Dc (4341102) - Problem Solver

Antigravity AI

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To my students,
who constantly inspire me to find new analogies,
break down complex theories, and make learning
an engineered science.

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Preface

About this Book
Welcome to **Computer Networks**! This textbook is meticulously designed to serve as a comprehensive problem solver and study companion. It provides a robust and intuitive foundation in the core principles of this subject, bridging the gap between theoretical models and practical applications. A unique feature of this book is its focus on decoding complex concepts using clear, step-by-step methodologies and real-world analogies.

Target Audience
This book is intended for students and professionals looking to master the concepts of Computer Networks. Whether you are revising for exams, seeking to solidify your fundamental understanding, or looking for practical examples to clarify abstract concepts, this book has been structured to support your learning journey. It serves as an accessible guide designed to remove the fear of complex topics, replacing it with an appreciation for the underlying mechanics.

Scope and Coverage
The coverage is divided logically to follow standard academic curriculums. Each chapter focuses on key topics, breaking them down into digestible sections:

- **Theoretical Insights** – Core concepts explained intuitively.
- **Method Blocks** – Standardized, step-by-step procedures for solving specific types of problems.
- **Worked Examples** – Fully solved problems that apply the methods in practice.

We hope this resource proves invaluable in your studies and helps you achieve excellence in **Computer Networks**. Happy learning!

Acknowledgments

Writing a comprehensive book that connects the fundamental forces of Chemistry with practical engineering applications requires more than just academic knowledge—it requires a community of support. I would like to express my sincere gratitude to everyone who contributed to the realization of this project.

Specially, I extend my heartfelt gratitude to the **Department of Technical Education (DTE), Government of Gujarat**, and **Government Polytechnic, Palanpur**, for providing an environment that fosters innovation, academic rigor, and continuous professional development.

I am deeply thankful to my family for their unwavering patience and support during the countless hours spent researching, formatting, and refining these chapters.

Finally, my deepest appreciation goes to my students. Your curiosity, challenging questions, and continuous feedback are the true driving force behind the unique analogies and streamlined structure of this book. This textbook was engineered for you.

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About the Author

Milav Jayeshkumar Dabgar is an engineering educator and R&D professional with over 9 years of experience in electronics hardware development, embedded systems, AI/ML, and full-stack software engineering. He currently serves as a **Lecturer (Class - II) & Innovation Leader** at Government Polytechnic, Palanpur, under the Education Department of the Government of Gujarat.

Milav holds a Master of Engineering (ME) in Communication Systems from L.D. College of Engineering and a Bachelor of Engineering (BE) in Electronics & Communication. He is also currently pursuing a **BS in Data Science and Applications from IIT Madras**, where he has completed both the **Diploma in Programming** and the **Diploma in Data Science** with distinction.

Most recently, he completed the **AICTE QIP PG Certificate Program in Deep Learning from SVNIT Surat** with a **perfect 10/10 CGPA**, topping his batch.

A dedicated mentor, Milav has guided numerous student teams in securing over **INR 45 lakhs** in innovation funding, including INR 25 lakhs from Shark Tank India and INR 20 lakhs through iHub Gujarat, resulting in two student patents. He is recognized as an **NPTEL Evangelist** and **NPTEL Discipline Star** in Computer Science, reflecting his commitment to continuous learning and academic excellence.

His technical expertise spans from embedded systems (8051, STM32, Arduino) to modern web technologies (Next.js, Python, PostgreSQL) and Data Science (TensorFlow, PyTorch). Through this textbook, he aims to simplify complex Engineering Chemistry concepts by integrating relatable, real-world analogies drawn from modern tech and engineering landscapes.

Milav is also an active content creator, running a popular **YouTube channel** (@milav.dabgar) where he shares technical tutorials and academic resources. He maintains a personal blog and resource hub at milav.in and can be reached for professional collaborations on LinkedIn.

CHAPTER 1

Elements of Digital Communication System

1.1 Information Content and Entropy

In a digital communication system, the amount of information carried by a message is inversely related to its probability of occurrence.

Methodology:

Calculating Information Content and Entropy **Step 1: Calculate Information Content of a Message** If a message x_i occurs with probability $P(x_i)$, the information content $I(x_i)$ is calculated as:

$$I(x_i) = \log_2\left(\frac{1}{P(x_i)}\right) = -\log_2(P(x_i)) \text{ bits}$$

Note: To calculate base-2 logarithms using standard calculators, use $\log_2(x) = \frac{\log_{10}(x)}{\log_{10}(2)}$ or $\frac{\ln(x)}{\ln(2)}$.
Step 2: Calculate Average Information (Entropy) For a source generating M independent messages with probabilities $P(x_1), P(x_2), \ldots, P(x_M)$, the entropy H (average information per symbol) is:

$$H = \sum_{i=1}^{M} P(x_i) \log_2\left(\frac{1}{P(x_i)}\right) \text{ bits/symbol}$$

Worked Example:

Information Content of Discrete Symbols **Problem:** A discrete memoryless source emits four symbols S_1, S_2, S_3 and S_4 with probabilities 0.5, 0.25, 0.125 and 0.125 respectively. Calculate the information content of each symbol and the entropy of the source.
Solution: Step 1: Calculate Information Content for each symbol

- For S_1: $I(S_1) = \log_2(1/0.5) = \log_2(2) = 1$ bit
- For S_2: $I(S_2) = \log_2(1/0.25) = \log_2(4) = 2$ bits
- For S_3: $I(S_3) = \log_2(1/0.125) = \log_2(8) = 3$ bits
- For S_4: $I(S_4) = \log_2(1/0.125) = \log_2(8) = 3$ bits

Step 2: Calculate Entropy

$$\begin{aligned}
H &= P(S_1)I(S_1) + P(S_2)I(S_2) + P(S_3)I(S_3) + P(S_4)I(S_4) \\
&= (0.5 \times 1) + (0.25 \times 2) + (0.125 \times 3) + (0.125 \times 3) \\
&= 0.5 + 0.5 + 0.375 + 0.375 \\
&= 1.75 \text{ bits/symbol}
\end{aligned}$$

1.2 Channel Capacity

The channel capacity is the maximum theoretical rate at which data can be transmitted over a communication channel without error.

Methodology:

Shannon Hartley Theorem and Channel Capacity **Step 1: Convert SNR from dB to Linear Ratio** If the Signal-to-Noise Ratio (SNR) is given in decibels (dB), convert it to a linear scale first:

$$\text{SNR}_{\text{dB}} = 10\log_{10}(\text{SNR}_{\text{linear}})$$

$$\text{SNR}_{\text{linear}} = 10^{(\text{SNR}_{\text{dB}}/10)}$$

Step 2: Apply the Shannon-Hartley Formula The maximum channel capacity C in bits per second (bps) is given by:

$$C = B\log_2(1 + \text{SNR}_{\text{linear}})$$

Where:

- B is the channel bandwidth in Hertz (Hz).
- $\text{SNR}_{\text{linear}}$ is the signal power to noise power ratio (dimensionless).

Worked Example:

Calculating Capacity of a Telephone Channel **Problem:** An analog voice frequency channel has a bandwidth of 3100 Hz and a signal-to-noise ratio of 30 dB. Calculate the maximum theoretical data rate (channel capacity) of this channel.
Solution: Step 1: Convert SNR from dB to linear scale Given $\text{SNR}_{\text{dB}} = 30$.

$$\begin{aligned} 30 &= 10\log_{10}(\text{SNR}_{\text{linear}}) \\ 3 &= \log_{10}(\text{SNR}_{\text{linear}}) \\ \text{SNR}_{\text{linear}} &= 10^3 = 1000 \end{aligned}$$

Step 2: Calculate Channel Capacity Given Bandwidth $B = 3100$ Hz.

$$\begin{aligned} C &= B\log_2(1 + \text{SNR}_{\text{linear}}) \\ C &= 3100 \times \log_2(1 + 1000) \\ C &= 3100 \times \log_2(1001) \\ C &\approx 3100 \times 9.967 \\ C &\approx 30898 \text{ bps or } 30.898 \text{ kbps} \end{aligned}$$

1.3 Nyquist Sampling Theorem

To accurately reconstruct a continuous analog signal into digital form without aliasing, it must be sampled at an adequate rate.

Methodology:

Nyquist Rate and Nyquist Interval **Step 1: Identify the Maximum Frequency Component** Determine the highest frequency component (f_m) present in the message signal $x(t)$.
Step 2: Calculate the Nyquist Rate The Nyquist rate (f_s) is the minimum sampling frequency required to avoid aliasing:

$$f_s = 2 \times f_m$$

Step 3: Calculate the Nyquist Interval The Nyquist interval (T_s) is the maximum time gap allowed between two consecutive samples:

$$T_s = \frac{1}{f_s} = \frac{1}{2f_m}$$

Worked Example:

Determining Sampling Parameters for a Signal **Problem:** Find the Nyquist rate and Nyquist interval for the continuous-time signal $x(t) = 10\sin(4000\pi t) + 5\cos(6000\pi t)$.
Solution: Step 1: Find the maximum frequency component The signal consists of two sinusoidal components. The general form is $\sin(2\pi ft)$ or $\cos(2\pi ft)$.

