# Awp (4341106) - Problem Solver

Antigravity AI

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**Awp**

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*To my students,*
*who constantly inspire me to find new analogies,*
*break down complex theories, and make learning*
*an engineered science.*

# Contents

# Preface

### About this Book

Welcome to **Computer Networks**! This textbook is meticulously designed to serve as a comprehensive problem solver and study companion. It provides a robust and intuitive foundation in the core principles of this subject, bridging the gap between theoretical models and practical applications. A unique feature of this book is its focus on decoding complex concepts using clear, step-by-step methodologies and real-world analogies.

### Target Audience

This book is intended for students and professionals looking to master the concepts of Computer Networks. Whether you are revising for exams, seeking to solidify your fundamental understanding, or looking for practical examples to clarify abstract concepts, this book has been structured to support your learning journey. It serves as an accessible guide designed to remove the fear of complex topics, replacing it with an appreciation for the underlying mechanics.

### Scope and Coverage

The coverage is divided logically to follow standard academic curriculums. Each chapter focuses on key topics, breaking them down into digestible sections:

- **Theoretical Insights** – Core concepts explained intuitively.
- **Method Blocks** – Standardized, step-by-step procedures for solving specific types of problems.
- **Worked Examples** – Fully solved problems that apply the methods in practice.

We hope this resource proves invaluable in your studies and helps you achieve excellence in **Computer Networks**. Happy learning!

# Acknowledgments

Writing a comprehensive book that connects the fundamental forces of Chemistry with practical engineering applications requires more than just academic knowledge—it requires a community of support. I would like to express my sincere gratitude to everyone who contributed to the realization of this project.

Specially, I extend my heartfelt gratitude to the **Department of Technical Education (DTE), Government of Gujarat**, and **Government Polytechnic, Palanpur**, for providing an environment that fosters innovation, academic rigor, and continuous professional development.

I am deeply thankful to my family for their unwavering patience and support during the countless hours spent researching, formatting, and refining these chapters.

Finally, my deepest appreciation goes to my students. Your curiosity, challenging questions, and continuous feedback are the true driving force behind the unique analogies and streamlined structure of this book. This textbook was engineered for you.

**Antigravity AI**

# About the Author

**Milav Jayeshkumar Dabgar** is an engineering educator and R&D professional with over 9 years of experience in electronics hardware development, embedded systems, AI/ML, and full-stack software engineering. He currently serves as a **Lecturer (Class - II) & Innovation Leader** at Government Polytechnic, Palanpur, under the Education Department of the Government of Gujarat.

Milav holds a Master of Engineering (ME) in Communication Systems from L.D. College of Engineering and a Bachelor of Engineering (BE) in Electronics & Communication. He is also currently pursuing a **BS in Data Science and Applications from IIT Madras**, where he has completed both the **Diploma in Programming** and the **Diploma in Data Science** with distinction.

Most recently, he completed the **AICTE QIP PG Certificate Program in Deep Learning from SVNIT Surat** with a **perfect 10/10 CGPA**, topping his batch.

A dedicated mentor, Milav has guided numerous student teams in securing over **INR 45 lakhs** in innovation funding, including INR 25 lakhs from Shark Tank India and INR 20 lakhs through iHub Gujarat, resulting in two student patents. He is recognized as an **NPTEL Evangelist** and **NPTEL Discipline Star** in Computer Science, reflecting his commitment to continuous learning and academic excellence.

His technical expertise spans from embedded systems (8051, STM32, Arduino) to modern web technologies (Next.js, Python, PostgreSQL) and Data Science (TensorFlow, PyTorch). Through this textbook, he aims to simplify complex Engineering Chemistry concepts by integrating relatable, real-world analogies drawn from modern tech and engineering landscapes.

Milav is also an active content creator, running a popular **YouTube channel** (@milav.dabgar) where he shares technical tutorials and academic resources. He maintains a personal blog and resource hub at milav.in and can be reached for professional collaborations on LinkedIn.

# CHAPTER 1

# Basics of Electromagnetic and Antenna

## 1.1 Fundamental Wave Parameters

**Methodology:**

Calculating Wavelength Frequency and Velocity The relationship between the velocity of propagation ($v$), frequency ($f$) and wavelength ($\lambda$) of an electromagnetic wave is fundamental in antenna engineering.

1. **Wave Velocity ($v$):** In free space the velocity is the speed of light $c = 3 \times 10^8$ m/s. In a dielectric medium $v = \frac{c}{\sqrt{\epsilon_r}}$ where $\epsilon_r$ is the relative permittivity of the medium.
2. **Wavelength ($\lambda$):** Calculated using the formula $\lambda = \frac{v}{f}$.
3. **Frequency ($f$):** Calculated using the formula $f = \frac{v}{\lambda}$.

**Worked Example:**

Determine Wavelength of an RF Signal An antenna transmits a radio frequency signal at 900 MHz in free space. Calculate the physical wavelength of the signal.
**Step 1: Identify the given parameters.** Frequency $f = 900$ MHz $= 900 \times 10^6$ Hz. Velocity $v = c = 3 \times 10^8$ m/s (since it is in free space).
**Step 2: Apply the wavelength formula.**

$$\lambda = \frac{c}{f}$$

**Step 3: Calculate the final value.**

$$\lambda = \frac{3 \times 10^8}{900 \times 10^6} = \frac{300}{900} = 0.333 \text{ meters}$$

## 1.2 Radiation Resistance and Antenna Efficiency

**Methodology:**

Calculating Antenna Efficiency and Gain An antenna can be modeled electrically as having a radiation resistance ($R_r$) and an ohmic loss resistance ($R_l$).

1. **Total Antenna Resistance ($R_t$):** $R_t = R_r + R_l$
2. **Radiation Efficiency ($\eta$):** The ratio of power radiated to the total input power accepted by the antenna.

$$\eta = \frac{R_r}{R_r + R_l}$$

3. **Percentage Efficiency:** Multiply $\eta$ by 100.

4. **Power Gain ($G$):** Power gain is related to Directivity ($D$) by the efficiency factor.

$$G = \eta \times D$$

**Worked Example:**

Calculate Antenna Efficiency and Gain A dipole antenna has a radiation resistance of 73 ohms and an ohmic loss resistance of 7 ohms. If the directivity of the antenna is 2.15 dBi (numeric value 1.64), find its radiation efficiency and power gain.
**Step 1: Identify given parameters.** $R_r = 73\ \Omega$ $R_l = 7\ \Omega$ $D = 1.64$ (dimensionless numeric ratio)
**Step 2: Calculate radiation efficiency.**

$$\eta = \frac{R_r}{R_r + R_l} = \frac{73}{73+7} = \frac{73}{80} = 0.9125$$

Percentage efficiency $= 91.25\%$.
**Step 3: Calculate power gain.**

$$G = \eta \times D = 0.9125 \times 1.64 \approx 1.496$$

## 1.3 Power Density and Electric Field

**Methodology:**

Relating Power Density and Electric Field Intensity In free space the power density ($P_d$) of an electromagnetic wave is directly related to the electric field intensity ($E$).

1. **Intrinsic Impedance ($\eta_0$):** In free space the impedance is $\eta_0 = 120\pi \approx 377\ \Omega$.
2. **Power Density ($P_d$):** Calculated using the root-mean-square (RMS) electric field $E$.

$$P_d = \frac{E^2}{\eta_0} = \frac{E^2}{377} \quad (\mathrm{W/m^2})$$

3. **Electric Field ($E$):** If power density is known the electric field can be found by:

$$E = \sqrt{P_d \times 377} \quad (\mathrm{V/m})$$

**Worked Example:**

Calculate Power Density from Electric Field The RMS electric field intensity of an electromagnetic wave in free space is 50 mV/m. Calculate the corresponding power density.
**Step 1: Identify given parameters.** $E = 50\ \mathrm{mV/m} = 50 \times 10^{-3}\ \mathrm{V/m}$. Intrinsic impedance $\eta_0 = 377\ \Omega$.
**Step 2: Apply the power density formula.**

$$P_d = \frac{E^2}{377}$$

**Step 3: Calculate the value.**

$$P_d = \frac{(50 \times 10^{-3})^2}{377} = \frac{2500 \times 10^{-6}}{377} \approx 6.63 \times 10^{-6}\ \mathrm{W/m^2} = 6.63\ \mu\mathrm{W/m^2}$$

## 1.4 Effective Aperture

**Methodology:**

Calculating Effective Aperture of an Antenna The effective aperture ($A_e$) or effective area describes how well a receiving antenna captures electromagnetic power from a passing wave.

1. **Relationship to Gain:** The effective area is proportional to the antenna gain $G$ and the square of the wavelength $\lambda$.
$$A_e = \frac{\lambda^2}{4\pi} G$$
2. **Maximum Effective Area:** If the antenna is lossless (100% efficiency) then $G = D$ and $A_{em} = \frac{\lambda^2}{4\pi} D$.
3. **Received Power ($P_r$):** The total power received by the antenna is the product of incident power density and its effective area.
$$P_r = P_d \times A_e$$

**Worked Example:**

Calculate Effective Aperture and Received Power An antenna operating at 2.4 GHz has a gain of 15 (dimensionless). The incident power density at the antenna is 10 mW/m$^2$. Calculate the effective aperture and the total received power.

