# Mwr (4351103) - Problem Solver

Antigravity AI

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**Mwr**

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**Author:** Antigravity AI
**Website:** milav.in
**YouTube:** @milav.dabgar

*To my students,*
*who constantly inspire me to find new analogies,*
*break down complex theories, and make learning*
*an engineered science.*

# Contents

# Preface

**About this Book**
Welcome to **Computer Networks**! This textbook is meticulously designed to serve as a comprehensive problem solver and study companion. It provides a robust and intuitive foundation in the core principles of this subject, bridging the gap between theoretical models and practical applications. A unique feature of this book is its focus on decoding complex concepts using clear, step-by-step methodologies and real-world analogies.

**Target Audience**
This book is intended for students and professionals looking to master the concepts of Computer Networks. Whether you are revising for exams, seeking to solidify your fundamental understanding, or looking for practical examples to clarify abstract concepts, this book has been structured to support your learning journey. It serves as an accessible guide designed to remove the fear of complex topics, replacing it with an appreciation for the underlying mechanics.

**Scope and Coverage**
The coverage is divided logically to follow standard academic curriculums. Each chapter focuses on key topics, breaking them down into digestible sections:

- **Theoretical Insights** – Core concepts explained intuitively.
- **Method Blocks** – Standardized, step-by-step procedures for solving specific types of problems.
- **Worked Examples** – Fully solved problems that apply the methods in practice.

We hope this resource proves invaluable in your studies and helps you achieve excellence in **Computer Networks**. Happy learning!

# Acknowledgments

Writing a comprehensive book that connects the fundamental forces of Chemistry with practical engineering applications requires more than just academic knowledge—it requires a community of support. I would like to express my sincere gratitude to everyone who contributed to the realization of this project.

Specially, I extend my heartfelt gratitude to the **Department of Technical Education (DTE), Government of Gujarat**, and **Government Polytechnic, Palanpur**, for providing an environment that fosters innovation, academic rigor, and continuous professional development.

I am deeply thankful to my family for their unwavering patience and support during the countless hours spent researching, formatting, and refining these chapters.

Finally, my deepest appreciation goes to my students. Your curiosity, challenging questions, and continuous feedback are the true driving force behind the unique analogies and streamlined structure of this book. This textbook was engineered for you.

**Antigravity AI**

# About the Author

**Milav Jayeshkumar Dabgar** is an engineering educator and R&D professional with over 9 years of experience in electronics hardware development, embedded systems, AI/ML, and full-stack software engineering. He currently serves as a **Lecturer (Class - II) & Innovation Leader** at Government Polytechnic, Palanpur, under the Education Department of the Government of Gujarat.

Milav holds a Master of Engineering (ME) in Communication Systems from L.D. College of Engineering and a Bachelor of Engineering (BE) in Electronics & Communication. He is also currently pursuing a **BS in Data Science and Applications from IIT Madras**, where he has completed both the **Diploma in Programming** and the **Diploma in Data Science** with distinction.

Most recently, he completed the **AICTE QIP PG Certificate Program in Deep Learning from SVNIT Surat** with a **perfect 10/10 CGPA**, topping his batch.

A dedicated mentor, Milav has guided numerous student teams in securing over **INR 45 lakhs** in innovation funding, including INR 25 lakhs from Shark Tank India and INR 20 lakhs through iHub Gujarat, resulting in two student patents. He is recognized as an **NPTEL Evangelist** and **NPTEL Discipline Star** in Computer Science, reflecting his commitment to continuous learning and academic excellence.

His technical expertise spans from embedded systems (8051, STM32, Arduino) to modern web technologies (Next.js, Python, PostgreSQL) and Data Science (TensorFlow, PyTorch). Through this textbook, he aims to simplify complex Engineering Chemistry concepts by integrating relatable, real-world analogies drawn from modern tech and engineering landscapes.

Milav is also an active content creator, running a popular **YouTube channel** (@milav.dabgar) where he shares technical tutorials and academic resources. He maintains a personal blog and resource hub at milav.in and can be reached for professional collaborations on LinkedIn.

CHAPTER 1

# Transmission Lines and Microwaves

## 1.1 Primary and Secondary Constants of Transmission Lines

A transmission line is described by its primary constants (distributed parameters): Resistance $R$ ($\Omega/m$), Inductance $L$ ($H/m$), Conductance $G$ ($S/m$), and Capacitance $C$ ($F/m$). From these, we derive the secondary constants: Characteristic Impedance $Z_0$ and Propagation Constant $\gamma$.

**Methodology:**

Secondary Constants Calculation **Step 1: Calculate Series Impedance ($Z$) and Shunt Admittance ($Y$)**

$$Z = R + j\omega L$$
$$Y = G + j\omega C$$

where $\omega = 2\pi f$ is the angular frequency.

**Step 2: Calculate Characteristic Impedance ($Z_0$)**

$$Z_0 = \sqrt{\frac{Z}{Y}} = \sqrt{\frac{R + j\omega L}{G + j\omega C}}$$

**Step 3: Calculate Propagation Constant ($\gamma$)**

$$\gamma = \sqrt{ZY} = \sqrt{(R + j\omega L)(G + j\omega C)}$$

The propagation constant is a complex quantity: $\gamma = \alpha + j\beta$, where:

- $\alpha$ is the attenuation constant (Nepers/m)
- $\beta$ is the phase constant (rad/m)

**Step 4: Lossless Line Approximation (if $R \approx 0$ and $G \approx 0$)**

$$Z_0 = \sqrt{\frac{L}{C}} \quad \text{(Purely real)}$$
$$\gamma = j\omega\sqrt{LC} \implies \alpha = 0, \beta = \omega\sqrt{LC}$$

**Worked Example:**

Calculating Secondary Constants A transmission line has the following primary constants at a frequency of 1 GHz: $R = 0.5\ \Omega/m$, $L = 0.2\ \mu H/m$, $G = 0.1\ mS/m$, $C = 100\ pF/m$. Calculate the characteristic impedance and propagation constant.

**Step 1: Determine $\omega$ and intermediate parameters**

$$f = 1 \times 10^9\ Hz \implies \omega = 2\pi f \approx 6.28 \times 10^9\ rad/s$$

**Step 2: Calculate Z and Y**

$$Z = R + j\omega L = 0.5 + j(6.28 \times 10^9)(0.2 \times 10^{-6}) = 0.5 + j1256\ \Omega/m$$
$$Y = G + j\omega C = 10^{-4} + j(6.28 \times 10^9)(100 \times 10^{-12}) = 10^{-4} + j0.628\ S/m$$

Since $\omega L \gg R$ and $\omega C \gg G$, the line can be approximated as low-loss or lossless.
**Step 3: Calculate Characteristic Impedance $Z_0$**

$$Z_0 = \sqrt{\frac{Z}{Y}} \approx \sqrt{\frac{j1256}{j0.628}} = \sqrt{2000} = 44.72\ \Omega$$

**Step 4: Calculate Propagation Constant $\gamma$**

$$\gamma = \sqrt{ZY} \approx \sqrt{(j1256)(j0.628)} = j\sqrt{788.768} = j28.08\ m^{-1}$$

Thus, $\alpha \approx 0$ Np/m and $\beta = 28.08$ rad/m.

## 1.2 Reflection Coefficient and Voltage Standing Wave Ratio

When a transmission line is terminated with a load impedance $Z_L$ that is not equal to its characteristic impedance $Z_0$, a portion of the incident wave is reflected back towards the source.

**Methodology:**

Reflection Coefficient and VSWR **Step 1: Calculate the Reflection Coefficient ($\Gamma$)** The voltage reflection coefficient at the load is given by:

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0}$$

$\Gamma$ is generally a complex number: $\Gamma = |\Gamma| e^{j\theta}$.

- Matched Load ($Z_L = Z_0$): $\Gamma = 0$
- Short Circuit ($Z_L = 0$): $\Gamma = -1$
- Open Circuit ($Z_L = \infty$): $\Gamma = 1$

**Step 2: Calculate the Voltage Standing Wave Ratio (VSWR)** The interference of incident and reflected waves creates a standing wave. VSWR (or simply $S$) is the ratio of maximum to minimum voltage magnitudes:

$$\text{VSWR} = \frac{1 + |\Gamma|}{1 - |\Gamma|}$$

Note that $1 \leq \text{VSWR} \leq \infty$.
**Step 3: Calculate Return Loss**

$$\text{Return Loss (dB)} = -20 \log_{10} |\Gamma|$$

**Worked Example:**

Load Matching and VSWR A lossless 50 $\Omega$ transmission line is terminated in a load impedance of $Z_L = (75 + j50)\ \Omega$. Calculate the reflection coefficient, VSWR, and Return Loss.

**Step 1: Calculate Reflection Coefficient** $(\Gamma)$

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0}$$
$$= \frac{(75 + j50) - 50}{(75 + j50) + 50}$$
$$= \frac{25 + j50}{125 + j50}$$

Convert to polar form to find the magnitude: Numerator magnitude $= \sqrt{25^2 + 50^2} = \sqrt{625 + 2500} = \sqrt{3125} \approx 55.9$ Denominator magnitude $= \sqrt{125^2 + 50^2} = \sqrt{15625 + 2500} = \sqrt{18125} \approx 134.6$

$$|\Gamma| = \frac{55.9}{134.6} \approx 0.415$$

**Step 2: Calculate VSWR**

$$\text{VSWR} = \frac{1 + |\Gamma|}{1 - |\Gamma|} = \frac{1 + 0.415}{1 - 0.415} = \frac{1.415}{0.585} \approx 2.42$$

**Step 3: Calculate Return Loss**

$$\text{Return Loss} = -20\log_{10}(0.415) = -20(-0.382) \approx 7.64 \text{ dB}$$

## 1.3 Input Impedance of a Lossless Line

The impedance seen looking into a transmission line of length $l$ depends on $Z_0$, $Z_L$, and the electrical length $\beta l$.

**Methodology:**

Input Impedance Formula **Step 1: Determine the Phase Constant** $(\beta)$ **and Line Length** $(l)$ Calculate $\beta = \frac{2\pi}{\lambda}$ or $\beta = \frac{\omega}{v_p}$. Find the electrical length $\beta l$ in radians.
**Step 2: Apply the Impedance Transformation Equation** For a lossless line, the input impedance $Z_{in}$ at a distance $l$ from the load is:

$$Z_{in} = Z_0 \frac{Z_L + jZ_0 \tan(\beta l)}{Z_0 + jZ_L \tan(\beta l)}$$

**Special Cases:**

- **Short-Circuited Line** $(Z_L = 0)$**:** $Z_{in} = jZ_0 \tan(\beta l)$ (purely reactive)
- **Open-Circuited Line** $(Z_L = \infty)$**:** $Z_{in} = -jZ_0 \cot(\beta l)$
- **Quarter-Wave Transformer** $(l = \lambda/4 \implies \beta l = \pi/2)$**:** As $\tan(\pi/2) \to \infty$, the formula simplifies to:
$$Z_{in} = \frac{Z_0^2}{Z_L} \implies Z_0 = \sqrt{Z_{in} Z_L}$$
- **Half-Wave Line** $(l = \lambda/2 \implies \beta l = \pi)$**:** Since $\tan(\pi) = 0$, $Z_{in} = Z_L$.