- For $10\sin(4000\pi t)$: $2\pi f_1 = 4000\pi \implies f_1 = 2000$ Hz
- For $5\cos(6000\pi t)$: $2\pi f_2 = 6000\pi \implies f_2 = 3000$ Hz

The maximum frequency component $f_m = \max(f_1, f_2) = 3000$ Hz.
Step 2: Calculate Nyquist Rate

$$f_s = 2 \times f_m = 2 \times 3000 = 6000 \text{ Hz} = 6 \text{ kHz}$$

Step 3: Calculate Nyquist Interval

$$T_s = \frac{1}{f_s} = \frac{1}{6000} \approx 0.1667 \text{ ms}$$

1.4 Bit Rate and Baud Rate

In digital transmission, it is crucial to distinguish between the rate at which bits are transmitted and the rate at which signal elements (symbols) are transmitted.

Methodology:

Calculating Bit Rate and Baud Rate **Step 1: Calculate the Bit Rate** The bit rate (R_b) is the number of bits transmitted per second (bps). If an analog signal is sampled at f_s samples per second and each sample is encoded using n bits, the bit rate is:

$$R_b = n \times f_s$$

Step 2: Calculate the Baud Rate The baud rate (Symbol Rate) is the number of signal elements transmitted per second. If M is the number of distinct signal levels (or symbols) and R_b is the bit rate:

$$\text{Baud Rate} = \frac{R_b}{\log_2(M)}$$

For a binary system ($M = 2$), $\log_2(2) = 1$, so Baud Rate is equal to the Bit Rate.

Worked Example:

Calculating Transmission Rates for PCM **Problem:** An analog signal with a bandwidth of 4 kHz is sampled at the Nyquist rate. Each sample is quantized into 256 levels and transmitted using a digital communication system. Calculate the bit rate. If the data is further transmitted using 16-QAM modulation (16 distinct symbols), calculate the baud rate.
Solution: Step 1: Calculate the Sampling Rate Given $f_m = 4$ kHz. The Nyquist rate $f_s = 2 \times f_m = 8$ kHz $= 8000$ samples/sec.
Step 2: Calculate Number of Bits per Sample Given the number of quantization levels $L = 256$. Since $L = 2^n \implies 256 = 2^n \implies n = 8$ bits/sample.

Step 3: Calculate Bit Rate

$$R_b = n \times f_s = 8 \times 8000 = 64,000 \text{ bps or } 64 \text{ kbps}$$

Step 4: Calculate Baud Rate for 16-QAM For 16-QAM, $M = 16$. Number of bits per symbol = $\log_2(16) = 4$ bits/symbol.

$$\text{Baud Rate} = \frac{R_b}{\log_2(M)} = \frac{64000}{4} = 16,000 \text{ baud (symbols/sec)}$$

CHAPTER 2

Digital Modulation Techniques

This chapter focuses on the computational aspects of digital modulation techniques, including bit rate, baud rate, channel capacity, and bandwidth requirements for various modulation schemes such as ASK, FSK, PSK, and QPSK.

2.1 Bit Rate and Baud Rate Calculations

Methodology:

Bit Rate and Baud Rate The relationship between bit rate and baud rate is determined by the number of bits encoded in each signal element (symbol).

1. **Bit Rate (R_b):** The number of bits transmitted per second (bps).
2. **Baud Rate (S):** The number of signal elements (symbols) transmitted per second (baud).
3. **Number of States (M):** The number of distinct signal elements. For binary signals, $M = 2$.
4. **Formula:** The relationship is given by:

$$R_b = S \times \log_2(M)$$

Alternatively, if n is the number of bits per symbol, then $n = \log_2(M)$ and $R_b = S \times n$.

Worked Example:

Bit Rate from Baud Rate A digital communication system uses a 16-QAM modulation scheme. If the baud rate is 2400 baud, calculate the bit rate of the system.
Solution:

1. Identify the given parameters:
   - Modulation scheme = 16-QAM. This means the number of states $M = 16$.
   - Baud rate $S = 2400$ baud.
2. Calculate the number of bits per symbol (n):

$$n = \log_2(M) = \log_2(16) = 4 \text{ bits/symbol}$$

3. Calculate the bit rate (R_b):

$$R_b = S \times n = 2400 \times 4 = 9600 \text{ bps}$$

The bit rate of the system is 9600 bps.

Worked Example:

Baud Rate from Bit Rate An audio signal is digitized at a bit rate of 64 kbps. If the system uses QPSK modulation, determine the required baud rate.

Solution:

1. Identify the given parameters:
   - Bit rate $R_b = 64$ kbps $= 64000$ bps.
   - Modulation scheme = QPSK (Quadrature Phase Shift Keying), so $M = 4$.
2. Calculate the number of bits per symbol (n):

$$n = \log_2(4) = 2 \text{ bits/symbol}$$

3. Calculate the baud rate (S):

$$S = \frac{R_b}{n} = \frac{64000}{2} = 32000 \text{ baud}$$

The required baud rate is 32000 baud (or 32 kbaud).

2.2 Shannon-Hartley Channel Capacity

Methodology:

Shannon Hartley Theorem The Shannon-Hartley theorem states the theoretical maximum data rate (capacity) of a communication channel in the presence of noise.

1. **Channel Capacity (C):** Maximum error-free data rate in bps.
2. **Bandwidth (B):** Channel bandwidth in Hz.
3. **Signal-to-Noise Ratio (SNR):** Ratio of signal power to noise power (linear scale).
4. **Formula:**

$$C = B \log_2 (1 + \text{SNR})$$

5. If SNR is given in decibels (dB), convert it to linear scale first:

$$\text{SNR}_{\text{linear}} = 10^{\frac{\text{SNR}_{\text{dB}}}{10}}$$

Worked Example:

Calculating Channel Capacity A standard telephone line has a bandwidth of 3.4 kHz and a signal-to-noise ratio of 30 dB. Calculate the maximum theoretical channel capacity.

Solution:

1. Identify the given parameters:
   - Bandwidth $B = 3400$ Hz.
   - $\text{SNR}_{\text{dB}} = 30$ dB.
2. Convert SNR from dB to linear scale:

$$\text{SNR} = 10^{\frac{30}{10}} = 10^3 = 1000$$

3. Apply the Shannon-Hartley formula:

$$C = B \log_2(1 + \text{SNR}) = 3400 \times \log_2(1 + 1000)$$

$$C = 3400 \times \log_2(1001)$$

4. Compute the value (using $\log_2(x) = \frac{\ln(x)}{\ln(2)}$):

$$\log_2(1001) \approx 9.967$$

$$C \approx 3400 \times 9.967 \approx 33888 \text{ bps} \approx 33.9 \text{ kbps}$$

The maximum channel capacity is approximately 33.9 kbps.

2.3 Bandwidth Requirements for Digital Modulation

Methodology:

Null-to-Null Bandwidth The bandwidth (BW) required for transmitting digital signals depends on the modulation technique and the bit rate (R_b). The most common estimation used in introductory digital communication is the null-to-null bandwidth.

1. **Amplitude Shift Keying (ASK) and Binary Phase Shift Keying (BPSK):**

$$BW = 2R_b$$

2. **Quadrature Phase Shift Keying (QPSK):** Because QPSK transmits 2 bits per symbol, its baud rate is half the bit rate.

$$BW = R_b$$

3. **Frequency Shift Keying (FSK):** FSK uses two different frequencies (f_1 and f_2) separated by a deviation $2\Delta f = |f_1 - f_2|$.

$$BW = 2\Delta f + 2R_b$$

4. **M-ary PSK and M-ary QAM:**

$$BW = \frac{2R_b}{\log_2(M)}$$

Worked Example:

Bandwidth of BPSK and QPSK A digital signal with a bit rate of 10 Mbps is to be transmitted. Calculate the minimum null-to-null bandwidth required if the modulation scheme is (a) BPSK and (b) QPSK.
Solution:

1. Identify the given bit rate:

$$R_b = 10 \text{ Mbps} = 10 \times 10^6 \text{ bps}$$

2. **(a) For BPSK:**

$$BW_{\text{BPSK}} = 2R_b = 2 \times 10 \text{ MHz} = 20 \text{ MHz}$$

3. **(b) For QPSK:**

$$BW_{\text{QPSK}} = R_b = 10 \text{ MHz}$$

The required bandwidth is 20 MHz for BPSK and 10 MHz for QPSK. QPSK is more bandwidth-efficient.

Worked Example:

Bandwidth of FSK An FSK system transmits data at a rate of 4800 bps. The mark and space frequencies are 50 kHz and 55 kHz respectively. Calculate the bandwidth of the FSK signal.
Solution:

1. Identify the given parameters:
   - Bit rate $R_b = 4800$ bps.
   - Mark frequency $f_1 = 50$ kHz.
   - Space frequency $f_2 = 55$ kHz.
2. Calculate the frequency separation $2\Delta f$:

$$2\Delta f = |f_1 - f_2| = |50 \text{ kHz} - 55 \text{ kHz}| = 5 \text{ kHz} = 5000 \text{ Hz}$$

3. Calculate the required bandwidth using the FSK formula:

$$BW = 2\Delta f + 2R_b = 5000 + 2(4800)$$

$$BW = 5000 + 9600 = 14600 \text{ Hz} = 14.6 \text{ kHz}$$

The bandwidth of the FSK signal is 14.6 kHz.

2.4 Energy per Bit to Noise Density Ratio

Methodology:

Energy per Bit Calculation The ratio E_b/N_0 is a normalized signal-to-noise ratio measure used to compare different digital modulation schemes independent of their specific bandwidths.

