Electronics Circuit Network & Measurement (DI03011031)

Complete Lecture Deck – All 45 Lectures

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Course Agenda I

Course Agenda II

8. Lecture 1.8: Mesh Analysis
9. Lecture 1.9: Network Theorems: Thevenin, Norton, Superposition
10. Lecture 1.10: Network Theorems: Max Power Transfer, Reciprocity, Duality
11. Lecture 2.1: Determine Quality factor of a Coil and Capacitor
12. Lecture 2.2: Series Resonant Circuits & Frequency Response
13. Lecture 2.3: Parallel Resonant Circuits & Frequency Response
14. Lecture 2.2: Calculation of Resonance Parameters

Course Agenda III

Course Agenda IV

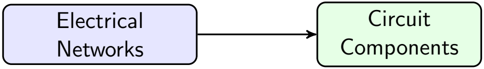
Course Agenda V

Course Agenda VI

Course Agenda VII

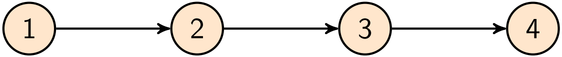
Welcome & Introduction

- Electronics Circuit Network & Measurement
- Unit 1: Network Elements and Network Analysis
- Lecture 1: Introduction to Network Elements
- Overview of electrical networks.
- Fundamental building blocks of circuits.
- Classification of network elements.



Agenda for Today

- What is a Network/Circuit?
- Active vs. Passive Elements
- Bilateral vs. Unilateral Elements
- Linear vs. Non-Linear Elements
- Lumped vs. Distributed Elements
- Quick Check & Summary



What is an Electrical Network?

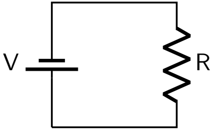
Definition

An interconnection of electrical elements (like resistors, inductors, capacitors, voltage sources) in any manner is called an electrical network.

Circuit vs. Network

- Network: Any interconnection, open or closed.
- Circuit: A network that contains at least one closed path for current to flow.
- Note: All circuits are networks, but not all networks are circuits.

Open Network



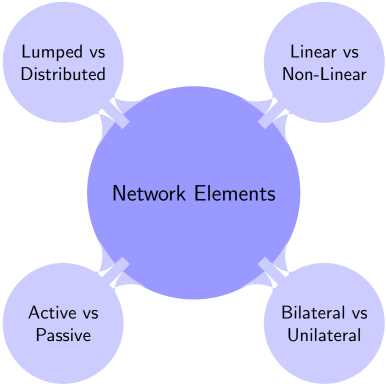
Closed Circuit

Classification of Network Elements

Network elements are the individual components in a circuit. They are classified based on their electrical properties and behavior:

- Energy handling: Active vs. Passive
- Directionality: Bilateral vs. Unilateral
- V-I Relationship: Linear vs. Non-Linear
- Physical size/distribution: Lumped vs. Distributed

These classifications help in choosing the right mathematical models for circuit analysis.



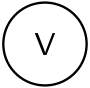
Active Elements

What are they?
Elements capable of continuously supplying energy to a network or amplifying a signal.
Characteristics:

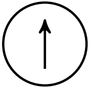
- Act as a source of electrical energy.
- Can control the flow of current (e.g., transistors).

Examples:

- Independent Sources: Batteries, DC/AC Voltage generators, Current sources.
- Dependent Sources: Voltage-controlled voltage sources (VCVS), etc.
- Electronic Devices: Transistors, Operational Amplifiers.



Voltage Source

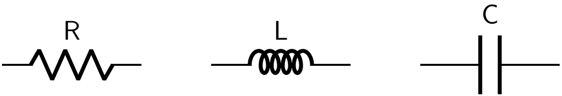


Current Source

Passive Elements

What are they?
Elements that can only receive, store, or dissipate energy. They cannot generate or amplify energy continuously.
Characteristics:
Consume energy (dissipate as heat) or store it temporarily in magnetic/electric fields.
The Big Three (R, L, C):

- Resistor (R): Dissipates energy as heat ($P = I^2R$).
- Inductor (L): Stores energy in a magnetic field ($E = \frac{1}{2}LI^2$).
- Capacitor (C): Stores energy in an electric field ($E = \frac{1}{2}CV^2$).



Bilateral Elements

Definition

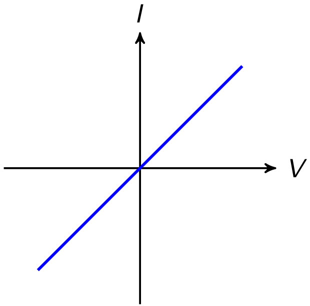
A bilateral element's Voltage-Current (V-I) relationship is the same for current flowing in either direction.

Key Property

- The magnitude of current is independent of the polarity of the applied voltage.
- If you reverse the component in the circuit, it behaves exactly the same.

Example

Resistor: Ohm's Law $V = IR$ holds true regardless of current direction. A 10V drop across a $1k\Omega$ resistor yields 10mA, whether current flows left-to-right or right-to-left.



Unilateral Elements

Definition

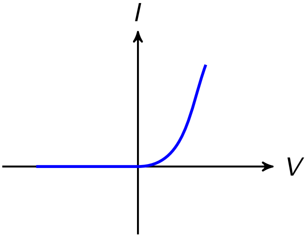
An element whose V-I characteristics depend on the direction of current flow or polarity of the applied voltage.

Key Property

It behaves differently if you reverse the voltage polarity.