**Step 1: Calculate the wavelength.** $f = 2.4 \times 10^9$ Hz

$$\lambda = \frac{c}{f} = \frac{3 \times 10^8}{2.4 \times 10^9} = 0.125 \text{ m}$$

**Step 2: Calculate effective aperture.**

$$A_e = \frac{\lambda^2}{4\pi} G = \frac{(0.125)^2}{4\pi} \times 15 = \frac{0.015625}{12.566} \times 15 \approx 0.0186 \text{ m}^2$$

**Step 3: Calculate received power.** $P_d = 10 \text{ mW/m}^2 = 10 \times 10^{-3} \text{ W/m}^2$.

$$P_r = P_d \times A_e = (10 \times 10^{-3}) \times 0.0186 = 0.000186 \text{ W} = 0.186 \text{ mW}$$

## 1.5 Friis Transmission Equation

**Methodology:**

Using the Friis Transmission Formula The Friis transmission equation computes the power received by an antenna under free-space idealized conditions over a distance.

1. **Identify Parameters:** Transmit power ($P_t$), Transmit gain ($G_t$), Receive gain ($G_r$), Distance between antennas ($R$), and Wavelength ($\lambda$).
2. **Apply Linear Friis Formula:**
$$P_r = P_t \frac{G_t G_r \lambda^2}{(4\pi R)^2}$$
3. **Decibel Form:** It is often easier to compute in logarithmic scales.
$$P_r(\text{dBm}) = P_t(\text{dBm}) + G_t(\text{dBi}) + G_r(\text{dBi}) - 20 \log_{10}\left(\frac{4\pi R}{\lambda}\right)$$

**Worked Example:**

Calculate Received Power over a Distance A transmitter sends 5 W of power at a frequency of 300 MHz. The transmit antenna has a numeric gain of 2 and the receive antenna has a numeric gain of 1.5. If the receiver is 10 km away in free space, calculate the received power.

**Step 1: Calculate the wavelength.**

$$\lambda = \frac{3 \times 10^8}{300 \times 10^6} = 1 \text{ m}$$

**Step 2: Identify and convert given terms.** $P_t = 5$ W, $G_t = 2$, $G_r = 1.5$, $R = 10000$ m.

**Step 3: Apply the Friis formula.**

$$P_r = P_t \frac{G_t G_r \lambda^2}{(4\pi R)^2}$$

$$P_r = 5 \times \frac{2 \times 1.5 \times (1)^2}{(4\pi \times 10000)^2}$$

$$P_r = 5 \times \frac{3}{(125663.7)^2} = \frac{15}{1.579 \times 10^{10}} \approx 9.5 \times 10^{-10} \text{ W} = 0.95 \text{ nW}$$

## 1.6 Effective Isotropic Radiated Power

**Methodology:**

Calculating EIRP Effective Isotropic Radiated Power (EIRP) is the apparent power transmitted towards the receiver if the transmitter were an isotropic radiator.

1. **Linear Calculation:**

$$\text{EIRP} = P_t \times G_t$$

where $P_t$ is the transmitted power and $G_t$ is the antenna gain (linear numeric ratio).

2. **Decibel Calculation:**

$$\text{EIRP (dBm)} = P_t(\text{dBm}) + G_t(\text{dBi}) - L_c(\text{dB})$$

where $L_c$ represents total cable and connector losses.

**Worked Example:**

Determine EIRP of a Transmitter A transmitter has a power output of 40 W. It is connected to an antenna with a gain of 12 dBi via a coaxial cable that introduces a 2 dB loss. Calculate the EIRP in dBW and dBm.

**Step 1: Convert transmit power to dBW and dBm.**

$$P_t(\text{dBW}) = 10 \log_{10}(40) = 16.02 \text{ dBW}$$

$$P_t(\text{dBm}) = 16.02 + 30 = 46.02 \text{ dBm}$$

**Step 2: Calculate EIRP in dBW.**

$$\text{EIRP (dBW)} = P_t(\text{dBW}) + G_t(\text{dBi}) - L_c(\text{dB})$$

$$\text{EIRP (dBW)} = 16.02 + 12 - 2 = 26.02 \text{ dBW}$$

**Step 3: Calculate EIRP in dBm.**

$$\text{EIRP (dBm)} = P_t(\text{dBm}) + G_t(\text{dBi}) - L_c(\text{dB})$$

$$\text{EIRP (dBm)} = 46.02 + 12 - 2 = 56.02 \text{ dBm}$$

Alternatively, EIRP (dBm) $= 26.02$ dBW $+ 30 = 56.02$ dBm.

CHAPTER 2

# Basic Antennas and Arrays

This chapter covers the fundamental computational aspects of wire antennas, loop antennas, and antenna arrays. The focus is on calculating dimensions, radiation resistance, and analyzing the performance parameters of broadside and end-fire uniform linear arrays.

## 2.1 Resonant and Non-Resonant Wire Antennas

**Methodology:**

Resonant Wire Antenna Calculations Resonant antennas are characterized by standing wave patterns of current and voltage. A half-wave dipole is the most common resonant antenna.

1. **Wavelength Calculation**: $\lambda = \frac{c}{f}$ where $c = 3 \times 10^8$ m/s.
2. **Physical Length**: For a half-wave dipole the exact theoretical length is $L = \frac{\lambda}{2}$. Due to end effects, the practical length is about 5% shorter, giving $L_{practical} = 0.95 \times \frac{\lambda}{2} = 0.475\lambda$.
3. **Radiation Resistance**: The radiation resistance of an ideal half-wave dipole in free space is approximately $R_r = 73\Omega$.

**Worked Example:**

Design of a Half-Wave Dipole Design a resonant half-wave dipole to operate at a frequency of 150 MHz. Calculate its physical length and state its theoretical radiation resistance.

**Step 1: Calculate the wavelength.**

$$\lambda = \frac{c}{f} = \frac{3 \times 10^8}{150 \times 10^6} = \frac{300}{150} = 2 \text{ meters}$$

**Step 2: Calculate the ideal physical length.**

$$L_{ideal} = \frac{\lambda}{2} = \frac{2}{2} = 1 \text{ meter}$$

**Step 3: Calculate the practical length considering end effects.**

$$L_{practical} = 0.95 \times \frac{\lambda}{2} = 0.95 \times 1 = 0.95 \text{ meters}$$

**Step 4: State the radiation resistance.** For a standard half-wave dipole, the theoretical radiation resistance is $73\Omega$.

## 2.2 Loop Antenna

**Methodology:**

Small Loop Antenna Parameters A small loop antenna has dimensions much smaller than a wavelength ($C < \lambda/3$ where $C$ is circumference).

1. **Area of Loop**: $A = \pi r^2$ (circular) or $A = a \times b$ (rectangular).
2. **Radiation Resistance**: For a small loop with $N$ turns and area $A$, the radiation resistance is given by:
$$R_r = 31200 \left(N \frac{A}{\lambda^2}\right)^2 \quad \text{Ohms}$$
3. **Induced Voltage**: The root-mean-square voltage induced in a loop antenna by an incident electric field $E$ is:
$$V_{rms} = \frac{2\pi ENA}{\lambda} \cos\theta$$
where $\theta$ is the angle of arrival of the wave relative to the loop's plane. Maximum voltage occurs when the wave arrives in the plane of the loop ($\theta = 0°$).

**Worked Example:**

Radiation Resistance of a Circular Loop A circular loop antenna with a radius of 10 cm and 10 turns is operated at a frequency of 100 MHz. Calculate its radiation resistance.
**Step 1: Calculate the wavelength.**

$$\lambda = \frac{c}{f} = \frac{3 \times 10^8}{100 \times 10^6} = 3 \text{ meters}$$

**Step 2: Calculate the area of the loop.**

$$r = 10 \text{ cm} = 0.1 \text{ meters}$$

$$A = \pi r^2 = \pi(0.1)^2 \approx 0.0314 \text{ m}^2$$

**Step 3: Calculate the radiation resistance.** Using the formula $R_r = 31200 \left(N \frac{A}{\lambda^2}\right)^2$:

$$N = 10$$

$$\frac{A}{\lambda^2} = \frac{0.0314}{3^2} = \frac{0.0314}{9} \approx 0.00349$$

$$R_r = 31200 \times (10 \times 0.00349)^2 = 31200 \times (0.0349)^2$$

$$R_r = 31200 \times 0.001218 \approx 38.0\Omega$$

The radiation resistance of the loop antenna is approximately $38.0\Omega$.

## 2.3 Folded Dipole Antenna

**Methodology:**

Folded Dipole Impedance Transformation A folded dipole consists of two parallel dipoles connected at the ends, forming a continuous loop.

1. **Impedance Step-Up**: The input impedance $Z_{in}$ of a folded dipole with conductors of equal radius is:
$$Z_{in} = n^2 \times Z_{dipole}$$
where $n$ is the number of parallel conductors and $Z_{dipole}$ is the impedance of a simple half-wave dipole

(approximately 73Ω).

2. **Standard Two-Wire Folded Dipole**: For $n = 2$, the radiation resistance is:

$$Z_{in} = 2^2 \times 73 = 4 \times 73 = 292\Omega \approx 300\Omega$$

This makes it an ideal match for 300Ω twin-lead transmission lines.