1. **Formula:**

$$\frac{E_b}{N_0} = \frac{S/R_b}{N/B} = \left(\frac{S}{N}\right) \times \left(\frac{B}{R_b}\right)$$

   Where:
   - E_b = Energy per bit (Joules)
   - N_0 = Noise power spectral density (Watts/Hz)
   - S/N = Signal-to-noise ratio (linear)
   - B = Bandwidth (Hz)
   - R_b = Bit rate (bps)
2. To calculate E_b/N_0 in dB directly from SNR in dB:

$$\left(\frac{E_b}{N_0}\right)_{\text{dB}} = \text{SNR}_{\text{dB}} + 10\log_{10}\left(\frac{B}{R_b}\right)$$

Worked Example:

Calculating Eb/N0 in dB A communication receiver has a bandwidth of 2 MHz and receives a signal with a bit rate of 1 Mbps. If the measured signal-to-noise ratio (SNR) is 12 dB, find the E_b/N_0 ratio in dB.
Solution:

1. Identify the given parameters:
   - Bandwidth $B = 2$ MHz $= 2 \times 10^6$ Hz.
   - Bit rate $R_b = 1$ Mbps $= 1 \times 10^6$ bps.
   - $\text{SNR}_{\text{dB}} = 12$ dB.

2. Calculate the bandwidth-to-bit rate ratio:

$$\frac{B}{R_b} = \frac{2 \times 10^6}{1 \times 10^6} = 2$$

3. Use the logarithmic formula for E_b/N_0:

$$\left(\frac{E_b}{N_0}\right)_{\text{dB}} = \text{SNR}_{\text{dB}} + 10\log_{10}\left(\frac{B}{R_b}\right)$$

$$\left(\frac{E_b}{N_0}\right)_{\text{dB}} = 12 + 10\log_{10}(2)$$

4. Compute $10\log_{10}(2) \approx 10 \times 0.301 = 3.01$ dB.

5. Add the values to find the final result:

$$\left(\frac{E_b}{N_0}\right)_{\text{dB}} = 12 + 3.01 = 15.01 \text{ dB}$$

The E_b/N_0 ratio is 15.01 dB.

CHAPTER 3

Information Theory and Coding

3.1 Information and Entropy

Methodology:

Calculation of Information and Entropy **1. Measure of Information (I):**
The amount of information carried by a discrete message x_i with probability $P(x_i)$ is given by:

$$I(x_i) = \log_2 \left(\frac{1}{P(x_i)} \right) \text{ bits}$$

2. Entropy (H):
Entropy is the average information per symbol produced by a source. For a source producing m symbols $x_1, x_2, \ldots, x_m$ with probabilities $P(x_1), P(x_2), \ldots, P(x_m)$:

$$H = \sum_{i=1}^{m} P(x_i) \log_2 \left(\frac{1}{P(x_i)} \right) \text{ bits/symbol}$$

Maximum entropy $H_{max} = \log_2(m)$ occurs when all symbols are equiprobable ($P(x_i) = 1/m$).
3. Information Rate (R):
If a source emits symbols at a rate of r symbols per second, the information rate is:

$$R = r \times H \text{ bits/second (bps)}$$

Worked Example:

Calculating Entropy and Information Rate A discrete memoryless source emits five symbols with probabilities $0.4, 0.2, 0.2, 0.1$, and 0.1. The symbol rate is 1000 symbols per second. Calculate the entropy of the source and the information rate.
Step 1: Calculate the Information for each symbol

- $I_1 = \log_2(1/0.4) = 1.322$ bits
- $I_2 = \log_2(1/0.2) = 2.322$ bits
- $I_3 = \log_2(1/0.2) = 2.322$ bits
- $I_4 = \log_2(1/0.1) = 3.322$ bits
- $I_5 = \log_2(1/0.1) = 3.322$ bits

Step 2: Calculate the Entropy (H)

$$H = \sum P(x_i) I_i$$
$$H = 0.4(1.322) + 0.2(2.322) + 0.2(2.322) + 0.1(3.322) + 0.1(3.322)$$
$$H = 0.5288 + 0.4644 + 0.4644 + 0.3322 + 0.3322$$
$$H = 2.122 \text{ bits/symbol}$$

Step 3: Calculate Information Rate (R)

$$R = r \times H = 1000 \times 2.122 = 2122 \text{ bits/sec}$$

3.2 Channel Capacity

Methodology:

Shannon Hartley Theorem The Shannon-Hartley theorem states the maximum capacity (C) of a channel subject to Additive White Gaussian Noise (AWGN):

$$C = B \log_2 \left(1 + \frac{S}{N}\right) \text{ bits/sec}$$

where:

- B = Channel Bandwidth in Hz
- S/N = Signal-to-Noise Power Ratio (linear scale)

Note: If SNR is given in decibels (dB), convert it to linear scale before using the formula:

$$\left(\frac{S}{N}\right)_{dB} = 10 \log_{10} \left(\frac{S}{N}\right) \implies \frac{S}{N} = 10^{\frac{SNR_{dB}}{10}}$$

Worked Example:

Calculating Maximum Channel Capacity A continuous channel has a bandwidth of 3 kHz and a signal-to-noise ratio of 30 dB. Calculate the maximum theoretical information rate (Channel Capacity).

Step 1: Convert SNR from dB to linear scale

$$SNR_{dB} = 30 \text{ dB}$$

$$\frac{S}{N} = 10^{\frac{30}{10}} = 10^3 = 1000$$

Step 2: Apply the Shannon-Hartley Formula

$$C = B \log_2 \left(1 + \frac{S}{N}\right)$$
$$C = 3000 \log_2 (1 + 1000)$$
$$C = 3000 \log_2 (1001)$$

Using the conversion $\log_2(x) = \frac{\log_{10}(x)}{\log_{10}(2)}$:

$$\log_2(1001) \approx \frac{3.0004}{0.3010} \approx 9.967$$

$$C = 3000 \times 9.967 = 29901 \text{ bits/sec}$$

3.3 Source Coding Methods

3.3.1 Shannon-Fano Coding

Methodology:

Shannon Fano Algorithm **1.** List the source symbols in order of decreasing probability.
2. Partition the set of symbols into two groups (upper and lower) such that the sum of probabilities in each group is as close to equal as possible.
3. Assign a '0' to all symbols in the upper group and a '1' to all symbols in the lower group.
4. Recursively apply steps 2 and 3 to each subgroup until every subgroup contains exactly one symbol.
5. Calculate Average Code Length (L) and Coding Efficiency (η):

$$L = \sum_{i=1}^{m} P(x_i) l_i \text{ bits/symbol}$$

$$\eta = \frac{H}{L} \times 100\%$$

where l_i is the length of the assigned codeword for symbol x_i.

Worked Example:

Applying Shannon Fano Coding A source generates 5 symbols with probabilities 0.3, 0.25, 0.2, 0.15, and 0.1. Generate the Shannon-Fano code and calculate its efficiency.
Step 1 & 2: Initial Partitioning
Symbols sorted by probability: $x_1(0.3), x_2(0.25), x_3(0.2), x_4(0.15), x_5(0.1)$.
Split into two groups aiming for equal sum:

- Group 1: x_1, x_2 (Sum $= 0.55$) $\rightarrow$ Assign bit '0'
- Group 2: x_3, x_4, x_5 (Sum $= 0.45$) $\rightarrow$ Assign bit '1'

Step 3: Sub-partitioning Group 1 (x_1, x_2)

- Subgroup 1.1: x_1 (Sum $= 0.3$) $\rightarrow$ Assign bit '0'. Code for $x_1 = 00$
- Subgroup 1.2: x_2 (Sum $= 0.25$) $\rightarrow$ Assign bit '1'. Code for $x_2 = 01$

Step 4: Sub-partitioning Group 2 (x_3, x_4, x_5)

- Subgroup 2.1: x_3 (Sum $= 0.2$) $\rightarrow$ Assign bit '0'. Code for $x_3 = 10$
- Subgroup 2.2: x_4, x_5 (Sum $= 0.25$) $\rightarrow$ Assign bit '1'.

Step 5: Sub-partitioning Subgroup 2.2 (x_4, x_5)

- Sub-subgroup 2.2.1: x_4 (Sum $= 0.15$) $\rightarrow$ Assign bit '0'. Code for $x_4 = 110$
- Sub-subgroup 2.2.2: x_5 (Sum $= 0.1$) $\rightarrow$ Assign bit '1'. Code for $x_5 = 111$

Summary Table:

Symbol	Probability	Codeword	Length (l_i)
x_1	0.3	00	2
x_2	0.25	01	2
x_3	0.2	10	2
x_4	0.15	110	3
x_5	0.1	111	3

Worked Example:

Calculating Efficiency for Shannon Fano **Step 1: Calculate Entropy (H)**

$$H = 0.3\log_2(1/0.3) + 0.25\log_2(1/0.25) + 0.2\log_2(1/0.2) + 0.15\log_2(1/0.15) + 0.1\log_2(1/0.1)$$
$$H = 0.3(1.737) + 0.25(2) + 0.2(2.322) + 0.15(2.737) + 0.1(3.322)$$
$$H = 0.521 + 0.5 + 0.464 + 0.411 + 0.332 = 2.228 \text{ bits/symbol}$$

Step 2: Calculate Average Length (L)

$$L = \sum P(x_i)l_i = 0.3(2) + 0.25(2) + 0.2(2) + 0.15(3) + 0.1(3)$$
$$L = 0.6 + 0.5 + 0.4 + 0.45 + 0.3 = 2.25 \text{ bits/symbol}$$

Step 3: Calculate Efficiency (η)

$$\eta = \frac{H}{L} \times 100\% = \frac{2.228}{2.25} \times 100\% = 99.02\%$$

3.3.2 Huffman Coding

Methodology:

Huffman Coding Algorithm **1.** Arrange the source symbols in decreasing order of probability.
2. Combine the two symbols with the lowest probabilities to form a new composite node. The probability of this new node is the sum of the two probabilities.
3. Re-sort the new list of probabilities in descending order.
4. Repeat steps 2 and 3 until only one node remains (with a total probability of 1.0).
5. Trace backward from the final node (1.0) to the original symbols. At each splitting branch, assign a '0' to the upper branch and a '1' to the lower branch (or vice versa, maintaining consistency).
6. The codeword for each symbol is generated by reading the bits from the root (1.0) to the original symbol leaf.

