Problem Solver (DI03011031) - Problem Solver

Antigravity AI

June 4, 2026

Problem Solver

First Edition: 2026

Author: Antigravity AI
Website: milav.in
YouTube: @milav.dabgar

To my students,
who constantly inspire me to find new analogies,
break down complex theories, and make learning
an engineered science.

Preface

About this Book
Welcome to **Computer Networks**! This textbook is meticulously designed to serve as a comprehensive problem solver and study companion. It provides a robust and intuitive foundation in the core principles of this subject, bridging the gap between theoretical models and practical applications. A unique feature of this book is its focus on decoding complex concepts using clear, step-by-step methodologies and real-world analogies.

Target Audience
This book is intended for students and professionals looking to master the concepts of Computer Networks. Whether you are revising for exams, seeking to solidify your fundamental understanding, or looking for practical examples to clarify abstract concepts, this book has been structured to support your learning journey. It serves as an accessible guide designed to remove the fear of complex topics, replacing it with an appreciation for the underlying mechanics.

Scope and Coverage
The coverage is divided logically to follow standard academic curriculums. Each chapter focuses on key topics, breaking them down into digestible sections:

- **Theoretical Insights** – Core concepts explained intuitively.
- **Method Blocks** – Standardized, step-by-step procedures for solving specific types of problems.
- **Worked Examples** – Fully solved problems that apply the methods in practice.

We hope this resource proves invaluable in your studies and helps you achieve excellence in **Computer Networks**. Happy learning!

Acknowledgments

Writing a comprehensive book that connects the fundamental forces of Chemistry with practical engineering applications requires more than just academic knowledge—it requires a community of support. I would like to express my sincere gratitude to everyone who contributed to the realization of this project.

Specially, I extend my heartfelt gratitude to the **Department of Technical Education (DTE), Government of Gujarat**, and **Government Polytechnic, Palanpur**, for providing an environment that fosters innovation, academic rigor, and continuous professional development.

I am deeply thankful to my family for their unwavering patience and support during the countless hours spent researching, formatting, and refining these chapters.

Finally, my deepest appreciation goes to my students. Your curiosity, challenging questions, and continuous feedback are the true driving force behind the unique analogies and streamlined structure of this book. This textbook was engineered for you.

Antigravity AI

About the Author

Milav Jayeshkumar Dabgar is an engineering educator and R&D professional with over 9 years of experience in electronics hardware development, embedded systems, AI/ML, and full-stack software engineering. He currently serves as a **Lecturer (Class - II) & Innovation Leader** at Government Polytechnic, Palanpur, under the Education Department of the Government of Gujarat.

Milav holds a Master of Engineering (ME) in Communication Systems from L.D. College of Engineering and a Bachelor of Engineering (BE) in Electronics & Communication. He is also currently pursuing a **BS in Data Science and Applications from IIT Madras**, where he has completed both the **Diploma in Programming** and the **Diploma in Data Science** with distinction.

Most recently, he completed the **AICTE QIP PG Certificate Program in Deep Learning from SVNIT Surat** with a **perfect 10/10 CGPA**, topping his batch.

A dedicated mentor, Milav has guided numerous student teams in securing over **INR 45 lakhs** in innovation funding, including INR 25 lakhs from Shark Tank India and INR 20 lakhs through iHub Gujarat, resulting in two student patents. He is recognized as an **NPTEL Evangelist** and **NPTEL Discipline Star** in Computer Science, reflecting his commitment to continuous learning and academic excellence.

His technical expertise spans from embedded systems (8051, STM32, Arduino) to modern web technologies (Next.js, Python, PostgreSQL) and Data Science (TensorFlow, PyTorch). Through this textbook, he aims to simplify complex Engineering Chemistry concepts by integrating relatable, real-world analogies drawn from modern tech and engineering landscapes.

Milav is also an active content creator, running a popular **YouTube channel** (@milav.dabgar) where he shares technical tutorials and academic resources. He maintains a personal blog and resource hub at milav.in and can be reached for professional collaborations on LinkedIn.

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CHAPTER 1

Network Elements and Network Analysis

1.1 Network Elements and Classifications

Network elements and networks are classified based on their characteristic properties. A strong grasp of these classifications is essential for selecting the correct analysis technique.

1.1.1 Types of Network Elements

- **Active and Passive:** Active elements deliver power to the circuit (e.g. voltage and current sources). Passive elements absorb or store energy (e.g. resistors, capacitors, inductors).
- **Linear and Non-Linear:** Linear elements have a strictly proportional relationship between voltage and current (constant resistance). Non-linear elements do not (e.g. diodes).
- **Unilateral and Bilateral:** Bilateral elements conduct current equally well in both directions (e.g. resistors). Unilateral elements conduct differently depending on polarity (e.g. diodes).
- **Lumped and Distributed:** Lumped elements are physically discrete components. Distributed elements have their resistance, capacitance, and inductance spread along their length (e.g. transmission lines).

1.1.2 Types of Networks

Networks are distinguished based on the elements they contain and their structural properties:

- **Active vs Passive:** A network is active if it contains at least one active element; otherwise, it is passive.
- **Linear vs Non-Linear:** A network is linear if it contains only linear elements.
- **Unilateral vs Bilateral:** A bilateral network contains only bilateral elements.
- **Lumped vs Distributed:** Refers to the physical manifestation of the network.
- **Symmetrical vs Asymmetrical:** A symmetrical network has identical input and output electrical characteristics when viewed from either port.
- **Single vs Two Port:** A single-port network has one pair of terminals for accessing the circuit. A two-port network has two pairs of terminals (input and output).

Methodology:

Classification of Network Elements To classify a network element:

1. **Active or Passive:** Check if the element generates power ($P = VI < 0$ continuously) or only dissipates/stores it.
2. **Linear or Non-Linear:** Determine if the V-I characteristic curve is a straight line passing through the origin.
3. **Bilateral or Unilateral:** Check if swapping the terminals alters the V-I relationship. If it does not alter it, it is bilateral.

Worked Example:

Classify a Resistor and a Diode **Problem:** Classify an ideal resistor and an ideal semiconductor diode in terms of Active/Passive, Linear/Non-Linear, and Bilateral/Unilateral.

Solution:

1. **Ideal Resistor:**
   - *Active/Passive:* It dissipates power ($P = I^2R$). Therefore, it is **Passive**.
   - *Linear/Non-Linear:* It follows Ohm's Law ($V = IR$), a straight line. Therefore, it is **Linear**.
   - *Bilateral/Unilateral:* Current flows equally well in both directions. Therefore, it is **Bilateral**.
2. **Ideal Diode:**
   - *Active/Passive:* It does not generate power. Therefore, it is **Passive**.
   - *Linear/Non-Linear:* Current is zero in reverse bias and exponential in forward bias. Therefore, it is **Non-Linear**.
   - *Bilateral/Unilateral:* It conducts only in one direction. Therefore, it is **Unilateral**.

1.2 Series and Parallel Resistors

Methodology:

Equivalent Resistance Calculation

1. **Series Combination:** Resistors are in series if the exact same current flows through them sequentially.
$$R_{eq} = R_1 + R_2 + \cdots + R_n$$
2. **Parallel Combination:** Resistors are in parallel if they are connected across the exact same two nodes (same voltage).
$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_n}$$
For two resistors in parallel: $R_{eq} = \frac{R_1R_2}{R_1+R_2}$

Worked Example:

Calculate Equivalent Resistance **Problem:** Find the equivalent resistance between terminals A and B if a $10\,\Omega$ resistor is in series with a parallel combination of a $20\,\Omega$ and a $30\,\Omega$ resistor.

Solution:

1. **Identify Parallel Part:** The $20\,\Omega$ and $30\,\Omega$ resistors are in parallel.
$$R_p = \frac{20 \times 30}{20 + 30} = \frac{600}{50} = 12\,\Omega$$
2. **Identify Series Part:** The $10\,\Omega$ resistor is in series with R_p.
$$R_{eq} = 10 + 12 = 22\,\Omega$$

Final Answer: The equivalent resistance is $22\,\Omega$.

1.3 Network Topology

Network topology deals with the graphical representation of circuits.

- **Node:** A junction where two or more circuit elements meet.
- **Branch:** A path that connects two nodes and contains exactly one element.
- **Loop:** Any closed path in a circuit.
- **Mesh:** A loop that does not contain any other loops within it.

Methodology:

Counting Nodes and Branches

1. **Nodes:** Identify all continuous conductive wires connecting components. A continuous wire, regardless of its shape, represents a single node.
2. **Branches:** Count the number of distinct two-terminal elements (resistors, sources, etc.) in the circuit.
3. **Relationship:** For a planar circuit, the number of independent meshes M, branches B, and nodes N is related by: $M = B - N + 1$.

Worked Example:

Determine Topology Metrics **Problem:** A circuit has 4 nodes and 6 branches. Calculate the number of independent meshes.
Solution:

1. **Given:** $N = 4$, $B = 6$.
2. **Formula:** $M = B - N + 1$
3. **Calculation:** $M = 6 - 4 + 1 = 3$ meshes.

Final Answer: There are 3 independent meshes.

1.4 Mesh and Node Analysis

Methodology:

Mesh Analysis Algorithm Mesh analysis uses Kirchhoff's Voltage Law (KVL) to find unknown currents.

1. **Identify Meshes:** Identify all M independent meshes in the circuit.
2. **Assign Currents:** Assign a mesh current (e.g. $i_1, i_2, \ldots, i_M$) to each mesh, typically in the clockwise direction.
3. **Write KVL Equations:** For each mesh, apply KVL ($\sum V = 0$). When a resistor is shared between two meshes, the current through it is the algebraic sum of the two mesh currents (e.g. $i_1 - i_2$).
4. **Solve System:** Solve the resulting system of simultaneous linear equations for the mesh currents.

Worked Example:

Solve Circuit using Mesh Analysis **Problem:** A two-mesh circuit has a 10V voltage source in mesh 1, and a 5V voltage source in mesh 2 (opposing polarity). Mesh 1 has a $2\,\Omega$ resistor, mesh 2 has a $4\,\Omega$ resistor, and they share a $3\,\Omega$ central resistor. Find mesh currents i_1 and i_2.
Solution:

1. **KVL for Mesh 1 (clockwise):** Starting from the voltage source: $-10 + 2i_1 + 3(i_1 - i_2) = 0$ Simplifying: $5i_1 - 3i_2 = 10 \quad$ — (Equation 1)

2. **KVL for Mesh 2 (clockwise):** Starting from the shared resistor: $3(i_2 - i_1) + 4i_2 + 5 = 0$ Simplifying: $-3i_1 + 7i_2 = -5 \quad$ — (Equation 2)

3. **Solve Equations:** From Eq 1: $i_1 = \frac{10+3i_2}{5} = 2 + 0.6i_2$ Substitute into Eq 2: $-3(2 + 0.6i_2) + 7i_2 = -5$ $-6 - 1.8i_2 + 7i_2 = -5$ $5.2i_2 = 1 \implies i_2 = \frac{1}{5.2} = 0.192\text{A}$ Back-substitute: $i_1 = 2 + 0.6(0.192) = 2.115\text{A}$

Final Answer: $i_1 = 2.115\text{A}$, $i_2 = 0.192\text{A}$.

Methodology:

Nodal Analysis Algorithm Nodal analysis uses Kirchhoff's Current Law (KCL) to find unknown node voltages.