**Worked Example:**

Impedance of a Multi Conductor Folded Dipole A folded dipole is constructed using three parallel wires of equal diameter. Calculate its theoretical input impedance and state its practical length at 200 MHz.
**Step 1: Determine the number of conductors ($n$).** The problem states there are 3 parallel wires, so $n = 3$.
**Step 2: Calculate the input impedance.**

$$Z_{in} = n^2 \times Z_{dipole} = 3^2 \times 73 = 9 \times 73 = 657\Omega$$

**Step 3: Calculate the wavelength and practical length.**

$$\lambda = \frac{c}{f} = \frac{3 \times 10^8}{200 \times 10^6} = 1.5 \text{ meters}$$

The practical length $L$ of the folded dipole is slightly less than a half wavelength:

$$L \approx 0.95 \times \frac{\lambda}{2} = 0.95 \times \frac{1.5}{2} = 0.7125 \text{ meters} = 71.25 \text{ cm}$$

## 2.4 Antenna Arrays and Pattern Multiplication

**Methodology:**

Principle of Pattern Multiplication The radiation pattern of an array of identical elements is found by multiplying the pattern of a single element by the Array Factor (AF).

1. **Total Pattern**: $E_{total} = E_{element} \times \text{Array Factor (AF)}$

2. **Phase Angle** $\psi$: The total phase difference between adjacent elements is:

$$\psi = \beta d \cos\theta + \alpha$$

where $\beta = \frac{2\pi}{\lambda}$ is the phase constant, $d$ is element spacing, $\theta$ is the spatial angle relative to the array axis, and $\alpha$ is the electrical phase difference between feeds.

3. **Array Factor for Uniform Array**: For $n$ identical elements carrying equal amplitude currents:

$$AF = \frac{\sin\left(n\frac{\psi}{2}\right)}{n\sin\left(\frac{\psi}{2}\right)}$$

## 2.5 Uniform Linear Arrays: Broadside and End-Fire

**Methodology:**

Broadside Array Calculations A Broadside Array has maximum radiation perpendicular to the array axis. Elements are fed in-phase ($\alpha = 0$).

1. **Maximum Direction**: Occurs at $\theta_m = 90°$ and $270°$.

2. **Null Directions**: The directions $\theta_{null}$ where radiation is zero:

$$\theta_{null} = \cos^{-1}\left(\pm\frac{k\lambda}{nd}\right) \quad \text{for } k = 1, 2, 3\ldots$$

3. **Beamwidth Between First Nulls BWFN**:

$$BWFN \approx \frac{2\lambda}{nd} \text{ radians} \quad \text{or} \quad \frac{114.6°}{nd/\lambda}$$

4. **Half Power Beamwidth HPBW**:

$$HPBW \approx \frac{BWFN}{2} \approx \frac{\lambda}{nd} \text{ radians} \quad \text{or} \quad \frac{57.3°}{nd/\lambda}$$

**Worked Example:**

Parameters of a Broadside Array A broadside array of 4 identical isotropic elements is spaced by $\lambda/2$. Calculate the directions of the first nulls and the Beamwidth Between First Nulls.
**Step 1: Identify given parameters.** Number of elements $n = 4$. Spacing $d = 0.5\lambda$. Since it is broadside, the phase excitation $\alpha = 0$.
**Step 2: Calculate the null directions.** The general formula for null directions is $\theta_{null} = \cos^{-1}\left(\pm\frac{k\lambda}{nd}\right)$. For the first nulls, $k = 1$:

$$\frac{\lambda}{nd} = \frac{\lambda}{4 \times 0.5\lambda} = \frac{1}{2} = 0.5$$

$$\theta_{null} = \cos^{-1}(\pm 0.5)$$

The angles are $\theta_1 = \cos^{-1}(0.5) = 60°$ and $\theta_2 = \cos^{-1}(-0.5) = 120°$.
**Step 3: Calculate the BWFN.** The BWFN is the angular width between the first nulls. Since maximum is at $90°$, the first nulls are symmetrically located at $60°$ and $120°$.

$$BWFN = 120° - 60° = 60°$$

Alternatively using the approximation formula:

$$BWFN = \frac{114.6°}{nd/\lambda} = \frac{114.6°}{4 \times 0.5} = \frac{114.6°}{2} = 57.3° \approx 60°$$

(The exact value is $60°$ calculated from the arc cosine directly).

**Methodology:**

End Fire Array Calculations An End-Fire Array has maximum radiation along the axis of the array. Elements are fed with a progressive phase shift $\alpha = -\beta d$.

1. **Maximum Direction**: Occurs at $\theta_m = 0°$ (and/or $180°$ depending on $\alpha$).

2. **Null Directions**: The directions $\theta_{null}$ where radiation is zero:

$$\theta_{null} = \cos^{-1}\left(1 - \frac{k\lambda}{nd}\right) \quad \text{for } k = 1, 2, 3 \ldots$$

3. **Beamwidth Between First Nulls BWFN**:

$$BWFN \approx 2\sqrt{\frac{2\lambda}{nd}} \text{ radians} \quad \text{or} \quad 114.6^\circ\sqrt{\frac{2}{nd/\lambda}}$$

4. **Half Power Beamwidth HPBW**:

$$HPBW \approx \frac{BWFN}{\sqrt{2}}$$

**Worked Example:**

Parameters of an End Fire Array An end-fire array consists of 8 isotropic elements spaced by $\lambda/4$. Determine the progressive phase shift required and calculate the exact direction of the first nulls.

**Step 1: Calculate the progressive phase shift $\alpha$.** For an end-fire array with maximum at $\theta = 0^\circ$, $\alpha = -\beta d$.

$$\beta = \frac{2\pi}{\lambda}$$

$$d = \frac{\lambda}{4}$$

$$\alpha = -\left(\frac{2\pi}{\lambda}\right)\left(\frac{\lambda}{4}\right) = -\frac{\pi}{2} \text{ radians} = -90^\circ$$

**Step 2: Calculate the direction of the first nulls.** Number of elements $n = 8$. Spacing $d = 0.25\lambda$. For first nulls, $k = 1$.

$$\theta_{null} = \cos^{-1}\left(1 - \frac{1 \cdot \lambda}{8 \cdot 0.25\lambda}\right)$$

$$\theta_{null} = \cos^{-1}\left(1 - \frac{1}{2}\right) = \cos^{-1}(0.5)$$

$$\theta_{null} = 60^\circ$$

Because the pattern is symmetric around the axis, there is a cone of nulls at $60^\circ$ from the array axis.

**Step 3: Calculate the exact BWFN.** Since the maximum is at $0^\circ$ and the first null is at $60^\circ$, the beamwidth between first nulls is:

$$BWFN = 2 \times 60^\circ = 120^\circ$$

## 2.6 Yagi-Uda Antenna

**Methodology:**

Yagi Uda Antenna Design Dimensions A Yagi-Uda antenna is a widely used high-gain parasitic array consisting of a driven element, a reflector, and one or more directors.

1. **Driven Element**: Usually a folded dipole. Length $L_{driven} \approx 0.47\lambda$.

2. **Reflector**: Placed behind the driven element. It is inductive and longer than resonance. Length $L_{reflector} \approx 0.5\lambda$ (often 5% longer than driven element).

3. **Directors**: Placed in front of the driven element. They are capacitive and shorter than resonance. Length $L_{director} \approx 0.44\lambda$ (often 5% shorter than driven element).

4. **Element Spacing**:
   - Spacing between reflector and driven element: $0.15\lambda$ to $0.25\lambda$.
   - Spacing between driven element and first director: $0.1\lambda$ to $0.2\lambda$.
   - Spacing between consecutive directors: $0.2\lambda$ to $0.2\lambda$.

**Worked Example:**

Design of a Three Element Yagi Uda Antenna Design a 3-element Yagi-Uda antenna for television reception at a frequency of 300 MHz. Calculate the lengths of the reflector, driven element, and director, and specify typical spacings.

**Step 1: Calculate the operating wavelength.**

$$\lambda = \frac{c}{f} = \frac{3 \times 10^8}{300 \times 10^6} = 1 \text{ meter}$$

**Step 2: Calculate lengths of the elements.** Using standard practical approximations:

- **Driven Element**: $L_{driven} = 0.47\lambda = 0.47 \times 1 = 0.47$ meters $= 47$ cm.
- **Reflector**: $L_{reflector} = 0.5\lambda = 0.5 \times 1 = 0.5$ meters $= 50$ cm.
- **Director**: $L_{director} = 0.44\lambda = 0.44 \times 1 = 0.44$ meters $= 44$ cm.

**Step 3: Specify the element spacings.** Select nominal spacing values from the standard ranges:

- **Reflector to Driven Element**: Select $0.2\lambda = 0.2 \times 1 = 0.2$ meters $= 20$ cm.
- **Driven Element to Director**: Select $0.15\lambda = 0.15 \times 1 = 0.15$ meters $= 15$ cm.

The total length of the antenna boom will be $20 \text{ cm} + 15 \text{ cm} = 35 \text{ cm}$.

CHAPTER 3

# Antennas for Special Applications

## 3.1 Yagi-Uda Antenna Design

The Yagi-Uda antenna is a highly directional antenna consisting of a driven element (usually a folded dipole), a reflector, and one or more directors. It is widely used for point-to-point communication and broadcasting.

**Methodology:**

Yagi-Uda Antenna Dimensions To design a standard Yagi-Uda array for a given operating frequency $f$:

1. Calculate the free-space wavelength: $\lambda = \frac{c}{f}$, where $c = 3 \times 10^8$ m/s.
2. **Reflector Length ($L_R$):** Typically 5% longer than the resonant half-wavelength.
$$L_R \approx 0.50\lambda$$
3. **Driven Element Length ($L$):** Usually a slightly shortened half-wave dipole for resonance.
$$L \approx 0.47\lambda$$
4. **Director Length ($L_D$):** Typically 5% shorter than the driven element.
$$L_D \approx 0.44\lambda$$
5. **Element Spacings:**
   - Reflector to Driven Element ($S_R$): $0.20\lambda$ to $0.25\lambda$.
   - Driven Element to First Director ($S_{D1}$): $0.20\lambda$ to $0.31\lambda$.
   - Director to Director ($S_D$): $0.20\lambda$.

