

Lecture 6.39

Binary trees: Complete Binary Tree

Agenda

- Recap of Binary Trees
- What is a Complete Binary Tree?
- Properties and Array Representation
- Tree Terminology: Level Number
- Tree Terminology: Degree (In-degree & Out-degree)
- Tree Terminology: Leaf Nodes
- Summary

Recap: What is a Binary Tree?

- A non-linear data structure.
- Consists of nodes connected by edges.
- Each node has at most two children: a left child and a right child.
- Used for hierarchical data storage and efficient searching algorithms.

What is a Complete Binary Tree?

- A binary tree where all levels are completely filled, except possibly the last level.
- In the last level, all nodes are as far left as possible.
- It is different from a Full Binary Tree (where every node has 0 or 2 children).
- It is an important concept for data structures like Heaps.

Visualizing a Complete Binary Tree

- Valid Complete Binary Tree: All levels filled, last level filled left-to-right.
- Invalid Complete Binary Tree: A gap exists in the last level before a node.

Properties of a Complete Binary Tree

- Height of a complete binary tree with n nodes is $\text{Floor}(\log_2(n))$.
- If a tree has depth d , it can have between 2^d and $(2^{(d+1)} - 1)$ nodes.
- Allows for highly memory-efficient array representation.
- Perfect for implementing priority queues using Binary Heaps.

Array Representation of Complete Binary Tree

- Nodes can be mapped to an array sequentially level by level.
- For a node at index i (0-based indexing):
- Left Child is at index $(2 * i + 1)$
- Right Child is at index $(2 * i + 2)$
- Parent is at index $\text{Floor}((i - 1) / 2)$

Basic Terminology: Level Number

- Level of a node represents its distance from the root.
- Root node is at Level 0 (or Level 1, depending on convention).
- Children of the root are at Level 1, their children at Level 2, and so on.
- The level number determines the maximum number of nodes at that depth.

Basic Terminology: Degree of a Node

- The degree of a node is the total number of its children.
- In a binary tree, the degree of any node can be 0, 1, or 2.
- Degree of a tree is the maximum degree of a node in the tree.
- For a binary tree, the degree of the tree is at most 2.

In-degree and Out-degree

- In-degree: The number of edges coming into a node.
- Out-degree: The number of edges going out of a node (equivalent to the node's degree).
- In a tree, the in-degree of the Root is always 0.
- The in-degree of all other nodes is exactly 1.

Basic Terminology: Leaf Node

- A leaf node (or terminal node) is a node with degree 0.
- It has no children (no left child and no right child).
- Leaf nodes represent the bottom-most endpoints of a tree.
- In a Complete Binary Tree, all leaf nodes are found at the last and second-to-last levels.

Summary & Next Steps

- Complete Binary Trees are filled level-by-level, left-to-right.
- They can be efficiently stored in arrays.
- Level number tracks distance from the root.
- Degree is the number of children; in-degree is 1 (except root), out-degree is 0-2.
- Leaf nodes have zero children.