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Antigravity AI

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To my students,
who constantly inspire me to find new analogies,
break down complex theories, and make learning
an engineered science.

Preface

About this Book

Welcome to **Computer Networks**! This textbook is meticulously designed to serve as a comprehensive problem solver and study companion. It provides a robust and intuitive foundation in the core principles of this subject, bridging the gap between theoretical models and practical applications. A unique feature of this book is its focus on decoding complex concepts using clear, step-by-step methodologies and real-world analogies.

Target Audience

This book is intended for students and professionals looking to master the concepts of Computer Networks. Whether you are revising for exams, seeking to solidify your fundamental understanding, or looking for practical examples to clarify abstract concepts, this book has been structured to support your learning journey. It serves as an accessible guide designed to remove the fear of complex topics, replacing it with an appreciation for the underlying mechanics.

Scope and Coverage

The coverage is divided logically to follow standard academic curriculums. Each chapter focuses on key topics, breaking them down into digestible sections:

- **Theoretical Insights** – Core concepts explained intuitively.
- **Method Blocks** – Standardized, step-by-step procedures for solving specific types of problems.
- **Worked Examples** – Fully solved problems that apply the methods in practice.

We hope this resource proves invaluable in your studies and helps you achieve excellence in **Computer Networks**. Happy learning!

Acknowledgments

Writing a comprehensive book that connects the fundamental forces of Chemistry with practical engineering applications requires more than just academic knowledge—it requires a community of support. I would like to express my sincere gratitude to everyone who contributed to the realization of this project.

Specially, I extend my heartfelt gratitude to the **Department of Technical Education (DTE), Government of Gujarat**, and **Government Polytechnic, Palanpur**, for providing an environment that fosters innovation, academic rigor, and continuous professional development.

I am deeply thankful to my family for their unwavering patience and support during the countless hours spent researching, formatting, and refining these chapters.

Finally, my deepest appreciation goes to my students. Your curiosity, challenging questions, and continuous feedback are the true driving force behind the unique analogies and streamlined structure of this book. This textbook was engineered for you.

Antigravity AI

About the Author

Milav Jayeshkumar Dabgar is an engineering educator and R&D professional with over 9 years of experience in electronics hardware development, embedded systems, AI/ML, and full-stack software engineering. He currently serves as a **Lecturer (Class - II) & Innovation Leader** at Government Polytechnic, Palanpur, under the Education Department of the Government of Gujarat.

Milav holds a Master of Engineering (ME) in Communication Systems from L.D. College of Engineering and a Bachelor of Engineering (BE) in Electronics & Communication. He is also currently pursuing a **BS in Data Science and Applications from IIT Madras**, where he has completed both the **Diploma in Programming** and the **Diploma in Data Science** with distinction.

Most recently, he completed the **AICTE QIP PG Certificate Program in Deep Learning from SVNIT Surat** with a **perfect 10/10 CGPA**, topping his batch.

A dedicated mentor, Milav has guided numerous student teams in securing over **INR 45 lakhs** in innovation funding, including INR 25 lakhs from Shark Tank India and INR 20 lakhs through iHub Gujarat, resulting in two student patents. He is recognized as an **NPTEL Evangelist** and **NPTEL Discipline Star** in Computer Science, reflecting his commitment to continuous learning and academic excellence.

His technical expertise spans from embedded systems (8051, STM32, Arduino) to modern web technologies (Next.js, Python, PostgreSQL) and Data Science (TensorFlow, PyTorch). Through this textbook, he aims to simplify complex Engineering Chemistry concepts by integrating relatable, real-world analogies drawn from modern tech and engineering landscapes.

Milav is also an active content creator, running a popular **YouTube channel** (@milav.dabgar) where he shares technical tutorials and academic resources. He maintains a personal blog and resource hub at milav.in and can be reached for professional collaborations on LinkedIn.

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CHAPTER 1

Introduction to Digital Marketing

Digital marketing uses digital channels to promote products and services. In this chapter, we focus on the quantitative aspects of digital marketing, emphasizing the formulas, metrics, and problem-solving methods used to evaluate marketing campaigns.

1.1 Basic of Marketing and the 4Ps

The 4Ps of marketing are Product, Price, Place, and Promotion. Quantifying the marketing mix helps in determining the profitability of a product.

Methodology:

Marketing Mix Profitability Analysis To evaluate a marketing strategy quantitatively, calculate the total costs and projected revenue:

1. Identify the **Price** (P) per unit.
2. Identify the Cost of Goods Sold (**Product** Cost, C_p).
3. Identify the Distribution Cost (**Place** Cost, C_d) per unit.
4. Identify the Total **Promotion** Cost (C_{promo}).
5. Given the projected number of units sold (Q), calculate Total Revenue (TR):
$$TR = P \times Q$$
6. Calculate Total Cost (TC):
$$TC = (C_p + C_d) \times Q + C_{promo}$$
7. Calculate Expected Profit (Π):
$$\Pi = TR - TC$$

Worked Example:

Evaluating a New Product Launch A company is launching a new software product. The marketing mix is defined as follows:

- Product Cost (C_p): \$15 per unit
- Place (Distribution) Cost (C_d): \$5 per unit
- Promotion Cost (C_{promo}): \$10000 (fixed)
- Price (P): \$50 per unit

If the company expects to sell 1000 units, calculate the Expected Profit.
Solution:

1. Total Revenue (TR):
$$TR = P \times Q = 50 \times 1000 = \$50000$$
2. Total Variable Cost per unit:
$$C_p + C_d = 15 + 5 = \$20$$
3. Total Cost (TC):
$$TC = (20 \times 1000) + 10000 = 20000 + 10000 = \$30000$$
4. Expected Profit (Π):
$$\Pi = TR - TC = 50000 - 30000 = \$20000$$

The expected profit from the product launch is \$20000.

1.2 Digital vs Traditional Marketing

Digital marketing offers precise tracking compared to traditional marketing. We compare them using the Cost Per Acquisition (CPA).

Methodology:

Cost Per Acquisition CPA Comparison

1. Identify the Total Campaign Cost (C_{total}) for both traditional and digital channels.
2. Identify the Total Number of Acquisitions (A) or customers gained from each channel.
3. Calculate Cost Per Acquisition (CPA):
$$CPA = \frac{C_{total}}{A}$$
4. Compare the CPA values. The channel with the lower CPA is generally more cost-effective.

Worked Example:

Billboard vs Social Media Ad A business spends \$5000 on a traditional billboard and acquires 50 customers. The same business spends \$2000 on a digital social media ad campaign and acquires 40 customers. Compare the CPA of both channels.
Solution:

1. **Traditional Marketing (Billboard):**
$$C_{total} = \$5000 \quad A = 50$$
$$CPA_{trad} = \frac{5000}{50} = \$100 \text{ per customer}$$
2. **Digital Marketing (Social Media):**
$$C_{total} = \$2000 \quad A = 40$$
$$CPA_{dig} = \frac{2000}{40} = \$50 \text{ per customer}$$
3. **Comparison:** Since $\$50 < \$100$, the digital marketing campaign is more cost-effective.

1.3 Importance of Digital Marketing

The importance of digital marketing is often quantified through Return on Investment (ROI) and Return on Ad Spend (ROAS).