**Worked Example:**

Yagi-Uda Antenna Design Design a 3-element Yagi-Uda antenna to operate at a frequency of 200 MHz. Calculate the lengths of all elements and the spacing between them. Assume standard empirical dimensions ($S_R = 0.2\lambda$ and $S_D = 0.2\lambda$).

**Solution: Step 1: Calculate the wavelength.**

$$\lambda = \frac{c}{f} = \frac{3 \times 10^8}{200 \times 10^6} = 1.5 \text{ m}$$

**Step 2: Calculate element lengths.**

- Reflector length ($L_R$):
$$L_R = 0.50\lambda = 0.50 \times 1.5 = 0.75 \text{ m}$$
- Driven element length ($L$):
$$L = 0.47\lambda = 0.47 \times 1.5 = 0.705 \text{ m}$$

- Director length ($L_D$):

$$L_D = 0.44\lambda = 0.44 \times 1.5 = 0.66 \text{ m}$$

**Step 3: Calculate element spacings.**

- Spacing between reflector and driven element ($S_R$):

$$S_R = 0.20\lambda = 0.20 \times 1.5 = 0.30 \text{ m}$$

- Spacing between driven element and director ($S_D$):

$$S_D = 0.20\lambda = 0.20 \times 1.5 = 0.30 \text{ m}$$

The designed 3-element Yagi-Uda antenna requires a 0.75 m reflector, a 0.705 m driven element, and a 0.66 m director. Each gap between adjacent elements is 0.30 m.

## 3.2 Helical Antenna

A helical antenna consists of a conducting wire wound in the form of a helix. When the circumference of one turn is approximately one wavelength, the antenna radiates in the axial mode, producing circularly polarized waves with high directivity.

**Methodology:**

Helical Antenna Parameters For a helical antenna with $N$ turns, diameter $D$, and turn spacing $S$:

1. **Circumference ($C$):** $C = \pi D$
2. **Pitch Angle ($\alpha$):**

$$\alpha = \tan^{-1}\left(\frac{S}{C}\right)$$

3. **Half-Power Beamwidth (HPBW):** In degrees,

$$\text{HPBW} \approx \frac{52\lambda}{C\sqrt{N\frac{S}{\lambda}}}$$

4. **First Null Beamwidth (FNBW):** In degrees,

$$\text{FNBW} \approx \frac{115\lambda}{C\sqrt{N\frac{S}{\lambda}}}$$

5. **Directivity ($D_0$):** Ratio (dimensionless),

$$D_0 \approx 15N\frac{C^2 S}{\lambda^3}$$

In decibels (dB), $D_0$ (dB) $= 10\log_{10}(D_0)$.

6. **Axial Ratio (AR):** Ratio of major to minor axes of polarization,

$$\text{AR} \approx \frac{2N+1}{2N}$$

**Worked Example:**

Helical Antenna Computation A helical antenna operating at 1.5 GHz has 10 turns, a pitch angle of 13°, and a diameter of 6.3 cm. Calculate the spacing between turns, the half-power beamwidth, and the directivity in dB.

**Solution: Step 1: Find the wavelength.**

$$\lambda = \frac{3 \times 10^8}{1.5 \times 10^9} = 0.2 \text{ m} = 20 \text{ cm}$$

**Step 2: Calculate circumference and spacing.**

$$C = \pi D = \pi \times 6.3 \approx 19.79 \text{ cm}$$

From the pitch angle $\alpha = 13°$:

$$\tan \alpha = \frac{S}{C} \implies S = C \tan(13°) = 19.79 \times 0.2309 \approx 4.57 \text{ cm}$$

**Step 3: Calculate the Half-Power Beamwidth (HPBW).**

$$\text{HPBW} = \frac{52\lambda}{C\sqrt{N\frac{S}{\lambda}}}$$

Using cm for all lengths:

$$\text{HPBW} = \frac{52 \times 20}{19.79\sqrt{10 \times \frac{4.57}{20}}} = \frac{1040}{19.79\sqrt{2.285}} = \frac{1040}{19.79 \times 1.5116} \approx 34.77°$$

**Step 4: Calculate the Directivity.**

$$D_0 = 15N\frac{C^2 S}{\lambda^3} = 15 \times 10 \times \frac{(19.79)^2 \times 4.57}{(20)^3} = 150 \times \frac{391.64 \times 4.57}{8000} \approx 33.56$$

In decibels:

$$D_0 \text{ (dB)} = 10 \log_{10}(33.56) \approx 15.26 \text{ dB}$$

The antenna has a turn spacing of 4.57 cm, a HPBW of 34.77°, and a directivity of 15.26 dB.

## 3.3 Log-Periodic Dipole Array

The Log-Periodic Dipole Array (LPDA) is a broadband, multi-element, directional antenna whose impedance and radiation characteristics repeat as a logarithmic function of frequency.

**Methodology:**

Log-Periodic Array Dimensions For an LPDA, the lengths of adjacent elements ($L_n$) and their spacings ($d_n$) are related by the scale factor $\tau$ and spacing factor $\sigma$:

1. **Scale Factor ($\tau$):**

$$\tau = \frac{L_{n+1}}{L_n} = \frac{R_{n+1}}{R_n} \quad (\tau < 1)$$

   where $L_n$ is the length of the $n$-th dipole and $R_n$ is its distance from the apex.

2. **Spacing Factor ($\sigma$):**

$$\sigma = \frac{d_n}{2L_n}$$

   where $d_n$ is the physical spacing between element $n$ and $n + 1$.

3. **Wedge Angle ($\alpha$):** The angle defining the envelope of the array elements.

$$\tan\left(\frac{\alpha}{2}\right) = \frac{1-\tau}{4\sigma}$$

4. **Element Lengths:** To cover a frequency band from $f_{min}$ to $f_{max}$:

   - Longest element (lowest frequency): $L_1 \approx \frac{c}{2f_{min}}$
   - Shortest element (highest frequency): $L_N \approx \frac{c}{2f_{max}}$

**Worked Example:**

LPDA Scale and Spacing Factor A Log-Periodic Dipole Array is designed to operate over the frequency band of 100 MHz to 300 MHz. The scale factor is $\tau = 0.8$ and the spacing factor is $\sigma = 0.15$. Find the wedge angle, the length of the longest element, and the length of the next adjacent element.

**Solution: Step 1: Calculate the longest element length ($L_1$).** The lowest operating frequency is $f_{min} = 100$ MHz.

$$\lambda_{max} = \frac{c}{f_{min}} = \frac{3 \times 10^8}{100 \times 10^6} = 3 \text{ m}$$

The longest element is a half-wavelength dipole at the lowest frequency:

$$L_1 = \frac{\lambda_{max}}{2} = \frac{3}{2} = 1.5 \text{ m}$$

**Step 2: Calculate the length of the second element ($L_2$).** Using the scale factor $\tau = 0.8$:

$$L_2 = \tau \times L_1 = 0.8 \times 1.5 = 1.2 \text{ m}$$

**Step 3: Calculate the wedge angle ($\alpha$).** Using the relationship between $\alpha, \tau$, and $\sigma$:

$$\tan\left(\frac{\alpha}{2}\right) = \frac{1-\tau}{4\sigma} = \frac{1-0.8}{4 \times 0.15} = \frac{0.2}{0.6} = 0.3333$$

$$\frac{\alpha}{2} = \tan^{-1}(0.3333) \approx 18.43^\circ \implies \alpha \approx 36.86^\circ$$

The longest element is 1.5 m, the next element is 1.2 m, and the wedge angle is 36.86°.

## 3.4 Parabolic Reflector Antennas

Parabolic reflectors are widely used for high-gain applications. They convert a spherical wave radiated by a feed antenna at the focal point into a plane wave.

**Methodology:**

Parabolic Reflector Formulas For a parabolic dish of physical diameter $D$ and operating at wavelength $\lambda$:

1. **Effective Area ($A_e$):**

$$A_e = \eta_a A_p = \eta_a \left(\frac{\pi D^2}{4}\right)$$

   where $\eta_a$ is the aperture efficiency (typically 0.5 to 0.7).

2. **Power Gain ($G$):**

$$G = \frac{4\pi}{\lambda^2} A_e = \eta_a \left(\frac{\pi D}{\lambda}\right)^2$$

3. **Beamwidths:** For a typical illumination profile, the approximate beamwidths in degrees are:

$$\text{HPBW} \approx 70^\circ \left(\frac{\lambda}{D}\right)$$

$$\text{FNBW} \approx 140^\circ \left(\frac{\lambda}{D}\right)$$

**Worked Example:**

Gain and Beamwidth of a Parabolic Dish A parabolic reflector antenna with a diameter of 3 m is used for a satellite link operating at 10 GHz. Assuming an aperture efficiency of 60%, calculate the effective area, the gain in dB, and the Half-Power Beamwidth (HPBW).