**Worked Example:**

Input Impedance of a Shorted Line A 50 $\Omega$ lossless transmission line is terminated with a short circuit $(Z_L = 0)$. If the operating frequency is 300 $MHz$ and the wave velocity on the line is $2 \times 10^8$ $m/s$, find the input impedance for a line length of 0.5 $m$.
**Step 1: Calculate Wavelength and Phase Constant**

$$\lambda = \frac{v_p}{f} = \frac{2 \times 10^8}{300 \times 10^6} = \frac{2}{3}\ m$$

$$\beta = \frac{2\pi}{\lambda} = \frac{2\pi}{2/3} = 3\pi \text{ rad/m}$$

**Step 2: Calculate Electrical Length**

$$\beta l = 3\pi \times 0.5 = 1.5\pi \text{ radians} = \frac{3\pi}{2} \text{ radians}$$

**Step 3: Calculate Input Impedance** For a short-circuited line, $Z_{in} = jZ_0 \tan(\beta l)$.

$$Z_{in} = j50 \tan(1.5\pi)$$

Since $\tan(1.5\pi)$ approaches infinity, the line acts as an open circuit (anti-resonance). Therefore, $Z_{in} \to \infty$ (Open Circuit).

**Worked Example:**

Quarter-Wave Impedance Matching We need to match a 73 $\Omega$ antenna to a 50 $\Omega$ main transmission line using a quarter-wave transformer. Determine the required characteristic impedance of the quarter-wave section.

**Step 1: Identify the Known Variables** Main line (input impedance required): $Z_{in} = 50\ \Omega$ Load (antenna): $Z_L = 73\ \Omega$

**Step 2: Apply the Quarter-Wave Transformer Formula** For a quarter-wave line ($l = \lambda/4$), the characteristic impedance $Z_{0,qw}$ must satisfy:

$$Z_{0,qw} = \sqrt{Z_{in} Z_L}$$

**Step 3: Calculate $Z_{0,qw}$**

$$Z_{0,qw} = \sqrt{50 \times 73} = \sqrt{3650} \approx 60.41\ \Omega$$

A quarter-wave line with $Z_0 = 60.41\ \Omega$ will perfectly match the 73 $\Omega$ load to the 50 $\Omega$ line.

## 1.4 Phase Velocity and Group Velocity

In microwave transmission lines and waveguides, wave propagation is characterized by phase velocity (the speed at which a constant phase point travels) and group velocity (the speed at which the envelope or energy of the wave travels).

**Methodology:**

Phase and Group Velocity Calculations **Step 1: Phase Velocity ($v_p$)** The phase velocity is defined as the ratio of angular frequency to the phase constant:

$$v_p = \frac{\omega}{\beta}$$

For a lossless transmission line, $v_p = \frac{1}{\sqrt{LC}}$.

**Step 2: Group Velocity ($v_g$)** The group velocity is the derivative of angular frequency with respect to the phase constant:

$$v_g = \frac{d\omega}{d\beta}$$

**Step 3: Relationship in Waveguides** In a dispersive medium like a waveguide, the velocities are related to the speed of light $c$ in the medium by:

$$v_p v_g = c^2$$

where $c = \frac{1}{\sqrt{\mu\epsilon}}$.

**Worked Example:**

Calculating Velocities A wave propagates in a medium such that the phase constant is given by $\beta = \omega\sqrt{\mu\epsilon}\sqrt{1-(\frac{\omega_c}{\omega})^2}$, where $\omega_c$ is the cutoff angular frequency. Find the expressions for phase and group velocity.

**Step 1: Calculate Phase Velocity**

$$\begin{aligned} v_p &= \frac{\omega}{\beta} \\ &= \frac{\omega}{\omega\sqrt{\mu\epsilon}\sqrt{1-(\frac{\omega_c}{\omega})^2}} \\ &= \frac{1}{\sqrt{\mu\epsilon}\sqrt{1-(\frac{\omega_c}{\omega})^2}} \end{aligned}$$

If $c = \frac{1}{\sqrt{\mu\epsilon}}$, then $v_p = \frac{c}{\sqrt{1-(\frac{\omega_c}{\omega})^2}}$.

**Step 2: Calculate Group Velocity** Using the relationship $v_p v_g = c^2$:

$$\begin{aligned} v_g &= \frac{c^2}{v_p} \\ &= c^2\left(\frac{\sqrt{1-(\frac{\omega_c}{\omega})^2}}{c}\right) \\ &= c\sqrt{1-(\frac{\omega_c}{\omega})^2} \end{aligned}$$

Notice that for $\omega > \omega_c$, $v_p > c$ and $v_g < c$.

## 1.5 Microwave Power and Decibel Calculations

In microwave engineering, power levels and attenuation are strictly handled using decibel (dB) and decibel-milliwatts (dBm) due to the large variation in magnitudes.

**Methodology:**

Power Ratios and dBm Conversion **Step 1: Power Ratio to dB** The gain or loss in dB between output power $P_{out}$ and input power $P_{in}$ is:

$$\text{Gain (dB)} = 10\log_{10}\left(\frac{P_{out}}{P_{in}}\right)$$

(If the result is negative, it indicates an attenuation or loss).

**Step 2: Absolute Power in dBm** Power can be expressed absolutely with respect to 1 mW:

$$P(\text{dBm}) = 10\log_{10}\left(\frac{P(\text{in mW})}{1\text{ mW}}\right)$$

To convert back to mW:

$$P(\text{mW}) = 10^{\frac{P(\text{dBm})}{10}}$$

**Step 3: System Calculations** If multiple microwave components are cascaded, total output power in dBm is:

$$P_{out}(\text{dBm}) = P_{in}(\text{dBm}) + \text{Gain}_1(\text{dB}) + \text{Gain}_2(\text{dB}) - \text{Loss}_3(\text{dB})\ldots$$

**Worked Example:**

Cascaded Microwave System A microwave transmitter generates 50 $mW$ of power. It is fed into a transmission line with 3 $dB$ of loss, followed by an amplifier with a 15 $dB$ gain. What is the output power in mW?

**Step 1: Convert Input Power to dBm**

$$P_{in} = 50\ mW$$

$$P_{in}(\text{dBm}) = 10\log_{10}(50) = 10(1.699) = 16.99\ \text{dBm}$$

**Step 2: Calculate Output Power in dBm** The total system consists of a loss ($-3\ dB$) and a gain ($+15\ dB$).

$$\begin{aligned} P_{out}(\text{dBm}) &= P_{in}(\text{dBm}) - \text{Loss} + \text{Gain} \\ &= 16.99 - 3 + 15 \\ &= 28.99\ \text{dBm} \end{aligned}$$

**Step 3: Convert Output Power back to mW**

$$P_{out}(\text{mW}) = 10^{\frac{28.99}{10}} = 10^{2.899} \approx 792.5\ mW$$

# CHAPTER 2

# Microwave Propagation and Components

## 2.1 Rectangular Waveguides

**Methodology:**

Rectangular Waveguide Parameters A rectangular waveguide has a width $a$ and height $b$ (where $a > b$). To compute the propagation characteristics for a specific transverse electric (TE) or transverse magnetic (TM) mode denoted by indices $m$ and $n$, follow these steps:

1. **Determine the Cutoff Wavelength ($\lambda_c$):** Calculate the cutoff wavelength for the $TE_{mn}$ or $TM_{mn}$ mode using:
$$\lambda_c = \frac{2}{\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}}$$
For the dominant mode ($TE_{10}$), $m = 1$ and $n = 0$, giving $\lambda_c = 2a$.

2. **Determine the Cutoff Frequency ($f_c$):** Calculate the cutoff frequency using the speed of light $c \approx 3 \times 10^8$ m/s (for air-filled guides):
$$f_c = \frac{c}{\lambda_c}$$

3. **Calculate the Free-Space Wavelength ($\lambda_0$):** For an operating frequency $f$:
$$\lambda_0 = \frac{c}{f}$$

4. **Calculate the Guide Wavelength ($\lambda_g$):** The wavelength of the signal inside the waveguide is given by:
$$\lambda_g = \frac{\lambda_0}{\sqrt{1 - \left(\frac{f_c}{f}\right)^2}}$$

5. **Calculate Phase Velocity ($v_p$) and Group Velocity ($v_g$):** The phase velocity (speed of the wave's phase) and group velocity (speed of energy propagation) are calculated as:
$$v_p = \frac{c}{\sqrt{1 - \left(\frac{f_c}{f}\right)^2}}$$
$$v_g = c\sqrt{1 - \left(\frac{f_c}{f}\right)^2}$$

6. **Calculate Wave Impedance ($Z_w$):** The wave impedance depends on the mode type. Using the intrinsic impedance of free space $\eta \approx 377\ \Omega$: For TE modes:
$$Z_{TE} = \frac{\eta}{\sqrt{1 - \left(\frac{f_c}{f}\right)^2}}$$

For TM modes:

$$Z_{TM} = \eta\sqrt{1 - \left(\frac{f_c}{f}\right)^2}$$

**Worked Example:**

Calculating Parameters for TE10 Mode An air-filled rectangular waveguide has dimensions $a = 2.286$ cm and $b = 1.016$ cm. It is operating in the dominant $TE_{10}$ mode at a frequency of 9 GHz. Calculate the cutoff frequency, guide wavelength, phase velocity, and wave impedance.

**Step 1: Identify dimensions and operating frequency.** Given: $a = 2.286$ cm $= 0.02286$ m $b = 1.016$ cm $= 0.01016$ m $f = 9$ GHz $= 9 \times 10^9$ Hz $c = 3 \times 10^8$ m/s

**Step 2: Determine Cutoff Wavelength ($\lambda_c$) and Frequency ($f_c$).** For the $TE_{10}$ dominant mode:

$$\lambda_c = 2a = 2(0.02286) = 0.04572 \text{ m}$$

$$f_c = \frac{c}{\lambda_c} = \frac{3 \times 10^8}{0.04572} \approx 6.56 \times 10^9 \text{ Hz} = 6.56 \text{ GHz}$$

**Step 3: Calculate Free-Space Wavelength ($\lambda_0$).**

$$\lambda_0 = \frac{c}{f} = \frac{3 \times 10^8}{9 \times 10^9} \approx 0.0333 \text{ m} = 3.33 \text{ cm}$$

**Step 4: Calculate the Guide Wavelength ($\lambda_g$).** Using the ratio $(f_c/f) = 6.56/9 = 0.7289$:

$$\lambda_g = \frac{\lambda_0}{\sqrt{1 - (f_c/f)^2}} = \frac{0.0333}{\sqrt{1 - (0.7289)^2}}$$

$$\lambda_g = \frac{0.0333}{\sqrt{1 - 0.5313}} = \frac{0.0333}{\sqrt{0.4687}} = \frac{0.0333}{0.6846} \approx 0.0486 \text{ m} = 4.86 \text{ cm}$$

**Step 5: Calculate Phase Velocity ($v_p$).**

$$v_p = \frac{c}{\sqrt{1 - (f_c/f)^2}} = \frac{3 \times 10^8}{0.6846} \approx 4.38 \times 10^8 \text{ m/s}$$

**Step 6: Calculate Wave Impedance ($Z_{TE}$).** For TE mode:

$$Z_{TE} = \frac{377}{\sqrt{1 - (f_c/f)^2}} = \frac{377}{0.6846} \approx 550.7\ \Omega$$

## 2.2 Circular Waveguides

**Methodology:**

Circular Waveguide Parameters For a circular waveguide of radius $r$, the cutoff parameters depend on the roots of Bessel functions.