Methodology:

Calculating ROI and ROAS

1. **Return on Ad Spend (ROAS):** Measures gross revenue generated for every dollar spent on advertising.
$$ROAS = \frac{\text{Revenue from Ad}}{\text{Cost of Ad}}$$
2. **Return on Investment (ROI):** Measures the net profit relative to the total marketing investment.
$$ROI(\%) = \left(\frac{\text{Revenue} - \text{Total Cost}}{\text{Total Cost}}\right) \times 100$$

Worked Example:

Measuring Digital Marketing Impact A company spends \$3000 on a Google Ads campaign. The campaign generates \$15000 in sales. The cost of producing the goods sold is \$8000. Calculate the ROAS and the ROI of the campaign.
Solution:

1. **ROAS Calculation:**
$$ROAS = \frac{15000}{3000} = 5$$
This means the company earns \$5 for every \$1 spent on ads (often written as 5:1).
2. **ROI Calculation:** Total Cost = Cost of Goods + Cost of Ad = $8000 + 3000 = \$11000$.
$$ROI = \left(\frac{15000 - 11000}{11000}\right) \times 100$$
$$ROI = \left(\frac{4000}{11000}\right) \times 100 \approx 36.36\%$$

The campaign yields a 36.36% return on the total investment.

1.4 Types of Digital Marketing

Different types of digital marketing use specific quantitative metrics for evaluation. We will look at SEO, PPC, and Email Marketing.

Methodology:

SEO Keyword Density Keyword density measures how often a keyword appears relative to total words.

1. Count the number of times the target keyword appears (K).
2. Count the total number of words in the text (W_{total}).
3. Calculate Keyword Density (KD):
$$KD(\%) = \left(\frac{K}{W_{total}}\right) \times 100$$

Note: Ideal density is usually between 1% and 2%.

Worked Example:

Calculating Keyword Density A blog post has 800 words. The primary keyword "digital marketing" appears 12 times. Find the keyword density.

Solution:

1. $K = 12$
2. $W_{total} = 800$
3. $KD = \left(\frac{12}{800}\right) \times 100 = 1.5\%$

The keyword density is 1.5%, which is within the optimal range.

Methodology:

Pay-Per-Click PPC Metrics PPC campaigns evaluate user engagement and cost efficiency.

1. **Click-Through Rate (CTR):** The percentage of ad impressions that resulted in clicks.

$$CTR(\%) = \left(\frac{\text{Total Clicks}}{\text{Total Impressions}}\right) \times 100$$

2. **Cost Per Click (CPC):** The average cost paid for each click.

$$CPC = \frac{\text{Total Ad Spend}}{\text{Total Clicks}}$$

3. **Conversion Rate (CR):** The percentage of clicks that result in a desired action (sale, signup).

$$CR(\%) = \left(\frac{\text{Total Conversions}}{\text{Total Clicks}}\right) \times 100$$

Worked Example:

Evaluating a Search Engine Ad An advertiser runs a PPC campaign with a budget of $500. The ad is shown 100000 times (impressions) and receives 2500 clicks. Out of these clicks, 50 users make a purchase. Calculate CTR, CPC, and CR.

Solution:

1. **Click-Through Rate (CTR):**

$$CTR = \left(\frac{2500}{100000}\right) \times 100 = 2.5\%$$

2. **Cost Per Click (CPC):**

$$CPC = \frac{500}{2500} = \$0.20 \text{ per click}$$

3. **Conversion Rate (CR):**

$$CR = \left(\frac{50}{2500}\right) \times 100 = 2.0\%$$

Methodology:

Email Marketing Metrics To measure the effectiveness of an email campaign:

1. **Open Rate (OR):**

$$OR(\%) = \left(\frac{\text{Unique Opens}}{\text{Total Delivered Emails}}\right) \times 100$$

2. **Click-to-Open Rate (CTOR):** Measures the effectiveness of the email content.

$$CTOR(\%) = \left(\frac{\text{Unique Clicks}}{\text{Unique Opens}}\right) \times 100$$

Worked Example:

Email Campaign Analysis A marketer sends an email to 5000 subscribers. 200 emails bounce, so 4800 are delivered. The tracking software shows 1200 unique opens and 180 unique clicks. Calculate the Open Rate and Click-to-Open Rate.

Solution:

1. **Open Rate (OR):**

$$OR = \left(\frac{1200}{4800}\right) \times 100 = 25\%$$

2. **Click-to-Open Rate (CTOR):**

$$CTOR = \left(\frac{180}{1200}\right) \times 100 = 15\%$$

The campaign achieved a 25% open rate and a 15% click-to-open rate.

CHAPTER 2

Search Engine Optimization

2.1 How Search Engines Work

Search engines use automated programs called spiders or crawlers to discover, process, and organize information on the web. The three primary functions are crawling, indexing, and ranking.

Methodology:

Search Engine Crawling and Indexing Process

1. **URL Discovery:** Start with a seed list of known URLs and extract new links from these pages to add to the crawl queue.
2. **Crawling:** Fetch web pages from the crawl queue while respecting the rules defined in `robots.txt` and the allocated crawl budget.
3. **Parsing:** Analyze the fetched HTML content to extract text content, metadata, and structural elements.
4. **Indexing:** Store the extracted content in an inverted index data structure. An inverted index maps individual words (tokens) to the list of document IDs where they appear.
5. **Ranking:** When a user enters a search query, retrieve the matching documents from the index and sort them based on relevance and authority algorithms.

Worked Example:

Building a Simple Inverted Index Given three short web pages (documents), construct an inverted index for the terms "SEO" and "Marketing".

- Document 1 (D_1): "SEO is part of digital marketing."
- Document 2 (D_2): "Content marketing improves SEO."
- Document 3 (D_3): "Website speed is important."

Step 1: Tokenize the documents Convert text to lowercase and remove punctuation to extract indexable terms.

- D_1: [seo, is, part, of, digital, marketing]
- D_2: [content, marketing, improves, seo]
- D_3: [website, speed, is, important]

Step 2: Map terms to document IDs Look for the occurrences of "seo" and "marketing".

- "seo" appears in D_1 and D_2.
- "marketing" appears in D_1 and D_2.

Step 3: Construct the Inverted Index The inverted index maps terms to a list of containing Document IDs.

Term	Document IDs
seo	D_1, D_2
marketing	D_1, D_2

Thus, a query for "seo marketing" will quickly retrieve D_1 and D_2 by intersecting the document lists.

2.2 Keyword Research

Keyword research involves identifying the phrases users search for. To mathematically prioritize keywords, SEO professionals often use the Keyword Effectiveness Index (KEI).

Methodology:

Calculating Keyword Effectiveness Index (KEI) KEI helps identify keywords with high search volume and low competition. A higher KEI indicates a more attractive keyword target.

1. **Find Search Volume (V):** The average number of monthly searches for the keyword.
2. **Find Competition (C):** The number of competing websites or the keyword difficulty score.
3. **Calculate KEI:** Use the formula:
$$KEI = \frac{V^2}{C}$$
4. **Rank Keywords:** Sort the keywords in descending order of their KEI values.

Worked Example:

Selecting the Best Keyword using KEI You have three potential keywords for an e-commerce store. Calculate the KEI for each and determine which keyword should be prioritized.