1. **Select Reference Node:** Choose one node as the reference (ground) node and assign it 0V.

2. **Assign Voltages:** Assign unknown voltages (e.g. $V_1, V_2, \ldots, V_{N-1}$) to the remaining non-reference nodes.

3. **Write KCL Equations:** Apply KCL ($\sum I_{leaving} = 0$) at each non-reference node. Express currents in terms of node voltages using Ohm's Law ($I = \frac{V_{node} - V_{adjacent}}{R}$).

4. **Solve System:** Solve the resulting system of linear equations for the node voltages.

Worked Example:

Solve Circuit using Nodal Analysis **Problem:** A circuit has two non-reference nodes, V_1 and V_2. Node 1 is connected to a 2A incoming current source, to ground via a $2\,\Omega$ resistor, and to Node 2 via a $4\,\Omega$ resistor. Node 2 is connected to ground via a $1\,\Omega$ resistor. Find V_1 and V_2.
Solution:

1. **KCL at Node 1:** Sum of currents leaving $= 0$ $-2 + \frac{V_1}{2} + \frac{V_1 - V_2}{4} = 0$ Multiply by 4: $-8 + 2V_1 + V_1 - V_2 = 0 \implies 3V_1 - V_2 = 8 \quad$ — (Equation 1)

2. **KCL at Node 2:** $\frac{V_2 - V_1}{4} + \frac{V_2}{1} = 0$ Multiply by 4: $V_2 - V_1 + 4V_2 = 0 \implies -V_1 + 5V_2 = 0 \implies V_1 = 5V_2$ — (Equation 2)

3. **Solve Equations:** Substitute Eq 2 into Eq 1: $3(5V_2) - V_2 = 8$ $14V_2 = 8 \implies V_2 = \frac{8}{14} = 0.571\text{V}$ Find V_1: $V_1 = 5(0.571) = 2.855\text{V}$

Final Answer: $V_1 = 2.855\text{V}$, $V_2 = 0.571\text{V}$.

1.5 Network Theorems

Methodology:

Thevenin Theorem Any linear bilateral network can be replaced by an equivalent circuit consisting of a single voltage source (V_{th}) in series with a single resistor (R_{th}).

1. **Remove Load:** Remove the load resistor R_L to create open terminals A and B.

2. **Find V_{th}:** Calculate the open-circuit voltage across terminals A and B. $V_{th} = V_{AB}$.

3. **Find R_{th}:** Turn off all independent sources (short-circuit voltage sources, open-circuit current sources). Calculate the equivalent resistance looking into terminals A and B.

4. **Equivalent Circuit:** Connect V_{th} in series with R_{th}, and reconnect R_L. $I_L = \frac{V_{th}}{R_{th} + R_L}$.

Worked Example:

Find Thevenin Equivalent **Problem:** A circuit has a 12V source in series with a $4\,\Omega$ resistor. This combination is in parallel with an $8\,\Omega$ resistor. Terminals A and B are across the $8\,\Omega$ resistor. Find the Thevenin equivalent circuit across A and B.

Solution:

1. **Find V_{th}:** The open-circuit voltage is the voltage across the $8\,\Omega$ resistor. Using voltage divider:

$$V_{th} = 12\text{V} \times \frac{8}{4+8} = 12 \times \frac{8}{12} = 8\text{V}$$

2. **Find R_{th}:** Short the 12V source. Looking back from A and B, the $4\,\Omega$ and $8\,\Omega$ resistors are in parallel.

$$R_{th} = \frac{4 \times 8}{4+8} = \frac{32}{12} = 2.67\,\Omega$$

Final Answer: $V_{th} = 8\text{V}$, $R_{th} = 2.67\,\Omega$.

Methodology:

Norton Theorem Any linear bilateral network can be replaced by an equivalent circuit consisting of a single current source (I_N) in parallel with a single resistor (R_N).

1. **Remove Load:** Remove the load resistor R_L and short-circuit the terminals A and B.
2. **Find I_N:** Calculate the short-circuit current flowing from A to B. $I_N = I_{SC}$.
3. **Find R_N:** Same calculation as R_{th}. Deactivate all independent sources and find equivalent resistance. $R_N = R_{th}$.
4. **Equivalent Circuit:** Connect I_N in parallel with R_N.

Worked Example:

Find Norton Equivalent **Problem:** For the same circuit as the previous example (12V source in series with $4\,\Omega$, both in parallel with $8\,\Omega$), find the Norton equivalent.

Solution:

1. **Find I_N:** Short terminals A and B. The $8\,\Omega$ resistor is bypassed. The short-circuit current is limited only by the $4\,\Omega$ resistor.

$$I_N = \frac{12\text{V}}{4\,\Omega} = 3\text{A}$$

2. **Find R_N:** As calculated previously, $R_N = R_{th} = 2.67\,\Omega$.

Final Answer: $I_N = 3\text{A}$, $R_N = 2.67\,\Omega$.

Methodology:

Superposition Theorem In a linear network with multiple independent sources, the response (voltage or current) in any branch is the algebraic sum of the responses caused by each independent source acting alone.

1. **Isolate Source:** Select one independent source to remain active. Turn off all other independent sources (short voltage sources, open current sources).
2. **Calculate Partial Response:** Calculate the current or voltage in the target branch due to this single source.
3. **Repeat:** Repeat steps 1 and 2 for every independent source in the network.

4. **Algebraic Sum:** Add all partial responses together algebraically (paying attention to polarities/directions) to find the total response.

Worked Example:

Solve using Superposition **Problem:** A resistor $R = 2\,\Omega$ is connected between a 10V voltage source (with a $1\,\Omega$ series resistor) and a 5A current source. Find the total current through R.
Solution:

1. **Response due to 10V source (I'):** Open the 5A current source. The circuit is just the 10V source in series with the $1\,\Omega$ and $2\,\Omega$ resistors.

$$I' = \frac{10}{1+2} = \frac{10}{3} = 3.33\text{A}$$

2. **Response due to 5A source (I''):** Short the 10V voltage source. The 5A current splits between the $1\,\Omega$ resistor and the $2\,\Omega$ resistor. Using current divider for current through the $2\,\Omega$ resistor:

$$I'' = 5 \times \frac{1}{1+2} = \frac{5}{3} = 1.67\text{A}$$

3. **Total Current (I):** Assuming both sources push current in the same direction through R.

$$I = I' + I'' = 3.33 + 1.67 = 5.0\text{A}$$

Final Answer: Total current is 5.0A.

Methodology:

Maximum Power Transfer Theorem A DC voltage source will deliver maximum power to a variable load resistor when the load resistance equals the Thevenin equivalent resistance of the network as viewed from the load terminals.

1. **Find R_{th}:** Remove the load resistor R_L and calculate the Thevenin resistance of the network.
2. **Set R_L:** Set $R_L = R_{th}$.
3. **Calculate Max Power:** Find V_{th}. The maximum power is $P_{max} = \frac{V_{th}^2}{4R_{th}}$.

Worked Example:

Maximum Power Transfer **Problem:** A network has $V_{th} = 20\text{V}$ and $R_{th} = 5\,\Omega$. Find the value of R_L for maximum power transfer and calculate the maximum power.
Solution:

1. **Condition for Max Power:** R_L must equal R_{th}.

$$R_L = 5\,\Omega$$

2. **Calculate Max Power:**

$$P_{max} = \frac{V_{th}^2}{4R_{th}} = \frac{20^2}{4 \times 5} = \frac{400}{20} = 20\text{W}$$

Final Answer: $R_L = 5\,\Omega$, $P_{max} = 20\text{W}$.

Methodology:

Reciprocity Theorem In any linear, bilateral network with a single excitation source, the ratio of excitation to response is constant even if the positions of excitation and response are interchanged.

1. **Initial Setup:** Place a voltage source V in branch A and measure current I in branch B.
2. **Interchange:** Move the voltage source V to branch B and measure the new current I' in branch A.
3. **Verify:** The theorem holds if $I = I'$.

Methodology:

Principle of Duality Two electrical networks are duals if the mesh equations of one have the same mathematical form as the nodal equations of the other.

1. **Identify Dual Elements:**

Element	Dual Element
Voltage (V)	Current (I)
Resistance (R)	Conductance (G)
Inductance (L)	Capacitance (C)
Series Connection	Parallel Connection
Mesh	Node
Open Circuit	Short Circuit

2. **Construction:** Place a node inside each mesh of the original circuit and a reference node outside. Draw branches between nodes, passing through original elements, replacing each element with its dual.

1.6 Two Port Network Parameters

A two-port network has four variables: V_1, I_1 (input port) and V_2, I_2 (output port). Two variables are independent and two are dependent, leading to different parameter sets.

Methodology:

Z Parameters (Impedance) Equations:

$$V_1 = Z_{11}I_1 + Z_{12}I_2$$
$$V_2 = Z_{21}I_1 + Z_{22}I_2$$

Calculation:

1. **Output Open-Circuited ($I_2 = 0$):** $Z_{11} = \frac{V_1}{I_1}$ (Input driving-point impedance)
   $Z_{21} = \frac{V_2}{I_1}$ (Forward transfer impedance)
2. **Input Open-Circuited ($I_1 = 0$):** $Z_{12} = \frac{V_1}{I_2}$ (Reverse transfer impedance)
   $Z_{22} = \frac{V_2}{I_2}$ (Output driving-point impedance)

Methodology:

Y Parameters (Admittance) Equations:

$$I_1 = Y_{11}V_1 + Y_{12}V_2$$
$$I_2 = Y_{21}V_1 + Y_{22}V_2$$

Calculation:

1. **Output Short-Circuited ($V_2 = 0$):** $Y_{11} = \frac{I_1}{V_1}, \quad Y_{21} = \frac{I_2}{V_1}$
2. **Input Short-Circuited ($V_1 = 0$):** $Y_{12} = \frac{I_1}{V_2}, \quad Y_{22} = \frac{I_2}{V_2}$

Methodology:

h Parameters (Hybrid) Equations:

$$V_1 = h_{11} I_1 + h_{12} V_2$$
$$I_2 = h_{21} I_1 + h_{22} V_2$$

Calculation:

1. **Output Short-Circuited ($V_2 = 0$):** $h_{11} = \frac{V_1}{I_1}, \quad h_{21} = \frac{I_2}{I_1}$
2. **Input Open-Circuited ($I_1 = 0$):** $h_{12} = \frac{V_1}{V_2}, \quad h_{22} = \frac{I_2}{V_2}$

Methodology:

ABCD Parameters (Transmission) Equations:

$$V_1 = AV_2 - BI_2$$
$$I_1 = CV_2 - DI_2$$

Calculation:

1. **Output Open-Circuited ($I_2 = 0$):** $A = \frac{V_1}{V_2}, \quad C = \frac{I_1}{V_2}$
2. **Output Short-Circuited ($V_2 = 0$):** $B = -\frac{V_1}{I_2}, \quad D = -\frac{I_1}{I_2}$

Worked Example:

Calculate Z Parameters for a T-Network **Problem:** A T-network consists of a series resistor $R_1 = 10\,\Omega$, another series resistor $R_2 = 20\,\Omega$, and a shunt resistor $R_3 = 30\,\Omega$ in the middle. Find the Z parameters.
Solution:

1. **Find Z_{11} and Z_{21} (Output Open, $I_2 = 0$):** The input current I_1 flows through R_1 and R_3. R_2 has no current. $V_1 = I_1(R_1 + R_3) \implies Z_{11} = \frac{V_1}{I_1} = 10 + 30 = 40\,\Omega$. V_2 is the voltage across R_3, so $V_2 = I_1 R_3$. $Z_{21} = \frac{V_2}{I_1} = 30\,\Omega$.
2. **Find Z_{12} and Z_{22} (Input Open, $I_1 = 0$):** The output current I_2 flows through R_2 and R_3. $V_2 = I_2(R_2 + R_3) \implies Z_{22} = \frac{V_2}{I_2} = 20 + 30 = 50\,\Omega$. V_1 is the voltage across R_3, so $V_1 = I_2 R_3$. $Z_{12} = \frac{V_1}{I_2} = 30\,\Omega$.