1. **Identify the Root for the Mode:** For TM modes, find the root $p_{mn}$ of the Bessel function $J_n(x) = 0$. For TE modes, find the root $p'_{mn}$ of the Bessel function derivative $J'_n(x) = 0$. Common roots include:
   - $TE_{11}$ (dominant mode): $p'_{11} = 1.841$
   - $TM_{01}$: $p_{01} = 2.405$

2. **Calculate Cutoff Wavelength ($\lambda_c$):** For TE modes:

$$\lambda_c = \frac{2\pi r}{p'_{mn}}$$

For TM modes:

$$\lambda_c = \frac{2\pi r}{p_{mn}}$$

3. **Calculate Cutoff Frequency ($f_c$):**

$$f_c = \frac{c}{\lambda_c}$$

4. **Calculate Other Parameters:** Use the same formulas as rectangular waveguides for guide wavelength ($\lambda_g$), phase velocity ($v_p$), and group velocity ($v_g$), substituting the $f_c$ found in Step 3.

**Worked Example:**

Calculating TE11 Cutoff in Circular Waveguide An air-filled circular waveguide has a radius of 2 cm. Calculate the cutoff frequency and cutoff wavelength for the dominant $TE_{11}$ mode. If the operating frequency is 6 GHz, find the guide wavelength.

**Step 1: Identify root and dimensions.** Radius $r = 2$ cm $= 0.02$ m. For $TE_{11}$ mode, root $p'_{11} = 1.841$. Operating frequency $f = 6$ GHz $= 6 \times 10^9$ Hz.

**Step 2: Calculate Cutoff Wavelength ($\lambda_c$).**

$$\lambda_c = \frac{2\pi r}{p'_{11}} = \frac{2 \times 3.1416 \times 0.02}{1.841} \approx 0.0682 \text{ m} = 6.82 \text{ cm}$$

**Step 3: Calculate Cutoff Frequency ($f_c$).**

$$f_c = \frac{3 \times 10^8}{0.0682} \approx 4.39 \times 10^9 \text{ Hz} = 4.39 \text{ GHz}$$

**Step 4: Calculate Guide Wavelength ($\lambda_g$).** First, find free-space wavelength $\lambda_0$:

$$\lambda_0 = \frac{3 \times 10^8}{6 \times 10^9} = 0.05 \text{ m} = 5 \text{ cm}$$

Now compute $(f_c/f) = 4.39/6 = 0.7317$.

$$\lambda_g = \frac{\lambda_0}{\sqrt{1 - (f_c/f)^2}} = \frac{0.05}{\sqrt{1 - (0.7317)^2}} = \frac{0.05}{\sqrt{1 - 0.5354}} = \frac{0.05}{\sqrt{0.4646}} = \frac{0.05}{0.6816} \approx 0.0733 \text{ m} = 7.33 \text{ cm}$$

## 2.3 Scattering Parameters and Microwave Components

**Methodology:**

S-Parameter Properties and Network Analysis The Scattering matrix (S-matrix) characterizes a microwave network. For an N-port network, $[S]$ is an $N \times N$ matrix.

1. **Check for Reciprocity:** A network is reciprocal if its S-matrix is symmetric. Check if $S_{ij} = S_{ji}$ for all $i, j$.

2. **Check for Losslessness:** A network is lossless if its S-matrix is unitary. The sum of the squares of the magnitudes of the elements in any row (or column) must equal exactly 1.

$$\sum_{k=1}^{N} |S_{ki}|^2 = 1 \quad \text{for all columns } i$$

3. **Calculate Return Loss and Insertion Loss:** To evaluate reflection at port $i$:

$$\text{Return Loss (RL)} = -20\log_{10}|S_{ii}| \text{ dB}$$

To evaluate transmission from port $j$ to port $i$:

$$\text{Insertion Loss (IL)} = -20\log_{10}|S_{ij}| \text{ dB}$$

**Worked Example:**

Analyzing a Two-Port S-Matrix The S-parameters of a two-port microwave network are given by:

$$[S] = \begin{bmatrix} 0.1 & 0.8j \\ 0.8j & 0.2 \end{bmatrix}$$

Determine whether the network is reciprocal and whether it is lossless. Calculate the return loss at port 1.
**Step 1: Check Reciprocity.** We inspect the off-diagonal elements: $S_{12} = 0.8j$ $S_{21} = 0.8j$ Since $S_{12} = S_{21}$, the matrix is symmetric, and the network is **reciprocal**.
**Step 2: Check Losslessness.** Check the sum of squared magnitudes for Column 1:

$$|S_{11}|^2 + |S_{21}|^2 = |0.1|^2 + |0.8j|^2 = 0.01 + 0.64 = 0.65$$

Since $0.65 \neq 1$, the network is **not lossless** (it is a lossy network).
**Step 3: Calculate Return Loss at Port 1.**

$$RL = -20\log_{10}|S_{11}| = -20\log_{10}(0.1) = -20 \times (-1) = 20 \text{ dB}$$

**Methodology:**

Directional Coupler Calculations A directional coupler is a 4-port device. Let Port 1 be the input, Port 2 the through port, Port 3 the coupled port, and Port 4 the isolated port. Given the powers $P_1, P_2, P_3, P_4$ at these ports, compute performance parameters in decibels (dB):

1. **Coupling Factor (C):** Measures how much power is transferred to the coupled port.

$$C = 10\log_{10}\left(\frac{P_1}{P_3}\right) \text{ dB}$$

2. **Directivity (D):** Measures the ability to separate forward and backward waves.

$$D = 10\log_{10}\left(\frac{P_3}{P_4}\right) \text{ dB}$$

3. **Isolation (I):** Measures power delivered to the uncoupled (isolated) port.

$$I = 10\log_{10}\left(\frac{P_1}{P_4}\right) \text{ dB}$$

Alternatively, $I = C + D$.

4. **Insertion Loss (IL):** Measures the power loss between the input and the through port.

$$IL = 10\log_{10}\left(\frac{P_1}{P_2}\right) \text{ dB}$$

**Worked Example:**

Directional Coupler Performance Parameters A 90 mW signal is applied to Port 1 of a directional coupler. The power measured at Port 2 is 70 mW, at Port 3 is 10 mW, and at Port 4 is 0.1 mW. Calculate the Coupling Factor, Directivity, Isolation, and Insertion Loss.

**Step 1: Identify port powers.** $P_1 = 90$ mW (Input) $P_2 = 70$ mW (Through) $P_3 = 10$ mW (Coupled) $P_4 = 0.1$ mW (Isolated)

**Step 2: Calculate Coupling Factor (C).**

$$C = 10\log_{10}\left(\frac{90}{10}\right) = 10\log_{10}(9) \approx 10 \times 0.954 = 9.54 \text{ dB}$$

**Step 3: Calculate Directivity (D).**

$$D = 10\log_{10}\left(\frac{10}{0.1}\right) = 10\log_{10}(100) = 10 \times 2 = 20 \text{ dB}$$

**Step 4: Calculate Isolation (I).**

$$I = 10\log_{10}\left(\frac{90}{0.1}\right) = 10\log_{10}(900) \approx 10 \times 2.954 = 29.54 \text{ dB}$$

*Check:* $I = C + D = 9.54 + 20 = 29.54$ dB. (Matches exactly)

**Step 5: Calculate Insertion Loss (IL).**

$$IL = 10\log_{10}\left(\frac{90}{70}\right) = 10\log_{10}(1.285) \approx 1.09 \text{ dB}$$

**Methodology:**

Magic Tee Matrix Properties A Magic Tee (or hybrid tee) is a 4-port microwave device constructed from a combination of an E-plane tee and an H-plane tee. The ports are typically numbered as: Port 1 and 2: Collinear arms Port 3: H-plane arm (sum port) Port 4: E-plane arm (difference port)

1. **Formulate Ideal S-Matrix:** An ideal, perfectly matched, and lossless Magic Tee has the following scattering matrix:

$$[S] = \frac{1}{\sqrt{2}}\begin{bmatrix} 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & -1 \\ 1 & 1 & 0 & 0 \\ 1 & -1 & 0 & 0 \end{bmatrix}$$

2. **Determine Output Powers for Given Inputs:** Given incident wave amplitudes $[a] = [a_1, a_2, a_3, a_4]^T$, compute the reflected/transmitted waves $[b] = [S][a]$.

$$b_1 = \frac{1}{\sqrt{2}}(a_3 + a_4)$$

$$b_2 = \frac{1}{\sqrt{2}}(a_3 - a_4)$$

$$b_3 = \frac{1}{\sqrt{2}}(a_1 + a_2)$$

$$b_4 = \frac{1}{\sqrt{2}}(a_1 - a_2)$$

3. **Calculate Output Power at Each Port:** The power emerging from port $i$ is $P_i = |b_i|^2$ (assuming $a_i$ values are given in $\sqrt{\text{W}}$).

**Worked Example:**

Calculating Power Flow in a Magic Tee Two coherent microwave signals of equal amplitude $a_1 = 5$ (which corresponds to an input power $P_1 = 25$ mW) and $a_2 = 3$ (which corresponds to $P_2 = 9$ mW) are fed into the collinear arms (Ports 1 and 2) of an ideal Magic Tee. Ports 3 and 4 are perfectly matched. Determine the power emerging from Port 3 and Port 4.