Example: The PN Junction Diode

- Forward Bias: Conducts current easily (low resistance).
- Reverse Bias: Blocks current (extremely high resistance).
- Therefore, a diode is strictly a unilateral element.



Linear Elements

Definition

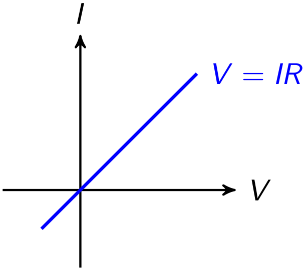
An element is linear if the relationship between voltage and current is strictly proportional and represented by a straight line passing through the origin.

Requirements

- Superposition: $f(x_1 + x_2) = f(x_1) + f(x_2)$
- Homogeneity (Scaling): $f(kx) = k \cdot f(x)$

Characteristics

- Parameters (Resistance, Inductance, Capacitance) remain constant, regardless of voltage or current magnitude.
- Example: An ideal resistor ($V = IR$).



Non-Linear Elements

Definition

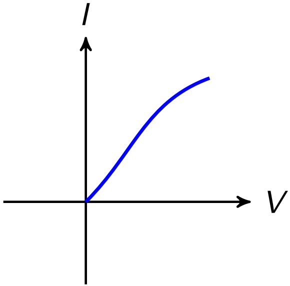
An element where the voltage-current relationship is not proportional. The V-I graph is a curve, not a straight line.

Characteristics

- The parameters change with applied voltage or current.
- Do not obey the principle of superposition.

Examples

- Diodes & Transistors: Current increases exponentially with voltage.
- Incandescent Lamp: As current increases, temperature rises, which increases the filament's resistance. Therefore, resistance is not constant.



Definition

Elements that are considered to be concentrated at a single point in space for the purpose of analysis.

Characteristics

- Physical size is much smaller than the wavelength of the electromagnetic signal passing through it (Size $\ll \lambda$).
- Voltage and current do not vary spatially across the component.
- Kirchhoff's Laws (KCL, KVL) apply perfectly.

Examples

Standard discrete resistors, capacitors, and inductors used in low-frequency circuits

Discrete Component

Size $\ll \lambda$

(e.g., audio, household AC).

Distributed Elements

Definition

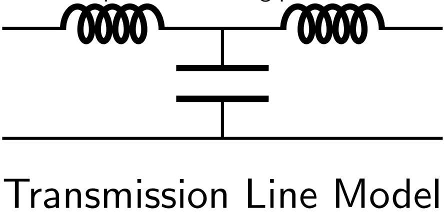
Elements where electrical properties (resistance, inductance, capacitance) are distributed continuously along the physical length of the circuit.

Characteristics

- Physical size is comparable to the signal wavelength (Size $\approx \lambda$).
- Current and voltage vary as a function of distance along the element.
- Cannot be separated into discrete parts; standard KCL/KVL do not apply globally.

Example

Transmission Lines: Coaxial cables, optical fibers, long power lines, PCB traces at microwave frequencies.



Summary of Classifications

Classification Pair	Basis of Distinction	Examples
Active / Passive	Energy generation/amplification	Battery (Active), Resistor (Passive)
Bilateral / Unilateral	Direction of current flow	Resistor (Bilateral), Diode (Unilateral)
Linear / Non-Linear	V-I proportional relationship	Ideal Resistor (Linear), Transistor (Non-Linear)
Lumped / Distributed	Physical size vs. Wavelength	Discrete Capacitor (Lumped), Coaxial Cable (Distributed)

Quick Knowledge Check

- **Question 1:** Is a standard PN junction diode a bilateral or unilateral element? Why?
- **Question 2:** Why do we consider a battery an active element?
- **Question 3:** If you are analyzing a 60Hz power circuit using standard components, are you dealing with lumped or distributed elements?

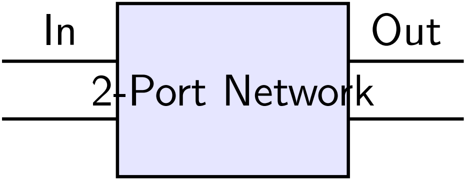
Key Takeaways

- **Network vs. Circuit:** A circuit is a network with a closed loop.
- **Energy:** Active elements supply energy; Passive elements consume or store it.
- **Direction:** Bilateral elements don't care about current direction; Unilateral elements do.
- **Relationship:** Linear elements have a straight-line V-I graph; Non-Linear elements have curves.
- **Scale:** Lumped models work for small sizes/low frequencies; Distributed models are needed for large sizes/high frequencies.

Next Lecture Preview

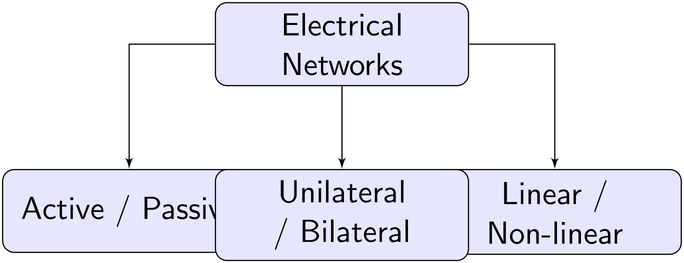
Distinguishing Various Networks

- In our next lecture, we will zoom out from individual elements to entire networks.
- We will apply these same concepts (Active/Passive, Linear/Non-Linear, etc.) to complex, multi-component circuits.
- We will introduce the concepts of Symmetrical vs. Asymmetrical networks.
- We will define Single-port and Two-port networks, setting the stage for advanced analysis.