Final Answer: $Z_{11} = 40\,\Omega$, $Z_{12} = 30\,\Omega$, $Z_{21} = 30\,\Omega$, $Z_{22} = 50\,\Omega$.

1.7 Various Network Impedances

In network analysis, several characteristic impedances are defined based on the configuration and termination of the network.

Methodology:

Types of Impedance

1. **Driving Point Impedance:** The impedance measured at a given port when all other ports are

terminated in specific ways (usually open or short). e.g. $Z_{11} = V_1/I_1$.

2. **Transfer Impedance:** The ratio of voltage at one port to current at another port. e.g. $Z_{21} = V_2/I_1$.
3. **Input Impedance (Z_{in}):** The impedance seen looking into the input port when the output port is terminated with a load Z_L.
4. **Output Impedance (Z_{out}):** The impedance seen looking into the output port when the input port is terminated with a source impedance Z_S.
5. **Image Impedances (Z_{I1}, Z_{I2}):** Two impedances such that if the output is terminated in Z_{I2}, the input impedance is Z_{I1}; and if the input is terminated in Z_{I1}, the output impedance is Z_{I2}.
6. **Terminating Impedance:** The actual load or source impedance connected to the ports of the network.

Worked Example:

Calculate Input Impedance **Problem:** A two-port network has Z-parameters: $Z_{11} = 40\,\Omega$, $Z_{12} = 30\,\Omega$, $Z_{21} = 30\,\Omega$, $Z_{22} = 50\,\Omega$. The output is terminated with a load $Z_L = 10\,\Omega$. Find the input impedance Z_{in}.
Solution:

1. **Set up the equations:** $V_1 = Z_{11}I_1 + Z_{12}I_2 = 40I_1 + 30I_2$ $V_2 = Z_{21}I_1 + Z_{22}I_2 = 30I_1 + 50I_2$
2. **Apply Termination Condition:** At the output port, $V_2 = -I_2Z_L = -10I_2$ (negative sign because I_2 is defined as entering the port).
3. **Substitute and solve for I_2 in terms of I_1:** $-10I_2 = 30I_1 + 50I_2$ $-60I_2 = 30I_1 \implies I_2 = -0.5I_1$
4. **Calculate $Z_{in} = \frac{V_1}{I_1}$:** $V_1 = 40I_1 + 30(-0.5I_1) = 40I_1 - 15I_1 = 25I_1$ $Z_{in} = \frac{V_1}{I_1} = 25\,\Omega$

Final Answer: The input impedance is $25\,\Omega$.

CHAPTER 2

Resonance Circuits and Passive Filters

2.1 Quality Factor of Coil and Capacitor

Methodology:

Calculating Quality Factor The Quality Factor (Q-factor) is a dimensionless parameter that describes how under-damped an oscillator or resonator is. It represents the ratio of energy stored to energy dissipated per cycle.

1. **Q-factor of a Coil (Inductor):** A practical inductor has a series resistance R_L.

$$Q_L = \frac{X_L}{R_L} = \frac{2\pi f L}{R_L}$$

   where X_L is the inductive reactance, f is the frequency, L is the inductance, and R_L is the internal resistance.

2. **Q-factor of a Capacitor:** A practical capacitor has a series equivalent resistance R_C (or shunt leakage resistance). Assuming the series resistance model:

$$Q_C = \frac{X_C}{R_C} = \frac{1}{2\pi f C R_C}$$

   where X_C is the capacitive reactance, C is the capacitance, and R_C is the internal series resistance.

Worked Example:

Quality Factor of a Practical Inductor A coil has an inductance of 50 mH and an internal resistance of 10 Ω. Calculate its Quality Factor at a frequency of 1 kHz.
Solution:

1. **Identify given parameters:** $L = 50 \text{ mH} = 50 \times 10^{-3}$ H
   $R_L = 10\ \Omega$
   $f = 1 \text{ kHz} = 1000$ Hz

2. **Calculate the inductive reactance X_L:**

$$X_L = 2\pi f L = 2 \times 3.1416 \times 1000 \times 50 \times 10^{-3} = 314.16\ \Omega$$

3. **Calculate the Q-factor:**

$$Q_L = \frac{X_L}{R_L} = \frac{314.16}{10} = 31.416$$

The Quality Factor of the coil is 31.42.

Worked Example:

Quality Factor of a Capacitor A practical capacitor has a capacitance of 2.2 μF and an equivalent series resistance of 0.5 Ω. Find its Q-factor at a frequency of 500 Hz.

Solution:

1. **Identify given parameters:** $C = 2.2\ \mu\text{F} = 2.2 \times 10^{-6}$ F
   $R_C = 0.5\ \Omega$
   $f = 500$ Hz
2. **Calculate the capacitive reactance X_C:**

$$X_C = \frac{1}{2\pi f C} = \frac{1}{2 \times 3.1416 \times 500 \times 2.2 \times 10^{-6}} = \frac{1}{0.00691} \approx 144.69\ \Omega$$

3. **Calculate the Q-factor:**

$$Q_C = \frac{X_C}{R_C} = \frac{144.69}{0.5} = 289.38$$

The Quality Factor of the capacitor is 289.38.

2.2 Series and Parallel Resonant Circuits

Methodology:

Series Resonance Parameters Calculation A series RLC circuit achieves resonance when the inductive reactance equals the capacitive reactance ($X_L = X_C$), resulting in a purely resistive total impedance.

1. **Resonance Frequency (f_r):** The frequency at which resonance occurs.

$$f_r = \frac{1}{2\pi\sqrt{LC}}$$

2. **Impedance at Resonance (Z):** At resonance, the reactive components cancel out.

$$Z = R$$

3. **Quality Factor at Resonance (Q_0):**

$$Q_0 = \frac{1}{R}\sqrt{\frac{L}{C}} = \frac{2\pi f_r L}{R}$$

4. **Bandwidth (BW):** The range of frequencies between the half-power points (f_1 and f_2).

$$BW = f_2 - f_1 = \frac{R}{2\pi L} = \frac{f_r}{Q_0}$$

Worked Example:

Analysis of a Series Resonant Circuit A series RLC circuit consists of a resistance of 5 Ω, an inductance of 20 mH, and a capacitance of 0.5 μF. Calculate the resonance frequency, the Q-factor at resonance, and the bandwidth.

Solution:

1. **Identify given parameters:** $R = 5\ \Omega$
   $L = 20\ \text{mH} = 20 \times 10^{-3}$ H
   $C = 0.5\ \mu\text{F} = 0.5 \times 10^{-6}$ F

2. **Calculate the resonance frequency f_r:**

$$f_r = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{(20 \times 10^{-3}) \times (0.5 \times 10^{-6})}} = \frac{1}{2\pi\sqrt{10 \times 10^{-9}}} = \frac{1}{2\pi \times 10^{-4}} \approx 1591.55 \text{ Hz}$$

3. **Calculate the Q-factor at resonance Q_0:**

$$Q_0 = \frac{1}{R}\sqrt{\frac{L}{C}} = \frac{1}{5}\sqrt{\frac{20 \times 10^{-3}}{0.5 \times 10^{-6}}} = \frac{1}{5}\sqrt{40000} = \frac{200}{5} = 40$$

4. **Calculate the Bandwidth (BW):**

$$BW = \frac{f_r}{Q_0} = \frac{1591.55}{40} \approx 39.79 \text{ Hz}$$

Methodology:

Parallel Resonance Parameters Calculation A practical parallel resonant circuit (Tank Circuit) consists of a coil (with inductance L and internal resistance R) in parallel with a capacitor C.

1. **Resonance Frequency (f_r):**

$$f_r = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$$

If the coil resistance is very small ($R^2/L^2 \ll 1/LC$), it approximates to $f_r \approx 1/(2\pi\sqrt{LC})$.

2. **Dynamic Impedance (Z_d):** The purely resistive total equivalent impedance of the circuit at resonance.

$$Z_d = \frac{L}{RC}$$

3. **Quality Factor at Resonance (Q_0):**

$$Q_0 = \frac{2\pi f_r L}{R}$$

Worked Example:

Analysis of a Parallel Tank Circuit A coil having a resistance of 20 Ω and an inductance of 0.2 H is connected in parallel with a 100 μF capacitor. Determine the resonant frequency and dynamic impedance of the circuit.

Solution:

1. **Identify given parameters:** $R = 20\ \Omega$
$L = 0.2$ H
$C = 100\ \mu\text{F} = 100 \times 10^{-6}$ F

2. **Calculate the resonant frequency f_r:**

$$\frac{1}{LC} = \frac{1}{0.2 \times 100 \times 10^{-6}} = \frac{1}{2 \times 10^{-5}} = 50000$$

$$\frac{R^2}{L^2} = \frac{20^2}{0.2^2} = \frac{400}{0.04} = 10000$$

$$f_r = \frac{1}{2\pi}\sqrt{50000 - 10000} = \frac{1}{2\pi}\sqrt{40000} = \frac{200}{2\pi} \approx 31.83 \text{ Hz}$$

3. **Calculate the dynamic impedance Z_d:**

$$Z_d = \frac{L}{RC} = \frac{0.2}{20 \times 100 \times 10^{-6}} = \frac{0.2}{0.002} = 100\ \Omega$$

2.3 Passive Filter Circuits Classification

Methodology:

Passive Filter Circuit Classification Filters are networks designed to pass signals of desired frequencies while blocking or attenuating others. Passive filters are built using only resistors, capacitors, and inductors. They are classified based on their frequency response:

1. **Low Pass Filter (LPF):** Passes all frequencies from zero up to a specified cut-off frequency (f_c) and heavily attenuates all frequencies above f_c.
2. **High Pass Filter (HPF):** Attenuates all frequencies below the cut-off frequency (f_c) and passes all frequencies above it.
3. **Band Pass Filter (BPF):** Passes a specific band of frequencies between a lower cut-off frequency (f_1) and an upper cut-off frequency (f_2) while attenuating frequencies outside this band.
4. **Band Stop Filter (BSF) or Notch Filter:** Attenuates a specific band of frequencies between f_1 and f_2 and passes all frequencies outside this range.

2.4 Design of Constant-k Low Pass Filter

Methodology:

Constant k Low Pass Filter Design A constant-k (or k-type) filter is designed such that the product of its series arm impedance (Z_1) and shunt arm impedance (Z_2) is a constant value R_0^2 independent of frequency. For a Low Pass Filter, $Z_1 = j\omega L_k$ and $Z_2 = 1/(j\omega C_k)$.