**Step 1: Identify incident waves.** $a_1 = 5$ $a_2 = 3$ $a_3 = 0$ (No input at Port 3) $a_4 = 0$ (No input at Port 4)

**Step 2: Calculate emergent wave amplitudes.** Using the Magic Tee S-matrix relations:

$$b_3 = \frac{1}{\sqrt{2}}(a_1 + a_2) = \frac{1}{\sqrt{2}}(5 + 3) = \frac{8}{\sqrt{2}} = 4\sqrt{2}$$

$$b_4 = \frac{1}{\sqrt{2}}(a_1 - a_2) = \frac{1}{\sqrt{2}}(5 - 3) = \frac{2}{\sqrt{2}} = \sqrt{2}$$

**Step 3: Calculate output powers.** Power at Port 3 (H-plane arm / sum port):

$$P_3 = |b_3|^2 = (4\sqrt{2})^2 = 16 \times 2 = 32 \text{ mW}$$

Power at Port 4 (E-plane arm / difference port):

$$P_4 = |b_4|^2 = (\sqrt{2})^2 = 2 \text{ mW}$$

*Check Power Conservation:* Total Input $= 25 + 9 = 34$ mW. Total Output $= 32 + 2 = 34$ mW.

CHAPTER 3

# Microwave Tubes and Measurement

## 3.1 Limitations of Vacuum Tubes at Microwave Frequencies

At microwave frequencies, conventional vacuum tubes face severe performance limitations primarily due to inter-electrode capacitance, lead inductance, and transit time effects.

**Methodology:**

Transit Time and Transit Angle Calculation The time taken by an electron to travel from the cathode to the anode is called transit time ($\tau$). At low frequencies, $\tau$ is negligible, but at microwave frequencies, it becomes comparable to the time period of the signal.

1. **DC Electron Velocity ($v_0$):** Calculated using kinetic energy $\frac{1}{2}mv_0^2 = eV_0$.

$$v_0 = \sqrt{\frac{2eV_0}{m}} \approx 0.593 \times 10^6 \sqrt{V_0} \text{ m/s}$$

   Where $V_0$ is the anode voltage, $e = 1.6 \times 10^{-19}$ C, and $m = 9.1 \times 10^{-31}$ kg.

2. **Transit Time ($\tau$):** For a planar spacing $d$ between cathode and anode:

$$\tau = \frac{d}{v_0} \text{ seconds}$$

3. **Transit Angle ($\theta_g$):** The phase shift occurring during the electron's transit is the transit angle:

$$\theta_g = \omega\tau = 2\pi f\tau \text{ radians}$$

**Worked Example:**

Evaluation of Transit Time Effect A conventional vacuum tube operates at an anode voltage of 100 V with a cathode-to-anode spacing of 2 cm. Calculate the electron velocity transit time and the transit angle at an operating frequency of 1 GHz.

**Step 1: Calculate electron velocity ($v_0$).**

$$v_0 = 0.593 \times 10^6 \sqrt{100} = 0.593 \times 10^6 \times 10 = 5.93 \times 10^6 \text{ m/s}$$

**Step 2: Calculate transit time ($\tau$).**

$$\tau = \frac{d}{v_0} = \frac{2 \times 10^{-2}}{5.93 \times 10^6} \approx 3.37 \times 10^{-9} \text{ seconds (or 3.37 ns)}$$

**Step 3: Calculate transit angle ($\theta_g$).** Given $f = 1 \times 10^9$ Hz:

$$\theta_g = 2\pi f\tau = 2\pi(10^9)(3.37 \times 10^{-9}) \approx 21.17 \text{ radians}$$

Since 21.17 rad is multiple full cycles (one RF cycle is $2\pi \approx 6.28$ rad), the electron takes more than 3 full RF cycles to reach the anode resulting in drastic efficiency loss.

## 3.2 Microwave Tube Amplifiers

### 3.2.1 Two-Cavity and Multi-Cavity Klystron

Klystrons use velocity modulation rather than density modulation. In a two-cavity klystron, an electron beam passes through a buncher cavity (which velocity-modulates the beam) and a catcher cavity (where energy is extracted from the bunched electrons). Multi-cavity klystrons add intermediate cavities to increase gain.

**Methodology:**

Two Cavity Klystron Computations

1. **Beam Velocity ($v_0$):** $v_0 = 0.593 \times 10^6 \sqrt{V_0}$ m/s.
2. **Gap Transit Angle ($\theta_g$):** $\theta_g = \frac{\omega d}{v_0}$, where $d$ is the gap spacing.
3. **Beam Coupling Coefficient ($\beta_i$):** Accounts for finite gap transit time.
$$\beta_i = \frac{\sin(\theta_g/2)}{\theta_g/2}$$
4. **Velocity Modulation Depth ($m$):**
$$m = \frac{\beta_i V_1}{V_0}$$
where $V_1$ is the RF input voltage at the buncher.
5. **Bunching Parameter ($X$):**
$$X = \frac{m}{2}\theta_0 = \frac{\beta_i V_1}{2V_0}\left(\frac{\omega L}{v_0}\right)$$
where $\theta_0$ is the DC transit angle between cavities and $L$ is the drift space length.

**Worked Example:**

Two Cavity Klystron Analysis A two-cavity klystron operates at 5 GHz with a beam voltage of 10 kV and cavity gap spacing of 2 mm. The drift space is 4 cm long. Calculate the gap transit angle and beam coupling coefficient.

**Step 1: Calculate DC electron velocity.**

$$v_0 = 0.593 \times 10^6 \sqrt{10000} = 5.93 \times 10^7 \text{ m/s}$$

**Step 2: Calculate gap transit angle ($\theta_g$).** Given $f = 5 \times 10^9$ Hz, $\omega = 2\pi(5 \times 10^9) = 3.14 \times 10^{10}$ rad/s, $d = 0.002$ m.

$$\theta_g = \frac{3.14 \times 10^{10} \times 0.002}{5.93 \times 10^7} \approx 1.059 \text{ radians}$$

**Step 3: Calculate beam coupling coefficient ($\beta_i$).**

$$\beta_i = \frac{\sin(1.059/2)}{1.059/2} = \frac{\sin(0.5295 \text{ rad})}{0.5295} \approx \frac{0.505}{0.5295} \approx 0.954$$

### 3.2.2 Travelling Wave Tube TWT

A TWT uses a slow-wave structure (usually a helix) to slow down the RF signal so its velocity matches the electron beam velocity. This allows continuous interaction and broadband amplification.

**Methodology:**

TWT Gain Equation The power gain of a Travelling Wave Tube is determined by the interaction length and the gain parameter.

1. **Gain Parameter ($C$):**

$$C = \left(\frac{I_0 Z_0}{4V_0}\right)^{1/3}$$

   Where $I_0$ is beam current, $V_0$ is beam voltage, and $Z_0$ is the characteristic impedance of the slow-wave structure.

2. **Circuit Length in Wavelengths ($N$):**

$$N = \frac{L}{\lambda_e} = \frac{fL}{v_0}$$

   where $L$ is the length of the slow wave structure and $v_0$ is the axial phase velocity.

3. **Power Gain ($A_p$):**

$$A_p\ (\text{dB}) = -9.54 + 47.3NC$$

**Worked Example:**

TWT Amplifier Gain A TWT operates with beam voltage $V_0 = 3$ kV, beam current $I_0 = 30$ mA, characteristic impedance $Z_0 = 10\ \Omega$, operating frequency $f = 10$ GHz, and helix length $L = 0.15$ m. Determine the power gain in dB.

**Step 1: Calculate the Gain Parameter ($C$).**

$$C = \left(\frac{30 \times 10^{-3} \times 10}{4 \times 3000}\right)^{1/3} = \left(\frac{0.3}{12000}\right)^{1/3} = (25 \times 10^{-6})^{1/3} \approx 0.0292$$

**Step 2: Calculate electron velocity and circuit length ($N$).**

$$v_0 = 0.593 \times 10^6 \sqrt{3000} \approx 3.24 \times 10^7 \text{ m/s}$$

$$N = \frac{fL}{v_0} = \frac{10 \times 10^9 \times 0.15}{3.24 \times 10^7} \approx 46.3$$

**Step 3: Calculate Power Gain.**

$$A_p\ (\text{dB}) = -9.54 + 47.3 \times 46.3 \times 0.0292 = -9.54 + 63.95 \approx 54.41 \text{ dB}$$

## 3.3 Microwave Tube Oscillators

### 3.3.1 Reflex Klystron

The reflex klystron contains a single cavity that acts as both buncher and catcher, and a repeller plate with a negative voltage that reflects electrons back to the cavity.

**Methodology:**

Reflex Klystron Mode and Frequency

1. **Mode of Operation ($n$):** For sustained oscillations, the electrons must return to the cavity in bunches when the RF field is in the maximum retarding phase. The optimum round trip transit time is:

$$T = \left(n + \frac{3}{4}\right) T_{rf} \quad \text{for } n = 0, 1, 2, \ldots$$

Where $T_{rf}$ is the time period of the RF signal. The mode number is often defined as $N = n + \frac{3}{4}$.

2. **Repeller Voltage ($V_R$):** The repeller voltage required for a specific mode depends on the cavity voltage $V_0$, spacing $L$ between cavity and repeller, and frequency $f$:

$$\frac{V_0 + |V_R|}{L} = \frac{4V_0 f}{(n + 3/4)v_0} = \frac{4V_0}{v_0 T}$$

### 3.3.2 Magnetron and Backward Wave Oscillator

The Magnetron is a crossed-field tube oscillator (electric and magnetic fields are perpendicular). Electrons travel in cycloidal paths. The Backward Wave Oscillator (BWO) is similar to a TWT but its wave travels opposite to the electron beam, providing voltage-tunable oscillations.

**Methodology:**

Magnetron Cutoff Equations To prevent electrons from reaching the anode without interacting with the RF field, the magnetic field must exceed a critical value, or the anode voltage must be below the Hull cut-off voltage.

1. **Hull Cut-off Magnetic Field ($B_c$):**

$$B_c = \frac{\sqrt{8V_0 \frac{m}{e}}}{b\left(1 - \frac{a^2}{b^2}\right)}$$

Where $V_0$ is anode voltage, $a$ is cathode radius, $b$ is anode radius, and $m/e \approx 5.68 \times 10^{-12}$ kg/C.

2. **Cyclotron Frequency ($f_c$):** The frequency of the circular motion of electrons:

$$f_c = \frac{eB}{2\pi m} \approx 2.8 \times 10^{10} B \text{ Hz}$$

**Worked Example:**

Magnetron Hull Cutoff Parameters A cylindrical magnetron has a cathode radius of 1.5 mm and an anode radius of 3 mm. The anode voltage is 10 kV. Calculate the Hull cut-off magnetic field required.

**Step 1: Identify given parameters.** $V_0 = 10000$ V, $a = 1.5 \times 10^{-3}$ m, $b = 3 \times 10^{-3}$ m. Constant $\frac{m}{e} = 5.68 \times 10^{-12}$ kg/C.