- Keyword A: Volume = 5000, Competition = 1000
- Keyword B: Volume = 1000, Competition = 50
- Keyword C: Volume = 10000, Competition = 8000

Step 1: Calculate KEI for Keyword A

$$KEI_A = \frac{5000^2}{1000} = \frac{25000000}{1000} = 25000$$

Step 2: Calculate KEI for Keyword B

$$KEI_B = \frac{1000^2}{50} = \frac{1000000}{50} = 20000$$

Step 3: Calculate KEI for Keyword C

$$KEI_C = \frac{10000^2}{8000} = \frac{100000000}{8000} = 12500$$

Step 4: Conclusion Ranking the keywords by KEI gives: A (25000), B (20000), C (12500). Keyword A has the highest KEI and should be prioritized.

2.3 On-Page SEO

On-page SEO involves optimizing individual web pages to rank higher. A fundamental concept in assessing page relevance is Term Frequency-Inverse Document Frequency (TF-IDF).

Methodology:

Term Frequency-Inverse Document Frequency Calculation TF-IDF evaluates how important a word is to a document within a collection (corpus).

1. **Calculate Term Frequency (TF):**

$$TF = \frac{\text{Number of times term } t \text{ appears in a document}}{\text{Total number of words in the document}}$$

2. **Calculate Inverse Document Frequency (IDF):**

$$IDF = \log_{10}\left(\frac{\text{Total number of documents}}{\text{Number of documents containing term } t}\right)$$

3. **Calculate TF-IDF Score:**

$$\text{TF-IDF} = TF \times IDF$$

Worked Example:

Evaluating On-Page Keyword Relevance A webpage has 400 total words. The target keyword "laptop" appears 8 times on this page. In the search engine's index of 10,000,000 documents, the term "laptop" appears in 100,000 documents. Calculate the TF-IDF score for the term "laptop" on this webpage.

Step 1: Calculate TF

$$TF = \frac{8}{400} = 0.02$$

Step 2: Calculate IDF

$$IDF = \log_{10}\left(\frac{10000000}{100000}\right) = \log_{10}(100) = 2$$

Step 3: Calculate TF-IDF

$$\text{TF-IDF} = 0.02 \times 2 = 0.04$$

The TF-IDF score for "laptop" on this webpage is 0.04, which the search engine uses to compare against the score of competing pages.

2.4 Off-Page SEO

Off-page SEO relies heavily on backlinks. The PageRank algorithm, originally developed by Google, computes the authority of a webpage based on the quantity and quality of incoming links.

Methodology:

Simplified PageRank Algorithm PageRank distributes authority among interlinked pages.

1. **Identify the Network:** Let pages be A, B, C, ...
2. **Determine Outbound Links:** For each page, count the total number of outbound links $L(x)$.
3. **Apply the PageRank Formula:**

$$PR(A) = (1-d) + d\left(\frac{PR(B)}{L(B)} + \frac{PR(C)}{L(C)} + \dots\right)$$

where d is the damping factor (usually set to 0.85).

4. **Iterate:** Initialize all pages with a PageRank of 1. Calculate the new PageRank for each page simultaneously until the values converge.

Worked Example:

Calculating First Iteration of PageRank Three pages A, B, and C link to each other as follows:

- Page A links to B and C. $(L(A) = 2)$
- Page B links only to A. $(L(B) = 1)$
- Page C links only to A. $(L(C) = 1)$

Assume an initial PageRank of 1 for all pages and a damping factor $d = 0.5$ for simplicity. Calculate the PageRank of page A after one iteration.
Step 1: Identify incoming links to A Page A receives links from Page B and Page C.
Step 2: Apply the formula for A

$$PR(A) = (1 - d) + d\left(\frac{PR(B)}{L(B)} + \frac{PR(C)}{L(C)}\right)$$

Step 3: Substitute the known values Initial values: $PR(B) = 1$, $PR(C) = 1$. $L(B) = 1$, $L(C) = 1$. $d = 0.5$.

$$PR(A) = (1 - 0.5) + 0.5\left(\frac{1}{1} + \frac{1}{1}\right)$$
$$PR(A) = 0.5 + 0.5(2)$$
$$PR(A) = 0.5 + 1.0 = 1.5$$

The PageRank of Page A after the first iteration is 1.5.

2.5 Technical SEO

Technical SEO ensures that search engines can easily crawl and render a website. A critical component is Page Speed, measured by Core Web Vitals, and the Crawl Budget.

Methodology:

Calculating Website Crawl Rate Limit Search engines assign a Crawl Budget to websites based on crawl capacity (server health) and crawl demand (popularity).

1. **Determine Server Capacity (C):** Maximum requests the server can handle per day without slowing down.
2. **Determine Search Engine Share (S):** The percentage of server capacity allocated to search engine bots (usually 10% to 20%).
3. **Calculate Daily Crawl Limit:**
$$\text{Crawl Limit} = C \times S$$
4. **Determine Indexing Time:** If a website has N total pages, the minimum time to crawl the entire site is $\frac{N}{\text{Crawl Limit}}$ days.

Worked Example:

Estimating Time for Complete Site Crawl An e-commerce website has 500,000 product pages. Its server can handle a maximum of 2,000,000 requests per day. The search engine allocates 15% of this capacity for its

crawl budget. How many days will it take for the search engine to crawl the entire site once?
Step 1: Calculate Daily Crawl Limit Server Capacity $C = 2,000,000$ requests/day. Search Engine Share $S = 0.15$.

$$\text{Crawl Limit} = 2000000 \times 0.15 = 300,000 \text{ pages/day}$$

Step 2: Calculate Indexing Time Total Pages $N = 500,000$.

$$\text{Time (days)} = \frac{N}{\text{Crawl Limit}} = \frac{500000}{300000} = 1.67 \text{ days}$$

It will take approximately 1.67 days for the search engine to crawl the entire website.

2.6 Local SEO

Local SEO optimizes a business's online presence to appear in local search results (like the Google Map Pack). Ranking is determined by a combination of Proximity, Relevance, and Prominence.

Methodology:

Evaluating Local Search Ranking Score We can model local search rankings using a weighted scoring system based on the three primary ranking factors.

1. **Score Proximity (P):** Scale of 1-10 based on the physical distance from the searcher to the business.
2. **Score Relevance (R):** Scale of 1-10 based on how well the business profile and website content match the search query.
3. **Score Prominence (Pr):** Scale of 1-10 based on reviews, backlinks, and brand mentions.
4. **Calculate Total Score:** Apply weights determined by the algorithm. For example:

$$\text{Local Score} = (W_P \times P) + (W_R \times R) + (W_{Pr} \times Pr)$$

Worked Example:

Determining the Map Pack Rankings Three restaurants are competing for the local search query "pizza near me". The search engine uses the following weights: Proximity (40%), Relevance (30%), Prominence (30%). Calculate their scores and determine the final ranking order.