Worked Example:

Applying Huffman Coding A source generates 5 symbols with probabilities 0.4, 0.2, 0.2, 0.1, and 0.1. Generate the Huffman code.
Step 1: Initial Sorting and Combination

- Initial: $x_1(0.4), x_2(0.2), x_3(0.2), x_4(0.1), x_5(0.1)$
- Combine two lowest: $x_4(0.1)$ and $x_5(0.1) \rightarrow$ New Node $N_1(0.2)$. (Branches: $x_4 = 0, x_5 = 1$)

Step 2: First Re-sort and Combination

- List: $x_1(0.4), x_2(0.2), x_3(0.2), N_1(0.2)$
- Combine two lowest: $x_3(0.2)$ and $N_1(0.2) \rightarrow$ New Node $N_2(0.4)$. (Branches: $x_3 = 0, N_1 = 1$)

Step 3: Second Re-sort and Combination

- List: $x_1(0.4), N_2(0.4), x_2(0.2)$
- Combine two lowest: $N_2(0.4)$ and $x_2(0.2) \rightarrow$ New Node $N_3(0.6)$. (Branches: $N_2 = 0, x_2 = 1$)

Step 4: Final Combination

- List: $N_3(0.6), x_1(0.4)$
- Combine two remaining: $N_3(0.6)$ and $x_1(0.4) \rightarrow$ Root (1.0). (Branches: $N_3 = 0, x_1 = 1$)

Step 5: Traceback to derive codes

- x_1 is directly connected to Root with branch '1' $\rightarrow$ Code: **1**
- x_2 traceback: Root $\rightarrow N_3(0) \rightarrow x_2(1) \rightarrow$ Code: **01**
- x_3 traceback: Root $\rightarrow N_3(0) \rightarrow N_2(0) \rightarrow x_3(0) \rightarrow$ Code: **000**
- x_4 traceback: Root $\rightarrow N_3(0) \rightarrow N_2(0) \rightarrow N_1(1) \rightarrow x_4(0) \rightarrow$ Code: **0010**
- x_5 traceback: Root $\rightarrow N_3(0) \rightarrow N_2(0) \rightarrow N_1(1) \rightarrow x_5(1) \rightarrow$ Code: **0011**

Summary Table:

Symbol	Probability	Codeword	Length (l_i)
x_1	0.4	1	1
x_2	0.2	01	2
x_3	0.2	000	3
x_4	0.1	0010	4
x_5	0.1	0011	4

3.4 Error Control Coding

Methodology:

Hamming Code Encoding The (n, k) Hamming Code encodes k data bits into an n-bit codeword by appending r parity bits ($n = k + r$). **1. Determine Parity Bits (r):** Find the smallest r satisfying $2^r \geq k + r + 1$.
2. Bit Positioning: Position bits from left to right starting at index 1. Positions that are powers of 2 (1, 2, 4, 8) are parity bits ($P_1, P_2, P_4, \dots$). Remaining positions are data bits ($D_3, D_5, D_6, D_7, \dots$).
3. Parity Calculation (Even Parity): Each parity bit checks specific bit positions where the binary representation of the position index has a '1' in the same place as the parity bit's index.

- P_1 (xxx1) checks positions 1, 3, 5, 7, 9, 11 ...
- P_2 (xx1x) checks positions 2, 3, 6, 7, 10, 11 ...
- P_4 (x1xx) checks positions 4, 5, 6, 7, 12, 13 ...

Set P_i such that the total number of 1s in its checked positions is even (i.e., XOR sum is 0).

Worked Example:

Generating a 7 4 Hamming Code Generate the (7,4) even-parity Hamming code for the 4-bit data word 1011.
Step 1: Identify bits
$k = 4$ data bits. Number of parity bits r: $2^3 \geq 4 + 3 + 1 \implies r = 3$.
Total length $n = 7$.
Step 2: Assign bit positions

- Position 1: P_1
- Position 2: P_2
- Position 3: $D_3 = 1$
- Position 4: P_4
- Position 5: $D_5 = 0$

- Position 6: $D_6 = 1$
- Position 7: $D_7 = 1$

Codeword format: $P_1, P_2, 1, P_4, 0, 1, 1$.
Step 3: Calculate Parity Bits

- For P_1 (checks 1, 3, 5, 7): $P_1 \oplus D_3 \oplus D_5 \oplus D_7 = 0$
  $P_1 \oplus 1 \oplus 0 \oplus 1 = 0 \implies P_1 \oplus 0 = 0 \implies P_1 = 0$.
- For P_2 (checks 2, 3, 6, 7): $P_2 \oplus D_3 \oplus D_6 \oplus D_7 = 0$
  $P_2 \oplus 1 \oplus 1 \oplus 1 = 0 \implies P_2 \oplus 1 = 0 \implies P_2 = 1$.
- For P_4 (checks 4, 5, 6, 7): $P_4 \oplus D_5 \oplus D_6 \oplus D_7 = 0$
  $P_4 \oplus 0 \oplus 1 \oplus 1 = 0 \implies P_4 \oplus 0 = 0 \implies P_4 = 0$.

Step 4: Final Codeword
Substituting the parity bits back into the format:
Codeword = **0 1 1 0 0 1 1**

Methodology:

Hamming Code Decoding and Error Correction **1.** Receive the n-bit codeword.
2. Calculate the **Syndrome word** $S = S_r \dots S_2 S_1$, where each S_i is the XOR sum of the bits checked by parity bit P_i.

- $S_1 = C_1 \oplus C_3 \oplus C_5 \oplus C_7$
- $S_2 = C_2 \oplus C_3 \oplus C_6 \oplus C_7$
- $S_4 = C_4 \oplus C_5 \oplus C_6 \oplus C_7$

3. Evaluate the Syndrome:

- If $S = 0$, there is no error.
- If $S \neq 0$, the decimal equivalent of the binary syndrome word indicates the position of the erroneous bit.

4. Correct the error by inverting the bit at the indicated position.

Worked Example:

Correcting a Single Bit Error A receiver receives the (7,4) Hamming code **0 1 0 0 0 1 1**. Determine if there is an error, and if so, correct it. Assume even parity.
Step 1: Extract received bits

- $C_1 = 0$
- $C_2 = 1$
- $C_3 = 0$
- $C_4 = 0$
- $C_5 = 0$
- $C_6 = 1$
- $C_7 = 1$

Step 2: Calculate Syndromes

- $S_1 = C_1 \oplus C_3 \oplus C_5 \oplus C_7 = 0 \oplus 0 \oplus 0 \oplus 1 = \mathbf{1}$
- $S_2 = C_2 \oplus C_3 \oplus C_6 \oplus C_7 = 1 \oplus 0 \oplus 1 \oplus 1 = \mathbf{1}$
- $S_4 = C_4 \oplus C_5 \oplus C_6 \oplus C_7 = 0 \oplus 0 \oplus 1 \oplus 1 = \mathbf{0}$

Step 3: Determine Error Position
The syndrome word is $S = S_4 S_2 S_1 = \mathbf{011}$.
The decimal equivalent of binary 011 is 3.
This indicates that an error has occurred at **Position 3**.
Step 4: Correct the Error
The bit at position 3 is $C_3 = 0$.
Invert it to correct the error: $C_3 = 1$.
The corrected codeword is **0 1 1 0 0 1 1**. The original data word (positions 3, 5, 6, 7) is **1011**.

CHAPTER 4

Multiplexing and Multiple Access Techniques

Multiplexing and multiple access techniques allow multiple signals or users to share a single communication medium. This chapter focuses on the computational aspects of Frequency Division Multiplexing (FDM), Time Division Multiplexing (TDM), and Code Division Multiple Access (CDMA).

4.1 Frequency Division Multiplexing (FDM)

In FDM, the total available bandwidth of a communication channel is divided into a series of non-overlapping frequency bands, each carrying a separate signal. Guard bands are inserted between adjacent channels to prevent cross-talk.

Methodology:

FDM Bandwidth Calculation To calculate the total bandwidth required for FDM or the number of channels that can be accommodated:

1. Identify the number of channels N.
2. Identify the bandwidth required per channel BW_c.
3. Identify the guard band bandwidth BW_g allocated between adjacent channels.
4. Calculate the total bandwidth BW_{total} using the formula:

$$BW_{total} = (N \times BW_c) + (N-1) \times BW_g$$

5. If the total available bandwidth BW_{avail} is given and you need to find the maximum number of channels N:

$$N = \lfloor \frac{BW_{avail} + BW_g}{BW_c + BW_g} \rfloor$$

Worked Example:

Calculate Total Bandwidth for Multiplexed Channels Five distinct voice signals are to be multiplexed using FDM. Each voice signal requires a bandwidth of 4 kHz. To prevent interference a guard band of 1 kHz is placed between adjacent channels. Calculate the total bandwidth required for the multiplexed signal.

