 - T_1)$ kJ/kg
- Turbine Work $(W_t) = c_p(T_3 - T_4)$ kJ/kg
- Net Work $(W_{net}) = W_t - W_c$
- Heat Added $(Q_{in}) = c_p(T_3 - T_2)$

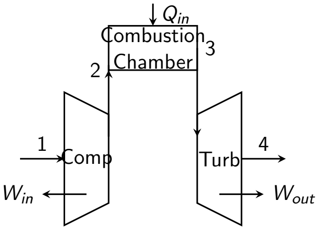
Ideal Thermal Efficiency

$$\eta_{th} = \frac{W_{net}}{Q_{in}} = 1 - \frac{1}{r_p^{(k-1)/k}}$$

Problem 1: Ideal Cycle Statement

Problem Statement

- A gas turbine power plant operates on an ideal Brayton cycle with a pressure ratio of 8.
- Air enters the compressor at 300 K and leaves the combustion chamber at 1300 K.
- Assume constant specific heats at room temperature ($c_p = 1.005$ kJ/kg·K, $k = 1.4$).



Determine:

a. The gas temperature at the exits of the compressor and the turbine.
b. The back work ratio.

Problem 1 Solution: States 1 and 2 (Compressor)

Given: $T_1 = 300$ K, $r_p = 8$, $k = 1.4$

Using the isentropic relation for the compressor:

$$T_2 = T_1 \cdot (r_p)^{(k-1)/k}$$

$$T_2 = 300 \cdot (8)^{(0.4/1.4)} = 300 \cdot (8)^{0.2857}$$

$$T_2 = 300 \cdot 1.811 = 543.3 \text{ K}$$

Compressor Work (W_c):

$$W_c = c_p(T_2 - T_1)$$

$$W_c = 1.005 \cdot (543.3 - 300)$$

$$W_c = \mathbf{244.5\ kJ/kg}$$

Problem 1 Solution: States 3 and 4 (Turbine)

Given: $T_3 = 1300$ K, $P_3/P_4 = r_p = 8$

Using the isentropic relation for the turbine:

$$T_4 = \frac{T_3}{(r_p)^{(k-1)/k}}$$

$$T_4 = \frac{1300}{(8)^{(0.4/1.4)}} = \frac{1300}{1.811}$$

$$T_4 = \mathbf{717.8\ K}$$

Turbine Work (W_t):

$$W_t = c_p(T_3 - T_4)$$

$$W_t = 1.005 \cdot (1300 - 717.8)$$

$$W_t = \mathbf{585.1\ kJ/kg}$$

Problem 1 Solution: Net Work, BWR, and Efficiency

Net Work Output (W_{net})

$W_{net} = W_t - W_c$
$W_{net} = 585.1 - 244.5$
$W_{net} = \mathbf{340.6}$ kJ/kg

Thermal Efficiency (η_{th})

$\eta_{th} = W_{net}/Q_{in}$
$\eta_{th} = 340.6/760.5$
$\eta_{th} = \mathbf{0.448}$ (or **44.8%**)

Heat Added (Q_{in})

$Q_{in} = c_p(T_3 - T_2)$
$Q_{in} = 1.005 \cdot (1300 - 543.3)$
$Q_{in} = \mathbf{760.5}$ kJ/kg

Back Work Ratio (BWR)

$BWR = W_c/W_t$
$BWR = 244.5/585.1$
$BWR = \mathbf{0.418}$ (or **41.8%**)

Problem 2: Actual Cycle Statement

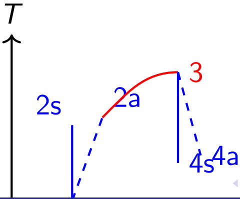
Problem Statement

- Consider the same gas turbine cycle as Problem 1.
- Now, assume the compressor and the turbine have isentropic efficiencies of 80% and 85%, respectively.
- Isentropic efficiencies account for irreversible losses (friction, turbulence) in practical components.

Determine:

(a) The back work ratio.

(b) The thermal efficiency.



Problem 2 Solution: Actual Component Work & Temperatures

Compressor ($\eta_c = 0.80$)

Actual Work ($W_{c,a}$):

$$W_{c,a} = W_{c,ideal}/\eta_c$$
$$W_{c,a} = 244.5/0.80$$
$$W_{c,a} = \mathbf{305.6}\ \text{kJ/kg}$$

Actual Exit Temp (T_{2a}):

$$T_{2a} = T_1 + (W_{c,a}/c_p)$$
$$T_{2a} = 300 + (305.6/1.005)$$
$$T_{2a} = \mathbf{604.1}\ \text{K}$$

Turbine ($\eta_t = 0.85$)

Actual Work ($W_{t,a}$):

$$W_{t,a} = W_{t,ideal} \cdot \eta_t$$
$$W_{t,a} = 585.1 \cdot 0.85$$
$$W_{t,a} = \mathbf{497.3}\ \text{kJ/kg}$$

Actual Exit Temp (T_{4a}):

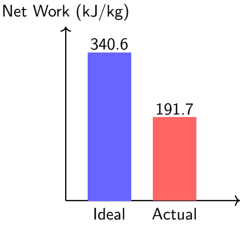
$$T_{4a} = T_3 - (W_{t,a}/c_p)$$
$$T_{4a} = 1300 - (497.3/1.005)$$
$$T_{4a} = \mathbf{805.2}\ \text{K}$$

Problem 2 Solution: Actual Net Work and Efficiency

- **Actual Net Work ($W_{net,a}$):**
  $W_{net,a} = W_{t,a} - W_{c,a} = 497.3 - 305.6 = \mathbf{191.7}$ kJ/kg
- **Actual Heat Added ($Q_{in,a}$):**
  $Q_{in,a} = c_p(T_3 - T_{2a}) = 1.005 \cdot (1300 - 604.1) = \mathbf{700.3}$ kJ/kg

Actual Thermal Efficiency ($\eta_{th,a}$)

$\eta_{th,a} = W_{net,a}/Q_{in,a} = 191.7/700.3 = \mathbf{0.274}$ (or **27.4%**)

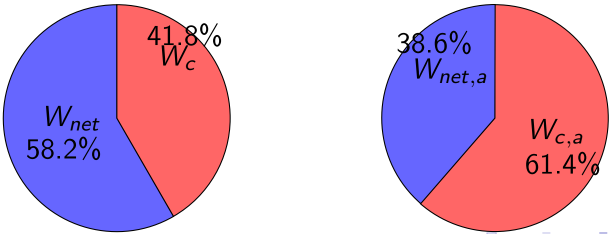


Actual Back Work Ratio (BWR_a)

$BWR_a = W_{c,a}/W_{t,a}$
$BWR_a = 305.6/497.3 = \mathbf{0.614}$ (or **61.4**%)

- Ideal BWR = 41.8%
- Actual BWR = 61.4%

Conclusion: In the actual plant, over 61% of the turbine's gross output is consumed by the compressor, leaving less net work for generating electricity.



Summary

- The ideal Brayton cycle's efficiency is strongly dependent on the pressure ratio across the compressor.
- Gas turbine cycles inherently suffer from high back work ratios because compressing a gas requires significant work compared to pumping a liquid.
- Real-world component inefficiencies (non-isentropic compression and expansion) drastically reduce cycle net work and thermal efficiency.
- Careful calculation of temperatures at each state point is the foundation for analyzing these cycles.

Next Lecture Preview

Introduction to Cogeneration Systems

- Combined Heat and Power (CHP) Principles
- Waste Heat Recovery from Gas Turbine Exhaust
- Thermodynamic Analysis of Cogeneration Cycles