Lecture 2: Distinguishing Various Networks

- Course: Electronics Circuit Network & Measurement (DI03011031)
- Unit 1: Network Elements and Network Analysis
- Topic: Classification and Distinction of Various Networks



Agenda

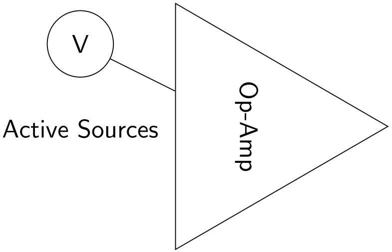
- Introduction to Network Classification
- Passive vs. Active Networks
- Linear vs. Nonlinear Networks
- Lumped vs. Distributed Networks
- Unilateral vs. Bilateral Networks
- Symmetrical vs. Asymmetrical Networks
- Single-port vs. Two-port Networks
- Summary & Next Steps

Introduction to Network Classification

- A network is an interconnection of various electrical elements like resistors, inductors, capacitors, voltage sources, and current sources.
- To analyze complex networks, it is crucial to classify them based on their physical and behavioral characteristics.
- Classification simplifies the selection of appropriate mathematical theorems (like Superposition, Thevenin's) for analysis.

Active Networks

- An active network contains at least one source of electromotive force (EMF) or energy source.
- It can supply electrical energy to the circuit continuously.
- Examples of active elements inside the network: Voltage sources (batteries, generators), Current sources, Transistors, Operational Amplifiers (Op-Amps).

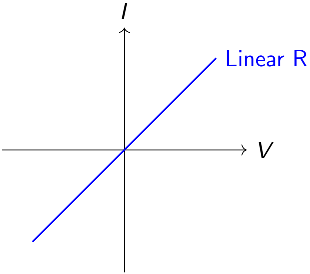


Passive Networks

- A passive network does not contain any source of EMF.
- It cannot generate electrical energy; it can only dissipate, store, or release energy previously supplied to it.
- Examples of passive elements: Resistors (dissipate), Capacitors (store electric field), Inductors (store magnetic field).

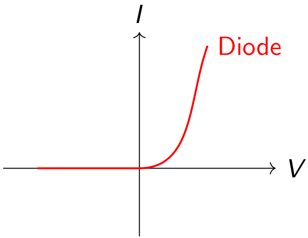
Linear Networks

- A linear network is one whose parameters (Resistance, Inductance, Capacitance) remain constant with changes in current or voltage.
- The relationship between voltage and current follows Ohm's Law ($V = IR$) and is strictly proportional.
- The V-I characteristic curve is a straight line passing through the origin.



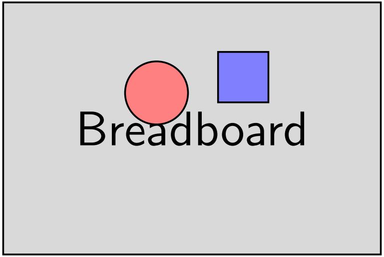
Non-Linear Networks

- A non-linear network is one whose parameters change with variations in voltage or current.
- The relationship between V and I is not strictly proportional.
- Examples: Diodes, Transistors, Iron-core inductors (due to magnetic saturation).



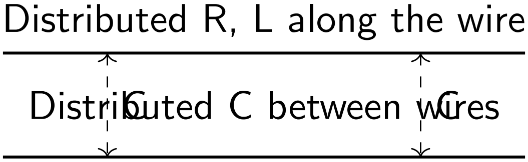
Lumped Networks

- In a lumped network, the electrical components (R, L, C) are physically concentrated at discrete points.
- The physical size of the network is small compared to the wavelength of the applied signal.
- We can easily identify and separate individual components. Kirchhoff's Laws (KVL, KCL) are fully applicable.



Distributed Networks

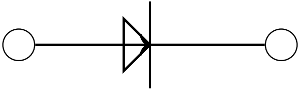
- In a distributed network, the electrical properties (R, L, C) are spread out along the entire length of the circuit.
- They cannot be physically separated or identified as discrete components.
- Examples: Transmission lines, coaxial cables, telephone lines.
- Analyzed using field theory and Maxwell's equations, as KVL/KCL are not directly applicable.



Unilateral Networks

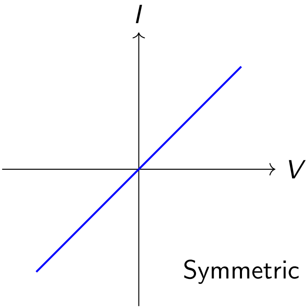
- A unilateral network has characteristics or behavior that depend on the direction of the current.
- The network's response is different if the supply polarity is reversed.
- Examples: Circuits containing diodes or rectifiers.

Current blocked in reverse



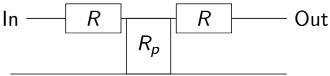
Bilateral Networks

- A bilateral network exhibits the same characteristics regardless of the direction of current flow.
- Reversing the voltage polarity simply reverses the current direction, but the magnitude and network properties remain unchanged.
- Examples: Circuits containing only resistors, standard inductors, and standard capacitors.

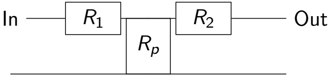


Symmetrical vs. Asymmetrical Networks

- **Symmetrical Network**: An electrical network that can be divided into two identical halves.
- The electrical properties are exactly the same when looking from either the input or output terminals.
- **Asymmetrical Network**: Cannot be divided into identical halves. The input and output impedances differ.
- Example of Symmetrical: T-network or Pi-network with equal series/shunt arms.