Solution:

1. Identify the given parameters: Number of channels $N = 5$ Channel bandwidth $BW_c = 4$ kHz Guard band $BW_g = 1$ kHz
2. The number of guard bands is always $N - 1$: Number of guard bands $= 5 - 1 = 4$
3. Calculate the total bandwidth:

$$BW_{total} = (N \times BW_c) + (N-1) \times BW_g$$

$$BW_{total} = (5 \times 4 \text{ kHz}) + (4 \times 1 \text{ kHz})$$

$$BW_{total} = 20 \text{ kHz} + 4 \text{ kHz} = 24 \text{ kHz}$$

The total bandwidth required is 24 kHz.

Worked Example:

Determine Maximum Number of FDM Channels A coaxial cable has an available bandwidth of 1.5 MHz. It is used to transmit multiple signals using FDM. Each signal requires a bandwidth of 100 kHz and a guard band of 10 kHz is required between adjacent channels. Find the maximum number of signals that can be transmitted simultaneously.

Solution:

1. Identify the given parameters: Available bandwidth $BW_{avail} = 1.5$ MHz $= 1500$ kHz Channel bandwidth $BW_c = 100$ kHz Guard band $BW_g = 10$ kHz
2. Apply the rearranged FDM bandwidth formula for N:

$$N = \lfloor \frac{BW_{avail} + BW_g}{BW_c + BW_g} \rfloor$$

$$N = \lfloor \frac{1500 + 10}{100 + 10} \rfloor$$

$$N = \lfloor \frac{1510}{110} \rfloor = \lfloor 13.72 \rfloor = 13$$

A maximum of 13 channels can be transmitted.

4.2 Time Division Multiplexing (TDM)

In TDM, multiple digital signals are transmitted over a single channel by dividing the time frame into slots, one slot for each signal.

Methodology:

TDM Link Data Rate and Frame Parameters For synchronous TDM multiplexing N input connections each with a data rate of R_{in} bps:

1. Determine the number of bits grouped per time slot for each connection (b bits/slot).
2. Calculate the frame rate f_{frame} (number of frames per second):

$$f_{frame} = \frac{R_{in}}{b} \text{ frames/second}$$

3. Calculate the frame duration T_{frame}:

$$T_{frame} = \frac{1}{f_{frame}} \text{ seconds}$$

4. Calculate the number of bits per frame B_{frame}, including synchronization (framing) bits S:

$$B_{frame} = (N \times b) + S \text{ bits/frame}$$

5. Calculate the total link data rate R_{link}:

$$R_{link} = B_{frame} \times f_{frame} \text{ bps}$$

Worked Example:

TDM with Synchronization Bits Four digital sources each transmitting at 250 kbps are to be multiplexed using synchronous TDM. Each output frame carries 1 bit from each source and adds 1 extra bit for synchronization. Calculate the frame rate frame duration number of bits per frame and total output data rate.

Solution:

1. Identify parameters: Number of sources $N = 4$ Input rate $R_{in} = 250$ kbps $= 250,000$ bps Bits per slot $b = 1$ bit Sync bits $S = 1$ bit

2. Calculate frame rate:
$$f_{frame} = \frac{R_{in}}{b} = \frac{250,000}{1} = 250,000 \text{ frames/sec}$$

3. Calculate frame duration:
$$T_{frame} = \frac{1}{f_{frame}} = \frac{1}{250,000} = 4 \times 10^{-6} \text{ s} = 4\mu\text{s}$$

4. Calculate bits per frame:
$$B_{frame} = (N \times b) + S = (4 \times 1) + 1 = 5 \text{ bits/frame}$$

5. Calculate total output data rate:
$$R_{link} = B_{frame} \times f_{frame} = 5 \times 250,000 = 1,250,000 \text{ bps} = 1.25 \text{ Mbps}$$

Worked Example:

Digital Telephony T1 Carrier System In the T1 carrier system 24 voice channels are multiplexed using TDM. Each voice channel is sampled at 8 kHz and each sample is quantized into 8 bits. One framing bit is added at the end of each frame for synchronization. Calculate the link data rate of the T1 system.

Solution:

1. Identify parameters: $N = 24$ channels Sampling rate $f_s = 8000$ samples/sec (this dictates the frame rate) Bits per sample $b = 8$ bits/channel/frame Framing bit $S = 1$ bit/frame

2. Calculate the frame rate (equal to the sampling rate of a single channel):
$$f_{frame} = 8000 \text{ frames/sec}$$

3. Calculate bits per frame:
$$B_{frame} = (N \times b) + S = (24 \times 8) + 1 = 192 + 1 = 193 \text{ bits/frame}$$

4. Calculate the total link data rate:
$$R_{link} = B_{frame} \times f_{frame} = 193 \times 8000 = 1,544,000 \text{ bps} = 1.544 \text{ Mbps}$$

4.3 TDMA Frame Efficiency

In Time Division Multiple Access (TDMA), a frame contains payload data as well as overhead (synchronization, guard times, control bits). Frame efficiency measures the proportion of a frame used for actual data.

Methodology:

TDMA Frame Efficiency Calculation To compute the efficiency of a TDMA frame:

1. Determine the total number of bits per frame B_T.
2. Determine the total number of overhead bits per frame B_{OH}. This includes guard bits, preamble, and synchronization sequences.
3. Calculate the TDMA frame efficiency η:

$$\eta = \left(1 - \frac{B_{OH}}{B_T}\right) \times 100\%$$

4. Alternatively, using data bits $B_{data} = B_T - B_{OH}$:

$$\eta = \left(\frac{B_{data}}{B_T}\right) \times 100\%$$

Worked Example:

Efficiency of a TDMA Frame A TDMA system uses a frame consisting of 8 time slots. Each time slot contains a 156.25-bit sequence, which is composed of 114 data bits, 8 training bits, 3 tail bits, and 31.25 guard bits. Calculate the overall frame efficiency.

Solution:

1. Identify the structure of a single time slot: Total bits per slot = 156.25 bits Data bits per slot = 114 bits Overhead bits per slot = $8 + 3 + 31.25 = 42.25$ bits
2. Calculate parameters for the entire frame (8 slots):

$$B_T = 8 \times 156.25 = 1250 \text{ bits/frame}$$

$$B_{OH} = 8 \times 42.25 = 338 \text{ bits/frame}$$

3. Calculate the frame efficiency:

$$\eta = \left(1 - \frac{B_{OH}}{B_T}\right) \times 100\%$$

$$\eta = \left(1 - \frac{338}{1250}\right) \times 100\%$$

$$\eta = (1 - 0.2704) \times 100\% = 72.96\%$$

The frame efficiency is 72.96%.

4.4 Code Division Multiple Access (CDMA)

CDMA allows multiple users to transmit simultaneously in the same frequency band by assigning unique orthogonal mathematical codes to each user.

Methodology:

Processing Gain Calculation Processing Gain (PG) indicates a system's ability to resist interference. It is the ratio of the spread bandwidth to the original data bandwidth.

1. Identify the baseband data bit rate R_b.
2. Identify the spread spectrum chip rate R_c.

3. Calculate the Processing Gain as a linear ratio:

$$PG = \frac{R_c}{R_b}$$

4. Calculate the Processing Gain in decibels (dB):

$$PG_{dB} = 10 \log_{10} \left(\frac{R_c}{R_b} \right)$$

Worked Example:

Calculate Processing Gain A Direct Sequence Spread Spectrum (DSSS) CDMA system transmits a digital audio signal with a baseband data rate of 9.6 kbps. The system uses a pseudo-noise (PN) sequence with a chip rate of 1.2288 Mbps. Determine the processing gain in both linear and dB scales.

Solution:

1. Identify rates: $R_b = 9.6$ kbps $= 9600$ bps $R_c = 1.2288$ Mbps $= 1,228,800$ bps

2. Calculate linear Processing Gain:

$$PG = \frac{R_c}{R_b} = \frac{1,228,800}{9600} = 128$$

3. Calculate Processing Gain in dB:

$$PG_{dB} = 10 \log_{10}(128) \approx 10 \times 2.107 = 21.07 \text{ dB}$$

Methodology:

Orthogonal Code Generation using Walsh Matrices Walsh matrices are used to generate perfectly orthogonal codes for Synchronous CDMA.

1. Start with the fundamental 1×1 matrix:

$$W_1 = \begin{bmatrix} 1 \end{bmatrix}$$

2. Generate higher-order matrices W_{2N} from W_N using the recursive formula:

$$W_{2N} = \begin{bmatrix} W_N & W_N \\ W_N & -W_N \end{bmatrix}$$

3. The rows of the resulting matrix represent mutually orthogonal codes assigned to different users.

Worked Example:

Generate W2 and W4 Walsh Matrices Using the recursive matrix method generate the Walsh matrices W_2 and W_4.

Solution:

1. Start with W_1:

$$W_1 = \begin{bmatrix} 1 \end{bmatrix}$$

2. Apply the recursive formula for W_2:

$$W_2 = \begin{bmatrix} W_1 & W_1 \\ W_1 & -W_1 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

3. Apply the formula again for W_4 using W_2:

$$W_4 = \begin{bmatrix} W_2 & W_2 \\ W_2 & -W_2 \end{bmatrix}$$

$$W_4 = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \end{bmatrix}$$

The rows of W_4 provide four unique 4-chip orthogonal sequences for four different users.