1. **Calculate Inductance (L_k) and Capacitance (C_k):** Given the design impedance R_0 and the cut-off frequency f_c:
$$L_k = \frac{R_0}{\pi f_c}$$
$$C_k = \frac{1}{\pi R_0 f_c}$$
2. **Form the T-section:**
   - Two series arms, each with an inductor of value $L_k/2$.
   - One shunt arm with a capacitor of value C_k.
3. **Form the π-section:**
   - One series arm with an inductor of value L_k.
   - Two shunt arms, each with a capacitor of value $C_k/2$.

Worked Example:

Design of Constant k LPF T and Pi Sections Design a constant-k low pass filter having a cut-off frequency of 2.5 kHz and a design impedance of 600 Ω. Determine the parameters for the T and π sections.
Solution:

1. **Identify given parameters:** $f_c = 2.5$ kHz $= 2500$ Hz
$R_0 = 600\ \Omega$

2. **Calculate basic components L_k and C_k:**

$$L_k = \frac{R_0}{\pi f_c} = \frac{600}{3.1416 \times 2500} = \frac{600}{7854} \approx 0.0764 \text{ H} = 76.4 \text{ mH}$$

$$C_k = \frac{1}{\pi R_0 f_c} = \frac{1}{3.1416 \times 600 \times 2500} = \frac{1}{4.712 \times 10^6} \approx 0.212 \times 10^{-6} \text{ F} = 0.212\ \mu\text{F}$$

3. **Design the T-section elements:** Series arm inductors: $\frac{L_k}{2} = \frac{76.4}{2} = 38.2$ mH
Shunt arm capacitor: $C_k = 0.212\ \mu$F

T-Section Layout

Series Arms:	Two 38.2 mH inductors
Shunt Arm:	One 0.212 μF capacitor

4. **Design the π-section elements:** Series arm inductor: $L_k = 76.4$ mH
Shunt arm capacitors: $\frac{C_k}{2} = \frac{0.212}{2} = 0.106\ \mu$F

π-Section Layout

Series Arm:	One 76.4 mH inductor
Shunt Arms:	Two 0.106 μF capacitors

2.5 Design of Constant-k High Pass Filter

Methodology:

Constant k High Pass Filter Design For a Constant-k High Pass Filter, the series arm is a capacitor ($Z_1 = 1/(j\omega C_k)$) and the shunt arm is an inductor ($Z_2 = j\omega L_k$).

1. **Calculate Inductance (L_k) and Capacitance (C_k):** Given the design impedance R_0 and the cut-off frequency f_c:

$$L_k = \frac{R_0}{4\pi f_c}$$

$$C_k = \frac{1}{4\pi R_0 f_c}$$

2. **Form the T-section:**
   - Two series arms, each with a capacitor of value $2C_k$.
   - One shunt arm with an inductor of value L_k.

3. **Form the π-section:**
   - One series arm with a capacitor of value C_k.
   - Two shunt arms, each with an inductor of value $2L_k$.

Worked Example:

Design of Constant k HPF T and Pi Sections Design a constant-k high pass filter to operate with a cut-off frequency of 10 kHz and a design load of 500 Ω. Determine the elements for both T and π sections.
Solution:

1. **Identify given parameters:** $f_c = 10$ kHz $= 10000$ Hz
$R_0 = 500\ \Omega$

2. **Calculate basic components L_k and C_k:**

$$L_k = \frac{R_0}{4\pi f_c} = \frac{500}{4 \times 3.1416 \times 10000} = \frac{500}{125664} \approx 0.00398 \text{ H} = 3.98 \text{ mH}$$

$$C_k = \frac{1}{4\pi R_0 f_c} = \frac{1}{4 \times 3.1416 \times 500 \times 10000} = \frac{1}{6.283 \times 10^7} \approx 15.9 \times 10^{-9} \text{ F} = 15.9 \text{ nF}$$

3. **Design the T-section elements:** Series arm capacitors: $2C_k = 2 \times 15.9$ nF $= 31.8$ nF
Shunt arm inductor: $L_k = 3.98$ mH

T-Section Layout	
Series Arms:	Two 31.8 nF capacitors
Shunt Arm:	One 3.98 mH inductor

4. **Design the π-section elements:** Series arm capacitor: $C_k = 15.9$ nF
Shunt arm inductors: $2L_k = 2 \times 3.98$ mH $= 7.96$ mH

π-Section Layout	
Series Arm:	One 15.9 nF capacitor
Shunt Arms:	Two 7.96 mH inductors

CHAPTER 3

Characteristic of Measurement and Bridges

3.1 Measurement Characteristics

In measurement systems, the following characteristics define the quality and performance of an instrument:

- **Instrument:** A device or mechanism used to determine the present value of the quantity under measurement.
- **Error:** The algebraic difference between the measured value and the true value of the quantity.
- **Accuracy:** The degree of closeness with which the instrument reading approaches the true value of the quantity being measured.
- **Reproducibility:** The degree of closeness with which a given value may be repeatedly measured under different conditions or by different operators.
- **Repeatability:** The degree of closeness with which a given value may be repeatedly measured under the same exact operating conditions.
- **Precision:** A measure of the consistency or repeatability of measurements. It denotes how tightly grouped a set of measurements are.
- **Sensitivity:** The ratio of the change in output of the instrument to a change of input or measured variable.
- **Resolution:** The smallest change in the measured value to which the instrument will respond.
- **Linearity:** The ability to reproduce the input characteristics symmetrically and proportionally. The maximum deviation from a straight-line response.
- **Response time:** The time elapsed between the application of an input and the time the instrument reaches its final steady-state reading.

3.2 Types of Error and Limiting Errors

Errors in measurement are broadly classified into three categories:

1. **Gross Errors:** Human mistakes in reading, recording, or calculating results.
2. **Systematic Errors:** Instrumental, environmental, and observational errors that occur consistently in a particular direction.
3. **Random Errors:** Unknown causes producing small variations in measurements.

Limiting Errors: The maximum allowable error limits guaranteed by the manufacturer. For example, if a resistor is rated at $100\,\Omega \pm 5\%$, the limiting error is 5%.

Methodology:

Calculation of Limiting Errors When computing derived quantities with individual limiting errors, apply the following computational rules to find the worst-case combined limiting error:

1. **Sum or Difference ($X = A \pm B$):** The absolute errors add up.

$$\delta X = \pm(\delta A + \delta B)$$

2. **Product or Quotient ($X = A \cdot B$ or $X = \frac{A}{B}$):** The relative (or percentage) errors add up.

$$\frac{\delta X}{X} = \pm\left(\frac{\delta A}{A} + \frac{\delta B}{B}\right)$$

3. **Power ($X = A^n$):** The relative error is multiplied by the power n.

$$\frac{\delta X}{X} = \pm n\frac{\delta A}{A}$$

Worked Example:

Calculating Combined Limiting Error A resistor has a voltage drop of $V = 100\text{ V} \pm 2\%$ and a current of $I = 10\text{ A} \pm 1\%$. Calculate the power P dissipated in the resistor and its limiting error in percentage and absolute watts.

Solution:

1. The formula for calculating power is $P = V \times I$.
2. Calculate the nominal value of P:

$$P = 100\text{ V} \times 10\text{ A} = 1000\text{ W}$$

3. Since P is a product of V and I, the percentage limiting errors must be added together:

$$\%\text{ Error in } P = \pm(\%\text{ Error in } V + \%\text{ Error in } I)$$

$$\%\text{ Error in } P = \pm(2\% + 1\%) = \pm 3\%$$

4. Calculate the absolute limiting error in watts (3% of 1000 W):

$$\delta P = \pm\left(\frac{3}{100} \times 1000\right) = \pm 30\text{ W}$$

Thus, the power dissipated is $1000\text{ W} \pm 3\%$ or equivalently $1000\text{ W} \pm 30\text{ W}$.

3.3 DC Bridges

DC bridges are primarily used to measure unknown DC resistances accurately.

Methodology:

Wheatstone Bridge Balance Condition A Wheatstone bridge consists of four resistance arms (R_1, R_2, R_3, and R_4), a voltage source, and a galvanometer. Let R_4 be the unknown resistance.

1. Structure the bridge with R_1 and R_2 as the ratio arms, R_3 as the standard variable resistance arm, and R_4 as the unknown resistance.
2. Observe the balance condition: The bridge is balanced when the galvanometer reads zero current.

3. Set up the balance equation by equating the products of opposite arms:

$$R_1 R_4 = R_2 R_3$$

4. Solve for the unknown resistance R_4:

$$R_4 = \frac{R_2 R_3}{R_1}$$

Worked Example:

Finding Unknown Resistance using Wheatstone Bridge In a Wheatstone bridge, the ratio arms R_1 and R_2 are $1000\,\Omega$ and $100\,\Omega$ respectively. The standard arm R_3 is adjusted to $456\,\Omega$ to obtain a balance. Determine the value of the unknown resistor R_4.

Solution:

1. Identify the given parameters: $R_1 = 1000\,\Omega$, $R_2 = 100\,\Omega$, $R_3 = 456\,\Omega$.
2. Apply the Wheatstone bridge balance equation:

$$R_4 = \frac{R_2 R_3}{R_1}$$

3. Substitute the values to evaluate R_4:

$$R_4 = \frac{100 \times 456}{1000} = \frac{45600}{1000} = 45.6\,\Omega$$

The unknown resistance is exactly $45.6\,\Omega$.

3.4 AC Bridges

AC bridges are excited by an alternating current source and use an AC detector. They are essential for measuring unknown inductances and capacitances.

Methodology:

General AC Bridge Balance Condition An AC bridge is formed by four complex impedance arms Z_1, Z_2, Z_3, and Z_4.

1. Express the impedances of the four arms in complex form ($Z = R + jX$).
2. Apply the general balance condition, where products of opposite arm impedances are equal:

$$Z_1 Z_4 = Z_2 Z_3$$

3. Decompose the condition. Both magnitude and phase equations must be satisfied:
   - Magnitude Condition: $|Z_1||Z_4| = |Z_2||Z_3|$
   - Phase Angle Condition: $\angle Z_1 + \angle Z_4 = \angle Z_2 + \angle Z_3$
4. Alternatively, distribute the terms and equate the real parts and imaginary parts of $Z_1 Z_4 = Z_2 Z_3$ to form two independent scalar balance equations.

3.4.1 Maxwell's Bridge

Maxwell's Inductance-Capacitance Bridge measures an unknown inductance in terms of a standard known capacitance.

Methodology:

Maxwells Inductance Capacitance Bridge Computations Let the standard arms be configured as follows: Arm 1: Resistor R_1 in parallel with Capacitor C_1. ($Y_1 = \frac{1}{R_1} + j\omega C_1$) Arm 2: Pure resistor R_2. ($Z_2 = R_2$) Arm 3: Pure resistor R_3. ($Z_3 = R_3$) Arm 4: Unknown inductor L_x in series with resistor R_x. ($Z_4 = R_x + j\omega L_x$)

1. Apply the balance equation rearranged for admittance: $Z_4 = Z_2 Z_3 Y_1$.
2. Substitute the arm expressions:

$$R_x + j\omega L_x = R_2 R_3 \left(\frac{1}{R_1} + j\omega C_1\right)$$

$$R_x + j\omega L_x = \frac{R_2 R_3}{R_1} + j\omega C_1 R_2 R_3$$

3. Equate the real parts to compute the unknown resistance R_x:

$$R_x = \frac{R_2 R_3}{R_1}$$

4. Equate the imaginary parts to compute the unknown inductance L_x:

$$L_x = R_2 R_3 C_1$$

5. Calculate the Quality Factor (Q) of the unknown coil:

$$Q = \frac{\omega L_x}{R_x} = \omega C_1 R_1$$

Worked Example:

Calculating Inductance with Maxwells Bridge A Maxwell bridge is balanced with the following constants: $C_1 = 0.5\,\mu\text{F}$, $R_1 = 1200\,\Omega$, $R_2 = 400\,\Omega$, $R_3 = 600\,\Omega$. The supply frequency is 1000 Hz. Determine the unknown resistance R_x, inductance L_x, and the Q-factor of the coil.