**Step 2: Calculate the numerator.**

$$\text{Numerator} = \sqrt{8 \times 10000 \times 5.68 \times 10^{-12}} = \sqrt{454.4 \times 10^{-9}} \approx 6.74 \times 10^{-4}$$

**Step 3: Calculate the denominator.**

$$1 - \frac{a^2}{b^2} = 1 - \frac{1.5^2}{3^2} = 1 - 0.25 = 0.75$$

$$\text{Denominator} = 3 \times 10^{-3} \times 0.75 = 2.25 \times 10^{-3}$$

**Step 4: Compute $B_c$.**

$$B_c = \frac{6.74 \times 10^{-4}}{2.25 \times 10^{-3}} \approx 0.30 \text{ Weber/m}^2 \text{ (Tesla)}$$

## 3.4 Microwave Measurement

Measurement of microwave signals requires specialized techniques for parameters like power, frequency, wavelength, Voltage Standing Wave Ratio (VSWR), attenuation, and Q-factor.

### 3.4.1 Frequency Wavelength and VSWR Measurement

**Methodology:**

Guided Wavelength and Frequency Calculation Inside a waveguide, the wavelength of the signal ($\lambda_g$) differs from the free-space wavelength ($\lambda_0$).

1. **Free-Space Wavelength ($\lambda_0$):** $\lambda_0 = \frac{c}{f}$ where $c = 3 \times 10^8$ m/s.
2. **Cutoff Wavelength ($\lambda_c$):** For the dominant $TE_{10}$ mode in a rectangular waveguide of width $a$:
$$\lambda_c = 2a$$
3. **Guided Wavelength ($\lambda_g$):** Relates $\lambda_0$ and $\lambda_c$:
$$\frac{1}{\lambda_g^2} = \frac{1}{\lambda_0^2} - \frac{1}{\lambda_c^2} \implies \lambda_g = \frac{\lambda_0}{\sqrt{1 - (\lambda_0/\lambda_c)^2}}$$

**Worked Example:**

Guided Wavelength Problem A rectangular waveguide has a width of 3 cm. Calculate the guided wavelength for a microwave signal of 6 GHz propagating through it.
**Step 1: Find free-space wavelength.**
$$\lambda_0 = \frac{3 \times 10^8}{6 \times 10^9} = 0.05 \text{ m} = 5 \text{ cm}$$
**Step 2: Find cutoff wavelength.** For $TE_{10}$ mode, $\lambda_c = 2a = 2 \times 3 \text{ cm} = 6 \text{ cm}$.
**Step 3: Calculate guided wavelength.**
$$\lambda_g = \frac{5}{\sqrt{1 - (5/6)^2}} = \frac{5}{\sqrt{1 - 0.694}} = \frac{5}{\sqrt{0.306}} = \frac{5}{0.553} \approx 9.04 \text{ cm}$$

**Methodology:**

VSWR and Reflection Coefficient Computations Voltage Standing Wave Ratio (VSWR or $S$) measures the impedance mismatch.

1. **From Voltage Measurements:**
$$S = \frac{V_{max}}{V_{min}}$$
2. **Reflection Coefficient Magnitude ($|\Gamma|$):**
$$|\Gamma| = \frac{S - 1}{S + 1}$$
3. **Return Loss (RL):**
$$RL = -20 \log_{10} |\Gamma| \text{ dB}$$

**Worked Example:**

VSWR Evaluation A slotted line section is used to measure a microwave network. The maximum probe voltage recorded is 12 mV and the minimum is 3 mV. Calculate the VSWR, reflection coefficient, and Return Loss.

**Step 1: Calculate VSWR.**

$$S = \frac{V_{max}}{V_{min}} = \frac{12}{3} = 4$$

**Step 2: Calculate reflection coefficient.**

$$|\Gamma| = \frac{4-1}{4+1} = \frac{3}{5} = 0.6$$

**Step 3: Calculate Return Loss.**

$$RL = -20\log_{10}(0.6) \approx -20 \times (-0.2218) \approx 4.44 \text{ dB}$$

### 3.4.2 Power Attenuation and Q Factor Measurement

**Methodology:**

Power Attenuation and Q Factor

1. **Microwave Power Measurement:** Low power ($< 10$ mW) is measured using bolometers (thermistors or barretters) in a bridge circuit. Medium/high power is measured using calorimeters. Power conversion often uses dBm:
$$P_{dBm} = 10\log_{10}\left(\frac{P_{mW}}{1 \text{ mW}}\right)$$
2. **Attenuation Measurement:** Measured by comparing power levels with and without the Device Under Test (DUT).
$$\text{Attenuation (dB)} = 10\log_{10}\left(\frac{P_{in}}{P_{out}}\right)$$
3. **Q-Factor of a Resonant Cavity:** Can be measured dynamically by finding the 3-dB (half-power) frequencies.
$$Q = \frac{f_0}{\Delta f} = \frac{f_0}{f_2 - f_1}$$
Where $f_0$ is the resonant frequency, and $f_1, f_2$ are the lower and upper half-power frequencies respectively.

**Worked Example:**

Cavity Q Factor Measurement A microwave resonant cavity has a measured resonant frequency of 9 GHz. The half-power frequencies are found to be 8.995 GHz and 9.005 GHz. Calculate the Q-factor of the cavity.
**Step 1: Identify parameters.** $f_0 = 9 \times 10^9$ Hz, $f_1 = 8.995 \times 10^9$ Hz, $f_2 = 9.005 \times 10^9$ Hz.
**Step 2: Calculate bandwidth ($\Delta f$).**

$$\Delta f = f_2 - f_1 = 9.005 - 8.995 = 0.010 \text{ GHz} = 10 \text{ MHz}$$

**Step 3: Calculate Q-factor.**

$$Q = \frac{f_0}{\Delta f} = \frac{9 \times 10^9}{10 \times 10^6} = \frac{9000}{10} = 900$$

## 3.5 Microwave Radiation Hazards and Protection

Microwave energy at high power levels poses severe health and structural risks due to thermal heating and induced voltages.

**Methodology:**

Hazard Classifications and Safe Distance

1. **HERP (Hazards of Electromagnetic Radiation to Personnel):** Biological hazards due to tissue heating. The safe exposure limit is typically 10 mW/cm$^2$.

2. **HERO (Hazards of Electromagnetic Radiation to Ordnance):** Accidental detonation of electro-explosive devices.

3. **HERF (Hazards of Electromagnetic Radiation to Fuel):** Spark ignition of volatile fuels.

4. **Power Density ($P_d$):** To ensure safety limits are not exceeded, the power density at a distance $R$ from an antenna is:
$$P_d = \frac{P_t G}{4\pi R^2}$$
Where $P_t$ is the transmitted power and $G$ is the antenna gain. This equation is rearranged to find the Safe Distance $R_{safe}$.

**Worked Example:**

HERP Safe Distance Calculation A radar system transmits an average power of 1 kW with an antenna gain of 1000 (numeric, not dB). If the HERP safe exposure limit is 10 mW/cm$^2$, calculate the minimum safe distance from the antenna.

**Step 1: Convert units to standard meters and watts.** $P_t = 1000$ W. Limit $P_d = 10\ \text{mW/cm}^2 = \frac{10\times10^{-3}\ \text{W}}{10^{-4}\ \text{m}^2} = 100\ \text{W/m}^2$.

**Step 2: Rearrange power density formula for distance $R$.**

$$P_d = \frac{P_t G}{4\pi R^2} \implies R = \sqrt{\frac{P_t G}{4\pi P_d}}$$

**Step 3: Calculate safe distance ($R$).**

$$R = \sqrt{\frac{1000 \times 1000}{4\pi \times 100}} = \sqrt{\frac{1,000,000}{1256.6}} = \sqrt{795.77} \approx 28.2 \text{ meters}$$

Personnel must stay at least 28.2 meters away from the transmitting antenna to remain safe.

# CHAPTER 4

# Microwave Semiconductor Devices

This chapter focuses on the computational aspects of microwave semiconductor devices. These devices, which include Gunn diodes, IMPATT diodes, PIN diodes, Varactor diodes, and Tunnel diodes, utilize unique physical phenomena at the semiconductor level to generate, amplify, or control microwave signals. The emphasis here is on the mathematical formulations governing their operation, such as frequency of oscillation, transit time, junction capacitance, and cut-off frequency.

## 4.1 Gunn Diode

The Gunn diode is a transferred-electron device (TED) that exhibits negative differential resistance due to the transfer of electrons from a high-mobility lower energy valley to a low-mobility higher energy valley.

**Methodology:**

Gunn Diode Calculations The operation of a Gunn diode in transit-time mode is determined by the length of the active region and the electron drift velocity.

1. **Electric Field ($E$):** The applied electric field across the active region is given by:

$$E = \frac{V}{L}$$

where $V$ is the applied voltage and $L$ is the length of the active region.

2. **Threshold Voltage ($V_{th}$):** The minimum voltage required for the onset of negative resistance is:

$$V_{th} = E_{th} \times L$$

where $E_{th}$ is the threshold electric field (typically around 3300 V/cm for GaAs).

3. **Transit Time ($\tau_t$):** The time taken by an electron domain to travel across the active region:

$$\tau_t = \frac{L}{v_d}$$

where $v_d$ is the drift velocity of electrons (typically around $10^7$ cm/s for GaAs).

4. **Frequency of Oscillation ($f$):** The natural frequency of oscillation in transit-time mode is inversely proportional to the transit time:

$$f = \frac{1}{\tau_t} = \frac{v_d}{L}$$

**Worked Example:**

Gunn Diode Oscillation Frequency An n-type GaAs Gunn diode has an active region length of 10 $\mu$m. If the electron drift velocity is $10^5$ m/s and the threshold electric field is 3.3 kV/cm, calculate the transit time, the natural frequency of oscillation, and the threshold voltage.

**Solution:**

1. **Identify the given parameters:** Length $L = 10\ \mu\text{m} = 10 \times 10^{-6}$ m $= 10^{-3}$ cm. Drift velocity $v_d = 10^5$ m/s $= 10^7$ cm/s. Threshold electric field $E_{th} = 3.3$ kV/cm $= 3300$ V/cm.

2. **Calculate the transit time ($\tau_t$):**

$$\tau_t = \frac{L}{v_d} = \frac{10 \times 10^{-6}\text{ m}}{10^5\text{ m/s}} = 10^{-10}\text{ s} = 100\text{ ps}$$

3. **Calculate the frequency of oscillation ($f$):**

$$f = \frac{1}{\tau_t} = \frac{1}{10^{-10}\text{ s}} = 10 \times 10^9\text{ Hz} = 10\text{ GHz}$$

4. **Calculate the threshold voltage ($V_{th}$):**

$$V_{th} = E_{th} \times L = 3300\text{ V/cm} \times 10^{-3}\text{ cm} = 3.3\text{ V}$$

## 4.2 IMPATT Diode

The IMPATT (Impact Ionization Avalanche Transit Time) diode is a high-power microwave generator that uses avalanche breakdown and carrier transit time delay to create negative resistance.