**Solution: Step 1: Calculate the wavelength.**

$$\lambda = \frac{c}{f} = \frac{3 \times 10^8}{10 \times 10^9} = 0.03 \text{ m}$$

**Step 2: Calculate the Effective Area ($A_e$).** The physical area $A_p$ is:

$$A_p = \frac{\pi D^2}{4} = \frac{\pi \times (3)^2}{4} \approx 7.068 \text{ m}^2$$

The effective area $A_e$ with $\eta_a = 0.6$:

$$A_e = 0.6 \times 7.068 \approx 4.241 \text{ m}^2$$

**Step 3: Calculate the Gain ($G$).**

$$G = \eta_a \left(\frac{\pi D}{\lambda}\right)^2 = 0.6 \left(\frac{\pi \times 3}{0.03}\right)^2 = 0.6\,(100\pi)^2 \approx 59217.6$$

Converting gain to dB:

$$G \text{ (dB)} = 10 \log_{10}(59217.6) \approx 47.72 \text{ dB}$$

**Step 4: Calculate the HPBW.**

$$\text{HPBW} \approx 70^\circ \left(\frac{\lambda}{D}\right) = 70^\circ \left(\frac{0.03}{3}\right) = 0.7^\circ$$

The parabolic dish has a gain of 47.72 dB and a very narrow beamwidth of 0.7°.

## 3.5 Microstrip Patch Antennas

Microstrip patch antennas are low-profile, lightweight antennas printed on a dielectric substrate. They are extensively used in mobile devices and wireless systems.

**Methodology:**

Rectangular Microstrip Patch Dimensions To compute the dimensions of a rectangular patch for resonance at frequency $f_r$, with a substrate of dielectric constant $\epsilon_r$ and thickness $h$:

1. **Patch Width ($W$):** Provides optimal radiation efficiency.

$$W = \frac{c}{2f_r}\sqrt{\frac{2}{\epsilon_r + 1}}$$

2. **Effective Dielectric Constant ($\epsilon_{reff}$):** Accounts for fringing fields spanning the substrate and the

air.

$$\epsilon_{reff} = \frac{\epsilon_r + 1}{2} + \frac{\epsilon_r - 1}{2}\left[1 + 12\frac{h}{W}\right]^{-1/2}$$

3. **Length Extension ($\Delta L$):** Accounts for the fringing effect extending the apparent electrical length.

$$\Delta L = 0.412h\frac{(\epsilon_{reff} + 0.3)\left(\frac{W}{h} + 0.264\right)}{(\epsilon_{reff} - 0.258)\left(\frac{W}{h} + 0.8\right)}$$

4. **Actual Length ($L$):** The effective length $L_{eff}$ minus the fringing extensions on both sides.

$$L_{eff} = \frac{c}{2f_r\sqrt{\epsilon_{reff}}}$$

$$L = L_{eff} - 2\Delta L$$

**Worked Example:**

Microstrip Patch Design Design a rectangular microstrip patch antenna to operate at 2.4 GHz. The substrate is FR4 with a dielectric constant $\epsilon_r = 4.4$ and a thickness $h = 1.6$ mm. Calculate the physical width and length of the patch.

**Solution: Step 1: Calculate the Width ($W$).**

$$W = \frac{3 \times 10^8}{2 \times 2.4 \times 10^9}\sqrt{\frac{2}{4.4 + 1}} = \frac{3}{4.8}\sqrt{\frac{2}{5.4}} \approx 0.625 \times 0.6086 \approx 0.380 \text{ m} = 38.0 \text{ mm}$$

**Step 2: Calculate the Effective Dielectric Constant ($\epsilon_{reff}$).** With $W = 38.0$ mm and $h = 1.6$ mm, the ratio $\frac{h}{W} \approx 0.0421$.

$$\epsilon_{reff} = \frac{4.4 + 1}{2} + \frac{4.4 - 1}{2}[1 + 12(0.0421)]^{-1/2} = 2.7 + 1.7[1.505]^{-1/2} \approx 4.086$$

**Step 3: Calculate the Length Extension ($\Delta L$).** The ratio $\frac{W}{h} = \frac{38.0}{1.6} = 23.75$.

$$\Delta L = 0.412 \times 1.6 \times \frac{(4.086 + 0.3)(23.75 + 0.264)}{(4.086 - 0.258)(23.75 + 0.8)} = 0.6592 \times \frac{(4.386)(24.014)}{(3.828)(24.55)}$$

$$\Delta L = 0.6592 \times \frac{105.325}{93.977} \approx 0.739 \text{ mm}$$

**Step 4: Calculate the Actual Patch Length ($L$).**

$$L_{eff} = \frac{c}{2f_r\sqrt{\epsilon_{reff}}} = \frac{3 \times 10^8}{2 \times 2.4 \times 10^9\sqrt{4.086}} \approx \frac{3}{4.8 \times 2.021} \approx 30.9 \text{ mm}$$

$$L = L_{eff} - 2\Delta L = 30.9 - 2(0.739) = 29.42 \text{ mm}$$

The required dimensions for the microstrip patch are $W = 38.0$ mm and $L = 29.42$ mm.

## 3.6 Horn Antennas

Horn antennas are formed by flaring the end of a waveguide to provide a gradual transition between the waveguide impedance and free space.

## Methodology:

Pyramidal Horn Parameters For a pyramidal horn with aperture dimensions $a_1$ (E-plane) and $b_1$ (H-plane), operating at wavelength $\lambda$:

1. **Aperture Area ($A_p$):**
$$A_p = a_1 b_1$$
2. **Directivity ($D_0$):**
$$D_0 = \frac{4\pi}{\lambda^2}\epsilon_{ap}(a_1 b_1)$$
where $\epsilon_{ap}$ is the aperture efficiency (typically around 0.5).
3. **Half-Power Beamwidths (HPBW):**
    - In the E-plane: $\theta_E \approx 56^\circ \left(\frac{\lambda}{b_1}\right)$
    - In the H-plane: $\theta_H \approx 67^\circ \left(\frac{\lambda}{a_1}\right)$

## Worked Example:

Directivity and Beamwidth of a Horn Antenna A pyramidal horn has an aperture measuring $a_1 = 12$ cm and $b_1 = 8$ cm. It operates at a frequency of 5 GHz. Assuming an aperture efficiency of 0.5, calculate the directivity in dB and the HPBW in both planes.
**Solution: Step 1: Calculate the wavelength.**

$$\lambda = \frac{c}{f} = \frac{3 \times 10^8}{5 \times 10^9} = 0.06 \text{ m} = 6 \text{ cm}$$

**Step 2: Calculate the Directivity ($D_0$).** Using the lengths in cm:

$$D_0 = \frac{4\pi}{\lambda^2}\epsilon_{ap}(a_1 b_1) = \frac{4\pi}{(6)^2}(0.5)(12 \times 8) = \frac{4\pi}{36} \times 48 = \frac{16\pi}{3} \approx 16.755$$

Converting directivity to dB:

$$D_0 \text{ (dB)} = 10 \log_{10}(16.755) \approx 12.24 \text{ dB}$$

**Step 3: Calculate the Beamwidths.** For the E-plane:

$$\theta_E \approx 56^\circ \left(\frac{\lambda}{b_1}\right) = 56^\circ \left(\frac{6}{8}\right) = 42^\circ$$

For the H-plane:

$$\theta_H \approx 67^\circ \left(\frac{\lambda}{a_1}\right) = 67^\circ \left(\frac{6}{12}\right) = 33.5^\circ$$

The horn provides a directivity of 12.24 dB with an E-plane HPBW of 42° and H-plane HPBW of 33.5°.

# CHAPTER 4

# Wave Propagation

## 4.1 Introduction

Electromagnetic wave propagation describes how radio waves travel from a transmitting antenna to a receiving antenna. Depending on the frequency of the wave and the environment, propagation occurs mainly via three modes: Ground Wave, Sky Wave, and Space Wave. This chapter focuses on the computational formulas and methods used to analyze these propagation modes.

## 4.2 Sky Wave Propagation

Sky wave propagation involves the reflection of radio waves by the Earth's ionosphere. It is primarily used for high-frequency (HF) communication over long distances.

**Methodology:**

Refractive Index and Critical Frequency The ionosphere acts as a varying dielectric medium. The refractive index ($\mu$) of the ionosphere depends on the electron density and the frequency of the transmitted wave.

1. **Refractive Index ($\mu$):**

$$\mu = \sqrt{1 - \frac{81N}{f^2}}$$

   Where:

   - $N$ = Electron density (electrons/$m^3$)
   - $f$ = Frequency of the electromagnetic wave (Hz)

2. **Critical Frequency ($f_c$):** The highest frequency that will be returned to Earth by an ionospheric layer when sent straight up (vertical incidence).

$$f_c = 9\sqrt{N_{max}}$$

   Where:

   - $N_{max}$ = Maximum electron density of the layer (electrons/$m^3$)
   - $f_c$ = Critical frequency in Hertz (Hz)

**Worked Example:**

Calculating Critical Frequency If the maximum electron density of the F2 layer is $4 \times 10^{12}$ electrons/$m^3$ calculate the critical frequency of the layer.
**Solution:**

1. **Identify given parameters:** Maximum electron density $N_{max} = 4 \times 10^{12}$ electrons/$m^3$.

2. **Apply the critical frequency formula:**

$$f_c = 9\sqrt{N_{max}}$$

3. **Substitute the value of $N_{max}$:**

$$f_c = 9\sqrt{4 \times 10^{12}}$$
$$f_c = 9 \times (2 \times 10^6)$$
$$f_c = 18 \times 10^6 \text{ Hz} = 18 \text{ MHz}$$

The critical frequency of the layer is 18 MHz.