Methodology:

CDMA Data Spreading and Despreading To multiplex and subsequently separate multiple users in a CDMA channel:

1. **Bit Representation:** Convert binary data bits (0 and 1) into polar form: Logic 0 $\rightarrow -1$ Logic 1 $\rightarrow +1$
2. **Spreading:** Multiply each user's polar data bit by their assigned Walsh code vector C_i to get the transmitted signal vector S_i.
3. **Combining:** Sum the individual signal vectors to create the composite signal vector transmitted over the channel:
$$S_{comp} = S_1 + S_2 + \cdots + S_N$$
4. **Despreading (Receiver):** To recover the data for User k, multiply (dot product) the composite signal S_{comp} by User k's code C_k.
5. **Integration and Decision:** Sum the result from step 4 over the code length L, and divide by L:
$$Data_k = \frac{1}{L} \sum (S_{comp} \cdot C_k)$$
If the result is $+1$, the received bit is 1. If -1, the received bit is 0. If 0, no data was transmitted.

Worked Example:

Signal Transmission and Recovery in CDMA A CDMA system has two users. User A is assigned code $C_A = [1, 1]$ and User B is assigned code $C_B = [1, -1]$. User A wishes to transmit bit 1, and User B wishes to transmit bit 0. Calculate the composite signal transmitted and mathematically demonstrate how User B's receiver recovers its data.

Solution:

1. **Bit Representation:** User A's data $= 1 \rightarrow D_A = +1$ User B's data $= 0 \rightarrow D_B = -1$
2. **Spreading:** Multiply data by codes: User A's signal: $S_A = D_A \times C_A = +1 \times [1, 1] = [1, 1]$ User B's signal: $S_B = D_B \times C_B = -1 \times [1, -1] = [-1, 1]$
3. **Combining:** Composite signal $S_{comp} = S_A + S_B$: $S_{comp} = [1 + (-1), 1 + 1] = [0, 2]$
4. **Despreading at User B's Receiver:** Multiply the composite signal by User B's code element-wise: $S_{comp} \cdot C_B = [0 \times 1, 2 \times (-1)] = [0, -2]$
5. **Integration and Decision:** Sum the elements and divide by code length $L = 2$:
$$Data_B = \frac{0 + (-2)}{2} = \frac{-2}{2} = -1$$
The result is -1, which translates back to binary 0. User B's data is successfully recovered.

CHAPTER 5

Applications of Digital Communication

Digital communication techniques are the foundation of modern technological infrastructure. They enable a vast array of applications from mobile networks and satellite links to high-speed optical fibers and radar systems. This chapter focuses on the computational aspects and problem-solving methodologies associated with these key applications. We will explore the mathematical models used to evaluate cellular system capacity, satellite link budgets, optical transmission parameters, radar range, and multiplexing data rates.

5.1 Cellular Systems and Frequency Reuse

The core concept of cellular communication is frequency reuse. This allows the same radio frequencies to be used in different geographical areas to maximize network capacity without causing unacceptable interference.

Methodology:

Cellular Capacity and Cluster Size Calculation The design of a cellular system involves determining the cluster size and the number of channels available per cell.

1. **Cluster Size (N):** The number of cells in a cluster is given by:

$$N = i^2 + ij + j^2$$

where i and j are non-negative integers representing the coordinate steps along the hexagonal grid.

2. **Total Capacity (C):** If a system has a total of S duplex channels and a cluster size of N, the number of channels per cell k is:

$$k = \frac{S}{N}$$

The total capacity of a network with M clusters is $C = M \times S$.

3. **Signal to Interference Ratio (SIR):** For an omnidirectional antenna system, the worst-case SIR is approximated by:

$$\text{SIR} = \frac{1}{K_I}\left(\frac{D}{R}\right)^n$$

where K_I is the number of co-channel interfering cells (typically 6 in a hexagonal grid), D is the distance between centers of nearest co-channel cells, R is the cell radius, and n is the path loss exponent.

4. **Co-channel Reuse Ratio (Q):** Defined as $Q = \frac{D}{R} = \sqrt{3N}$. Substituting this into the SIR equation:

$$\text{SIR} = \frac{(\sqrt{3N})^n}{6}$$

5. **SIR in Decibels (dB):** $\text{SIR}_{\text{dB}} = 10\log_{10}(\text{SIR})$.

Worked Example:

Calculating Cluster Size and Channels per Cell A cellular system is allocated a total of 1001 radio channels. If the network is designed using a frequency reuse pattern with coordinate steps $i = 2$ and $j = 1$:

1. Calculate the cluster size N.
2. Determine the number of channels available per cell.

Solution:

1. **Step 1: Calculate the cluster size N.** Using the formula $N = i^2 + ij + j^2$ with $i = 2$ and $j = 1$:

$$N = 2^2 + (2)(1) + 1^2 = 4 + 2 + 1 = 7$$

The cluster size is 7 cells.

2. **Step 2: Determine channels per cell.** Given the total number of channels $S = 1001$ and $N = 7$. The number of channels per cell k is:

$$k = \frac{S}{N} = \frac{1001}{7} = 143 \text{ channels/cell}$$

Worked Example:

Signal to Interference Ratio Calculation In a cellular network employing a 7-cell reuse pattern with omnidirectional antennas, the path loss exponent is $n = 4$. Calculate the worst-case signal to interference ratio (SIR) in linear scale and in decibels.

Solution:

1. **Step 1: Identify given parameters.** Cluster size $N = 7$, path loss exponent $n = 4$. The number of co-channel interferers for a standard hexagonal grid is $K_I = 6$.

2. **Step 2: Calculate the Co-channel Reuse Ratio (Q).**

$$Q = \sqrt{3N} = \sqrt{3 \times 7} = \sqrt{21} \approx 4.583$$

3. **Step 3: Calculate the SIR in linear scale.**

$$\text{SIR} = \frac{Q^n}{K_I} = \frac{(\sqrt{21})^4}{6} = \frac{21^2}{6} = \frac{441}{6} = 73.5$$

4. **Step 4: Convert SIR to decibels (dB).**

$$\text{SIR}_{\text{dB}} = 10 \log_{10}(73.5) \approx 10 \times 1.866 = 18.66 \text{ dB}$$

5.2 Satellite Link Budget Analysis

Satellite communications require precise calculation of signal strengths over vast distances. A link budget accounts for all the power gains and losses that a communication signal experiences from the transmitter to the receiver.

Methodology:

Satellite Link Budget Calculation The primary objective is to ensure that the received signal power is sufficient for reliable data demodulation.

1. **Free Space Path Loss (FSPL):** The loss of signal strength over distance d (in meters) and frequency

f (in Hertz) is:

$$\text{FSPL} = \left(\frac{4\pi d}{\lambda}\right)^2 = \left(\frac{4\pi d f}{c}\right)^2$$

where $c = 3 \times 10^8$ m/s is the speed of light and λ is the wavelength. In decibels (dB), using distance d in kilometers and frequency f in Megahertz (MHz):

$$\text{FSPL}_{\text{dB}} = 32.44 + 20\log_{10}(d) + 20\log_{10}(f)$$

2. **Effective Isotropic Radiated Power (EIRP):** The total effective power transmitted in the direction of maximum antenna gain:

$$\text{EIRP}_{\text{dBW}} = P_{T,\text{dBW}} + G_{T,\text{dBi}} - L_{T,\text{dB}}$$

where P_T is transmit power, G_T is transmit antenna gain, and L_T covers miscellaneous transmitter losses.

3. **Received Power (P_R):** The power available at the receiving antenna terminals:

$$P_{R,\text{dBW}} = \text{EIRP}_{\text{dBW}} - \text{FSPL}_{\text{dB}} + G_{R,\text{dBi}} - L_{\text{misc,dB}}$$

where G_R is the receiver antenna gain and L_{misc} encompasses additional losses such as atmospheric attenuation or polarization mismatch.

Worked Example:

Free Space Path Loss Calculation A geostationary satellite is located at a distance of $36,000$ km from an earth station. It operates at a downlink frequency of 4 GHz. Calculate the free space path loss in decibels.
Solution:

1. **Step 1: Convert units to standard magnitudes.** Distance $d = 36,000$ km. Frequency $f = 4$ GHz $= 4000$ MHz.
2. **Step 2: Apply the FSPL formula in dB.**

$$\text{FSPL}_{\text{dB}} = 32.44 + 20\log_{10}(d) + 20\log_{10}(f)$$

3. **Step 3: Compute the logarithmic values.**

$$20\log_{10}(36000) = 20 \times 4.5563 \approx 91.13 \text{ dB}$$

$$20\log_{10}(4000) = 20 \times 3.6021 \approx 72.04 \text{ dB}$$

4. **Step 4: Calculate total FSPL.**

$$\text{FSPL}_{\text{dB}} = 32.44 + 91.13 + 72.04 = 195.61 \text{ dB}$$

Worked Example:

Satellite Link Received Power An earth station transmits a signal with a power of 500 W. The transmit antenna has a gain of 45 dBi, and transmitter losses are 2 dB. The path loss to the satellite is 200 dB. The satellite receiver antenna has a gain of 30 dBi, and there are 3 dB of miscellaneous atmospheric losses. Calculate the received power at the satellite in dBW.
Solution:

1. **Step 1: Convert transmit power to dBW.**

$$P_{T,\text{dBW}} = 10\log_{10}(500) \approx 26.99 \text{ dBW}$$

2. **Step 2: Calculate the EIRP.**

$$\text{EIRP}_{\text{dBW}} = P_{T,\text{dBW}} + G_{T,\text{dBi}} - L_{T,\text{dB}}$$

$$\text{EIRP}_{\text{dBW}} = 26.99 + 45 - 2 = 69.99 \text{ dBW}$$

3. **Step 3: Calculate the received power (P_R).**

$$P_{R,\text{dBW}} = \text{EIRP}_{\text{dBW}} - \text{FSPL}_{\text{dB}} + G_{R,\text{dBi}} - L_{\text{misc,dB}}$$

$$P_{R,\text{dBW}} = 69.99 - 200 + 30 - 3 = -103.01 \text{ dBW}$$

5.3 Optical Fiber Transmission Parameters

Optical fibers offer immense bandwidth and low attenuation, making them ideal for high-speed digital communications. Key computational parameters define how efficiently light propagates through the fiber.