Solution:

1. Calculate the unknown resistance R_x:

$$R_x = \frac{R_2 R_3}{R_1} = \frac{400 \times 600}{1200} = \frac{240000}{1200} = 200\,\Omega$$

2. Calculate the unknown inductance L_x:

$$L_x = R_2 R_3 C_1 = 400 \times 600 \times (0.5 \times 10^{-6})$$

$$L_x = 240000 \times 0.5 \times 10^{-6} = 0.12 \text{ H} = 120 \text{ mH}$$

3. Compute the angular frequency ω:

$$\omega = 2\pi f = 2 \times 3.1416 \times 1000 = 6283.2 \text{ rad/s}$$

4. Calculate the Q-factor:

$$Q = \omega C_1 R_1 = 6283.2 \times (0.5 \times 10^{-6}) \times 1200 = 3.77$$

The unknown resistance is $200\,\Omega$, inductance is 120 mH, and the Q-factor is 3.77.

3.4.2 Schering Bridge

The Schering bridge is extensively utilized for measuring capacitance and the dissipation factor of capacitors.

Methodology:

Schering Bridge Computations Let the typical standard configuration be: Arm 1: Unknown capacitor C_x in series with internal resistor R_x. ($Z_1 = R_x + \frac{1}{j\omega C_x}$) Arm 2: Standard loss-free standard capacitor C_2. ($Z_2 = \frac{1}{j\omega C_2}$) Arm 3: Non-inductive standard resistor R_3. ($Z_3 = R_3$) Arm 4: Resistor R_4 in parallel with variable capacitor C_4. ($Y_4 = \frac{1}{R_4} + j\omega C_4$)

1. Use the generalized admittance balance equation: $Z_1 = Z_2 Z_3 Y_4$.
2. Substitute the parameters for each arm:

$$R_x + \frac{1}{j\omega C_x} = \left(\frac{1}{j\omega C_2}\right) R_3 \left(\frac{1}{R_4} + j\omega C_4\right)$$

$$R_x - j\frac{1}{\omega C_x} = \frac{R_3 C_4}{C_2} - j\frac{R_3}{\omega C_2 R_4}$$

3. Equate the real parts to find the equivalent series resistance R_x:

$$R_x = R_3 \frac{C_4}{C_2}$$

4. Equate the imaginary parts to find the unknown capacitance C_x:

$$\frac{1}{\omega C_x} = \frac{R_3}{\omega C_2 R_4} \implies C_x = C_2 \frac{R_4}{R_3}$$

5. Calculate the Dissipation Factor (D) of the capacitor:

$$D = \omega C_x R_x = \omega\left(C_2 \frac{R_4}{R_3}\right)\left(R_3 \frac{C_4}{C_2}\right) = \omega C_4 R_4$$

Worked Example:

Measuring Capacitance using Schering Bridge A Schering bridge is used to measure an unknown capacitor at a frequency of 50 Hz. At balance, the standard capacitor is $C_2 = 100$ pF. Arm 3 is a pure resistor $R_3 = 300\,\Omega$. Arm 4 has a parallel combination of $R_4 = 1000\,\Omega$ and $C_4 = 0.5\,\mu$F. Find the unknown capacitance C_x, equivalent series resistance R_x, and the dissipation factor.

Solution:

1. Identify the parameters: $f = 50$ Hz, $C_2 = 100 \times 10^{-12}$ F, $R_3 = 300\,\Omega$, $R_4 = 1000\,\Omega$, $C_4 = 0.5 \times 10^{-6}$ F.
2. Evaluate the unknown capacitance C_x:

$$C_x = C_2 \frac{R_4}{R_3} = 100 \text{ pF} \times \frac{1000}{300} = 333.33 \text{ pF}$$

3. Evaluate the equivalent series resistance R_x:

$$R_x = R_3 \frac{C_4}{C_2} = 300 \times \frac{0.5 \times 10^{-6}}{100 \times 10^{-12}} = 300 \times 5000 = 1.5 \times 10^6\,\Omega = 1.5\text{ M}\Omega$$

4. Compute the angular frequency ω:

$$\omega = 2\pi f = 2 \times 3.1416 \times 50 = 314.16 \text{ rad/s}$$

5. Evaluate the Dissipation Factor D:

$$D = \omega C_4 R_4 = 314.16 \times (0.5 \times 10^{-6}) \times 1000 = 0.157$$

The unknown capacitance is 333.33 pF, equivalent series resistance is 1.5 MΩ, and the dissipation factor is 0.157.

CHAPTER 4

Basic Electrical Parameter Measurement

4.1 Moving Iron and Moving Coil Instruments

Methodology:

Calculation for Ammeter Shunt and Voltmeter Multiplier To extend the range of Permanent Magnet Moving Coil (PMMC) instruments: 1. **Ammeter Shunt:** A low resistance R_{sh} connected in parallel.

$$R_{sh} = \frac{R_m}{m-1}$$

where the multiplying factor $m = \frac{I}{I_m}$, I is the total current to be measured, I_m is the full-scale deflection current of the meter, and R_m is the meter internal resistance. 2. **Voltmeter Multiplier:** A high resistance R_s connected in series.

$$R_s = R_m(m-1)$$

where the multiplying factor $m = \frac{V}{V_m}$, V is the total voltage to be measured, and V_m is the full-scale voltage of the meter.

Worked Example:

Calculating Shunt Resistance for PMMC Ammeter A PMMC instrument has a coil resistance of 100 Ω and gives full-scale deflection for 1 mA. Calculate the value of shunt resistance required to extend its range to measure up to 1 A.

Step 1: Identify given parameters. Meter resistance $R_m = 100\ \Omega$. Full-scale current $I_m = 1\text{ mA} = 0.001\text{ A}$. Required range $I = 1$ A.

Step 2: Calculate the multiplying factor m.

$$m = \frac{I}{I_m} = \frac{1}{0.001} = 1000$$

Step 3: Calculate the shunt resistance R_{sh}.

$$R_{sh} = \frac{R_m}{m-1} = \frac{100}{1000-1} = \frac{100}{999} \approx 0.1001\ \Omega$$

Methodology:

Torque Equations for MI and PMMC Instruments 1. **PMMC Instrument (Moving Coil):** Deflecting Torque: $T_d = B \cdot I \cdot N \cdot A$ Where B is the magnetic flux density, I is the current, N is the number of turns, and A is the area of the coil. 2. **Moving Iron (MI) Instrument:** Deflecting Torque: $T_d = \frac{1}{2}I^2\frac{dL}{d\theta}$ Where I is the current and $\frac{dL}{d\theta}$ is the rate of change of inductance with respect to deflection angle θ. At steady state, the Deflecting Torque T_d equals the Controlling Torque $T_c = K\theta$, where K is the spring constant.

Worked Example:

Calculating Deflection Angle for Moving Iron Instrument The inductance of a moving iron ammeter is given by $L = (10 + 5\theta - \theta^2)\ \mu\text{H}$, where θ is the deflection in radians. The spring constant of the instrument is 12×10^{-6} N-m/rad. Calculate the steady-state deflection for a current of 5 A.
Step 1: Differentiate the inductance equation with respect to θ.

$$L = (10 + 5\theta - \theta^2) \times 10^{-6} \text{ H}$$

$$\frac{dL}{d\theta} = (5 - 2\theta) \times 10^{-6} \text{ H/rad}$$

Step 2: Apply the moving iron deflecting torque equation.

$$T_d = \frac{1}{2} I^2 \frac{dL}{d\theta}$$

$$T_d = \frac{1}{2}(5)^2 \cdot (5 - 2\theta) \times 10^{-6} = 12.5(5 - 2\theta) \times 10^{-6} \text{ N-m}$$

Step 3: Equate deflecting torque to controlling torque to find θ.

$$T_d = T_c$$

$$12.5(5 - 2\theta) \times 10^{-6} = 12 \times 10^{-6} \cdot \theta$$

$$62.5 - 25\theta = 12\theta$$

$$37\theta = 62.5$$

$$\theta \approx 1.689 \text{ rad}$$

4.2 Electronic Multimeters and Digital Panel Meters

Methodology:

Meter Sensitivity and Voltmeter Loading Effect 1. **Sensitivity (S):** Expressed in Ohms per Volt (Ω/V), it indicates the input resistance per volt of the selected range.

$$S = \frac{1}{I_{fsd}}$$

where I_{fsd} is the full-scale deflection current of the internal movement. 2. **Voltmeter Resistance (R_v):**

$$R_v = S \times V_{range}$$

3. **Loading Effect Error:** When measuring voltage across a resistor R_x in a circuit, the voltmeter acts in parallel, creating an equivalent resistance $R_{eq} = \frac{R_x R_v}{R_x + R_v}$. This lowers the overall resistance and changes the circuit dynamics, causing an artificially low voltage reading.

$$\text{Error Percentage} = \frac{\text{Measured Value} - \text{True Value}}{\text{True Value}} \times 100\%$$

Worked Example:

Calculating Loading Effect Error of an Electronic Multimeter Two resistors $R_1 = 10\ \text{k}\Omega$ and $R_2 = 10\ \text{k}\Omega$ are connected in series across a 100 V DC supply. A multimeter with a sensitivity of 10 kΩ/V on its 50 V range is used to measure the voltage across R_2. Find the reading and the percentage error.

Step 1: Calculate the true voltage across R_2 without the meter.

$$V_{true} = 100 \times \frac{R_2}{R_1 + R_2} = 100 \times \frac{10}{20} = 50 \text{ V}$$

Step 2: Calculate the voltmeter internal resistance.

$$R_v = S \times V_{range} = 10 \text{ k}\Omega/\text{V} \times 50 \text{ V} = 500 \text{ k}\Omega$$

Step 3: Calculate the equivalent resistance of R_2 and R_v in parallel.

$$R_{eq} = \frac{10 \text{ k}\Omega \times 500 \text{ k}\Omega}{10 \text{ k}\Omega + 500 \text{ k}\Omega} = \frac{5000}{510} \approx 9.804 \text{ k}\Omega$$

Step 4: Calculate the measured voltage with the meter connected.

$$V_{measured} = 100 \times \frac{R_{eq}}{R_1 + R_{eq}} = 100 \times \frac{9.804}{10 + 9.804} = 100 \times \frac{9.804}{19.804} \approx 49.505 \text{ V}$$

Step 5: Calculate the percentage loading error.

$$\% \text{ Error} = \frac{49.505 - 50}{50} \times 100\% \approx -0.99\%$$

Methodology:

Digital Panel Meter Resolution and Range A Digital Panel Meter (DPM) converts analog signals to a digital display. 1. **Maximum Count:** For a $3\frac{1}{2}$ digit meter, the full digits span $0-9$, while the half digit displays '0' or '1'. The maximum display is 1999. 2. **Resolution (R):** The smallest measurable change in input.

$$R = \frac{\text{Full Scale Range}}{\text{Total Number of Counts}}$$

3. **Digit Relationship:** An N-digit meter generally has a resolution of $1/10^N$.