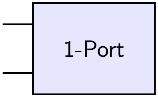
Symmetrical T-Network

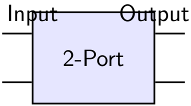


Asymmetrical T-Network

Single-port vs. Two-port Networks

- **Port**: A pair of terminals through which a signal may enter or leave a network.
- **Single-port Network**: Has only one pair of terminals (two wires). Energy can only enter or leave through this one port. Example: A simple load resistor or a driving source.
- **Two-port Network**: Has two pairs of terminals (four wires). Usually designated as an input port and an output port.
- **Example of Two-port**: Transformers, amplifiers, filters, transmission lines.





Quick Check

- **Question 1:** Which type of network's characteristics change when the direction of current is reversed?
  - A) Bilateral B) Unilateral C) Linear D) Passive
- **Question 2:** A transmission line is an example of which type of network?
  - A) Lumped B) Distributed C) Active D) Symmetrical

Quick Check

- **Question 1:** Which type of network's characteristics change when the direction of current is reversed?
  - A) Bilateral B) Unilateral C) Linear D) Passive
- **Question 2:** A transmission line is an example of which type of network?
  - A) Lumped B) Distributed C) Active D) Symmetrical

Answers

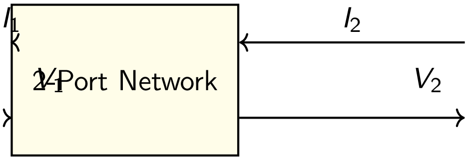
Q1: B) Unilateral
Q2: B) Distributed

Summary

Classification	Key Characteristic
Active / Passive	Presence or absence of an energy source
Linear / Non-linear	Constant parameters vs. changing parameters (V-I linearity)
Lumped / Distributed	Physical concentration of elements vs. spread over a length
Unilateral / Bilateral	Direction-dependent behavior vs. direction-independent behavior
Symmetrical / Asymmetrical	Structural and electrical symmetry from input to output
Single / Two-port	One pair of terminals vs. two pairs (input and output)

Next Lecture Preview

- Topic: Two Port Network: Z & Y Parameters
- We will dive deep into two-port networks.
- Learn how to represent complex networks using mathematical parameters.
- Focus on Open Circuit Impedance (Z) and Short Circuit Admittance (Y) parameters.



Recap of Previous Lecture

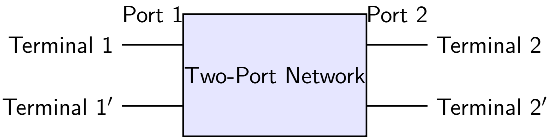
- Active vs Passive Networks: Presence or absence of energy sources.
- Linear vs Non-Linear: Superposition and homogeneity applicability.
- Unilateral vs Bilateral: V-I characteristics independence of current direction.
- Lumped vs Distributed: Physical separability of elements.
- Symmetrical vs Asymmetrical: Electrical properties unchanged if input/output reversed.

Learning Objectives

- Understand the structure and variables of a two-port network.
- Learn Z-parameters (Open-circuit impedance parameters).
- Construct equivalent circuits using Z-parameters.
- Learn Y-parameters (Short-circuit admittance parameters).
- Construct equivalent circuits using Y-parameters.
- Understand the matrix relationship between Z and Y parameters.

Introduction to Ports

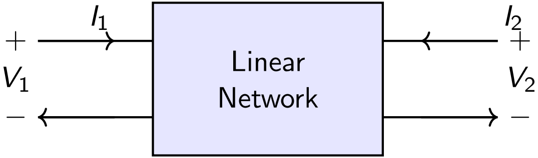
- Port: A pair of terminals through which a current may enter or leave a network.
- One-Port Network: A single pair of terminals. Example: A simple resistor, or a Thevenin equivalent circuit.
- Two-Port Network: Two pairs of terminals (Input port and Output port). Current entering one terminal of a port equals the current leaving the other terminal of the same port.



Variables in a Two-Port Network

A two-port network is characterized by four primary variables:

- V_1: Voltage across Port 1 (Input)
- I_1: Current entering Port 1
- V_2: Voltage across Port 2 (Output)
- I_2: Current entering Port 2
- By convention, both I_1 and I_2 are assumed to enter the network at the upper terminals.



Z-Parameters: Introduction

- Also known as Impedance Parameters or Open-Circuit Parameters.
- We choose currents I_1 and I_2 as independent variables.
- We choose voltages V_1 and V_2 as dependent variables.
- The relationships are linear: V_1 and V_2 are expressed as linear combinations of I_1 and I_2.

Z-Parameter Equations

The fundamental equations for Z-parameters are:

$$V_1 = Z_{11} I_1 + Z_{12} I_2$$
$$V_2 = Z_{21} I_1 + Z_{22} I_2$$

In Matrix Form:

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

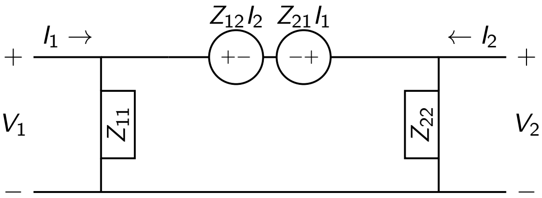
Defining the Z-Parameters

To find individual parameters, we open-circuit one of the ports (set current to zero).