**Methodology:**

IMPATT Diode Calculations The operating frequency of an IMPATT diode depends on the drift transit time through the depletion region. For maximum negative resistance, the total phase delay across the diode must be approximately 180° (or $\pi$ radians), which consists of a 90° delay from the avalanche process and a 90° delay from the transit time.

1. **Drift Time ($\tau_d$):** The time required for carriers to drift across the depletion region of length $L$:

$$\tau_d = \frac{L}{v_d}$$

where $v_d$ is the saturated drift velocity.

2. **Transit Angle ($\theta$):** The phase delay introduced by the drift region at an angular frequency $\omega$:

$$\theta = \omega\tau_d = 2\pi f \frac{L}{v_d}$$

3. **Optimum Operating Frequency ($f$):** For a $\pi/2$ (or 90°) transit angle delay to achieve total $\pi$ delay:

$$\omega\tau_d = \pi \implies 2\pi f \frac{L}{v_d} = \pi \implies f = \frac{v_d}{2L}$$

4. **Avalanche Breakdown Voltage ($V_b$):** Approximate breakdown voltage for an abrupt p-n junction:

$$V_b = \frac{\epsilon E_m^2}{2qN_d}$$

where $E_m$ is the maximum electric field, $\epsilon$ is the permittivity, $q$ is electron charge, and $N_d$ is doping concentration.

**Worked Example:**

IMPATT Diode Operating Frequency An IMPATT diode has a drift length of 2 $\mu$m. The drift velocity of the carriers is $10^7$ cm/s. Determine the optimum operating frequency of the diode and the drift time.
**Solution:**

1. **Identify the given parameters:** Drift length $L = 2\ \mu\text{m} = 2\times10^{-4}$ cm. Drift velocity $v_d = 10^7$ cm/s.
2. **Calculate the drift time ($\tau_d$):**
$$\tau_d = \frac{L}{v_d} = \frac{2\times10^{-4}\text{ cm}}{10^7\text{ cm/s}} = 2\times10^{-11}\text{ s} = 20\text{ ps}$$
3. **Calculate the optimum operating frequency ($f$):**
$$f = \frac{v_d}{2L} = \frac{10^7}{2\times2\times10^{-4}} = \frac{10^7}{4\times10^{-4}} = 0.25\times10^{11}\text{ Hz}$$
$$f = 25\times10^9\text{ Hz} = 25\text{ GHz}$$

## 4.3 Varactor Diode

A varactor diode operates as a voltage-variable capacitor, widely used in parametric amplifiers, frequency multipliers, and voltage-controlled oscillators.

**Methodology:**

Varactor Diode Parameters The operation relies on the variation of the depletion layer capacitance with the applied reverse bias voltage.

1. **Junction Capacitance ($C_j$):** The capacitance at a reverse bias voltage $V_R$ is given by:
$$C_j = \frac{C_0}{\left(1+\frac{V_R}{V_\phi}\right)^n}$$
where: $C_0$ = junction capacitance at zero bias ($V_R = 0$) $V_R$ = applied reverse bias voltage (positive value) $V_\phi$ = built-in potential or contact potential (typically 0.7 V for Silicon) $n$ = doping gradient factor (1/2 for abrupt junctions, 1/3 for linearly graded junctions).
2. **Cut-off Frequency ($f_c$):** The maximum frequency up to which the diode acts as a useful capacitor, limited by the series resistance $R_s$:
$$f_c = \frac{1}{2\pi R_s C_j}$$
3. **Quality Factor ($Q$):** At a given operating frequency $f$:
$$Q = \frac{f_c}{f} = \frac{1}{2\pi f R_s C_j}$$

**Worked Example:**

Varactor Diode Capacitance and Cutoff Frequency A silicon varactor diode has a zero-bias capacitance of 20 pF and a series resistance of 1.5 $\Omega$. The built-in potential is 0.7 V, and the diode has an abrupt junction. Calculate the junction capacitance, cut-off frequency, and quality factor at an operating frequency of 1 GHz for a reverse bias voltage of 5 V.
**Solution:**

1. **Identify the given parameters:** $C_0 = 20$ pF, $R_s = 1.5\ \Omega$, $V_\phi = 0.7$ V. $V_R = 5$ V, $f = 1$ GHz =

$10^9$ Hz. Abrupt junction implies $n = 1/2$.

2. **Calculate the junction capacitance ($C_j$):**

$$C_j = \frac{20 \text{ pF}}{\left(1 + \frac{5}{0.7}\right)^{1/2}} = \frac{20}{(1 + 7.14)^{1/2}} = \frac{20}{\sqrt{8.14}} = \frac{20}{2.853} \approx 7.01 \text{ pF}$$

3. **Calculate the cut-off frequency ($f_c$):**

$$f_c = \frac{1}{2\pi R_s C_j} = \frac{1}{2\pi \times 1.5 \times 7.01 \times 10^{-12}}$$

$$f_c = \frac{1}{66.07 \times 10^{-12}} \approx 15.14 \times 10^9 \text{ Hz} = 15.14 \text{ GHz}$$

4. **Calculate the quality factor ($Q$):**

$$Q = \frac{f_c}{f} = \frac{15.14 \text{ GHz}}{1 \text{ GHz}} = 15.14$$

## 4.4 PIN Diode

A PIN diode consists of an intrinsic (i) layer sandwiched between heavily doped p-type and n-type layers. It acts as a variable resistor at microwave frequencies, controlled by DC bias current.

**Methodology:**

PIN Diode Equivalent Circuit At microwave frequencies, the PIN diode is modeled based on its forward and reverse bias states.

1. **Reverse Bias Capacitance ($C_j$):** Under reverse bias, the intrinsic region is fully depleted, acting as a dielectric.

$$C_j = \frac{\epsilon A}{W}$$

where $\epsilon$ is the permittivity, $A$ is the junction area, and $W$ is the intrinsic region width.

2. **Forward Bias RF Resistance ($R_f$):** Under forward bias, carriers are injected into the intrinsic region, reducing its resistance.

$$R_f = \frac{W^2}{2\mu I_0 \tau} = \frac{W}{2\mu Q} \propto \frac{1}{I_0}$$

where $I_0$ is the DC forward current, $\tau$ is carrier lifetime, $\mu$ is effective mobility, and $Q = I_0\tau$ is the stored charge.

3. **Lower Frequency Limit ($f_{min}$):** For the PIN diode to behave as a pure resistor (rather than a rectifier), the RF period must be much smaller than the carrier lifetime.

$$f_{min} = \frac{1}{2\pi\tau}$$

**Worked Example:**

PIN Diode RF Resistance A PIN diode has an intrinsic region width of 20 $\mu$m, an effective carrier lifetime of 1.5 $\mu$s, and an effective mobility of 600 $\text{cm}^2/\text{V}\cdot\text{s}$. Determine the lower frequency limit and the forward RF resistance when a DC current of 50 mA is applied.

**Solution:**

1. **Identify the given parameters:** Width $W = 20\ \mu\text{m} = 20 \times 10^{-4}$ cm. Lifetime $\tau = 1.5\ \mu\text{s} =$

$1.5 \times 10^{-6}$ s. Mobility $\mu = 600$ cm$^2$/V $\cdot$ s. DC current $I_0 = 50$ mA $= 50 \times 10^{-3}$ A.

2. **Calculate the lower frequency limit ($f_{min}$):**

$$f_{min} = \frac{1}{2\pi\tau} = \frac{1}{2\pi \times 1.5 \times 10^{-6}} = \frac{10^6}{9.42} \approx 106.1 \text{ kHz}$$

The diode must be operated at frequencies well above 106.1 kHz to act as an RF resistor.

3. **Calculate the forward RF resistance ($R_f$):**

$$R_f = \frac{W^2}{2\mu I_0 \tau}$$

$$W^2 = (20 \times 10^{-4})^2 = 4 \times 10^{-6} \text{ cm}^2$$

$$R_f = \frac{4 \times 10^{-6}}{2 \times 600 \times 50 \times 10^{-3} \times 1.5 \times 10^{-6}}$$

$$R_f = \frac{4 \times 10^{-6}}{90 \times 10^{-6}} = \frac{4}{90} \approx 0.044 \ \Omega$$

## 4.5 Tunnel Diode

The Tunnel Diode utilizes quantum mechanical tunneling to provide a negative resistance region, useful for microwave oscillators and amplifiers.

**Methodology:**

Tunnel Diode Parameters The tunnel diode is characterized by its high doping levels which lead to a very narrow depletion region, enabling tunneling.

1. **Negative Resistance ($-R_n$):** The value of the dynamic negative resistance is found from the inverse of the slope of the $V - I$ curve in the negative differential resistance region:

$$R_n = -\frac{\Delta V}{\Delta I} = -\frac{V_v - V_p}{I_v - I_p}$$

where $(V_p, I_p)$ are the peak voltage and current, and $(V_v, I_v)$ are the valley voltage and current.

2. **Resistive Cut-off Frequency ($f_r$):** The frequency at which the real part of the diode impedance becomes zero. Above this frequency, the diode no longer exhibits negative resistance.

$$f_r = \frac{1}{2\pi |R_n| C_j} \sqrt{\frac{|R_n|}{R_s} - 1}$$

where $R_s$ is the series resistance and $C_j$ is the junction capacitance.

3. **Self-Resonant Frequency ($f_x$):** The frequency at which the imaginary part of the diode impedance becomes zero due to lead inductance $L_s$.

$$f_x = \frac{1}{2\pi} \sqrt{\frac{1}{L_s C_j} - \left(\frac{1}{|R_n| C_j}\right)^2}$$

**Worked Example:**

Tunnel Diode Cutoff Frequencies A tunnel diode has a negative resistance of $-30\ \Omega$, a series resistance of $2\ \Omega$, a junction capacitance of 1.5 pF, and a series inductance of 1 nH. Calculate its resistive cut-off

frequency and self-resonant frequency.
**Solution:**

1. **Identify the given parameters:** $|R_n| = 30\ \Omega$, $R_s = 2\ \Omega$. $C_j = 1.5$ pF $= 1.5 \times 10^{-12}$ F. $L_s = 1$ nH $= 10^{-9}$ H.