**Methodology:**

Maximum Usable Frequency and Secant Law The Maximum Usable Frequency (MUF) is the highest frequency that can be used for communication between two specific points via sky wave propagation at a given angle of incidence.

1. **Secant Law (Flat Earth Approximation):**

$$f_{MUF} = f_c \sec\theta_i$$

Where:

- $f_{MUF}$ = Maximum Usable Frequency
- $f_c$ = Critical Frequency
- $\theta_i$ = Angle of incidence at the ionospheric layer

**Worked Example:**

Determining the MUF Calculate the Maximum Usable Frequency for a critical frequency of 10 MHz if the angle of incidence of the radio wave at the ionospheric layer is 60°.
**Solution:**

1. **Identify given parameters:** Critical frequency $f_c = 10$ MHz. Angle of incidence $\theta_i = 60°$.

2. **Apply the Secant Law:**

$$f_{MUF} = f_c \sec\theta_i = \frac{f_c}{\cos\theta_i}$$

3. **Substitute the values:**

$$f_{MUF} = \frac{10 \text{ MHz}}{\cos 60°}$$
$$f_{MUF} = \frac{10}{0.5} = 20 \text{ MHz}$$

The Maximum Usable Frequency is 20 MHz.

**Methodology:**

Optimum Working Frequency The Optimum Working Frequency (OWF) or Optimum Traffic Frequency (FOT) is the frequency normally used for reliable ionospheric communication. It provides a safety margin against ionospheric variations.

1. **OWF Formula:**

$$f_{OWF} = 0.85 \times f_{MUF}$$

Where $f_{MUF}$ is the Maximum Usable Frequency.

**Worked Example:**

Calculating the Optimum Working Frequency A sky wave communication link is operating with a critical frequency of 8 MHz and an angle of incidence of 45°. Determine the Optimum Working Frequency.

**Solution:**

1. **Calculate the MUF first:**

$$f_{MUF} = f_c \sec\theta_i = \frac{8}{\cos 45°}$$

$$f_{MUF} = \frac{8}{0.707} \approx 11.31 \text{ MHz}$$

2. **Calculate the OWF:**

$$f_{OWF} = 0.85 \times f_{MUF}$$

$$f_{OWF} = 0.85 \times 11.31 \approx 9.61 \text{ MHz}$$

The Optimum Working Frequency is 9.61 MHz.

**Methodology:**

Skip Distance Skip distance ($D_{skip}$) is the minimum distance from the transmitter at which a sky wave of a given frequency is returned to Earth by the ionosphere.

1. **Skip Distance Formula (Flat Earth Approximation):**

$$D_{skip} = 2h\sqrt{\left(\frac{f_{MUF}}{f_c}\right)^2 - 1}$$

Where:

- $h$ = Virtual height of the ionospheric layer
- $f_{MUF}$ = Operating frequency (or MUF for minimum distance)
- $f_c$ = Critical frequency

**Worked Example:**

Calculating Skip Distance Find the skip distance for a radio link operating at 15 MHz. The critical frequency of the layer is 5 MHz and its virtual height is 300 km.

**Solution:**

1. **Identify given parameters:** Operating frequency $f_{MUF} = 15$ MHz. Critical frequency $f_c = 5$ MHz. Virtual height $h = 300$ km.

2. **Apply the skip distance formula:**

$$D_{skip} = 2h\sqrt{\left(\frac{f_{MUF}}{f_c}\right)^2 - 1}$$

3. **Substitute the values:**

$$D_{skip} = 2(300)\sqrt{\left(\frac{15}{5}\right)^2 - 1}$$

$$D_{skip} = 600\sqrt{3^2 - 1} = 600\sqrt{9 - 1} = 600\sqrt{8}$$

$$D_{skip} = 600 \times 2.828 \approx 1696.8 \text{ km}$$

The skip distance is approximately 1697 km.

## 4.3 Space Wave Propagation

Space wave propagation (Line of Sight or LOS) is primarily used for VHF UHF and microwave communications. The radio waves travel in a straight line from the transmitting antenna to the receiving antenna.

**Methodology:**

Radio Horizon and Maximum Range Because the Earth is spherical space waves are limited by the curvature of the Earth. The concept of the radio horizon accounts for standard atmospheric refraction.

1. **Distance to Radio Horizon ($d$):**

$$d \approx 4.12\sqrt{h_t}$$

   Where:

   - $d$ = Distance to the horizon in kilometers (km)
   - $h_t$ = Height of the transmitting antenna in meters (m)

2. **Maximum Line of Sight Distance ($d_{max}$):** The maximum communication distance between a transmitting and a receiving antenna.

$$d_{max} = 4.12\left(\sqrt{h_t} + \sqrt{h_r}\right)$$

   Where:

   - $d_{max}$ = Maximum line of sight distance in km
   - $h_t$ = Height of transmitting antenna in m
   - $h_r$ = Height of receiving antenna in m

**Worked Example:**

Maximum Line of Sight Range A transmitting antenna has a height of 50 meters and the receiving antenna has a height of 20 meters. Calculate the maximum line of sight distance between them.
**Solution:**

1. **Identify given parameters:** $h_t = 50$ m $h_r = 20$ m.

2. **Apply the maximum range formula:**

$$d_{max} = 4.12\left(\sqrt{h_t} + \sqrt{h_r}\right)$$

3. **Calculate the square roots:**

$$\sqrt{50} \approx 7.07$$
$$\sqrt{20} \approx 4.47$$

4. **Substitute and compute the final distance:**

$$d_{max} = 4.12\,(7.07 + 4.47) = 4.12 \times 11.54 \approx 47.54 \text{ km}$$

The maximum line of sight distance is approximately 47.54 km.

**Worked Example:**

Determining Required Antenna Height It is desired to establish a line of sight microwave link over a distance of 80 km. If the receiving antenna is fixed at a height of 36 meters what must be the minimum height of

the transmitting antenna?
**Solution:**

1. **Identify given parameters:** $d_{max} = 80$ km $h_r = 36$ m.
2. **Rearrange the maximum range formula to solve for $h_t$:**

$$d_{max} = 4.12\left(\sqrt{h_t} + \sqrt{h_r}\right)$$

$$\frac{d_{max}}{4.12} = \sqrt{h_t} + \sqrt{h_r}$$

$$\sqrt{h_t} = \frac{d_{max}}{4.12} - \sqrt{h_r}$$

3. **Substitute the values:**

$$\sqrt{h_t} = \frac{80}{4.12} - \sqrt{36}$$

$$\sqrt{h_t} \approx 19.42 - 6 = 13.42$$

4. **Square the result to find $h_t$:**

$$h_t = (13.42)^2 \approx 180.1 \text{ m}$$

The minimum required height for the transmitting antenna is approximately 180.1 meters.

CHAPTER 5

# Antennas for Advanced Communication

Modern communication systems such as 5G LTE Wi-Fi and satellite links demand high performance from their radiating elements. This requires advanced designs like Microstrip Patch Antennas MIMO systems and Smart Antennas capable of dynamic beamforming. This chapter provides the essential computational methods and design algorithms to evaluate and engineer these advanced antenna systems.

## 5.1 Microstrip Patch Antenna Design

Microstrip patch antennas are ubiquitous in mobile and aerospace communications due to their low profile lightweight and ease of fabrication. Designing a rectangular microstrip patch requires calculating its physical width and length based on the desired resonant frequency and the properties of the dielectric substrate.

**Methodology:**

Rectangular Microstrip Patch Antenna Design Given the resonant frequency ($f_r$), the relative dielectric constant of the substrate ($\varepsilon_r$), and the substrate thickness ($h$), the design steps are:

1. **Calculate the Patch Width ($W$):** The width is calculated to ensure good radiation efficiency:

$$W = \frac{c}{2f_r}\sqrt{\frac{2}{\varepsilon_r + 1}}$$

   where $c = 3 \times 10^8$ m/s is the speed of light in free space.

2. **Calculate the Effective Dielectric Constant ($\varepsilon_{reff}$):** Due to fringing fields at the edges of the patch, the effective dielectric constant is slightly less than $\varepsilon_r$:

$$\varepsilon_{reff} = \frac{\varepsilon_r + 1}{2} + \frac{\varepsilon_r - 1}{2}\left[1 + 12\frac{h}{W}\right]^{-1/2}$$

3. **Calculate the Effective Length ($L_{eff}$):** The effective length dictates the resonance condition taking fringing into account:

$$L_{eff} = \frac{c}{2f_r\sqrt{\varepsilon_{reff}}}$$

4. **Calculate the Length Extension ($\Delta L$):** The fringing fields make the patch appear electrically longer than its physical length by a distance of $\Delta L$ on each side:

$$\Delta L = 0.412h\frac{(\varepsilon_{reff} + 0.3)\left(\frac{W}{h} + 0.264\right)}{(\varepsilon_{reff} - 0.258)\left(\frac{W}{h} + 0.8\right)}$$

5. **Calculate the Actual Physical Length ($L$):**

$$L = L_{eff} - 2\Delta L$$

**Worked Example:**

Design of 2.4 GHz Microstrip Patch Antenna **Problem:** Design a rectangular microstrip patch antenna operating at a resonant frequency of 2.4 GHz. The chosen substrate is RT/Duroid with a relative dielectric constant $\varepsilon_r = 2.2$ and a thickness $h = 1.588$ mm.