- Restaurant X: Proximity=9, Relevance=7, Prominence=6
- Restaurant Y: Proximity=5, Relevance=9, Prominence=9
- Restaurant Z: Proximity=8, Relevance=8, Prominence=7

Step 1: Calculate Score for Restaurant X

$$\text{Score}_X = (0.4 \times 9) + (0.3 \times 7) + (0.3 \times 6) = 3.6 + 2.1 + 1.8 = 7.5$$

Step 2: Calculate Score for Restaurant Y

$$\text{Score}_Y = (0.4 \times 5) + (0.3 \times 9) + (0.3 \times 9) = 2.0 + 2.7 + 2.7 = 7.4$$

Step 3: Calculate Score for Restaurant Z

$$\text{Score}_Z = (0.4 \times 8) + (0.3 \times 8) + (0.3 \times 7) = 3.2 + 2.4 + 2.1 = 7.7$$

Step 4: Rank the Businesses Comparing the final scores: Z (7.7) > X (7.5) > Y (7.4). The local map pack will display the businesses in this order: Restaurant Z, Restaurant X, Restaurant Y.

CHAPTER 3

Web Analytics

Web analytics involves the collection, measurement, and analysis of web data to understand and optimize web usage. In this chapter, we focus on the practical methodologies of setting up tracking, extracting metrics, and calculating performance indicators such as bounce rate and conversion metrics.

3.1 Importance of Web Analytics

Web analytics translates raw visitor data into actionable business intelligence. Rather than relying on theory, we use specific formulas to measure the impact of web traffic through metrics like Conversion Rate and Return on Investment (ROI).

Methodology:

Calculating Conversion Rate and ROI To quantify the importance and effectiveness of a web presence, two primary formulas are utilized based on analytics data:

1. **Conversion Rate (CR):** The percentage of total visitors who take a desired action (e.g. purchasing a product or filling out a form).

$$\text{CR} = \left(\frac{\text{Total Conversions}}{\text{Total Visitors}} \right) \times 100$$

2. **Return on Investment (ROI):** The profitability of web marketing efforts.

$$\text{ROI} = \left(\frac{\text{Revenue from Web} - \text{Cost of Web Marketing}}{\text{Cost of Web Marketing}} \right) \times 100$$

Worked Example:

Computing Web Effectiveness Metrics A local business tracks its website traffic over one month. The analytics dashboard reports 15000 total visitors. Out of these visitors, 450 completed a purchase. The total cost spent on web marketing and hosting was \$500, and the revenue generated from the 450 purchases was \$2500. Calculate the Conversion Rate and the Return on Investment.

Solution:

1. **Identify given values:**
   - Total Visitors = 15000
   - Total Conversions = 450
   - Cost = \$500
   - Revenue = \$2500
2. **Calculate Conversion Rate (CR):**

$$\text{CR} = \left(\frac{450}{15000} \right) \times 100 = 0.03 \times 100 = 3\%$$

3. **Calculate Return on Investment (ROI):**

$$\text{ROI} = \left(\frac{2500 - 500}{500}\right) \times 100 = \left(\frac{2000}{500}\right) \times 100 = 400\%$$

The website operates with a 3% conversion rate and yields a 400% return on investment.

3.2 Google Analytics Setup

Google Analytics (GA) is the industry standard for tracking web metrics. To collect data, a unique tracking code must be embedded into the HTML structure of the target website.

Methodology:

GA Tracking Code Implementation Implementing Google Analytics involves generating a Measurement ID and embedding the Global Site Tag (gtag.js) into your web pages. Follow these steps:

1. **Generate Measurement ID:** Create a property in the Google Analytics dashboard to receive a unique identifier formatted as `G-XXXXXXXXXX`.
2. **Locate the `<head>` Section:** Open the HTML document of the web page you wish to track.
3. **Inject the Script:** Paste the GA snippet immediately after the opening `<head>` tag. The script ensures that every page load fires an event to Google's servers.

Worked Example:

Embedding GA in a HTML Document You are tasked with setting up web analytics for a single-page portfolio. The Google Analytics dashboard provides you with the Measurement ID `G-ABC123XYZ9`. Inject the necessary tracking script into the provided raw HTML structure.
Raw HTML:

```
<!DOCTYPE html>
<html>
<head>
    <title>My Portfolio</title>
</head>
<body>
    <h1>Welcome to my Portfolio</h1>
</body>
</html>
```

Solution: We place the global site tag script directly below the `<head>` tag, utilizing the provided Measurement ID.

```
<!DOCTYPE html>
<html>
<head>
    <!-- Google tag (gtag.js) -->
    <script async src="https://www.googletagmanager.com/gtag/js?id=G-ABC123XYZ9"></script>
    <script>
      window.dataLayer = window.dataLayer || [];
      function gtag(){dataLayer.push(arguments);}
      gtag('js', new Date());

      gtag('config', 'G-ABC123XYZ9');
    </script>
```

```
    <title>My Portfolio</title>
</head>
<body>
    <h1>Welcome to my Portfolio</h1>
</body>
</html>
```

The page is now configured to report page views and session data to Google Analytics.

3.3 Metrics Like Traffic Bounce Rate Goals

Once analytics is set up, raw data is aggregated into key metrics. The most critical metrics for evaluating site performance are Traffic (Sessions and Pageviews), Bounce Rate, and Goal Completions.

Methodology:

Formulas for Key Analytics Metrics Understanding user engagement requires calculating the following interaction metrics:

1. **Pages per Session:** The average number of pages viewed during a single session.

$$\text{Pages/Session} = \frac{\text{Total Pageviews}}{\text{Total Sessions}}$$

2. **Bounce Rate:** The percentage of single-page sessions in which there was no interaction with the page. A bounced session has a duration of 0 seconds.

$$\text{Bounce Rate} = \left(\frac{\text{Total Bounces}}{\text{Total Entrances}}\right) \times 100$$

3. **Goal Conversion Rate:** The efficiency of a website in fulfilling specific objectives (e.g. newsletter signups).

$$\text{Goal Conversion Rate} = \left(\frac{\text{Total Goal Completions}}{\text{Total Sessions}}\right) \times 100$$

Worked Example:

Calculating Traffic Metrics and Bounce Rate An e-commerce landing page has been active for one week. The analytics dashboard provides the following raw data for this specific page:

Metric	Value
Total Entrances (Sessions starting on this page)	800
Total Bounces (Left without interaction)	460
Total Pageviews (Across all 800 sessions)	2400
Total Goal Completions (Add to Cart)	64

Calculate the Pages per Session, Bounce Rate, and the Goal Conversion Rate.
Solution:

1. **Calculate Pages per Session:**

$$\text{Pages/Session} = \frac{2400}{800} = 3.0 \text{ pages}$$

2. **Calculate Bounce Rate:**

$$\text{Bounce Rate} = \left(\frac{460}{800}\right) \times 100 = 0.575 \times 100 = 57.5\%$$

3. **Calculate Goal Conversion Rate:**

$$\text{Goal Conversion Rate} = \left(\frac{64}{800}\right) \times 100 = 0.08 \times 100 = 8\%$$

The landing page has an average depth of 3 pages per session, a bounce rate of 57.5%, and successfully converts 8% of its sessions into "Add to Cart" actions.

Methodology:

Goal Funnel Analysis A Goal Funnel represents the series of pages a user must navigate to complete a goal. Analyzing drop-off rates between funnel steps helps identify friction points in the user journey.