Worked Example:

Calculating Resolution for a Digital Panel Meter A $3\frac{1}{2}$ digit Digital Panel Meter is set to a voltage range of 200 mV. Calculate its resolution in volts and determine the maximum displayable voltage.
Step 1: Determine the total number of counts. A $3\frac{1}{2}$ digit meter counts from 0000 to 1999. Maximum count reading = 1999. Total number of distinct states (counts) = 2000.
Step 2: Calculate the maximum displayed voltage. While the nominal range is 200 mV, the display stops at 1999. Therefore, the maximum voltage it can show is 199.9 mV.
Step 3: Calculate the resolution.

$$R = \frac{200 \text{ mV}}{2000} = 0.1 \text{ mV} = 10^{-4} \text{ V}$$

Methodology:

Digital Frequency Meter Computations A digital panel meter operating in frequency mode works by counting the number of cycles (pulses) of the input signal over a precisely defined time interval called the gate time (T_g). 1. **Measured Frequency (f_{in}):**

$$f_{in} = \frac{N}{T_g}$$

where N is the number of pulses registered by the counter. 2. **Quantization Error:** Because the counter increments in whole numbers, there is an inherent ± 1 count ambiguity.

$$\text{Percentage Error} = \pm \left(\frac{1}{N} \times 100 \right) \%$$

Worked Example:

Frequency Measurement Accuracy Calculation A digital frequency meter is used to measure a 50 Hz signal. If the instrument's gate time base is configured to 10 seconds, calculate the number of pulses accumulated and the maximum percentage error caused by the ± 1 count ambiguity.
Step 1: Calculate the number of pulses counted (N).

$$N = f_{in} \times T_g = 50 \text{ Hz} \times 10 \text{ s} = 500 \text{ pulses}$$

Step 2: Calculate the ± 1 count error percentage. The final count might be 499, 500, or 501 depending on synchronization between the signal and the gate.

$$\text{Percentage Error} = \pm \left(\frac{1}{500} \times 100 \right) \% = \pm 0.2\%$$

4.3 Power and Energy Measurement

Methodology:

Three Phase Power Measurement using Two Wattmeters Using the Two-Wattmeter method on a balanced three-phase load, wattmeters W_1 and W_2 yield the total power and power factor. 1. **Total Active Power (P):**

$$P = W_1 + W_2 = \sqrt{3} V_L I_L \cos\phi$$

2. **Power Factor Angle (ϕ):**

$$\tan\phi = \sqrt{3} \frac{W_1 - W_2}{W_1 + W_2}$$

(Assuming $W_1 > W_2$). 3. **Power Factor (pf):**

$$pf = \cos\phi$$

Worked Example:

Calculating Power Factor from Two Wattmeter Readings The power drawn by a three-phase induction motor is measured using the two-wattmeter method. The recorded readings are $W_1 = 5.2$ kW and $W_2 = -1.7$ kW. Calculate the total power consumption and the power factor of the motor.
Step 1: Calculate the total power.

$$P = W_1 + W_2 = 5.2 + (-1.7) = 3.5 \text{ kW}$$

Step 2: Calculate the phase angle using the tangent formula.

$$\tan\phi = \sqrt{3} \frac{W_1 - W_2}{W_1 + W_2} = \sqrt{3} \frac{5.2 - (-1.7)}{5.2 + (-1.7)} = \sqrt{3} \frac{6.9}{3.5} \approx 1.732 \times 1.971 \approx 3.414$$

Step 3: Calculate the phase angle ϕ and corresponding power factor.

$$\phi = \tan^{-1}(3.414) \approx 73.67^\circ$$

$$\text{Power Factor} = \cos(73.67^\circ) \approx 0.281 \text{ (lagging)}$$

Methodology:

Energy Meter Constant and Error Calculation 1. **Energy Meter Constant (K):** Represents the number of revolutions the disc makes per kilowatt-hour.

$$K = \frac{N}{E}$$

where N is the number of revolutions and E is the true energy in kWh. 2. **True Energy Consumed (E):**

$$E = \frac{P \times t}{1000} \text{ kWh}$$

where P is true power in Watts ($P = VI\cos\phi$) and t is time in hours. 3. **Percentage Error:**

$$\% \text{ Error} = \frac{\text{Measured Revs} - \text{True Expected Revs}}{\text{True Expected Revs}} \times 100\%$$

Worked Example:

Calculating Energy Meter Registration Error An energy meter has a rating of 1200 rev/kWh. It is connected to a load drawing 20 A at 230 V with a power factor of 0.8. The meter is observed to make 730 revolutions over a period of 10 minutes. Determine the percentage error of the meter.

Step 1: Calculate the true power consumed by the load.

$$P = V \times I \times \cos\phi = 230 \times 20 \times 0.8 = 3680 \text{ W} = 3.68 \text{ kW}$$

Step 2: Calculate true energy consumed in 10 minutes.

$$t = \frac{10}{60} \text{ hours} = \frac{1}{6} \text{ hours}$$

$$E = P \times t = 3.68 \times \frac{1}{6} \approx 0.6133 \text{ kWh}$$

Step 3: Calculate the true number of revolutions expected.

$$\text{True Revs} = K \times E = 1200 \times 0.6133 = 736 \text{ revolutions}$$

Step 4: Calculate the percentage error.

$$\% \text{ Error} = \frac{730 - 736}{736} \times 100\% = \frac{-6}{736} \times 100\% \approx -0.815\%$$

The meter is running slower than expected by 0.815%.

4.4 Clamp on Ammeters

Methodology:

Clamp on Ammeter Multiplier Calculation A clamp-on ammeter uses a current transformer (CT) to measure alternating current without physically breaking the circuit. 1. **Current Ratio:**

$$\frac{I_p}{I_s} = \frac{N_s}{N_p}$$

where I_p is the primary line current, I_s is the secondary meter current, N_s is secondary turns, and N_p is primary turns. Usually, a single wire passes through, meaning $N_p = 1$. 2. **Current Amplification via Looping:** To measure currents below the meter's threshold, the conductor can be looped through the clamp jaws N times. The effective primary turns become $N_p = N$.

$$\text{Actual Current} = \frac{\text{Meter Reading}}{\text{Number of Loops } (N)}$$

Worked Example:

Measuring Low Current with a Clamp on Ammeter A clamp-on ammeter has a minimum effective scale of 10 A. A technician needs to measure a small AC current of approximately 1.5 A with better precision. To achieve this, the technician loops the current-carrying wire 5 times through the clamp jaws. If the meter displays 7.6 A, what is the actual current in the wire?
Step 1: Identify given parameters. Number of loops (effective primary turns) $N = 5$. Meter reading (apparent primary current) $I_{reading} = 7.6$ A.
Step 2: Apply the turns multiplier formula. Because the wire passes through the core 5 times, the induced magnetic field is 5 times stronger. The meter measures this amplified field.

$$\text{Actual Current} = \frac{\text{Meter Reading}}{N}$$

Step 3: Calculate the actual current.

$$\text{Actual Current} = \frac{7.6}{5} = 1.52 \text{ A}$$

4.5 Digital Storage Oscilloscope (DSO)

Methodology:

Waveform Parameters using DSO DSO traces offer a visual representation of signals which can be interpreted mathematically using scale factors. 1. **Peak-to-Peak Voltage (V_{pp}):**

$$V_{pp} = (\text{Number of Vertical Divisions}) \times (\text{Volts/Div Setting})$$

2. **Time Period (T):**

$$T = (\text{Number of Horizontal Divisions for One Cycle}) \times (\text{Time/Div Setting})$$

3. **Frequency (f):**

$$f = \frac{1}{T}$$

Worked Example:

Calculating Frequency and Amplitude from DSO Screen A sinusoidal waveform displayed on a DSO spans 4.5 vertical divisions from peak to peak. A single full cycle spans 8 horizontal divisions. The vertical scale dial is set to 2 V/div and the horizontal time base dial is at 50 μs/div. Determine the RMS voltage and the frequency of the waveform.
Step 1: Calculate Peak-to-Peak and RMS Voltage.

$$V_{pp} = 4.5 \text{ div} \times 2 \text{ V/div} = 9 \text{ V}$$

Peak voltage $V_p = \frac{V_{pp}}{2} = 4.5$ V.

$$V_{rms} = \frac{V_p}{\sqrt{2}} = \frac{4.5}{1.414} \approx 3.18 \text{ V}$$

Step 2: Calculate Time Period.

$$T = 8 \text{ div} \times 50\ \mu\text{s/div} = 400\ \mu\text{s} = 400 \times 10^{-6} \text{ s}$$

Step 3: Calculate Frequency.

$$f = \frac{1}{T} = \frac{1}{400 \times 10^{-6}} = 2500 \text{ Hz} = 2.5 \text{ kHz}$$

Methodology:

DSO Memory Depth and Sampling Rate In the block diagram of a DSO, the Analog-to-Digital Converter (ADC) samples the waveform and stores it into digital memory. 1. **Acquisition Time:** The total time span captured across the entire display.

$$\text{Acquisition Time} = (\text{Time/Div Setting}) \times (\text{Total Horizontal Divisions})$$

(A standard DSO grid has 10 horizontal divisions). 2. **Record Length (Memory Depth):** The total number of digital points or samples the DSO must store to recreate the trace.

$$\text{Memory Depth} = \text{Sampling Rate} \times \text{Acquisition Time}$$

Worked Example:

Calculating Required Memory Depth for DSO A Digital Storage Oscilloscope needs to capture a signal with a time base set to 2 ms/div. The screen consists of 10 horizontal divisions. If the Analog-to-Digital Converter's sampling rate is 50 MSa/s (Mega Samples per second), what is the minimum memory depth required to store the complete visible trace?

Step 1: Calculate the total Acquisition Time.

$$\text{Acquisition Time} = 2 \text{ ms/div} \times 10 \text{ div} = 20 \text{ ms} = 20 \times 10^{-3} \text{ s}$$

Step 2: Formulate the Sampling Rate.

$$\text{Sampling Rate} = 50 \times 10^{6} \text{ Samples/s}$$

Step 3: Calculate the Memory Depth.

$$\text{Memory Depth} = (50 \times 10^{6}) \text{ Samples/s} \times (20 \times 10^{-3}) \text{ s}$$

$$\text{Memory Depth} = 1000 \times 10^{3} \text{ Samples} = 1 \text{ Mpoints (Mega points)}$$

CHAPTER 5

Transducers and Sensors

5.1 Transducer Classification and Performance

Transducers are devices that convert a physical quantity into an electrical signal. They are broadly classified into active transducers (generating their own voltage/current) and passive transducers (requiring an external power source). The computational aspect of transducer selection primarily relies on parameters such as sensitivity and resolution.

Methodology:

Sensitivity and Resolution Evaluation 1. **Sensitivity Formulation:** Sensitivity (S) is the ratio of the change in output to the change in input.

$$S = \frac{\Delta \text{Output}}{\Delta \text{Input}}$$

2. **Resolution Calculation:** The resolution is the smallest input increment that produces a measurable change in the output. If the measurement system has a minimum detectable output ΔV_{min}, the resolution is:

$$\text{Resolution} = \frac{\Delta V_{min}}{S}$$

Worked Example:

Pressure Transducer Sensitivity Analysis A passive pressure transducer requires an external excitation of 10 V and produces an output voltage ranging from 0 mV to 50 mV for an applied pressure of 0 bar to 200 bar.