If Port 2 is open-circuited ($I_2 = 0$):	**If Port 1 is open-circuited ($I_1 = 0$):**
$Z_{11} = \frac{V_1}{I_1}$: Open-circuit Input Impedance	$Z_{12} = \frac{V_1}{I_2}$: Open-circuit Reverse Transfer Impedance
$Z_{21} = \frac{V_2}{I_1}$: Open-circuit Forward Transfer Impedance	$Z_{22} = \frac{V_2}{I_2}$: Open-circuit Output Impedance

Z-Parameter Equivalent Circuit

- Based on the equations $V_1 = Z_{11}I_1 + Z_{12}I_2$ and $V_2 = Z_{21}I_1 + Z_{22}I_2$:
- The network can be modeled using dependent voltage sources.
- Port 1 has an impedance Z_{11} in series with a voltage source $Z_{12}I_2$.
- Port 2 has an impedance Z_{22} in series with a voltage source $Z_{21}I_1$.
- For a bilateral network, $Z_{12} = Z_{21}$, and a simpler T-network equivalent can be used.



Quick Check: Z-Parameters

Question

If a two-port network has $V_1 = 10I_1 + 5I_2$ and $V_2 = 5I_1 + 20I_2$, what is the open-circuit reverse transfer impedance, Z_{12}?

- A) $10\,\Omega$
- B) $20\,\Omega$
- C) $5\,\Omega$ **(Correct)**
- D) $15\,\Omega$

Y-Parameters: Introduction

- Also known as Admittance Parameters or Short-Circuit Parameters.
- We choose voltages V_1 and V_2 as independent variables.
- We choose currents I_1 and I_2 as dependent variables.
- The relationships are linear: I_1 and I_2 are expressed as linear combinations of V_1 and V_2.

Y-Parameter Equations

The fundamental equations for Y-parameters are:

$$I_1 = Y_{11}V_1 + Y_{12}V_2$$
$$I_2 = Y_{21}V_1 + Y_{22}V_2$$

In Matrix Form:

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

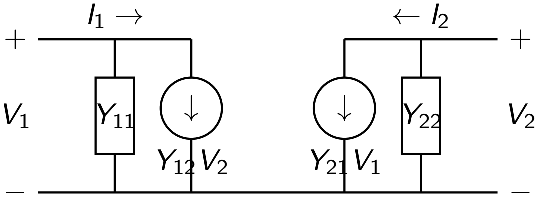
Defining the Y-Parameters

To find individual parameters, we short-circuit one of the ports (set voltage to zero).

If Port 2 is short-circuited ($V_2 = 0$):	**If Port 1 is short-circuited ($V_1 = 0$):**
$Y_{11} = \frac{I_1}{V_1}$: Short-circuit Input Admittance	$Y_{12} = \frac{I_1}{V_2}$: Short-circuit Reverse Transfer Admittance
$Y_{21} = \frac{I_2}{V_1}$: Short-circuit Forward Transfer Admittance	$Y_{22} = \frac{I_2}{V_2}$: Short-circuit Output Admittance

Y-Parameter Equivalent Circuit

- Based on the equations $I_1 = Y_{11}V_1 + Y_{12}V_2$ and $I_2 = Y_{21}V_1 + Y_{22}V_2$:
- The network can be modeled using dependent current sources in parallel with admittances.
- Port 1 has admittance Y_{11} in parallel with a current source $Y_{12}V_2$.
- Port 2 has admittance Y_{22} in parallel with a current source $Y_{21}V_1$.
- For a bilateral network, $Y_{12} = Y_{21}$, and a Π (Pi) network equivalent can be used.



Relationship between Z and Y Parameters

- Since $[V] = [Z][I]$ and $[I] = [Y][V]$, it follows that:
- $[Y] = [Z]^{-1}$ and $[Z] = [Y]^{-1}$
- To find Y from Z:
- Let $\Delta Z = Z_{11}Z_{22} - Z_{12}Z_{21}$ (Determinant of Z-matrix)
- $Y_{11} = \frac{Z_{22}}{\Delta Z}$, $Y_{12} = \frac{-Z_{12}}{\Delta Z}$
- $Y_{21} = \frac{-Z_{21}}{\Delta Z}$, $Y_{22} = \frac{Z_{11}}{\Delta Z}$

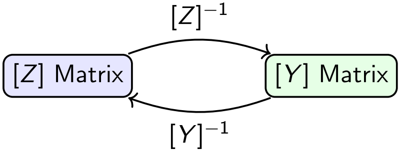
Quick Check: Reciprocity and Symmetry

Property	Z-parameters	Y-parameters
Condition for Reciprocity	$Z_{12} = Z_{21}$	$Y_{12} = Y_{21}$
Condition for Symmetry	$Z_{11} = Z_{22}$	$Y_{11} = Y_{22}$

Why is a network reciprocal?
Because it consists purely of linear bilateral passive elements (no dependent sources).

Summary & Key Takeaways

- Two-Port Networks have 4 variables (V_1, I_1, V_2, I_2).
- Z-parameters (Open-circuit): V_1, V_2 expressed in terms of I_1, I_2.
- Y-parameters (Short-circuit): I_1, I_2 expressed in terms of V_1, V_2.
- The Y-matrix is the inverse of the Z-matrix.
- Reciprocal networks have equal cross-parameters ($Z_{12} = Z_{21}$, $Y_{12} = Y_{21}$).