1. **Step Completion Rate:** The percentage of users who move from Step i to Step $i+1$.

$$\text{Completion Rate (Step } i) = \left(\frac{\text{Users reaching Step } i+1}{\text{Users reaching Step } i}\right) \times 100$$

2. **Step Abandonment Rate:** The percentage of users who leave the funnel at Step i.

$$\text{Abandonment Rate (Step } i) = 100\% - \text{Completion Rate (Step } i)$$

Worked Example:

Evaluating a Multi Step Goal Funnel A website has a 3-step checkout funnel. The analytics tool reports the following user volume at each step for a particular day:

Funnel Step	Number of Users
Step 1: View Shopping Cart	1200
Step 2: Enter Shipping Details	600
Step 3: Confirm Payment (Goal Completed)	150

Determine the Abandonment Rate at Step 1 and Step 2 to identify where the most users are dropping off.
Solution:

1. **Analyze Step 1 (Cart to Shipping):**
   - Completion Rate $= \left(\frac{600}{1200}\right) \times 100 = 50\%$
   - Abandonment Rate $= 100\% - 50\% = 50\%$
2. **Analyze Step 2 (Shipping to Payment):**
   - Completion Rate $= \left(\frac{150}{600}\right) \times 100 = 25\%$
   - Abandonment Rate $= 100\% - 25\% = 75\%$

While 50% of users abandon the cart immediately, the highest friction point is Step 2, where 75% of users who start entering shipping details fail to complete the payment.

CHAPTER 4

Social Media and Video Marketing

4.1 Importance of SMM

Social Media Marketing (SMM) relies heavily on quantitative metrics to evaluate its importance and effectiveness. Rather than subjective analysis, performance is measured using engagement and financial return models.

Methodology:

Calculating Social Media Metrics To measure the importance and success of an SMM campaign mathematically, apply the following formulas:

1. **Engagement Rate (ER):** Measures active audience interaction.

$$ER = \left(\frac{\text{Total Engagements}}{\text{Total Followers or Impressions}} \right) \times 100$$

2. **Conversion Rate (CR):** Measures the percentage of clicks resulting in a desired action.

$$CR = \left(\frac{\text{Total Conversions}}{\text{Total Clicks}} \right) \times 100$$

3. **Return on Investment (ROI):** Determines the financial viability of the campaign.

$$ROI = \left(\frac{\text{Revenue from SMM} - \text{Cost of SMM}}{\text{Cost of SMM}} \right) \times 100$$

Worked Example:

Evaluate Campaign ROI and Engagement A recent SMM campaign reached an audience of 50000 followers. It generated 3500 likes and comments (engagements), and 1200 link clicks. Out of those clicks, 150 users purchased a product. The campaign cost $400 and generated $2000 in revenue. Calculate the Engagement Rate based on followers, the Conversion Rate, and the ROI.

Step 1: Calculate Engagement Rate (ER)

$$ER = \frac{3500}{50000} \times 100 = 7\%$$

Step 2: Calculate Conversion Rate (CR)

$$CR = \frac{150}{1200} \times 100 = 12.5\%$$

Step 3: Calculate Return on Investment (ROI)

$$ROI = \frac{2000 - 400}{400} \times 100 = \frac{1600}{400} \times 100 = 400\%$$

The campaign yielded a 7% engagement rate and a highly profitable 400% ROI.

4.2 SMM Platforms

Selecting the right social media platform requires evaluating multiple platform attributes against campaign objectives using a weighted scoring algorithm.

Methodology:

Weighted Platform Selection Model Use a multi-criteria decision approach to select an SMM platform:

1. Identify n evaluation criteria (e.g. Target Demographic Match D, Average Cost Per Click C, Expected Engagement E).
2. Assign a weight W_i to each criterion such that $\sum W_i = 1.0$.
3. Score each platform j on a scale of 1 to 10 for each criterion, denoted as S_{ij}. For cost criteria use inverse scoring (lower cost gets a higher score).
4. Calculate the total weighted score T_j for each platform:

$$T_j = \sum_{i=1}^{n} (W_i \times S_{ij})$$

5. Select the platform with the highest T_j.

Worked Example:

Select the Optimal SMM Platform A brand is choosing between Platform X and Platform Y. The criteria and weights are: Demographic Match ($W_1 = 0.5$), Cost Efficiency ($W_2 = 0.3$), and Ad Format Options ($W_3 = 0.2$). The scores out of 10 are: Platform X: Demographic = 8, Cost = 6, Ad Formats = 9 Platform Y: Demographic = 6, Cost = 9, Ad Formats = 7
Determine which platform should be selected.

Step 1: Calculate Total Score for Platform X

$$T_X = (0.5 \times 8) + (0.3 \times 6) + (0.2 \times 9)$$

$$T_X = 4.0 + 1.8 + 1.8 = 7.6$$

Step 2: Calculate Total Score for Platform Y

$$T_Y = (0.5 \times 6) + (0.3 \times 9) + (0.2 \times 7)$$

$$T_Y = 3.0 + 2.7 + 1.4 = 7.1$$

Step 3: Compare and Select Since $7.6 > 7.1$, the brand should select Platform X.

4.3 Best practices

A critical best practice in SMM is algorithmic scheduling to maximize reach. This requires determining the optimal time window for posting based on historical interaction data.

Methodology:

Optimal Posting Time Algorithm To systematically find the best time to post:

1. Divide the day into k discrete time slots (e.g. 1-hour intervals).
2. Collect historical engagement count E_t for posts published in time slot t.
3. Calculate the historical average engagement $\bar{E}_t = \frac{1}{N_t} \sum E_t$, where N_t is the number of posts in slot t.

4. Identify the time slot t_{opt} that maximizes $\bar{E}_t$:

$$t_{opt} = \arg\max_t(\bar{E}_t)$$

Worked Example:

Determine Best Posting Window A company has tracked engagement for posts over three time slots: Morning (M), Afternoon (A), and Evening (E). Morning: 4 posts with engagements 120, 150, 130, 100. Afternoon: 3 posts with engagements 200, 180, 220. Evening: 5 posts with engagements 80, 90, 110, 85, 95.
Find the optimal posting window.

Step 1: Calculate Average Engagement for Morning

$$\bar{E}_M = \frac{120 + 150 + 130 + 100}{4} = \frac{500}{4} = 125$$

Step 2: Calculate Average Engagement for Afternoon

$$\bar{E}_A = \frac{200 + 180 + 220}{3} = \frac{600}{3} = 200$$

Step 3: Calculate Average Engagement for Evening

$$\bar{E}_E = \frac{80 + 90 + 110 + 85 + 95}{5} = \frac{460}{5} = 92$$

Step 4: Identify Maximum Since $\bar{E}_A$ (200) is the maximum, the Afternoon time slot is the optimal posting window.

4.4 Content creation

Systematic content creation utilizes proportional distribution rules to ensure a balanced feed that maintains audience interest while achieving business goals.

Methodology:

Content Allocation using Rule of Thirds The Rule of Thirds formulaically divides content C_{total} into three equal categories:

1. C_{promo}: Promotional content (driving sales or traffic).
2. C_{share}: Curated content (sharing industry news or other creators).
3. C_{engage}: Interactive content (personal stories or community building).

Given a total number of posts planned for a period N:

$$C_{promo} = \lceil N \times \frac{1}{3} \rceil$$

$$C_{share} = \lfloor N \times \frac{1}{3} \rfloor$$

$$C_{engage} = N - (C_{promo} + C_{share})$$

Note: The ceiling and floor functions manage fractional posts to ensure exact totals.