Step 1: Identify given parameters.

- Input range: $\Delta \text{Input} = 200 - 0 = 200$ bar
- Output range: $\Delta \text{Output} = 50 - 0 = 50$ mV

Step 2: Calculate sensitivity.

$$S = \frac{\Delta \text{Output}}{\Delta \text{Input}} = \frac{50 \text{ mV}}{200 \text{ bar}} = 0.25 \text{ mV/bar}$$

Step 3: Determine resolution. If the voltmeter reading the output has a minimum measurable voltage of 0.1 mV, calculate the resolution.

$$\text{Resolution} = \frac{\Delta V_{min}}{S} = \frac{0.1 \text{ mV}}{0.25 \text{ mV/bar}} = 0.4 \text{ bar}$$

The smallest detectable pressure change is 0.4 bar.

5.2 Linear Variable Differential Transformer (LVDT)

The LVDT is an inductive transducer used to measure linear displacement. It consists of one primary winding and two secondary windings connected in series opposition.

Methodology:

LVDT Output Voltage Calculation 1. **Core Displacement:** Let x be the linear displacement of the core from the null position. 2. **Secondary Voltages:** The voltages induced in the two secondary coils are V_{s1} and V_{s2}. 3. **Differential Output:** The net output voltage V_{out} is the difference between the two secondary voltages.

$$V_{out} = V_{s1} - V_{s2}$$

4. **Sensitivity Equation:** If the LVDT operates in its linear range, the output voltage is proportional to the displacement.

$$V_{out} = S \times x$$

where S is the sensitivity, usually expressed in mV/mm or V/mm.

Worked Example:

LVDT Displacement Measurement An LVDT has a sensitivity of 50 mV/mm. The secondary voltages are $V_{s1} = 2.4$ V and $V_{s2} = 2.1$ V. Determine the core displacement.

Step 1: Calculate the net output voltage.

$$V_{out} = |V_{s1} - V_{s2}| = |2.4 \text{ V} - 2.1 \text{ V}| = 0.3 \text{ V} = 300 \text{ mV}$$

Step 2: Calculate the displacement.

$$x = \frac{V_{out}}{S} = \frac{300 \text{ mV}}{50 \text{ mV/mm}} = 6 \text{ mm}$$

The core has moved 6 mm from the null position towards the secondary coil S_1 (since $V_{s1} > V_{s2}$).

5.3 Inductive and Capacitive Transducers

Inductive and capacitive transducers measure physical quantities by converting them into changes of inductance or capacitance.

5.3.1 Inductive Transducers

Methodology:

Inductance Variation Analysis 1. **Self-Inductance Equation:** The inductance L of a coil with an air gap is:

$$L = \frac{N^2 \mu_0 A}{l_g}$$

where N is the number of turns, μ_0 is the permeability of free space, A is the cross-sectional area, and l_g is the length of the air gap. 2. **Change in Inductance:** A change in the air gap Δl_g causes an inversely proportional change in inductance.

Worked Example:

Air Gap Inductive Sensor An inductive sensor has a nominal inductance of 10 mH when the air gap is 2 mm. If the air gap is reduced to 1.5 mm, calculate the new inductance, assuming all other parameters remain constant.

Step 1: Set up the proportionality. Since $L \propto \frac{1}{l_g}$, we can write:

$$L_1 \cdot l_{g1} = L_2 \cdot l_{g2}$$

Step 2: Calculate the new inductance L_2.

$$10 \text{ mH} \times 2 \text{ mm} = L_2 \times 1.5 \text{ mm}$$

$$L_2 = \frac{20}{1.5} \text{ mH} \approx 13.33 \text{ mH}$$

5.3.2 Capacitive Transducer for Pressure Measurement

Methodology:

Capacitance Variation with Plate Separation 1. **Parallel Plate Capacitance:**

$$C = \frac{\varepsilon_0 \varepsilon_r A}{d}$$

where $\varepsilon_0 = 8.854 \times 10^{-12}$ F/m, ε_r is the relative permittivity, A is the overlapping area, and d is the distance between plates. 2. **Pressure Induced Change:** Applied pressure P changes the distance d by Δd. The new distance is $d_{new} = d - \Delta d$. 3. **New Capacitance:**

$$C_{new} = \frac{\varepsilon_0 \varepsilon_r A}{d - \Delta d}$$

Worked Example:

Capacitive Pressure Sensor Evaluation A capacitive pressure sensor has two parallel circular plates of radius 10 mm separated by an air gap ($\varepsilon_r = 1$) of 1 mm. A pressure application reduces the gap by 0.2 mm. Calculate the change in capacitance.

Step 1: Calculate the area of the plates.

$$A = \pi r^2 = \pi (10 \times 10^{-3} \text{ m})^2 = 3.14 \times 10^{-4} \text{ m}^2$$

Step 2: Calculate initial capacitance C_0 (with $d = 1$ mm $= 10^{-3}$ m).

$$C_0 = \frac{(8.854 \times 10^{-12}) \times 1 \times (3.14 \times 10^{-4})}{10^{-3}} \approx 2.78 \times 10^{-12} \text{ F} = 2.78 \text{ pF}$$

Step 3: Calculate the new capacitance C_{new} (with $d_{new} = 0.8$ mm $= 0.8 \times 10^{-3}$ m).

$$C_{new} = \frac{(8.854 \times 10^{-12}) \times 1 \times (3.14 \times 10^{-4})}{0.8 \times 10^{-3}} \approx 3.48 \times 10^{-12} \text{ F} = 3.48 \text{ pF}$$

Step 4: Calculate the change in capacitance ΔC.

$$\Delta C = C_{new} - C_0 = 3.48 \text{ pF} - 2.78 \text{ pF} = 0.70 \text{ pF}$$

5.4 Temperature Transducers

Temperature transducers convert thermal energy into an electrical signal. The most common types evaluated computationally are RTDs, Thermistors, Thermocouples, and integrated ICs like LM35.

5.4.1 Resistance Temperature Detector (RTD)

Methodology:

RTD Linear Resistance Model 1. **Linear Resistance Equation:**

$$R_T = R_0[1 + \alpha(T - T_0)]$$

where R_T is the resistance at temperature T, R_0 is the nominal resistance at reference temperature T_0 (usually 0°C), and α is the temperature coefficient of resistance. 2. **Temperature Calculation:**

$$T = T_0 + \frac{R_T - R_0}{\alpha R_0}$$

Worked Example:

Pt100 RTD Temperature Measurement A Pt100 sensor has a resistance of 100 Ω at 0°C and a temperature coefficient $\alpha = 0.00385\ °\mathrm{C}^{-1}$. If the measured resistance is 138.5 Ω, compute the measured temperature.
Step 1: Identify parameters. $R_0 = 100\ \Omega$, $T_0 = 0°\mathrm{C}$, $\alpha = 0.00385\ °\mathrm{C}^{-1}$, $R_T = 138.5\ \Omega$.
Step 2: Apply the temperature formula.

$$T = 0 + \frac{138.5 - 100}{0.00385 \times 100} = \frac{38.5}{0.385} = 100°\mathrm{C}$$

The measured temperature is 100°C.

5.4.2 Thermistor

Methodology:

Thermistor Beta Parameter Equation 1. **Exponential Resistance Equation:**

$$R_T = R_0 \exp\left[\beta\left(\frac{1}{T} - \frac{1}{T_0}\right)\right]$$

where temperatures T and T_0 must be in Kelvin ($K =^\circ \mathrm{C} + 273.15$). R_0 is resistance at T_0 (often 25°C), and β is the material constant. 2. **Temperature Calculation from Resistance:**

$$T = \left(\frac{1}{T_0} + \frac{1}{\beta}\ln\left(\frac{R_T}{R_0}\right)\right)^{-1}$$

Worked Example:

Thermistor Resistance Computation An NTC thermistor has a nominal resistance of 10 kΩ at 25°C and a beta value (β) of 3950 K. Calculate its resistance at 50°C.
Step 1: Convert temperatures to Kelvin.

$$T_0 = 25 + 273.15 = 298.15\ \mathrm{K}$$

$$T = 50 + 273.15 = 323.15\ \mathrm{K}$$

Step 2: Calculate the exponential term argument.

$$\beta\left(\frac{1}{T} - \frac{1}{T_0}\right) = 3950\left(\frac{1}{323.15} - \frac{1}{298.15}\right)$$

$$= 3950(0.0030945 - 0.0033540) = 3950(-0.0002595) = -1.025$$

Step 3: Compute final resistance.

$$R_T = 10\ \mathrm{k}\Omega \times \exp(-1.025) = 10\ \mathrm{k}\Omega \times 0.3588 = 3.588\ \mathrm{k}\Omega$$

As expected for an NTC, resistance decreases as temperature increases.

5.4.3 Thermocouple and LM35

Methodology:

Thermocouple and IC Sensor Calculations 1. **Thermocouple Seebeck Equation:** The thermoelectric voltage V_{th} generated is proportional to the temperature difference between the hot and cold junctions.

$$V_{th} = S \times (T_{hot} - T_{cold})$$

where S is the Seebeck sensitivity coefficient (μV/°C). 2. **LM35 Output Equation:** The LM35 provides a linear analog voltage output strictly proportional to Celsius temperature.

$$V_{out} = 10 \text{ mV/}^\circ\text{C} \times T_{celsius}$$

Worked Example:

LM35 and Thermocouple Analysis **Part A: LM35** Calculate the required ADC reference voltage if an LM35 sensor is used to measure up to 150°C.
Step 1: Compute the maximum output voltage.

$$V_{out(max)} = 10 \text{ mV/}^\circ\text{C} \times 150^\circ\text{C} = 1500 \text{ mV} = 1.5 \text{ V}$$

The ADC reference voltage must be at least 1.5 V.
Part B: Thermocouple A K-type thermocouple ($S = 41$ μV/°C) has its cold junction at 25°C. The measured voltage is 4.1 mV. Find the hot junction temperature.
Step 1: Rearrange the Seebeck equation.

$$T_{hot} = T_{cold} + \frac{V_{th}}{S}$$

Step 2: Substitute the values (ensure units match, 4.1 mV = 4100 μV).

$$T_{hot} = 25^\circ\text{C} + \frac{4100\ \mu\text{V}}{41\ \mu\text{V}/^\circ\text{C}} = 25 + 100 = 125^\circ\text{C}$$

5.5 Gas and Humidity Sensors

Methodology:

MQ2 Gas Sensor Resistor Divider 1. **Voltage Divider Output:** The MQ2 sensor has a variable resistance R_s based on gas concentration. It is used in a voltage divider with a load resistor R_L.

$$V_{out} = V_{cc} \times \frac{R_L}{R_s + R_L}$$

2. **Sensor Resistance Extraction:**

$$R_s = R_L \times \left(\frac{V_{cc}}{V_{out}} - 1\right)$$

The ratio $\frac{R_s}{R_0}$ (where R_0 is resistance in clean air) is then used to find ppm from the sensor's logarithmic datasheet graph.