2. **Calculate the resistive cut-off frequency ($f_r$):**

$$f_r = \frac{1}{2\pi |R_n| C_j} \sqrt{\frac{|R_n|}{R_s} - 1}$$

$$f_r = \frac{1}{2\pi \times 30 \times 1.5 \times 10^{-12}} \sqrt{\frac{30}{2} - 1}$$

$$f_r = \frac{1}{282.74 \times 10^{-12}} \sqrt{15 - 1} = \frac{\sqrt{14}}{282.74 \times 10^{-12}} \approx \frac{3.742}{282.74 \times 10^{-12}}$$

$$f_r \approx 13.23 \times 10^9 \text{ Hz} = 13.23 \text{ GHz}$$

3. **Calculate the self-resonant frequency ($f_x$):**

$$f_x = \frac{1}{2\pi} \sqrt{\frac{1}{L_s C_j} - \left(\frac{1}{|R_n| C_j}\right)^2}$$

First, find the individual terms:

$$\frac{1}{L_s C_j} = \frac{1}{10^{-9} \times 1.5 \times 10^{-12}} = \frac{1}{1.5 \times 10^{-21}} = 6.667 \times 10^{20}$$

$$\frac{1}{|R_n| C_j} = \frac{1}{30 \times 1.5 \times 10^{-12}} = \frac{1}{45 \times 10^{-12}} = 2.22 \times 10^{10}$$

$$\left(\frac{1}{|R_n| C_j}\right)^2 = (2.22 \times 10^{10})^2 = 4.938 \times 10^{20}$$

Now, substitute these back:

$$f_x = \frac{1}{2\pi} \sqrt{6.667 \times 10^{20} - 4.938 \times 10^{20}} = \frac{1}{2\pi} \sqrt{1.729 \times 10^{20}}$$

$$f_x = \frac{1}{2\pi} \times 1.315 \times 10^{10} \approx 2.09 \times 10^9 \text{ Hz} = 2.09 \text{ GHz}$$

## CHAPTER 5

# RADAR Systems

## 5.1 The Radar Range Equation

The Radar Range Equation is the fundamental mathematical model used to determine the maximum distance at which a radar system can detect a target. It relates the maximum range to the radar's transmitter power, antenna gain, operating wavelength, target size (radar cross-section), and receiver sensitivity.

**Methodology:**

Radar Range Equation To calculate the maximum range or minimum detectable signal, follow these formulas:

1. **Standard Form:** The maximum detection range $R_{max}$ is given by:

$$R_{max} = \left[ \frac{P_t G^2 \lambda^2 \sigma}{(4\pi)^3 S_{min}} \right]^{1/4}$$

   Where:

   - $P_t$ = Transmitted peak power (Watts)
   - $G$ = Antenna gain (dimensionless)
   - $\lambda$ = Operating wavelength (meters)
   - $\sigma$ = Radar cross-section of the target (square meters)
   - $S_{min}$ = Minimum detectable signal of the receiver (Watts)

2. **Using Effective Aperture:** If the effective area of the antenna $A_e$ is given instead of gain ($G = \frac{4\pi A_e}{\lambda^2}$):

$$R_{max} = \left[ \frac{P_t A_e^2 \sigma}{4\pi \lambda^2 S_{min}} \right]^{1/4}$$

3. **Decibel Conversion:** Often, power and gain are given in decibels (dB or dBW). Convert them to linear scale before plugging them into the equation:

$$\text{Linear Value} = 10^{\frac{\text{Value in dB}}{10}}$$

**Worked Example:**

Calculating Maximum Radar Range A radar system operates at a frequency of 3 GHz with a peak transmitted power of 500 kW. The antenna has a gain of 30 dB. The receiver's minimum detectable signal is $10^{-12}$ W. Calculate the maximum range of the radar for a target with a radar cross-section of 5 $\text{m}^2$.

**Step 1: Identify given parameters and convert units.**

- Frequency $f = 3$ GHz $= 3 \times 10^9$ Hz
- Peak power $P_t = 500$ kW $= 5 \times 10^5$ W

- Antenna gain in dB $G_{dB} = 30$ dB $\implies G = 10^{30/10} = 10^3 = 1000$
- Minimum detectable signal $S_{min} = 10^{-12}$ W
- Target cross-section $\sigma = 5$ m$^2$

**Step 2: Calculate the wavelength $\lambda$.**

$$\lambda = \frac{c}{f} = \frac{3 \times 10^8}{3 \times 10^9} = 0.1 \text{ m}$$

**Step 3: Apply the Radar Range Equation.**

$$R_{max} = \left[ \frac{P_t G^2 \lambda^2 \sigma}{(4\pi)^3 S_{min}} \right]^{1/4}$$

Substitute the values:

$$R_{max} = \left[ \frac{(5 \times 10^5) \times (1000)^2 \times (0.1)^2 \times 5}{(4\pi)^3 \times 10^{-12}} \right]^{1/4}$$

$$R_{max} = \left[ \frac{(5 \times 10^5) \times 10^6 \times 0.01 \times 5}{1984.4 \times 10^{-12}} \right]^{1/4}$$

$$R_{max} = \left[ \frac{25 \times 10^9}{1984.4 \times 10^{-12}} \right]^{1/4}$$

$$R_{max} = \left[ 1.259 \times 10^{19} \right]^{1/4} \approx 59.7 \times 10^3 \text{ m}$$

**Step 4: State the final answer.** The maximum range of the radar is 59.7 km.

## 5.2 Pulsed Radar Characteristics

Pulsed radar systems transmit short bursts (pulses) of electromagnetic energy and listen for echoes during the rest time. Understanding the timing and power characteristics is essential for analyzing range and resolution.

**Methodology:**

Pulsed Radar Parameters To calculate parameters for a pulsed radar, use the following formulas:

1. **Pulse Repetition Time (PRT) and Frequency (PRF):**

$$T = \frac{1}{f_r}$$

   Where $T$ is the PRT (seconds) and $f_r$ is the PRF (Hz or pulses per second).

2. **Duty Cycle (D):** The ratio of active transmission time to the total time period.

$$D = \tau \times f_r = \frac{\tau}{T}$$

   Where $\tau$ is the pulse width (seconds).

3. **Average Power ($P_{avg}$):**

$$P_{avg} = P_t \times D = P_t \times \tau \times f_r$$

   Where $P_t$ is the peak power.

4. **Maximum Unambiguous Range ($R_{un}$):** The maximum range a target can be located without the echo overlapping into the next pulse period.

$$R_{un} = \frac{c \times T}{2} = \frac{c}{2 f_r}$$

Where $c = 3 \times 10^8$ m/s (speed of light).

5. **Range Resolution ($\Delta R$):** The ability of the radar to distinguish between two closely spaced targets on the same line of bearing.
$$\Delta R = \frac{c \times \tau}{2}$$

**Worked Example:**

Calculating Pulsed Radar Parameters A pulsed radar operates with a peak power of 100 kW, a pulse width of 2$\mu$s, and a Pulse Repetition Frequency (PRF) of 1000 Hz. Determine the duty cycle, average power, maximum unambiguous range, and range resolution.

**Step 1: Identify given parameters.**

- Peak power $P_t = 100$ kW $= 10^5$ W
- Pulse width $\tau = 2\mu\text{s} = 2 \times 10^{-6}$ s
- PRF $f_r = 1000$ Hz

**Step 2: Calculate the Duty Cycle ($D$).**

$$D = \tau \times f_r = (2 \times 10^{-6}) \times 1000 = 2 \times 10^{-3} = 0.002$$

**Step 3: Calculate the Average Power ($P_{avg}$).**

$$P_{avg} = P_t \times D = (10^5) \times 0.002 = 200 \text{ W}$$

**Step 4: Calculate the Maximum Unambiguous Range ($R_{un}$).**

$$R_{un} = \frac{c}{2f_r} = \frac{3 \times 10^8}{2 \times 1000} = 1.5 \times 10^5 \text{ m} = 150 \text{ km}$$

**Step 5: Calculate the Range Resolution ($\Delta R$).**

$$\Delta R = \frac{c \times \tau}{2} = \frac{(3 \times 10^8) \times (2 \times 10^{-6})}{2} = 300 \text{ m}$$

**Final Answers:** Duty cycle is 0.002 (0.2%), average power is 200 W, unambiguous range is 150 km, and range resolution is 300 m.

## 5.3 Continuous Wave (CW) and Doppler Radar

Continuous Wave radars transmit a continuous signal and rely on the Doppler effect to measure the radial velocity of a moving target.

**Methodology:**

Doppler Frequency Shift The Doppler frequency shift ($f_d$) is the difference between the received frequency and the transmitted frequency, caused by target motion.

1. **Doppler Frequency Formula:**
$$f_d = \frac{2v_r f_0}{c} = \frac{2v_r}{\lambda}$$
Where:
   - $f_d$ = Doppler shift frequency (Hz)
   - $v_r$ = Relative radial velocity of the target (m/s). (Approaching targets have positive velocity, receding have negative).

- $f_0$ = Transmitted frequency (Hz)
- $\lambda$ = Wavelength of the transmitted signal (m)
- $c$ = Speed of light ($3 \times 10^8$ m/s)

2. **Target Velocity Calculation:** Rearranging to solve for velocity:

$$v_r = \frac{f_d \lambda}{2}$$

**Worked Example:**

Determining Target Velocity from Doppler Shift A CW radar operating at a frequency of 10 GHz measures a Doppler frequency shift of 2.5 kHz from a target. Calculate the radial velocity of the target in m/s and km/h.

**Step 1: Identify given parameters.**

- Transmitted frequency $f_0 = 10$ GHz $= 10^{10}$ Hz
- Doppler shift $f_d = 2.5$ kHz $= 2500$ Hz

**Step 2: Calculate the wavelength ($\lambda$).**

$$\lambda = \frac{c}{f_0} = \frac{3 \times 10^8}{10^{10}} = 0.03 \text{ m}$$

**Step 3: Calculate the radial velocity in m/s.**

$$v_r = \frac{f_d \lambda}{2} = \frac{2500 \times 0.03}{2} = \frac{75}{2} = 37.5 \text{ m/s}$$

**Step 4: Convert velocity to km/h.**

$$v_{km/h} = v_r \times \frac{3600}{1000} = 37.5 \times 3.6 = 135 \text{ km/h}$$

**Final Answer:** The radial velocity of the target is 37.5 m/s or 135 km/h.

## 5.4 MTI Radar and Blind Speeds

Moving Target Indicator (MTI) radars utilize the Doppler shift to distinguish moving targets from stationary clutter. Since MTI radars are typically pulsed, they suffer from a phenomenon known as "blind speeds", where a moving target produces a Doppler shift exactly equal to a multiple of the Pulse Repetition Frequency (PRF), causing it to appear stationary.

**Methodology:**

MTI Blind Speeds To find the blind speeds of an MTI radar system:

1. **Condition for Blind Speed:** A target is "blind" when the phase shift between consecutive pulses is a multiple of $2\pi$. This occurs when the Doppler frequency equals a multiple of the PRF:

$$f_d = n \cdot f_r$$

Where $n = 1, 2, 3, \ldots$

2. **Blind Speed Formula:** Substituting the Doppler equation ($f_d = 2v_n/\lambda$), the $n$-th blind speed $v_n$ is:

$$v_n = \frac{n\lambda f_r}{2} = \frac{n\lambda}{2T}$$

Where:

- $v_n$ = $n$-th blind speed (m/s)
- $n$ = Integer (1 for the first blind speed, 2 for the second, etc.)
- $\lambda$ = Operating wavelength (m)
- $f_r$ = Pulse Repetition Frequency (Hz)
- $T$ = Pulse Repetition Time (s)

**Worked Example:**

Calculating MTI Blind Speeds An MTI radar operates at 5 GHz with a Pulse Repetition Frequency (PRF) of 800 Hz. Determine the first and second blind speeds in km/h.