Worked Example:

Develop a Weekly Content Plan A brand plans to publish 14 social media posts this week. Use the Rule of Thirds allocation method to determine exactly how many posts should belong to each category.

Step 1: Calculate Promotional Posts

$$C_{promo} = \lceil 14 \times \frac{1}{3} \rceil = \lceil 4.66 \rceil = 5 \text{ posts}$$

Step 2: Calculate Curated Posts

$$C_{share} = \lfloor 14 \times \frac{1}{3} \rfloor = \lfloor 4.66 \rfloor = 4 \text{ posts}$$

Step 3: Calculate Interactive Posts

$$C_{engage} = 14 - (5 + 4) = 14 - 9 = 5 \text{ posts}$$

The final plan requires 5 promotional posts, 4 curated posts, and 5 interactive posts.

4.5 Video marketing on YouTube

YouTube's recommendation algorithm relies heavily on initial view velocity and audience retention. Marketing performance on YouTube is computed by measuring these early-stage signals.

Methodology:

Calculating YouTube View Velocity and Retention To mathematically assess a YouTube video's algorithmic potential:

1. **View Velocity (V):** Measures the initial traction of a video over the first H hours.

$$V = \frac{\text{Total Views in first } H \text{ hours}}{H}$$

2. **Audience Retention Rate (ARR):** Measures how well the video holds attention.

$$ARR = \left(\frac{\text{Average View Duration (in seconds)}}{\text{Total Video Length (in seconds)}} \right) \times 100$$

Worked Example:

Analyze YouTube Video Metrics A new YouTube marketing video is 8 minutes and 20 seconds long. After the first 24 hours, it has accumulated 12000 views. Analytics show the average view duration is 4 minutes and 10 seconds. Calculate the View Velocity and Audience Retention Rate.

Step 1: Calculate View Velocity

$$V = \frac{12000 \text{ views}}{24 \text{ hours}} = 500 \text{ views per hour}$$

Step 2: Convert times to seconds Total Video Length $= (8 \times 60) + 20 = 500$ seconds. Average View Duration $= (4 \times 60) + 10 = 250$ seconds.

Step 3: Calculate ARR

$$ARR = \left(\frac{250}{500} \right) \times 100 = 50\%$$

The video has a velocity of 500 views/hour and an ARR of 50%.

4.6 Types of videos

Different video types (e.g. Tutorials, Product Demos, Vlogs) perform differently depending on the audience. A computational approach using A/B testing helps select the highest converting video type for marketing.

Methodology:

AB Testing Video Formats To evaluate the effectiveness of two different video types (Type A and Type B):

1. Run both video types simultaneously with an equal ad budget or impression target.
2. Record Impressions (I) and Clicks (C) for both videos.
3. Calculate the Click-Through Rate (CTR) for each:

$$CTR_A = \frac{C_A}{I_A} \times 100, \quad CTR_B = \frac{C_B}{I_B} \times 100$$

4. Select the video type with the highest CTR as the standard format for future campaigns.

Worked Example:

Evaluate AB Test Results for Video Types A software company tests two video types to promote a new tool: a Screen-recording Tutorial (Type A) and an Animated Explainer (Type B). Type A received 15000 impressions and 450 clicks. Type B received 18000 impressions and 630 clicks. Determine which video type has a better Click-Through Rate.

Step 1: Calculate CTR for Type A (Tutorial)

$$CTR_A = \frac{450}{15000} \times 100 = 3\%$$

Step 2: Calculate CTR for Type B (Explainer)

$$CTR_B = \frac{630}{18000} \times 100 = 3.5\%$$

Step 3: Conclusion The Animated Explainer (Type B) achieved a higher CTR (3.5% vs 3%). The company should allocate more budget to animated explainer videos.

CHAPTER 5

Digital Promotions

5.1 Email Marketing

Email marketing involves sending commercial messages to a targeted audience using email. It is used to build loyalty, trust, and brand awareness. From a computational perspective, the success of email marketing relies heavily on tracking campaign performance using precise quantitative metrics.

5.1.1 Email Marketing Types and Campaigns

Common campaign types include promotional emails (sales, discounts), newsletters (information, updates), welcome sequences, and retention emails. Executing a campaign involves audience segmentation, creating targeted content, and iteratively measuring effectiveness to improve future sending behavior.

5.1.2 Email Marketing Metrics

Methodology:

Email Marketing Metrics Calculation To evaluate the success of an email marketing campaign, analyze the following Key Performance Indicators (KPIs):

1. **Delivery Rate**: The percentage of emails that successfully reach the recipient's inbox.

$$\text{Delivery Rate} = \left(\frac{\text{Emails Delivered}}{\text{Emails Sent}}\right) \times 100$$

Note: Emails Delivered = Emails Sent − Bounces

2. **Open Rate**: The percentage of delivered emails that were opened by recipients.

$$\text{Open Rate} = \left(\frac{\text{Emails Opened}}{\text{Emails Delivered}}\right) \times 100$$

3. **Click-Through Rate (CTR)**: The percentage of delivered emails that resulted in at least one click.

$$\text{CTR} = \left(\frac{\text{Total Clicks}}{\text{Emails Delivered}}\right) \times 100$$

4. **Click-to-Open Rate (CTOR)**: Measures content effectiveness by comparing clicks to opens.

$$\text{CTOR} = \left(\frac{\text{Total Clicks}}{\text{Emails Opened}}\right) \times 100$$

5. **Conversion Rate**: The percentage of delivered emails that resulted in a desired action (e.g., a purchase).

$$\text{Conversion Rate} = \left(\frac{\text{Total Conversions}}{\text{Emails Delivered}}\right) \times 100$$

6. **Campaign Return on Investment (ROI)**: The financial return relative to campaign cost.

$$\text{ROI} = \left(\frac{\text{Total Revenue} - \text{Total Campaign Cost}}{\text{Total Campaign Cost}}\right) \times 100$$

Worked Example:

Calculating Email Campaign Metrics A tech startup sends out a promotional email campaign to a list of $25,000$ subscribers. The total cost of designing and running the campaign is \$800. The email marketing platform reports the following performance data:

- Bounces: $1,000$
- Emails Opened: $6,000$
- Clicks on the call-to-action link: 900
- Purchases made (Conversions): 120
- Total Revenue generated: $\$4,800$

Calculate the Delivery Rate, Open Rate, CTR, CTOR, Conversion Rate, and Campaign ROI.

Step 1: Calculate Delivery Rate

$$\text{Emails Delivered} = 25,000 - 1,000 = 24,000$$

$$\text{Delivery Rate} = \left(\frac{24,000}{25,000}\right) \times 100 = 96\%$$

Step 2: Calculate Open Rate

$$\text{Open Rate} = \left(\frac{6,000}{24,000}\right) \times 100 = 25\%$$

Step 3: Calculate Click-Through Rate (CTR)

$$\text{CTR} = \left(\frac{900}{24,000}\right) \times 100 = 3.75\%$$

Step 4: Calculate Click-to-Open Rate (CTOR)

$$\text{CTOR} = \left(\frac{900}{6,000}\right) \times 100 = 15\%$$

Step 5: Calculate Conversion Rate

$$\text{Conversion Rate} = \left(\frac{120}{24,000}\right) \times 100 = 0.5\%$$

Step 6: Calculate Campaign ROI

$$\text{ROI} = \left(\frac{4,800 - 800}{800}\right) \times 100 = \left(\frac{4,000}{800}\right) \times 100 = 500\%$$

5.2 Search Engine Marketing (SEM)

SEM aims to increase website visibility on Search Engine Results Pages (SERPs) primarily through paid advertising campaigns, such as Google Ads. While Search Engine Optimization (SEO) focuses on acquiring organic traffic over time without paying per click, Pay-Per-Click (PPC) advertising guarantees immediate visibility at a cost.