Worked Example:

MQ2 Sensor Resistance Calculation An MQ2 gas sensor is powered by $V_{cc} = 5$ V and uses a load resistor $R_L = 10$ kΩ. If the microcontroller measures an output of 3.3 V, calculate the sensor resistance R_s.

Step 1: Apply the extraction formula.

$$R_s = 10\ \text{k}\Omega \times \left(\frac{5\ \text{V}}{3.3\ \text{V}} - 1\right)$$

Step 2: Compute the result.

$$R_s = 10\ \text{k}\Omega \times (1.515 - 1) = 10\ \text{k}\Omega \times 0.515 = 5.15\ \text{k}\Omega$$

Methodology:

DHT11 Humidity Sensor Data Parsing 1. **Data Format:** The DHT11 transmits a 40-bit data packet.

- 8-bit integral RH data
- 8-bit decimal RH data (always 0 for DHT11)
- 8-bit integral Temperature data
- 8-bit decimal Temperature data (always 0 for DHT11)
- 8-bit Checksum

2. **Checksum Validation:**

$$\text{Checksum} = (\text{Integral RH} + \text{Decimal RH} + \text{Integral Temp} + \text{Decimal Temp})\ \text{AND 0xFF}$$

Worked Example:

DHT11 Data Verification A microcontroller receives the following 40 bits (in hex) from a DHT11 sensor: `32 00 1C 00 4E`. Verify the checksum and extract the physical readings.
Step 1: Convert hex to decimal.

- Integral RH = `0x32` = 50
- Decimal RH = `0x00` = 0
- Integral Temp = `0x1C` = 28
- Decimal Temp = `0x00` = 0
- Checksum = `0x4E` = 78

Step 2: Validate the Checksum.

$$\text{Sum} = 50 + 0 + 28 + 0 = 78$$

Since 78 matches the Checksum byte (`0x4E`), the data is valid.
Step 3: State the readings. Humidity = 50% RH, Temperature = 28°C.

5.6 Ultrasonic Sensors

Ultrasonic sensors (like the HC-SR04) measure distance by emitting high-frequency sound waves and timing the reflection (echo).

Methodology:

Time of Flight Distance Calculation 1. **Speed of Sound:** At room temperature, the speed of sound v in air is approximately 343 m/s, which is 0.0343 cm/μs. 2. **Distance Formula:** The total time measured t is for the round trip (ping to object and echo back). The distance d is:

$$d = \frac{v \times t}{2}$$

Using the speed in cm/μs:

$$d \text{ (cm)} = \frac{t\ (\mu\text{s})}{58.3} \approx \frac{t\ (\mu\text{s})}{58}$$

Worked Example:

Obstacle Distance Measurement An ultrasonic sensor's echo pin stays high for 1450 μs. Calculate the distance to the obstacle in cm.

Step 1: Apply the precise time-of-flight equation.

$$d = \frac{0.0343 \text{ cm}/\mu\text{s} \times 1450\ \mu\text{s}}{2}$$

Step 2: Compute the distance.

$$d = \frac{49.735}{2} = 24.8675 \text{ cm}$$

Alternatively, using the standard constant approximation: $d = 1450/58 = 25$ cm.

5.7 Optical Encoders

Optical encoders provide digital outputs corresponding to angular position. They are classified into absolute encoders (providing a unique code per position) and incremental encoders (providing pulses for relative movement).

Methodology:

Encoder Resolution and Rotational Speed 1. **Resolution Calculation:** For an encoder with N pulses per revolution (PPR), the angular resolution θ_{res} is:

$$\theta_{res} = \frac{360°}{N}$$

2. **Rotational Speed from Frequency:** If the encoder channel outputs pulses at a frequency f (in Hz or pulses per second), the rotational speed N_{rpm} is:

$$N_{rpm} = \frac{f \times 60}{N}$$

3. **Quadrature A B Z Waveform Processing:**

- **Channel A and B** are offset by 90° (quadrature). The phase relationship determines direction (A leading B, or B leading A). By counting both rising and falling edges of A and B, the effective resolution increases by 4× ($4N$).
- **Channel Z (Index)** pulses once per revolution, used for homing or precise revolution counting.

Worked Example:

Incremental Encoder Speed and Position Analysis An incremental optical encoder has a rating of 500 PPR. A microcontroller measures the frequency of Channel A pulses as 2500 Hz. Furthermore, an edge detection algorithm counts 125 pulses. Calculate the rotational speed of the shaft and the mechanical angle rotated.

Step 1: Calculate Angular Resolution.

$$\theta_{res} = \frac{360°}{500} = 0.72° \text{ per pulse}$$

Step 2: Calculate Rotational Speed (RPM). Using the frequency $f = 2500$ Hz:

$$\text{Speed in RPM} = \frac{f \times 60}{N} = \frac{2500 \times 60}{500} = \frac{150000}{500} = 300 \text{ RPM}$$

Step 3: Calculate Angle Rotated.

$$\text{Angle Rotated} = \text{Pulse Count} \times \theta_{res} = 125 \times 0.72° = 90°$$

The shaft has rotated by 90°, operating at a constant speed of 300 RPM.

Appendix: Key Formulae

This appendix provides a quick reference to the key symbolic formulae and mathematical relationships discussed in this book.

Unit 1: Basic Electrical Circuits and Theorems

Equivalent Resistance

- **Series Combination:**

$$R_{eq} = R_1 + R_2 + \cdots + R_n$$

- **Parallel Combination:**

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_n}$$

Network Theorems

- **Maximum Power Transfer Theorem:** The maximum power P_{max} delivered to the load is given by

$$P_{max} = \frac{V_{th}^2}{4R_{th}}$$

  where V_{th} is the Thevenin voltage and R_{th} is the Thevenin resistance.

Two-Port Networks

- **Z-parameters (Impedance parameters):**

$$V_1 = Z_{11}I_1 + Z_{12}I_2$$
$$V_2 = Z_{21}I_1 + Z_{22}I_2$$

- **Y-parameters (Admittance parameters):**

$$I_1 = Y_{11}V_1 + Y_{12}V_2$$
$$I_2 = Y_{21}V_1 + Y_{22}V_2$$

- **h-parameters (Hybrid parameters):**

$$V_1 = h_{11}I_1 + h_{12}V_2$$
$$I_2 = h_{21}I_1 + h_{22}V_2$$

- **ABCD-parameters (Transmission parameters):**

$$V_1 = AV_2 - BI_2$$
$$I_1 = CV_2 - DI_2$$

Unit 2: Filters and Resonance

Reactance and Quality Factor

- **Inductive Reactance:** $X_L = 2\pi f L$
- **Capacitive Reactance:** $X_C = \frac{1}{2\pi f C}$
- **Quality Factor of a Coil:** $Q_L = \frac{X_L}{R_L} = \frac{2\pi f L}{R_L}$
- **Quality Factor of a Capacitor:** $Q_C = \frac{X_C}{R_C} = \frac{1}{2\pi f C R_C}$

Resonance

- **Series Resonant Frequency:**

$$f_r = \frac{1}{2\pi\sqrt{LC}}$$

- **Parallel (Tank Circuit) Resonant Frequency:**

$$f_r = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$$

- **Dynamic Impedance (Parallel Resonance):**

$$Z_d = \frac{L}{RC}$$

- **Quality Factor at Resonance:**

$$Q_0 = \frac{1}{R}\sqrt{\frac{L}{C}} = \frac{2\pi f_r L}{R}$$

- **Bandwidth:**

$$BW = f_2 - f_1 = \frac{R}{2\pi L} = \frac{f_r}{Q_0}$$

Constant-k Filters

- **Low Pass Filter Components:**

$$L_k = \frac{R_0}{\pi f_c}, \quad C_k = \frac{1}{\pi R_0 f_c}$$

- **High Pass Filter Components:**

$$L_k = \frac{R_0}{4\pi f_c}, \quad C_k = \frac{1}{4\pi R_0 f_c}$$

Unit 3: Bridges and Measurement

- **AC Bridge Balance Condition:**

$$Z_1 Z_4 = Z_2 Z_3 \quad \text{or} \quad Z_1 = Z_2 Z_3 Y_4$$

  where Z represents impedance and Y represents admittance.

- **Impedance of Series RC Circuit:**

$$Z = R_x + \frac{1}{j\omega C_x}$$

Unit 4: Instrumentation and Meters

- **Deflecting Torque (PMMC Instruments):**

$$T_d = B \cdot I \cdot N \cdot A$$

where B is flux density, I is current, N is number of turns, and A is the area of the coil.

- **Deflecting Torque (Moving Iron Instruments):**

$$T_d = \frac{1}{2} I^2 \frac{dL}{d\theta}$$

where I is the operating current and L is the inductance at deflection θ.

- **Voltmeter Multiplier Factor:**

$$m = \frac{V}{V_m}$$

where V is the total voltage to be measured and V_m is the full-scale voltage of the meter.

- **Peak Voltage from Peak-to-Peak:**

$$V_p = \frac{V_{pp}}{2}$$

Unit 5: Sensors and Transducers

General Characteristics

- **Sensitivity:**

$$S = \frac{\Delta \text{Output}}{\Delta \text{Input}}$$

- **Resolution:**

$$\text{Resolution} = \frac{\Delta V_{min}}{S}$$

Temperature Sensors

- **RTD Resistance-Temperature Relationship:**

$$R_T = R_0[1 + \alpha(T - T_0)]$$

Inverse relationship: $T = T_0 + \frac{R_T - R_0}{\alpha R_0}$

- **Thermistor (NTC) Equation:**

$$R_T = R_0 \exp\left[\beta\left(\frac{1}{T} - \frac{1}{T_0}\right)\right]$$

where T and T_0 must be in Kelvin.

- **Thermocouple Output Voltage:**

$$V_{th} = S \times (T_{hot} - T_{cold})$$

- **LM35 Output Voltage:**

$$V_{out} = 10 \text{ mV}/^\circ\text{C} \times T_{celsius}$$

Displacement and Proximity Sensors

- **Parallel Plate Capacitance:**

$$C = \frac{\varepsilon_0 \varepsilon_r A}{d}$$

- **Inductance of a Magnetic Circuit:**

$$L = \frac{N^2 \mu_0 A}{l_g}$$

where l_g is the air gap length. If l_g changes while other parameters are constant: $L_1 \cdot l_{g1} = L_2 \cdot l_{g2}$

- **LVDT Output Voltage:**

$$V_{out} = |V_{s1} - V_{s2}| \quad \text{or} \quad V_{out} = S \times x$$

- **Ultrasonic Distance Measurement:**

$$d = \frac{v \times t}{2}$$

where v is the speed of sound and t is the total time of flight (echo time).

Encoders and Signal Conditioning

- **Encoder Angular Resolution:**

$$\theta_{res} = \frac{360°}{N}$$

where N is the number of pulses per revolution.

- **Speed Measurement using Encoder:**

$$N_{rpm} = \frac{f \times 60}{N}$$

where f is the pulse frequency.

- **Voltage Divider Output:**

$$V_{out} = V_{cc} \times \frac{R_L}{R_s + R_L}$$

- **Pull-up Resistor Calculation:**

$$R_s = R_L \times \left(\frac{V_{cc}}{V_{out}} - 1 \right)$$

- **DHT11 Checksum Validation:**

$$\text{Checksum} = (\text{Integral RH} + \text{Decimal RH} + \text{Integral Temp} + \text{Decimal Temp}) \text{ AND } 0\text{xFF}$$