Methodology:

Optical Fiber Waveguiding Parameters The light gathering ability and internal reflection conditions of an optical fiber are governed by its refractive indices.

1. **Critical Angle (θ_c):** The minimum angle of incidence for total internal reflection to occur at the core-cladding boundary:

$$\theta_c = \sin^{-1}\left(\frac{n_2}{n_1}\right)$$

where n_1 is the core refractive index and n_2 is the cladding refractive index ($n_1 > n_2$).

2. **Numerical Aperture (NA):** A dimensionless measure of the light-gathering capability of the fiber:

$$\text{NA} = \sqrt{n_1^2 - n_2^2}$$

3. **Acceptance Angle (θ_a):** The maximum angle at which light can enter the fiber and still undergo total internal reflection. Assuming the external medium is air ($n_0 \approx 1$):

$$\theta_a = \sin^{-1}(\text{NA})$$

Worked Example:

Calculating Numerical Aperture and Acceptance Angle A step-index optical fiber has a core refractive index of 1.50 and a cladding refractive index of 1.48. Determine the critical angle, numerical aperture, and the acceptance angle for light entering from air.

Solution:

1. **Step 1: Identify indices.** Core $n_1 = 1.50$, Cladding $n_2 = 1.48$.

2. **Step 2: Calculate the Critical Angle (θ_c).**

$$\theta_c = \sin^{-1}\left(\frac{1.48}{1.50}\right) = \sin^{-1}(0.9867) \approx 80.6^\circ$$

3. **Step 3: Calculate the Numerical Aperture (NA).**

$$\text{NA} = \sqrt{n_1^2 - n_2^2} = \sqrt{1.50^2 - 1.48^2} = \sqrt{2.25 - 2.1904} = \sqrt{0.0596} \approx 0.244$$

4. **Step 4: Calculate the Acceptance Angle (θ_a).**

$$\theta_a = \sin^{-1}(\text{NA}) = \sin^{-1}(0.244) \approx 14.1^\circ$$

Methodology:

Optical Fiber Attenuation Calculation Attenuation measures the loss of optical power as light travels through the fiber, typically expressed in decibels per kilometer (dB/km).

1. **Attenuation Coefficient (α):**

$$\alpha = \frac{10}{L} \log_{10}\left(\frac{P_{\text{in}}}{P_{\text{out}}}\right) \text{ dB/km}$$

   where L is the fiber length in km, P_{in} is the input power, and P_{out} is the output power.

2. **Output Power (P_{out}):** Rearranging the formula to find the output power given the attenuation:

$$P_{\text{out}} = P_{\text{in}} \cdot 10^{-\frac{\alpha L}{10}}$$

Worked Example:

Fiber Optic Attenuation Calculation An optical signal with an input power of 2 mW is injected into a 15 km long optical fiber. The fiber has an attenuation coefficient of 0.4 dB/km. Calculate the optical output power emerging from the fiber.
Solution:

1. **Step 1: Identify given parameters.** $P_{\text{in}} = 2$ mW, $L = 15$ km, $\alpha = 0.4$ dB/km.

2. **Step 2: Calculate the total attenuation.**

$$\text{Total Attenuation} = \alpha \times L = 0.4 \times 15 = 6 \text{ dB}$$

3. **Step 3: Calculate the output power using the rearranged formula.**

$$P_{\text{out}} = P_{\text{in}} \cdot 10^{-\frac{\text{Total Attenuation}}{10}}$$

$$P_{\text{out}} = 2 \times 10^{-\frac{6}{10}} = 2 \times 10^{-0.6} \approx 2 \times 0.251 = 0.502 \text{ mW}$$

   The output power is approximately 0.502 mW or $502\,\mu$W.

5.4 Radar System Fundamentals

Radio Detection and Ranging (Radar) utilizes digital signal processing to determine the position and velocity of targets. Important system metrics include maximum unambiguous range and the fundamental detection range equation.

Methodology:

Radar Range Equation and Unambiguous Range

1. **Maximum Unambiguous Range ($R_{\text{max,unambig}}$):** This is the maximum distance to a target such that the radar echo is received before the next pulse is transmitted.

$$R_{\text{max,unambig}} = \frac{c}{2 \cdot \text{PRF}} = \frac{c \cdot T_p}{2}$$

   where $c = 3 \times 10^8$ m/s is the speed of light, PRF is the Pulse Repetition Frequency, and T_p is the Pulse Repetition Time (where $T_p = 1/\text{PRF}$).

2. **Maximum Detection Range (R_{max}):** Based on the radar equation, this represents the maximum distance at which a target can be reliably detected given the minimum detectable signal power (S_{min}).

$$R_{\text{max}} = \left[\frac{P_t G^2 \lambda^2 \sigma}{(4\pi)^3 S_{\text{min}}}\right]^{1/4}$$

where P_t is the peak transmitted power, G is the antenna gain (assuming identical transmit and receive antennas), λ is the operating wavelength, and σ is the radar cross-section of the target.

Worked Example:

Maximum Unambiguous Range A pulsed radar system operates with a Pulse Repetition Frequency (PRF) of 1200 Hz. Determine the maximum unambiguous range of the radar in kilometers.

Solution:

1. **Step 1: Identify given parameters.** PRF = 1200 Hz, $c = 3 \times 10^8$ m/s.
2. **Step 2: Apply the maximum unambiguous range formula.**
$$R_{\text{max,unambig}} = \frac{c}{2 \cdot \text{PRF}}$$
3. **Step 3: Perform the calculation.**
$$R_{\text{max,unambig}} = \frac{3 \times 10^8}{2 \times 1200} = \frac{3 \times 10^8}{2400} = 125,000 \text{ m}$$
4. **Step 4: Convert to kilometers.**
$$125,000 \text{ m} = 125 \text{ km}$$

Worked Example:

Maximum Radar Detection Range Calculate the maximum range of a radar system operating at a wavelength of 3 cm. The peak transmit power is 500 kW, the antenna gain is 5000, and the minimum detectable signal power is 10^{-13} W. Assume a target radar cross-section of 5 m^2.

Solution:

1. **Step 1: Identify given parameters.** $P_t = 500$ kW $= 5 \times 10^5$ W, $G = 5000$, $\lambda = 3$ cm $= 0.03$ m, $\sigma = 5$ m^2, $S_{\text{min}} = 10^{-13}$ W.
2. **Step 2: Apply the radar range equation.**
$$R_{\text{max}} = \left[\frac{P_t G^2 \lambda^2 \sigma}{(4\pi)^3 S_{\text{min}}}\right]^{1/4}$$
3. **Step 3: Substitute the values.**
$$R_{\text{max}}^4 = \frac{(5 \times 10^5) \times (5000)^2 \times (0.03)^2 \times 5}{(4\pi)^3 \times 10^{-13}}$$
4. **Step 4: Compute the numerator.**
$$\begin{aligned} \text{Numerator} &= (5 \times 10^5) \times (25 \times 10^6) \times (0.0009) \times 5 \\ &= (5 \times 10^5) \times (2.5 \times 10^7) \times 0.0045 \\ &= 1.25 \times 10^{13} \times 0.0045 = 5.625 \times 10^{10} \end{aligned}$$
5. **Step 5: Compute the denominator.**
$$(4\pi)^3 \approx 1984.4$$
$$\text{Denominator} = 1984.4 \times 10^{-13}$$
6. **Step 6: Calculate R_{max}^4 and R_{max}.**
$$R_{\text{max}}^4 = \frac{5.625 \times 10^{10}}{1984.4 \times 10^{-13}} \approx \frac{5.625}{1984.4} \times 10^{23} \approx 2.834 \times 10^{20}$$
$$R_{\text{max}} = (2.834 \times 10^{20})^{1/4} \approx 129.7 \times 10^3 \text{ m} = 129.7 \text{ km}$$

5.5 Digital Multiplexing and Carrier Systems

Digital telephony and broad data communications rely heavily on time-division multiplexing (TDM) to combine multiple digital signals onto a single transmission medium. The standard architectures are the North American T-carrier system and the European E-carrier system.

Methodology:

T Carrier and E Carrier System Bit Rates These multiplexing hierarchies define structured frame sizes and bit rates.