5.2.1 Types of SEM and Ad Extensions

SEM broadly covers various formats:

- **Search Ads**: Text ads appearing at the top or bottom of Google search results.
- **Display Ads**: Visual banner ads displayed on third-party websites within an ad network.
- **Shopping Ads**: Product-based ads featuring images, prices, and merchant names.

Advertisers often use **Ad Extensions** (such as Sitelink Extensions, Call Extensions, and Location Extensions) to append additional useful information to their ads, increasing overall screen real estate and click-through rates.

5.2.2 Key Metrics in SEM

Methodology:

SEM Performance Metrics When managing a Pay-Per-Click campaign, advertising budget is mathematically modeled using several cost and efficiency metrics:

1. **Cost Per Click (CPC)**: The average amount paid each time a user clicks the ad.

$$\text{CPC} = \frac{\text{Total Ad Cost}}{\text{Total Clicks}}$$

2. **Cost Per Acquisition (CPA)**: The average cost required to generate one conversion (sale or lead).

$$\text{CPA} = \frac{\text{Total Ad Cost}}{\text{Total Conversions}}$$

3. **Return on Ad Spend (ROAS)**: Gross revenue generated for every dollar spent on advertising.

$$\text{ROAS} = \left(\frac{\text{Total Ad Revenue}}{\text{Total Ad Cost}}\right) \times 100$$

4. **Ad Rank Calculation**: The formula search engines use to determine ad position on the SERP. A higher Ad Rank secures a better position.

$$\text{Ad Rank} = \text{Maximum Bid (Max CPC)} \times \text{Quality Score}$$

5. **Actual CPC**: Under a second-price auction model, an advertiser only pays enough to beat the competitor immediately below them.

$$\text{Actual CPC} = \frac{\text{Ad Rank of Competitor Below}}{\text{Your Quality Score}} + 0.01$$

Worked Example:

Calculating CPA and ROAS A digital marketing agency runs an SEM campaign for an online shoe retailer. Over a two-week period, they spend $\$2,500$ on search ads. The campaign dashboard reports $1,250$ clicks and 50 finalized sales. The total revenue driven directly by these ad clicks is $\$8,750$.
Calculate the Average CPC, Conversion Rate from clicks, CPA, and ROAS.
Step 1: Calculate Average CPC

$$\text{CPC} = \frac{\text{Total Ad Cost}}{\text{Total Clicks}} = \frac{2,500}{1,250} = \$2.00$$

Step 2: Calculate Conversion Rate

$$\text{Conversion Rate} = \left(\frac{\text{Total Sales}}{\text{Total Clicks}}\right) \times 100 = \left(\frac{50}{1,250}\right) \times 100 = 4\%$$

Step 3: Calculate Cost Per Acquisition (CPA)

$$\text{CPA} = \frac{\text{Total Ad Cost}}{\text{Total Sales}} = \frac{2,500}{50} = \$50.00$$

Step 4: Calculate Return on Ad Spend (ROAS)

$$\text{ROAS} = \left(\frac{8,750}{2,500}\right) \times 100 = 350\%$$

Conclusion: For every $1 *spent on advertising, the retailer generated* $3.50 *in revenue.*

Worked Example:

Determining Ad Rank and Actual CPC Four advertisers are bidding on the exact match keyword "wireless headphones." The search engine uses Ad Rank to determine ad positions and the Actual CPC. The bids and assigned Quality Scores (on a scale from 1 to 10) are as follows:

Advertiser	Max Bid ($)	Quality Score
Advertiser A	4.00	6
Advertiser B	3.00	10
Advertiser C	5.00	4
Advertiser D	2.50	6

Determine the ad position for each advertiser and calculate the Actual CPC they will pay.
Step 1: Calculate Ad Rank for each Advertiser Using the formula Ad Rank = Max Bid × Quality Score:

- Advertiser A: $4.00 \times 6 = 24$
- Advertiser B: $3.00 \times 10 = 30$

- Advertiser C: $5.00 \times 4 = 20$
- Advertiser D: $2.50 \times 6 = 15$

Step 2: Determine Ad Positions Rank the advertisers in descending order of their Ad Rank:

- **Position 1**: Advertiser B (Ad Rank = 30)
- **Position 2**: Advertiser A (Ad Rank = 24)
- **Position 3**: Advertiser C (Ad Rank = 20)
- **Position 4**: Advertiser D (Ad Rank = 15)

Step 3: Calculate Actual CPC Using the formula Actual CPC $= \frac{\text{Ad Rank of Advertiser Below}}{\text{Own Quality Score}} + 0.01$:

- **For Advertiser B (Position 1):** The advertiser directly below is A (Ad Rank 24).

$$\text{Actual CPC}_B = \frac{24}{10} + 0.01 = 2.40 + 0.01 = \$2.41$$

- **For Advertiser A (Position 2):** The advertiser directly below is C (Ad Rank 20).

$$\text{Actual CPC}_A = \frac{20}{6} + 0.01 = 3.333 + 0.01 \approx \$3.34$$

- **For Advertiser C (Position 3):** The advertiser directly below is D (Ad Rank 15).

$$\text{Actual CPC}_C = \frac{15}{4} + 0.01 = 3.75 + 0.01 = \$3.76$$

- **For Advertiser D (Position 4):** Since there is no advertiser below, Advertiser D pays the platform's minimum required CPC threshold (e.g., $0.10).

Observation: Even though Advertiser C had the highest Maximum Bid ($5.00), they ended up in 3rd place and pay more per click ($3.76) than Advertiser B ($2.41), who bid less ($3.00) but had a superior Quality Score. This emphasizes the computational importance of Quality Score in SEM.

Appendix: Key Formulae

This appendix provides a quick reference to the key formulae and mathematical relationships discussed in this book.