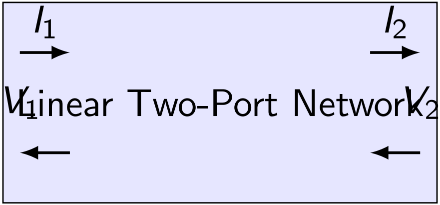


Next Lecture Preview

- Topic: Two Port Network: h & ABCD Parameters
- We will explore Hybrid (h) parameters, which use a mix of voltages and currents as independent variables.
- We will introduce Transmission (ABCD) parameters, essential for cascaded network analysis.
- We will see how they relate to the Z and Y parameters we learned today.

Two Port Network: h & ABCD Parameters

- Course: Electronics Circuit Network & Measurement (DI03011031)
- Unit 1: Network Elements and Network Analysis
- Topic: Representation of Two Port Parameter: h-parameter and ABCD parameter
- Focus: Understanding hybrid and transmission parameters, their derivations, and applications.

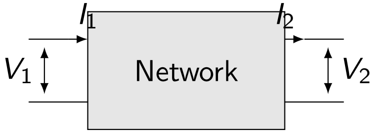


Agenda

- ✓ Recap: Two-Port Networks, Z and Y parameters
- ✓ Introduction to Hybrid (h) Parameters
- ✓ Equations and Definitions of h-parameters
- ✓ Equivalent Circuit and Applications of h-parameters
- ✓ Introduction to Transmission (ABCD) Parameters
- ✓ Equations and Definitions of ABCD parameters
- ✓ Conditions for Reciprocity and Symmetry
- ✓ Quick Knowledge Check

Recap: Two-Port Networks

- Variables: Four port variables exist: V_1, I_1 (Input) and V_2, I_2 (Output).
- Z-Parameters: V_1, V_2 expressed in terms of independent variables I_1, I_2. (Open-circuit parameters)
- Y-Parameters: I_1, I_2 expressed in terms of independent variables V_1, V_2. (Short-circuit parameters)
- Need for other parameters: Z and Y parameters are not always practical for all circuits (e.g., transistor modeling or cascading networks).



Introduction to Hybrid (h) Parameters

- Concept: In h-parameters, we choose I_1 (input current) and V_2 (output voltage) as independent variables.
- Dependent Variables: V_1 (input voltage) and I_2 (output current).
- Why "Hybrid"? The resulting parameters have mixed dimensions:
  - One is an impedance (ohms)
  - One is an admittance (mhos or siemens)
  - Two are dimensionless ratios.

Variable Type	**Port 1 (Input)**	**Port 2 (Output)**
Independent	I_1	V_2
Dependent	V_1	I_2

h-Parameter Equations

Fundamental Linear Equations

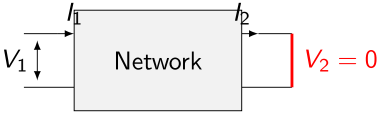
$$V_1 = h_{11} I_1 + h_{12} V_2$$
$$I_2 = h_{21} I_1 + h_{22} V_2$$

Matrix Form:

$$\underbrace{\begin{bmatrix} V_1 \\ I_2 \end{bmatrix}}_{\text{Dependent}} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \underbrace{\begin{bmatrix} I_1 \\ V_2 \end{bmatrix}}_{\text{Independent}}$$

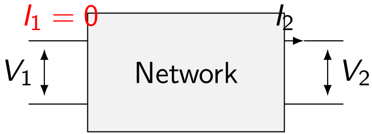
Defining h11 and h21 (Short-Circuit at Output)

- Condition: Set $V_2 = 0$ (Short-circuit the output port).
- h_{11} - Short-Circuit Input Impedance:
  - $h_{11} = \frac{V_1}{I_1}\Big|_{V_2=0}$ (Unit: Ohms, Ω)
- h_{21} - Short-Circuit Forward Current Gain:
  - $h_{21} = \frac{I_2}{I_1}\Big|_{V_2=0}$ (Dimensionless)



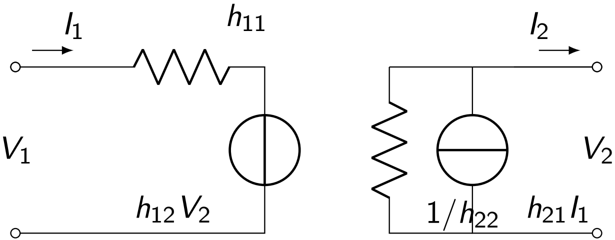
Defining h12 and h22 (Open-Circuit at Input)

- Condition: Set $I_1 = 0$ (Open-circuit the input port).
- h_{12} - Open-Circuit Reverse Voltage Gain:
  - $h_{12} = \left.\frac{V_1}{V_2}\right|_{I_1=0}$ (Dimensionless)
- h_{22} - Open-Circuit Output Admittance:
  - $h_{22} = \left.\frac{I_2}{V_2}\right|_{I_1=0}$ (Unit: Siemens or Mhos, ℧)



Equivalent Circuit Model for h-Parameters

- The h-parameter equations can be directly translated into an electrical circuit.
- $V_1 = h_{11}I_1 + h_{12}V_2$ represents KVL at the input port.
- $I_2 = h_{21}I_1 + h_{22}V_2$ represents KCL at the output port.
- Input side: Resistor h_{11} in series with a dependent voltage source $h_{12}V_2$.
- Output side: Dependent current source $h_{21}I_1$ in parallel with an admittance h_{22}.