1. **Sampling Rate:** Both systems use a sampling rate of 8000 samples/second (to properly sample voice frequencies up to 4 kHz) and encode each sample into 8 bits.
2. **T1 Carrier System:** Multiplexes 24 independent voice channels.

$$\text{Bits per frame} = (24 \text{ channels} \times 8 \text{ bits/channel}) + 1 \text{ framing bit}$$

$$\text{Total bit rate} = \text{Bits per frame} \times 8000 \text{ frames/second}$$

3. **E1 Carrier System:** Multiplexes 32 channels (30 for voice/data, 1 for framing/synchronization, 1 for signaling). There are no additional framing bits appended to the total block.

$$\text{Bits per frame} = 32 \text{ channels} \times 8 \text{ bits/channel}$$

$$\text{Total bit rate} = \text{Bits per frame} \times 8000 \text{ frames/second}$$

Worked Example:

T1 Carrier Bit Rate Calculation Verify the total transmission bit rate of the standard T1 carrier system used in North America. Determine what percentage of this total rate is dedicated to framing overhead.
Solution:

1. **Step 1: Calculate the number of bits in one T1 frame.**

$$\text{Bits per frame} = (24 \times 8) + 1 = 192 + 1 = 193 \text{ bits/frame}$$

2. **Step 2: Calculate the total transmission bit rate.** Since 8000 frames are transmitted per second:

$$\text{Bit Rate} = 193 \text{ bits/frame} \times 8000 \text{ frames/sec} = 1,544,000 \text{ bits/sec} = 1.544 \text{ Mbps}$$

3. **Step 3: Calculate the overhead percentage.** The framing overhead is 1 bit per 193-bit frame.

$$\text{Overhead } \% = \left(\frac{1}{193}\right) \times 100 \approx 0.518\%$$

Worked Example:

E1 Carrier Bit Rate Calculation Determine the total bit rate for a standard European E1 carrier system and calculate its useful data (payload) capacity in Mbps if two channels are entirely dedicated to synchronization and signaling.
Solution:

1. **Step 1: Calculate the total number of bits per frame.** The E1 system utilizes 32 channels of 8 bits each.

$$\text{Bits per frame} = 32 \times 8 = 256 \text{ bits/frame}$$

2. **Step 2: Calculate the total bit rate.** With a sampling frequency of 8000 Hz (8000 frames per

second):

$$\text{Total Bit Rate} = 256 \text{ bits/frame} \times 8000 \text{ frames/sec} = 2,048,000 \text{ bps} = 2.048 \text{ Mbps}$$

3. **Step 3: Calculate the payload capacity.** Since 30 channels are used for user data (payload):

$$\text{Payload Bits per frame} = 30 \times 8 = 240 \text{ bits/frame}$$

$$\text{Payload Capacity} = 240 \times 8000 = 1,920,000 \text{ bps} = 1.92 \text{ Mbps}$$

Formulae

Appendix: Key Formulae

This appendix provides a categorized, quick reference to the key formulae and mathematical relationships discussed in this book.

Unit 1: Information Theory and Signals

Information and Entropy

The fundamental measures of information content and average information (entropy) per symbol:

$$I(x_i) = \log_2 \left(\frac{1}{P(x_i)} \right) = -\log_2(P(x_i)) \text{ bits}$$

$$H = \sum_{i=1}^{M} P(x_i) \log_2 \left(\frac{1}{P(x_i)} \right) \text{ bits/symbol}$$

Signal-to-Noise Ratio (SNR)

Conversion between linear and decibel scales for SNR:

$$\text{SNR}_{\text{dB}} = 10 \log_{10}(\text{SNR}_{\text{linear}})$$

$$\text{SNR}_{\text{linear}} = 10^{(\text{SNR}_{\text{dB}}/10)}$$

Sampling Theorem

Nyquist criteria for sampling without aliasing:

$$f_s = 2 \times f_m$$

$$T_s = \frac{1}{f_s} = \frac{1}{2 f_m}$$

Channel Capacity and Data Rate

Shannon's Channel Capacity and standard transmission rates:

$$C = B \log_2(1 + \text{SNR}_{\text{linear}}) \text{ bps}$$

$$R_b = n \times f_s \text{ bps}$$

$$\text{Baud Rate} = \frac{R_b}{\log_2(M)} \text{ symbols/sec}$$

where $n = \log_2(M)$ is the number of bits per symbol.

Unit 2: Digital Modulation

Energy per Bit

Relationship between E_b/N_0 and SNR:

$$\frac{E_b}{N_0} = \frac{S/R_b}{N/B} = \left(\frac{S}{N}\right) \times \left(\frac{B}{R_b}\right)$$
$$\left(\frac{E_b}{N_0}\right)_{\text{dB}} = \text{SNR}_{\text{dB}} + 10\log_{10}\left(\frac{B}{R_b}\right)$$

Bandwidth Requirements

Bandwidth estimations for different modulation schemes:

$$BW_{\text{PSK}} = \frac{2R_b}{\log_2(M)}$$
$$BW_{\text{FSK}} = 2\Delta f + 2R_b$$
$$2\Delta f = |f_1 - f_2|$$

Unit 3: Source Coding

Coding Efficiency

Metrics for evaluating variable-length source coding (e.g., Huffman, Shannon-Fano):

$$L = \sum_{i=1}^{m} P(x_i) l_i \text{ bits/symbol}$$
$$\eta = \frac{H}{L} \times 100\%$$
$$R = r \times H \text{ bits/sec}$$

where L is the average code length, l_i is the length of codeword i, and r is the symbol rate.

Unit 4: Multiplexing and Multiple Access

Frequency Division Multiplexing (FDM)

$$N = \left\lfloor \frac{BW_{avail} + BW_g}{BW_c + BW_g} \right\rfloor$$
$$BW_{total} = (N \times BW_c) + (N-1) \times BW_g$$

Time Division Multiplexing (TDM)

Frame composition and data rates:

$$B_{frame} = (N \times b) + S \text{ bits/frame}$$
$$f_{frame} = \frac{R_{in}}{b} \text{ frames/sec}$$
$$T_{frame} = \frac{1}{f_{frame}} \text{ seconds}$$
$$R_{link} = B_{frame} \times f_{frame} \text{ bps}$$

Efficiency metrics:

$$\eta = \left(\frac{B_{data}}{B_T}\right) \times 100\% = \left(1 - \frac{B_{OH}}{B_T}\right) \times 100\%$$

Code Division Multiple Access (CDMA)

Processing gain and Walsh code generation:

$$PG = \frac{R_c}{R_b} \implies PG_{dB} = 10\log_{10}\left(\frac{R_c}{R_b}\right)$$

$$W_1 = \begin{bmatrix}1\end{bmatrix}, \quad W_{2N} = \begin{bmatrix} W_N & W_N \\ W_N & -W_N \end{bmatrix}$$

$$S_{comp} = S_1 + S_2 + \cdots + S_N$$

$$Data_k = \frac{1}{L}\sum(S_{comp} \cdot C_k)$$

Unit 5: Advanced Communication Systems

Cellular Systems

Frequency reuse and interference:

$$N = i^2 + ij + j^2$$

$$Q = \sqrt{3N}$$

$$k = \frac{S}{N} \text{ channels/cell}$$

$$\text{SIR} = \frac{1}{K_I}\left(\frac{D}{R}\right)^n = \frac{Q^n}{K_I}$$

where K_I is the number of co-channel interferers (e.g., 6 in the first tier for omnidirectional antennas).

Optical Fiber Communication

Light propagation and loss:

$$\text{NA} = \sqrt{n_1^2 - n_2^2}$$

$$\theta_a = \sin^{-1}(\text{NA})$$

$$\theta_c = \sin^{-1}\left(\frac{n_2}{n_1}\right)$$

$$\alpha = \frac{10}{L}\log_{10}\left(\frac{P_{\text{in}}}{P_{\text{out}}}\right) \text{ dB/km}$$

$$P_{\text{out}} = P_{\text{in}} \cdot 10^{-\frac{\alpha L}{10}}$$

Satellite Communication

Link budget components:

$$\text{EIRP}_{\text{dBW}} = P_{T,\text{dBW}} + G_{T,\text{dBi}} - L_{T,\text{dB}}$$

$$\text{FSPL} = \left(\frac{4\pi d}{\lambda}\right)^2 = \left(\frac{4\pi df}{c}\right)^2$$

$$\text{FSPL}_{\text{dB}} = 32.44 + 20\log_{10}(d) + 20\log_{10}(f)$$

$$P_{R,\text{dBW}} = \text{EIRP}_{\text{dBW}} - \text{FSPL}_{\text{dB}} + G_{R,\text{dBi}} - L_{\text{misc,dB}}$$

Radar Systems

Range equations:

$$R_{\text{max}} = \left[\frac{P_t G^2 \lambda^2 \sigma}{(4\pi)^3 S_{\text{min}}}\right]^{1/4}$$

$$R_{\text{max,unambig}} = \frac{c}{2 \cdot \text{PRF}} = \frac{c \cdot T_p}{2}$$

T-Carrier and E-Carrier Systems

Standard PCM carrier formats:

$$\begin{aligned}
\text{T1 Bits per frame} &= (24 \times 8) + 1 = 193 \text{ bits/frame} \\
\text{T1 Bit Rate} &= 193 \times 8000 = 1.544 \text{ Mbps} \\
\text{E1 Bits per frame} &= 32 \times 8 = 256 \text{ bits/frame} \\
\text{E1 Bit Rate} &= 256 \times 8000 = 2.048 \text{ Mbps}
\end{aligned}$$