Lecture 2.25

Simple Numerical on Rankine Cycle and Efficiency

Agenda

- Brief Recap of the Ideal Rankine Cycle
- Key Formulas and Thermodynamic State Postulates
- Numerical Problem 1: Ideal Rankine Cycle Calculation
- Step-by-Step Solution: Enthalpy Evaluation and Efficiency
- Numerical Problem 2: Incorporating Isentropic Efficiencies
- Analysis of Deviations in Actual Cycles
- Summary and Key Takeaways

Recap: The Ideal Rankine Cycle

- Process 1-2: Isentropic compression in a pump. The working fluid enters as saturated liquid and is pumped to the operating pressure of the boiler.
- Process 2-3: Constant pressure heat addition in a boiler. The subcooled liquid is heated to the saturation temperature, vaporized, and often superheated.
- Process 3-4: Isentropic expansion in a turbine. The superheated vapor or saturated vapor expands to condenser pressure, producing work.
- Process 4-1: Constant pressure heat rejection in a condenser. The wet steam is condensed back to saturated liquid.

Key Thermodynamic Formulas

- Pump Work Input (W_p,in): h2 - h1 ˜ v1(P2 - P1), where v1 is specific volume of saturated liquid at P1.
- Heat Added in Boiler (q_in): h3 - h2
- Turbine Work Output (W_t,out): h3 - h4
- Heat Rejected in Condenser (q_out): h4 - h1
- Net Work Output (W_net): W_t,out - W_p,in = q_in - q_out
- Thermal Efficiency (eta_th): W_net / q_in = 1 - (q_out / q_in)

Evaluating State Properties (Steam Tables)

- State 1 (Pump Inlet): Saturated liquid at condenser pressure P1. h1 = h_f@P1, v1 = v_f@P1.
- State 2 (Boiler Inlet): Compressed liquid at boiler pressure P2. h2 = h1 + W_p,in.
- State 3 (Turbine Inlet): Determined by boiler pressure P2 and temperature T3. If superheated, use superheated steam tables to find h3 and s3.
- State 4 (Turbine Exit): Isentropic expansion means s4 = s3. Determine steam quality x4 at pressure P1: x4 = (s4 - s_f@P1) / s_fg@P1.
- Calculate h4: h4 = h_f@P1 + x4 * h_fg@P1.

Problem 1: Ideal Rankine Cycle

- Statement: Consider a steam power plant operating on the simple ideal Rankine cycle.
- Steam enters the turbine at 3 MPa and 350\ensuremath{^\circ}C.
- Steam is condensed in the condenser at a pressure of 10 kPa.
- Determine the thermal efficiency of this cycle.
- Assume standard atmospheric conditions and steady-flow operation.

Problem 1: Pump and Boiler Analysis

- State 1 (P1 = 10 kPa, Sat. Liquid): From steam tables, h1 = 191.81 kJ/kg, v1 = 0.00101 m3/kg.
- Pump Work (W_p,in): v1(P2 - P1) = 0.00101 m3/kg * (3000 - 10) kPa = 3.02 kJ/kg.
- State 2 (P2 = 3 MPa): h2 = h1 + W_p,in = 191.81 + 3.02 = 194.83 kJ/kg.
- State 3 (P3 = 3 MPa, T3 = 350\ensuremath{^\circ}C): From superheated tables, h3 = 3116.1 kJ/kg, s3 = 6.7450 kJ/kg.K.
- Heat Input (q_in): h3 - h2 = 3116.1 - 194.83 = 2921.27 kJ/kg.

Problem 1: Turbine Analysis and Efficiency

- State 4 ($P4 = 10$ kPa, $s4 = s3 = 6.7450$ kJ/kg.K):
- Quality (x4): $(s4 - s_f) / s_{fg} = (6.7450 - 0.6492) / 7.4996 = 0.8128$ (81.28%).
- Enthalpy (h4): $h_f + x4 * h_{fg} = 191.81 + 0.8128 * 2392.1 = 2136.1$ kJ/kg.
- Turbine Work ($W_{t,out}$): $h3 - h4 = 3116.1 - 2136.1 = 980.0$ kJ/kg.
- Net Work (W_{net}): $W_{t,out} - W_{p,in} = 980.0 - 3.02 = 976.98$ kJ/kg.
- Thermal Efficiency (eta_{th}): $W_{net} / q_{in} = 976.98 / 2921.27 = 0.334$ (33.4%).

Problem 2: Non-Ideal Components

- Statement: Re-evaluate the previous power plant cycle considering component inefficiencies.
- Assume the turbine has an isentropic efficiency of 87%.
- Assume the pump has an isentropic efficiency of 85%.
- All other operating conditions (3 MPa, 350\ensuremath{^\circ}C inlet; 10 kPa condenser) remain the same.
- Determine the actual net work output and the new thermal efficiency.

Problem 2: Actual Pump and Boiler Properties

- Actual Pump Work (W_p,a): W_p,s / eta_p = 3.02 kJ/kg / 0.85 = 3.55 kJ/kg.
- Actual State 2 Enthalpy (h2_a): h1 + W_p,a = 191.81 + 3.55 = 195.36 kJ/kg.
- State 3 remains unchanged (Boiler outlet): h3 = 3116.1 kJ/kg.
- Actual Heat Input (q_in,a): h3 - h2_a = 3116.1 - 195.36 = 2920.74 kJ/kg.
- Note: The pump requires more work, slightly increasing the enthalpy entering the boiler and marginally decreasing required heat input.

Problem 2: Actual Turbine Work and Efficiency

- Ideal Turbine Work (W_t,s): 980.0 kJ/kg (from previous problem).
- Actual Turbine Work (W_t,a): W_t,s * eta_t = 980.0 * 0.87 = 852.6 kJ/kg.
- Actual Net Work (W_net,a): W_t,a - W_p,a = 852.6 - 3.55 = 849.05 kJ/kg.
- Actual Thermal Efficiency (eta_th,a): W_net,a / q_in,a = 849.05 / 2920.74 = 0.2907 (29.1%).
- Impact: The efficiency dropped from 33.4% to 29.1% due to component irreversibilities.

T-s Diagram: Ideal vs. Actual Cycle

- Deviation 1: Turbine expansion (3 to 4_actual) shifts to the right (increase in entropy) due to irreversibilities.
- Deviation 2: Pump compression (1 to 2_actual) also shifts slightly to the right, indicating higher work requirement.
- The area enclosed by the cycle decreases, visually representing the loss in net work.
- Actual exhaust steam quality (x4_actual) is higher than ideal, which is marginally beneficial for reducing turbine blade erosion.

Summary

- Accurate property evaluation using steam tables is fundamental to analyzing the Rankine cycle.
- Ideal cycle analysis sets the theoretical upper limit for thermal efficiency.
- Incorporating isentropic efficiencies for pumps and turbines provides realistic performance metrics.
- Turbine irreversibility has a dominant negative impact on plant efficiency compared to the pump.
- Understanding these calculations is critical for thermal plant design and optimization.

Next Lecture Preview

- Topic: Methods to Increase the Efficiency of the Rankine Cycle.
- Concept 1: Lowering condenser pressure and its limitations (moisture content).
- Concept 2: Superheating steam to high temperatures.
- Concept 3: Increasing boiler pressure.
- We will explore how these modifications affect the T-s diagram and overall plant economics.