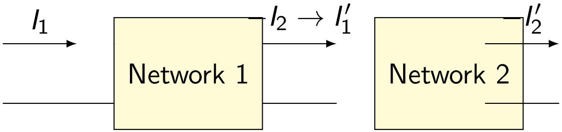


Applications of h-Parameters

- Transistor Modeling (BJT): h-parameters are the standard for modeling Bipolar Junction Transistors at low frequencies.
- Practical Measurement:
  - Transistor input impedance is low; output impedance is high.
  - It is easy to open-circuit the high-impedance output and short-circuit the low-impedance input without causing instability.
- Nomenclature in Electronics:
  - $h_{11} \rightarrow h_{ie}$ (input impedance)
  - $h_{12} \rightarrow h_{re}$ (reverse voltage ratio)
  - $h_{21} \rightarrow h_{fe}$ (forward current gain, β)
  - $h_{22} \rightarrow h_{oe}$ (output admittance)

Introduction to Transmission (ABCD) Parameters

- Concept: Express the input port variables (V_1, I_1) in terms of the output port variables $(V_2, -I_2)$.
- Current Direction: Note the use of $-I_2$. By convention, I_2 flows into the port. In ABCD parameters, we consider the current leaving the port to enter the next stage, hence $-I_2$.
- Alternative Names: Also known as T-parameters (Transmission) or General Circuit Parameters.



ABCD Parameter Equations

Fundamental Linear Equations

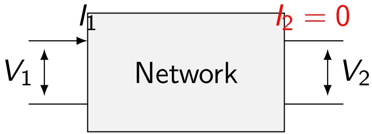
$$V_1 = A \cdot V_2 - B \cdot I_2$$
$$I_1 = C \cdot V_2 - D \cdot I_2$$

Matrix Form:

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

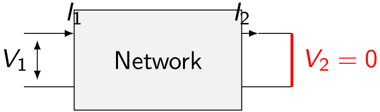
Defining A and C (Open-Circuit at Output)

- Condition: Set $I_2 = 0$ (Open-circuit the output port).
- A - Open-Circuit Reverse Voltage Ratio:
  - $A = \left.\frac{V_1}{V_2}\right|_{I_2=0}$ (Dimensionless)
- C - Open-Circuit Transfer Admittance:
  - $C = \left.\frac{I_1}{V_2}\right|_{I_2=0}$ (Unit: Siemens or Mhos, ℧)



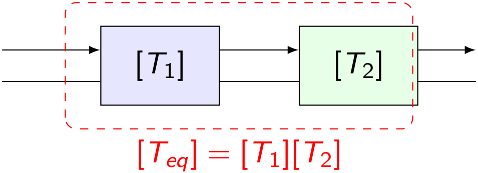
Defining B and D (Short-Circuit at Output)

- Condition: Set $V_2 = 0$ (Short-circuit the output port).
- B - Short-Circuit Transfer Impedance:
  - $B = -\frac{V_1}{I_2}\Big|_{V_2=0}$ (Unit: Ohms, Ω)
- D - Short-Circuit Reverse Current Ratio:
  - $D = -\frac{I_1}{I_2}\Big|_{V_2=0}$ (Dimensionless)



Significance of ABCD Parameters

- Cascaded Networks: ABCD parameters are incredibly powerful when networks are connected in cascade (series/tandem).
- The Rule: When two networks (Network 1 and Network 2) are cascaded, the overall ABCD matrix is simply the matrix product of the individual ABCD matrices.
- $[ABCD]_{total} = [ABCD]_1 \times [ABCD]_2$
- Applications: Power transmission lines, cascading multiple stages of amplifiers, microwave engineering.



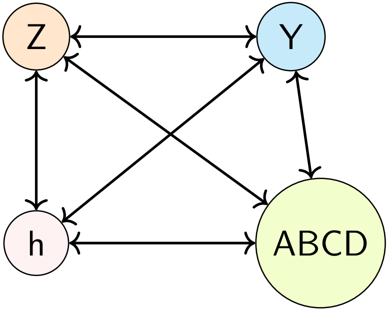
Reciprocal and Symmetrical Conditions

- A two-port network has specific properties if it is reciprocal (composed only of linear passive components) or symmetrical (input and output ports can be interchanged).

Parameters	**Reciprocity Condition**	**Symmetry Condition**
Z-parameters	$Z_{12} = Z_{21}$	$Z_{11} = Z_{22}$
Y-parameters	$Y_{12} = Y_{21}$	$Y_{11} = Y_{22}$
h-parameters	$h_{12} = -h_{21}$	$h_{11}h_{22} - h_{12}h_{21} = \Delta h = 1$
ABCD parameters	$AD - BC = 1$	$A = D$

Interrelation of Parameters

- Since all parameter sets (Z, Y, h, ABCD) describe the same physical network, they can be converted into one another.
- Conversions are derived purely through algebraic manipulation of the fundamental equations.
- Example (Z to h):
  - $h_{11} = \frac{\Delta Z}{Z_{22}}$ $\quad h_{12} = \frac{Z_{12}}{Z_{22}}$
  - $h_{21} = -\frac{Z_{21}}{Z_{22}}$ $\quad h_{22} = \frac{1}{Z_{22}}$
- Standard conversion tables are available for quick reference.