Unit 1: Digital Marketing Fundamentals

- **Return on Ad Spend (ROAS):** Measures the gross revenue generated for every dollar spent on advertising.
$$ROAS = \frac{\text{Revenue from Ad}}{\text{Cost of Ad}}$$
- **Keyword Density (KD):** The percentage of times a keyword or phrase appears on a web page compared to the total number of words on the page.
$$KD(\%) = \left(\frac{K}{W_{total}}\right) \times 100$$
where K is the number of times the keyword appears, and W_{total} is the total word count.
- **Total Revenue (TR):**
$$TR = P \times Q$$
where P is the price per unit and Q is the quantity sold.
- **Conversion Rate (CR):**
$$CR(\%) = \left(\frac{\text{Total Conversions}}{\text{Total Clicks}}\right) \times 100$$
- **Return on Investment (ROI):** Measures the profitability of an investment.
$$ROI(\%) = \left(\frac{\text{Revenue} - \text{Total Cost}}{\text{Total Cost}}\right) \times 100$$
- **Click-to-Open Rate (CTOR):** Evaluates the effectiveness of email content.
$$CTOR(\%) = \left(\frac{\text{Unique Clicks}}{\text{Unique Opens}}\right) \times 100$$
- **Open Rate (OR):**
$$OR(\%) = \left(\frac{\text{Unique Opens}}{\text{Total Delivered Emails}}\right) \times 100$$
- **Total Cost (TC):**
$$TC = (C_p + C_d) \times Q + C_{promo}$$
where C_p is production cost, C_d is distribution cost, Q is quantity, and C_{promo} is promotional cost.
- **Profit (Π):**
$$\Pi = TR - TC$$

- **Cost Per Click (CPC):**
$$CPC = \frac{\text{Total Ad Spend}}{\text{Total Clicks}}$$
- **Cost Per Acquisition (CPA):**
$$CPA = \frac{C_{total}}{A}$$
where C_{total} is the total campaign cost and A is the total number of acquisitions (or conversions).
- **Click-Through Rate (CTR):**
$$CTR(\%) = \left(\frac{\text{Total Clicks}}{\text{Total Impressions}} \right) \times 100$$

Unit 2: Search Engine Optimization (SEO)

- **Keyword Effectiveness Index (KEI):** Used to determine the potential of a keyword.
$$KEI = \frac{V^2}{C}$$
where V is the search volume and C is the competition.
- **Term Frequency (TF):**
$$TF = \frac{\text{Number of times term } t \text{ appears in a document}}{\text{Total number of words in the document}}$$
- **Inverse Document Frequency (IDF):**
$$IDF = \log_{10} \left(\frac{\text{Total number of documents}}{\text{Number of documents containing term } t} \right)$$
- **TF-IDF Score:**
$$\text{TF-IDF} = TF \times IDF$$
- **PageRank (PR):** Algorithm used to rank web pages.
$$PR(A) = (1-d) + d \left(\frac{PR(B)}{L(B)} + \frac{PR(C)}{L(C)} + \dots \right)$$
where d is the damping factor, $PR(B)$ is the PageRank of page B linking to A, and $L(B)$ is the number of outbound links from page B.
- **Local Search Score:**
$$\text{Local Score} = (W_P \times P) + (W_R \times R) + (W_{Pr} \times Pr)$$
where W_i represent weights, and P, R, Pr correspond to Prominence, Relevance, and Proximity.
- **Crawl Limit:**
$$\text{Crawl Limit} = C \times S$$
where C is crawl rate and S is server response capability. (Note: Often calculated based on allowed requests over time.)

Unit 3: Web Analytics

- **Bounce Rate:**
$$\text{Bounce Rate} = \left(\frac{\text{Total Bounces}}{\text{Total Entrances}}\right) \times 100$$
- **Pages per Session:**
$$\text{Pages/Session} = \frac{\text{Total Pageviews}}{\text{Total Sessions}}$$
- **Conversion Rate (CR):**
$$\text{CR} = \left(\frac{\text{Total Conversions}}{\text{Total Visitors}}\right) \times 100$$
- **Goal Conversion Rate:**
$$\text{Goal Conversion Rate} = \left(\frac{\text{Total Goal Completions}}{\text{Total Sessions}}\right) \times 100$$
- **Step Completion Rate:**
$$\text{Completion Rate (Step } i) = \left(\frac{\text{Users reaching Step } i+1}{\text{Users reaching Step } i}\right) \times 100$$
- **Step Abandonment Rate:**
$$\text{Abandonment Rate (Step } i) = 100\% - \text{Completion Rate (Step } i)$$
- **Web Marketing ROI:**
$$\text{ROI} = \left(\frac{\text{Revenue from Web} - \text{Cost of Web Marketing}}{\text{Cost of Web Marketing}}\right) \times 100$$

Unit 4: Social Media Marketing (SMM)

- **Weighted Score Matrix:**
$$T_j = \sum_{i=1}^{n} (W_i \times S_{ij})$$
where T_j is the total score for platform j, W_i is the weight of criterion i, and S_{ij} is the score of platform j for criterion i.
- **Engagement Rate (ER):**
$$ER = \left(\frac{\text{Total Engagements}}{\text{Total Followers or Impressions}}\right) \times 100$$
- **Audience Retention Rate (ARR):**
$$ARR = \left(\frac{\text{Average View Duration (in seconds)}}{\text{Total Video Length (in seconds)}}\right) \times 100$$
- **View Velocity (V):**
$$V = \frac{\text{Total Views in first } H \text{ hours}}{H}$$
- **Rule of Thirds for Content Distribution:** Given a total of N posts:
$$C_{promo} = \lceil N \times \frac{1}{3} \rceil \quad \text{(Promotional Content)}$$
$$C_{share} = \lfloor N \times \frac{1}{3} \rfloor \quad \text{(Shared Content)}$$
$$C_{engage} = N - (C_{promo} + C_{share}) \quad \text{(Engaging Content)}$$

- **Optimal Posting Time (t_{opt}):**
$$t_{opt} = \arg\max_t(\bar{E}_t)$$
where $\bar{E}_t$ is the average engagement at time t.

- **Click-Through Rate (CTR) for A/B Testing:**
$$CTR_A = \frac{C_A}{I_A} \times 100, \quad CTR_B = \frac{C_B}{I_B} \times 100$$

- **SMM Return on Investment (ROI):**
$$ROI = \left(\frac{\text{Revenue from SMM} - \text{Cost of SMM}}{\text{Cost of SMM}}\right) \times 100$$

Unit 5: Search Engine Marketing (SEM) & Email Marketing

- **Ad Rank:**
$$\text{Ad Rank} = \text{Maximum Bid (Max CPC)} \times \text{Quality Score}$$

- **Actual Cost Per Click (CPC):**
$$\text{Actual CPC} = \frac{\text{Ad Rank of Competitor Below}}{\text{Your Quality Score}} + 0.01$$

- **Return on Ad Spend (ROAS):**
$$\text{ROAS} = \left(\frac{\text{Total Ad Revenue}}{\text{Total Ad Cost}}\right) \times 100$$

- **Delivery Rate:**
$$\text{Delivery Rate} = \left(\frac{\text{Emails Delivered}}{\text{Emails Sent}}\right) \times 100$$

- **Open Rate:**
$$\text{Open Rate} = \left(\frac{\text{Emails Opened}}{\text{Emails Delivered}}\right) \times 100$$

- **Click-to-Open Rate (CTOR):**
$$\text{CTOR} = \left(\frac{\text{Total Clicks}}{\text{Emails Opened}}\right) \times 100$$

- **Email Campaign Click-Through Rate (CTR):**
$$\text{CTR} = \left(\frac{\text{Total Clicks}}{\text{Emails Delivered}}\right) \times 100$$

- **Cost Per Acquisition (CPA):**
$$\text{CPA} = \frac{\text{Total Ad Cost}}{\text{Total Sales or Conversions}}$$

- **Campaign ROI:**
$$\text{ROI} = \left(\frac{\text{Total Revenue} - \text{Total Campaign Cost}}{\text{Total Campaign Cost}}\right) \times 100$$