**Solution: Step 1: Calculate the Patch Width ($W$)**

$$W = \frac{3 \times 10^8}{2 \times 2.4 \times 10^9}\sqrt{\frac{2}{2.2+1}}$$

$$W = \frac{3 \times 10^8}{4.8 \times 10^9}\sqrt{\frac{2}{3.2}} = 0.0625 \times \sqrt{0.625}$$

$$W = 0.0625 \times 0.79057 = 0.04941 \text{ m} = 49.41 \text{ mm}$$

**Step 2: Calculate Effective Dielectric Constant ($\varepsilon_{reff}$)**

$$\varepsilon_{reff} = \frac{2.2+1}{2} + \frac{2.2-1}{2}\left[1 + 12\left(\frac{1.588}{49.41}\right)\right]^{-1/2}$$

$$\varepsilon_{reff} = 1.6 + 0.6\left[1 + 12(0.03214)\right]^{-1/2}$$

$$\varepsilon_{reff} = 1.6 + 0.6[1.3857]^{-1/2} = 1.6 + 0.6(0.8495) = 2.1097$$

**Step 3: Calculate Effective Length ($L_{eff}$)**

$$L_{eff} = \frac{3 \times 10^8}{2 \times 2.4 \times 10^9\sqrt{2.1097}} = \frac{0.0625}{1.4525} = 0.04303 \text{ m} = 43.03 \text{ mm}$$

**Step 4: Calculate Length Extension ($\Delta L$)**

$$\Delta L = 0.412(1.588)\frac{(2.1097+0.3)(49.41/1.588+0.264)}{(2.1097-0.258)(49.41/1.588+0.8)}$$

$$\Delta L = 0.654\frac{(2.4097)(31.115+0.264)}{(1.8517)(31.115+0.8)} = 0.654\frac{2.4097 \times 31.379}{1.8517 \times 31.915}$$

$$\Delta L = 0.654\frac{75.614}{59.097} = 0.654 \times 1.279 = 0.836 \text{ mm}$$

**Step 5: Calculate Actual Length ($L$)**

$$L = L_{eff} - 2\Delta L = 43.03 - 2(0.836) = 43.03 - 1.672 = 41.358 \text{ mm}$$

The required dimensions for the patch are $W = 49.41$ mm and $L = 41.36$ mm.

## 5.2 MIMO System Channel Capacity

Multiple Input Multiple Output (MIMO) technology uses multiple antennas at both the transmitter and receiver to multiply the capacity of a radio link. This exploitation of multipath propagation is critical in modern standards like Wi-Fi 6 and 5G.

**Methodology:**

Ergodic Channel Capacity for MIMO Systems For a system with a channel bandwidth $B$ and a given Signal-to-Noise Ratio (SNR), the Shannon capacity can be extended for MIMO configurations.

1. **Convert SNR to Linear Scale:** If SNR is given in decibels (dB), convert it to a linear ratio:

$$\text{SNR}_{linear} = 10^{\frac{\text{SNR}_{dB}}{10}}$$

2. **Calculate Baseline SISO Capacity:** For a Single Input Single Output (SISO) system:

$$C_{SISO} = B \log_2(1 + \text{SNR}_{linear}) \quad \text{(in bps)}$$

3. **Determine Spatial Multiplexing Gain ($K$):** The maximum theoretical number of independent spatial streams is limited by the minimum number of transmit ($N_T$) or receive ($N_R$) antennas:

$$K = \min(N_T, N_R)$$

4. **Calculate MIMO Capacity:** Assuming a high-scattering rich multipath environment with independent fading paths, the theoretical channel capacity scales linearly with $K$:

$$C_{MIMO} \approx K \times B \log_2(1 + \text{SNR}_{linear})$$

*Note:* This is an idealized upper bound assuming perfect spatial multiplexing without inter-stream interference.

**Worked Example:**

Capacity Calculation for 4x4 MIMO System **Problem:** A cellular base station uses a 20 MHz channel bandwidth. The Signal-to-Noise Ratio (SNR) at the receiver is 15 dB. Calculate and compare the maximum theoretical data rates (channel capacity) for a standard SISO system and a $4 \times 4$ MIMO system.

**Solution: Step 1: Convert SNR to Linear Scale**

$$\text{SNR}_{linear} = 10^{15/10} = 10^{1.5} = 31.62$$

**Step 2: Calculate Baseline SISO Capacity**

$$C_{SISO} = B \log_2(1 + \text{SNR}_{linear})$$

$$C_{SISO} = 20 \times 10^6 \times \log_2(1 + 31.62)$$

$$C_{SISO} = 20 \times 10^6 \times \log_2(32.62)$$

Using the change of base formula: $\log_2(32.62) = \frac{\ln(32.62)}{\ln 2} \approx \frac{3.4849}{0.6931} \approx 5.028$

$$C_{SISO} = 20 \times 10^6 \times 5.028 = 100.56 \text{ Mbps}$$

**Step 3: Determine Spatial Multiplexing Gain** For a $4 \times 4$ MIMO system, $N_T = 4$ and $N_R = 4$.

$$K = \min(4, 4) = 4$$

**Step 4: Calculate MIMO Capacity**

$$C_{MIMO} \approx K \times C_{SISO} = 4 \times 100.56 \text{ Mbps} = 402.24 \text{ Mbps}$$

The $4 \times 4$ MIMO system effectively quadruples the capacity of the link to 402.24 Mbps under ideal rich-scattering conditions.

## 5.3 Smart Antenna Beam Steering

Smart antennas and phased arrays can electronically steer the main radiation beam in a desired direction without physically moving the antenna. This is achieved by controlling the phase of the signal fed to each individual element in the array.

## Methodology:

Progressive Phase Shift for Beam Steering For a uniform linear array (ULA) of $N$ elements placed along the z-axis with an inter-element spacing $d$, the beam can be steered to a desired elevation angle $\theta_0$ (where $\theta = 90°$ is broadside).

1. **Determine Wavenumber ($k$):**
$$k = \frac{2\pi}{\lambda}$$
where $\lambda$ is the operating wavelength.

2. **Identify Required Array Spacing ($d$):** Typically expressed in terms of wavelength (e.g., $d = 0.5\lambda$).

3. **Calculate the Progressive Phase Shift ($\beta$):** To direct the main beam maximum towards angle $\theta_0$, the total phase $\psi$ of the array factor must be zero at $\theta_0$.
$$\psi = kd\cos(\theta) + \beta = 0$$
$$\beta = -kd\cos(\theta_0)$$
*Note:* A negative phase shift implies a delay applied to successive elements, whereas a positive shift implies a phase advance.

## Worked Example:

Phase Shift Calculation for Beam Steering **Problem:** A uniform linear array consists of 5 elements spaced evenly at a distance of $d = \lambda/2$. Calculate the progressive phase shift $\beta$ required to electronically steer the main lobe to: (a) $\theta_0 = 60°$ (b) $\theta_0 = 120°$

**Solution: Step 1: Calculate Wavenumber and Spacing Product**

$$kd = \left(\frac{2\pi}{\lambda}\right)(0.5\lambda) = \pi \text{ radians} = 180°$$

**Step 2: Calculate Phase Shift for Case (a)** $\theta_0 = 60°$

$$\beta = -kd\cos(\theta_0)$$
$$\beta = -180° \times \cos(60°)$$
$$\beta = -180° \times 0.5 = -90° \text{ (or } -\frac{\pi}{2} \text{ radians)}$$

To steer the beam to 60°, each successive antenna element must be delayed by 90° relative to the previous one.

**Step 3: Calculate Phase Shift for Case (b)** $\theta_0 = 120°$

$$\beta = -kd\cos(120°)$$
$$\beta = -180° \times (-0.5) = +90° \text{ (or } +\frac{\pi}{2} \text{ radians)}$$

To steer the beam to 120°, a progressive phase advance of 90° is applied to each successive element.

# Formulae

# Appendix: Key Formulae

This appendix provides a quick reference to the key formulae and mathematical relationships discussed in this book, categorized by unit.

## Unit 1: Fundamentals of Antennas

- **Wavelength and Frequency:**
$$\lambda = \frac{c}{f}$$
Where $\lambda$ is wavelength, $c$ is the speed of light, and $f$ is frequency.

- **Radiated Power Density:**
$$P_d = \frac{E^2}{\eta_0} = \frac{E^2}{377}$$
Where $P_d$ is the power density, $E$ is the electric field strength, and $\eta_0 \approx 377\,\Omega$ is the intrinsic impedance of free space.

- **Antenna Efficiency:**
$$\eta = \frac{R_r}{R_r + R_l}$$
Where $R_r$ is radiation resistance and $R_l$ is ohmic loss resistance.

- **Antenna Gain:**
$$G = \eta \times D$$
Where $G$ is gain, $\eta$ is efficiency, and $D$ is directivity.

- **Effective Aperture (Area):**
$$A_e = \frac{\lambda^2}{4\pi} G$$
Where $A_e$ is the effective aperture and $G$ is the antenna gain.

- **Equivalent Isotropically Radiated Power (EIRP):**
$$\text{EIRP} = P_t \times G_t$$
$$\text{EIRP (dBm or dBW)} = P_t(\text{dBm or dBW}) + G_t(\text{dBi}) - L_c(\text{dB})$$
Where $P_t$ is transmitter power, $G_t$ is transmitter antenna gain, and $L_c$ is cable loss.

- **Received Power:**
$$P_r = P_d \times A_e$$
Where $P_r$ is received power, $P_d$ is incident power density, and $A_e$ is effective aperture of the receiving antenna.