**Step 1: Identify given parameters.**

- Operating frequency $f = 5\text{ GHz} = 5 \times 10^9\text{ Hz}$
- PRF $f_r = 800\text{ Hz}$

**Step 2: Calculate the wavelength ($\lambda$).**

$$\lambda = \frac{c}{f} = \frac{3 \times 10^8}{5 \times 10^9} = 0.06\text{ m}$$

**Step 3: Calculate the first blind speed ($n = 1$) in m/s.**

$$v_1 = \frac{1 \cdot \lambda \cdot f_r}{2} = \frac{0.06 \times 800}{2} = 24\text{ m/s}$$

**Step 4: Convert the first blind speed to km/h.**

$$v_{1(km/h)} = 24 \times 3.6 = 86.4\text{ km/h}$$

**Step 5: Calculate the second blind speed ($n = 2$).** Since $v_n = n \cdot v_1$:

$$v_2 = 2 \times v_1 = 2 \times 86.4 = 172.8\text{ km/h}$$

**Final Answer:** The first blind speed is 86.4 km/h and the second blind speed is 172.8 km/h.

# Formulae

# Appendix: Key Formulae

This appendix provides a quick reference to the key formulae and mathematical relationships discussed in this book.

## Unit 1: Transmission Lines and Basics

### Power and Decibels

- **Power in dBm:** $P(\text{dBm}) = 10\log_{10}\left(\frac{P(\text{mW})}{1\text{ mW}}\right)$
- **Power from dBm:** $P(\text{mW}) = 10^{\frac{P(\text{dBm})}{10}}$
- **Gain/Attenuation in dB:** $\text{Gain (dB)} = 10\log_{10}\left(\frac{P_{out}}{P_{in}}\right)$

### Transmission Line Parameters

- **Propagation Constant ($\gamma$):** $\gamma = \sqrt{ZY} = \sqrt{(R + j\omega L)(G + j\omega C)}$
- **Characteristic Impedance ($Z_0$):** $Z_0 = \sqrt{\frac{Z}{Y}} = \sqrt{\frac{R+j\omega L}{G+j\omega C}}$
- **Lossless Characteristic Impedance:** $Z_0 = \sqrt{\frac{L}{C}}$
- **Phase Velocity ($v_p$):** $v_p = \frac{\omega}{\beta}$
- **Wavelength ($\lambda$):** $\lambda = \frac{v_p}{f} = \frac{2\pi}{\beta}$

### Reflection and Impedance

- **Reflection Coefficient ($\Gamma$):** $\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0}$
- **Voltage Standing Wave Ratio (VSWR):** $\text{VSWR} = \frac{1+|\Gamma|}{1-|\Gamma|}$
- **Return Loss:** $\text{RL (dB)} = -20\log_{10}|\Gamma|$
- **Input Impedance ($Z_{in}$):** $Z_{in} = Z_0\frac{Z_L + jZ_0\tan(\beta l)}{Z_0 + jZ_L\tan(\beta l)}$
- **Quarter-Wave Transformer:** $Z_{in} = \frac{Z_0^2}{Z_L} \implies Z_{0,qw} = \sqrt{Z_{in}Z_L}$

## Unit 2: Microwave Components and Waveguides

### Scattering Parameters (S-Parameters)

- **Return Loss (RL):** $\text{RL} = -20\log_{10}|S_{ii}|\text{ dB}$
- **Insertion Loss (IL):** $\text{IL} = -20\log_{10}|S_{ij}|\text{ dB}$
- **Unitary Property (Lossless Network):** $\sum_{k=1}^{N}|S_{ki}|^2 = 1 \quad \text{for all columns } i$

## Directional Couplers

- **Coupling Factor (C):** $C = 10\log_{10}\left(\frac{P_1}{P_3}\right)$ dB
- **Directivity (D):** $D = 10\log_{10}\left(\frac{P_3}{P_4}\right)$ dB
- **Isolation (I):** $I = 10\log_{10}\left(\frac{P_1}{P_4}\right)$ dB

## Magic Tee

- **Scattering Matrix:** $[S] = \frac{1}{\sqrt{2}}\begin{bmatrix} 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & -1 \\ 1 & 1 & 0 & 0 \\ 1 & -1 & 0 & 0 \end{bmatrix}$
- **Port Relationships:** $b_3 = \frac{1}{\sqrt{2}}(a_1 + a_2)$, $b_4 = \frac{1}{\sqrt{2}}(a_1 - a_2)$

## Waveguides

- **Cutoff Wavelength (Rectangular):** $\lambda_c = \frac{2}{\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}}$
- **Cutoff Frequency:** $f_c = \frac{c}{\lambda_c}$
- **Guide Wavelength ($\lambda_g$):** $\lambda_g = \frac{\lambda_0}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}}$ (Derived from $\frac{1}{\lambda_g^2} = \frac{1}{\lambda_0^2} - \frac{1}{\lambda_c^2}$)
- **Phase Velocity ($v_p$):** $v_p = \frac{c}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}}$
- **Group Velocity ($v_g$):** $v_g = c\sqrt{1-\left(\frac{f_c}{f}\right)^2}$ (Note: $v_p v_g = c^2$)
- **Wave Impedance (TE Mode):** $Z_{TE} = \frac{\eta}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}}$
- **Wave Impedance (TM Mode):** $Z_{TM} = \eta\sqrt{1-\left(\frac{f_c}{f}\right)^2}$

# Unit 3: Microwave Tubes and Measurements

## Klystrons

- **Electron Velocity ($v_0$):** $v_0 = \sqrt{\frac{2eV_0}{m}} \approx 0.593 \times 10^6\sqrt{V_0}$ m/s
- **Gap Transit Angle ($\theta_g$):** $\theta_g = \omega\tau = \frac{\omega d}{v_0} = 2\pi f\tau$
- **Beam Coupling Coefficient ($\beta_i$):** $\beta_i = \frac{\sin(\theta_g/2)}{\theta_g/2}$
- **Bunching Parameter ($X$):** $X = \frac{\beta_i V_1}{2V_0}\theta_0 = \frac{\beta_i V_1}{2V_0}\left(\frac{\omega L}{v_0}\right)$
- **Reflex Klystron Transit Time ($T$):** $T = \frac{n+3/4}{f} = \left(n + \frac{3}{4}\right)T_{rf}$
- **Reflex Klystron Repeller Voltage ($V_R$):** $\frac{V_0+|V_R|}{L} = \frac{4V_0 f}{(n+3/4)v_0}$

## Magnetrons

- **Cyclotron Frequency ($f_c$):** $f_c = \frac{eB}{2\pi m}$
- **Hull Cutoff Magnetic Field ($B_c$):** $B_c = \frac{\sqrt{8V_0 \frac{m}{e}}}{b\left(1-\frac{a^2}{b^2}\right)}$
- **Hull Cutoff Voltage ($V_c$):** $V_c = \frac{eB^2b^2}{8m}\left(1 - \frac{a^2}{b^2}\right)^2$

## Traveling Wave Tubes (TWT)

- **Gain Parameter ($C$):** $C = \left(\frac{I_0 Z_0}{4V_0}\right)^{1/3}$
- **Number of Wavelengths ($N$):** $N = \frac{L}{\lambda_e} = \frac{fL}{v_0}$
- **Power Gain ($A_p$):** $A_p\ (\text{dB}) = -9.54 + 47.3NC$

## Microwave Measurements

- **VSWR from Shift:** $S = \frac{\lambda_g}{\pi \Delta x}$
- **Cavity Quality Factor ($Q$):** $Q = \frac{f_0}{\Delta f}$
- **Cavity Resonance Frequency (Rectangular):** $f_r = \frac{c}{2}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2 + \left(\frac{p}{d}\right)^2}$

# Unit 4: Microwave Solid-State Devices

## Tunnel Diodes

- **Negative Resistance ($R_n$):** $R_n = -\frac{\Delta V}{\Delta I} = -\frac{V_v - V_p}{I_v - I_p}$
- **Resistive Cutoff Frequency ($f_r$):** $f_r = \frac{1}{2\pi |R_n| C_j}\sqrt{\frac{|R_n|}{R_s} - 1}$
- **Self-Resonance Frequency ($f_x$):** $f_x = \frac{1}{2\pi}\sqrt{\frac{1}{L_s C_j} - \left(\frac{1}{|R_n| C_j}\right)^2}$

## IMPATT and Read Diodes

- **Drift Time ($\tau_d$):** $\tau_d = \frac{L}{v_d}$
- **Drift Angle ($\theta$):** $\theta = \omega \tau_d = 2\pi f \frac{L}{v_d}$
- **Operating Frequency ($f$):** $f = \frac{v_d}{2L}$
- **Avalanche Zone Resistance ($R_f$):** $R_f = \frac{W^2}{2\mu I_0 \tau}$
- **Breakdown Voltage ($V_b$):** $V_b = \frac{\epsilon E_m^2}{2qN_d}$

## Device Capacitance

- **Junction Capacitance ($C_j$):** $C_j = \frac{\epsilon A}{W}$ or $C_j = \frac{C_0}{\left(1+\frac{V_R}{V_\phi}\right)^n}$

# Unit 5: Radar Systems

## Radar Range Equation

- **Maximum Range ($R_{max}$):** $R_{max} = \left[\frac{P_t G^2 \lambda^2 \sigma}{(4\pi)^3 S_{min}}\right]^{1/4}$ or $R_{max} = \left[\frac{P_t A_e^2 \sigma}{4\pi \lambda^2 S_{min}}\right]^{1/4}$
- **Received Power Density ($P_d$):** $P_d = \frac{P_t G}{4\pi R^2}$

## Radar Timing and Power

- **Pulse Repetition Time ($T$):** $T = \frac{1}{f_r}$
- **Duty Cycle ($D$):** $D = \tau \times f_r = \frac{\tau}{T}$
- **Average Power ($P_{avg}$):** $P_{avg} = P_t \times D = P_t \times \tau \times f_r$

## Radar Range Performance

- **Unambiguous Range ($R_{un}$):** $R_{un} = \frac{c \times T}{2} = \frac{c}{2 f_r}$
- **Range Resolution ($\Delta R$):** $\Delta R = \frac{c \times \tau}{2}$

## Doppler Radar

- **Doppler Frequency Shift ($f_d$):** $f_d = \frac{2 v_r f_0}{c} = \frac{2 v_r}{\lambda}$
- **Radial Velocity ($v_r$):** $v_r = \frac{f_d \lambda}{2}$
- **Blind Speeds ($v_n$):** $v_n = \frac{n \lambda f_r}{2} = \frac{n\lambda}{2T}$ (for $n = 1, 2, 3, \dots$)