- **Friis Transmission Equation:**
$$P_r = P_t \frac{G_t G_r \lambda^2}{(4\pi R)^2}$$
$$P_r(\text{dBm}) = P_t(\text{dBm}) + G_t(\text{dBi}) + G_r(\text{dBi}) - 20\log_{10}\left(\frac{4\pi R}{\lambda}\right)$$
Where $R$ is the distance between antennas.

# Unit 2: Antenna Arrays and Wire Antennas

- **Phase Constant:**
$$\beta = \frac{2\pi}{\lambda}$$

- **Total Phase Difference:**
$$\psi = \beta d \cos\theta + \alpha$$
Where $d$ is the element spacing, $\theta$ is the observation angle, and $\alpha$ is the progressive phase shift between elements.

- **Array Factor (AF) for 2 Elements:**
$$AF = \cos\left(\frac{\psi}{2}\right) = \cos\left(\frac{\beta d \cos\theta + \alpha}{2}\right)$$

- **Array Factor (AF) for N Elements:**
$$AF = \frac{\sin\left(n\frac{\psi}{2}\right)}{n \sin\left(\frac{\psi}{2}\right)}$$
Where $n$ (or $N$) is the number of elements.

- **Pattern Multiplication:**
$$E_t = E_e \times AF$$
Where $E_t$ is the total field and $E_e$ is the element field pattern.

- **Radiated Power:**
$$P_{rad} = I_{rms}^2 R_{rad}$$

- **Directivity of Arrays:**
$$D = \frac{4Nd}{\lambda} \quad \text{(Endfire Array)}$$
$$D = \frac{2Nd}{\lambda} \quad \text{(Broadside Array)}$$

- **Beamwidth for Arrays:**
$$\text{BWFN} = \frac{2\lambda}{Nd}\ \text{rad} \approx \frac{114.6^\circ}{\frac{Nd}{\lambda}} \quad \text{(Broadside)}$$
$$\text{BWFN} = 2\sqrt{\frac{2\lambda}{Nd}}\ \text{rad} \approx 114.6^\circ\sqrt{\frac{2\lambda}{Nd}} \quad \text{(Endfire)}$$
$$\text{HPBW} \approx \frac{\text{BWFN}}{2}$$

- **Null Directions:**
$$\theta_{null} = \cos^{-1}\left(\pm\frac{k\lambda}{Nd}\right) \quad \text{for } k = 1, 2, 3\ldots \quad \text{(Broadside)}$$
$$\theta_{null} = \cos^{-1}\left(1 - \frac{k\lambda}{Nd}\right) \quad \text{for } k = 1, 2, 3\ldots \quad \text{(Endfire)}$$

- **Folded Dipole Impedance:**
$$Z_{in} = n^2 \times Z_{dipole}$$
Where $n$ is the number of parallel conductors (for a standard 2-wire folded dipole, $n = 2$, $Z_{in} \approx 4 \times 73 = 292\,\Omega$).

- **Practical Length of Half-Wave Dipole:**
$$L_{practical} \approx 0.95\frac{\lambda}{2}$$

- **Radiation Resistance of Loop Antenna:**

$$R_r = 31200 \left( N \frac{A}{\lambda^2} \right)^2 \quad (\Omega)$$

Where $N$ is the number of turns and $A$ is the loop area.

- **Induced Voltage in a Loop Antenna:**

$$V_{rms} = \frac{2\pi E N A}{\lambda} \cos\theta$$

# Unit 3: Special Antennas

- **Microstrip Patch Antenna - Width ($W$):**

$$W = \frac{c}{2f_r} \sqrt{\frac{2}{\epsilon_r + 1}}$$

Where $f_r$ is resonant frequency and $\epsilon_r$ is the relative permittivity of the substrate.

- **Microstrip Patch Antenna - Effective Dielectric Constant ($\epsilon_{reff}$):**

$$\epsilon_{reff} = \frac{\epsilon_r + 1}{2} + \frac{\epsilon_r - 1}{2} \left[ 1 + 12 \frac{h}{W} \right]^{-1/2}$$

Where $h$ is the substrate thickness.

- **Microstrip Patch Antenna - Length Extension ($\Delta L$):**

$$\Delta L = 0.412h \frac{(\epsilon_{reff} + 0.3) \left( \frac{W}{h} + 0.264 \right)}{(\epsilon_{reff} - 0.258) \left( \frac{W}{h} + 0.8 \right)}$$

- **Microstrip Patch Antenna - Effective and Actual Length ($L$):**

$$L_{eff} = \frac{c}{2f_r \sqrt{\epsilon_{reff}}}$$

$$L = L_{eff} - 2\Delta L$$

- **Helical Antenna Dimensions:**

$$C = \pi D, \quad \alpha = \tan^{-1} \left( \frac{S}{C} \right)$$

Where $C$ is circumference, $D$ is diameter, $S$ is spacing between turns, and $\alpha$ is pitch angle.

- **Helical Antenna (Axial Mode) Characteristics:**

$$\text{HPBW} \approx \frac{52\lambda}{C\sqrt{N\frac{S}{\lambda}}} \quad \text{degrees}$$

$$\text{FNBW} \approx \frac{115\lambda}{C\sqrt{N\frac{S}{\lambda}}} \quad \text{degrees}$$

$$D_0 \approx 15N \frac{C^2 S}{\lambda^3}$$

$$\text{Axial Ratio (AR)} \approx \frac{2N + 1}{2N}$$

Where $N$ is the number of turns.

- **Parabolic Reflector Antenna:**

$$A_p = \frac{\pi D^2}{4}, \quad A_e = \eta_a A_p$$

$$\text{HPBW} \approx 70^\circ \left(\frac{\lambda}{D}\right), \quad \text{FNBW} \approx 140^\circ \left(\frac{\lambda}{D}\right)$$

Where $D$ is the dish diameter and $\eta_a$ is aperture efficiency.

- **Horn Antenna Directivity:**

$$D_0 = \frac{4\pi}{\lambda^2} \epsilon_{ap}(a_1 b_1)$$

Where $a_1$ and $b_1$ are the physical dimensions of the aperture.

- **Horn Antenna Beamwidths:**

$$\theta_H \approx 67^\circ \left(\frac{\lambda}{a_1}\right), \quad \theta_E \approx 56^\circ \left(\frac{\lambda}{b_1}\right)$$

- **Yagi-Uda Antenna Practical Lengths:**

$$L_{Reflector} \approx 0.50\lambda, \quad L_{Driven} \approx 0.47\lambda, \quad L_{Director} \approx 0.44\lambda$$

- **Log Periodic Dipole Array (LPDA):**

$$\tau = \frac{L_{n+1}}{L_n} = \frac{d_{n+1}}{d_n}, \quad \sigma = \frac{d_n}{2L_n}$$

$$\tan\left(\frac{\alpha}{2}\right) = \frac{1-\tau}{4\sigma}$$

Where $\tau$ is the scale factor, $\sigma$ is the spacing factor, and $\alpha$ is the apex angle.

# Unit 4: Wave Propagation

- **Critical Frequency ($f_c$):**

$$f_c = 9\sqrt{N_{max}}$$

Where $N_{max}$ is the maximum electron density in the ionosphere.

- **Maximum Usable Frequency (MUF):**

$$f_{MUF} = f_c \sec\theta_i = \frac{f_c}{\cos\theta_i}$$

Where $\theta_i$ is the angle of incidence.

- **Optimum Working Frequency (OWF):**

$$f_{OWF} = 0.85 \times f_{MUF}$$

- **Skip Distance ($D_{skip}$):**

$$D_{skip} = 2h\sqrt{\left(\frac{f_{MUF}}{f_c}\right)^2 - 1}$$

Where $h$ is the virtual height of the ionospheric layer.

- **Refractive Index of the Ionosphere ($\mu$):**

$$\mu = \sqrt{1 - \frac{81N}{f^2}}$$

Where $N$ is electron density and $f$ is the operating frequency.

- **Line-of-Sight (LOS) Distance:**

$$d \approx 4.12\sqrt{h_t} \quad \text{(in km, for height in meters)}$$

$$d_{max} = 4.12\left(\sqrt{h_t} + \sqrt{h_r}\right)$$

Where $h_t$ and $h_r$ are the heights of the transmitting and receiving antennas, respectively.

# Unit 5: Smart Antennas and MIMO

- **Phase Shift for Beam Steering ($\beta$):**

$$\beta = -kd\cos(\theta_0)$$

  Where $k = \frac{2\pi}{\lambda}$ and $\theta_0$ is the desired main beam direction.

- **Total Phase Shift ($\psi$):**

$$\psi = kd\cos(\theta) + \beta = 0 \quad \text{(for main beam)}$$

- **Signal-to-Noise Ratio (Linear to dB conversion):**

$$\text{SNR}_{linear} = 10^{\frac{\text{SNR}_{dB}}{10}}$$

- **SISO Channel Capacity (Shannon Capacity):**

$$C_{SISO} = B\log_2(1 + \text{SNR}_{linear}) \quad \text{(in bps)}$$

  Where $B$ is the channel bandwidth.

- **MIMO Channel Capacity:**

$$C_{MIMO} \approx K \times C_{SISO} = K \times B\log_2(1 + \text{SNR}_{linear})$$

$$K = \min(N_T, N_R)$$

  Where $N_T$ is the number of transmitting antennas and $N_R$ is the number of receiving antennas.