Quick Check

- Which parameter set is ideal for modeling cascaded networks?
  - A) Z-parameters B) Y-parameters
  - C) h-parameters **D) ABCD parameters**
- In an h-parameter model, what is the physical meaning of h_{21}?
  - A) Input Impedance **B) Forward Current Gain**
  - C) Reverse Voltage Ratio D) Output Admittance
- What is the reciprocity condition for ABCD parameters?
  - A) $A = D$ B) $h_{12} = h_{21}$
  - **C)** $AD - BC = 1$ D) $\Delta h = 1$

Summary & Next Lecture Preview I

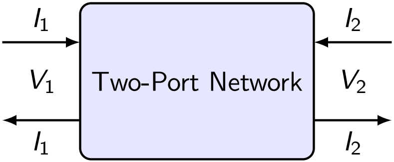
Summary

- h-parameters: Mix of impedance and admittance, highly suited for transistor (BJT) modeling.
- Derived via a mix of open-circuit and short-circuit conditions.
- ABCD parameters: Relate input variables directly to output variables.
- Essential for analyzing cascaded (tandem) networks.
- Established the mathematical conditions for reciprocity and symmetry for both.

Next Lecture

- We will define and explore various two-port impedances such as Transfer Impedance, Driving-point Impedance, Image Impedance, and Terminating Impedance.

- Welcome to Lecture 5.
- Today, we explore the different types of impedances associated with two-port networks.
- We will define Transfer, Driving Point, Image, Terminating, Input, and Output Impedances.

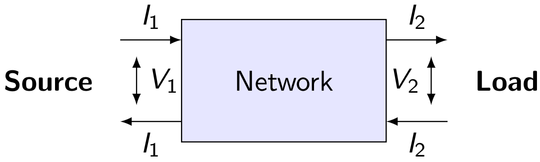


Agenda

- Introduction to Network Impedances
- Driving Point Impedance
- Transfer Impedance
- Terminating Impedances
- Input and Output Impedances
- Image Impedance Concept & Derivations
- Quick Check
- Summary & Next Lecture Preview

Introduction to Network Impedances

- Two-port networks are characterized by voltages and currents at two distinct ports.
- The relationship between voltage and current at any port, or between ports, defines specific impedances.
- Impedances help determine how the network loads a source or transfers power to a load.
- Categories of impedances in a two-port network:
  - Internal parameters: Driving Point, Transfer
  - Operational parameters: Input, Output
  - Matching parameters: Image, Characteristic



Driving Point Impedance

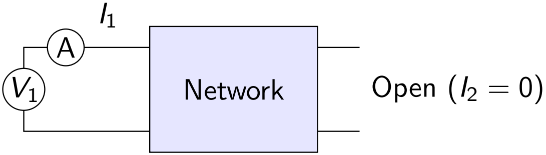
- Definition: The ratio of the voltage applied at a port to the resulting current entering the SAME port, with all other independent sources set to zero.
- Port 1 Driving Point Impedance (Z_{11}):

$$Z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0} \quad \text{(Port 2 open-circuited)}$$

- Port 2 Driving Point Impedance (Z_{22}):

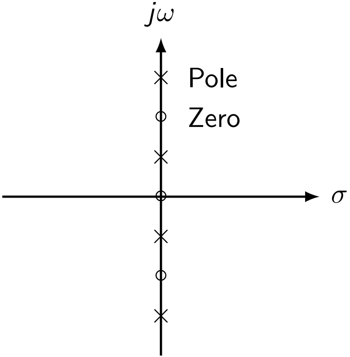
$$Z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0} \quad \text{(Port 1 open-circuited)}$$

- Also known as input impedance when the other port is completely open.



Properties of Driving Point Impedance

- Must be a positive real function for passive networks.
- Poles and zeros must alternate on the imaginary axis for lossless LC networks.
- Real part must be ≥ 0 for all frequencies.
- Represents the opposition offered by the network to the source directly at the terminals of excitation.
- Admittance counterpart: Driving Point Admittances (Y_{11}, Y_{22}) when the opposite port is short-circuited.



Transfer Impedance

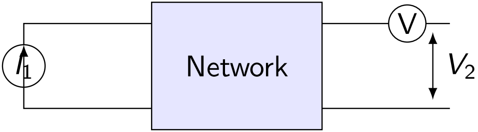
- Definition: The ratio of the voltage response observed at one port to the current excitation applied at the OTHER port.
- Forward Transfer Impedance (Z_{21}):

$$Z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0} \quad \text{(Voltage at port 2 due to current at port 1)}$$

- Reverse Transfer Impedance (Z_{12}):

$$Z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0} \quad \text{(Voltage at port 1 due to current at port 2)}$$

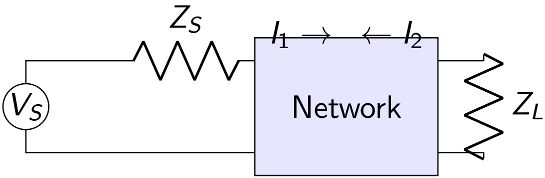
- For reciprocal (passive linear bilateral) networks, $Z_{12} = Z_{21}$.



$$Z_{12} = Z_{21} \ \textbf{(Reciprocal Networks)}$$

Terminating Impedance

- Two-port networks are rarely used in isolation; they connect a source to a load.
- Load Impedance (Z_L): The external impedance connected across the output terminals (Port 2).
- Forces the condition: $V_2 = -I_2 Z_L$
- Source Impedance (Z_S): The internal impedance of the voltage/current source connected to the input terminals (Port 1).
- Forces the condition: $V_1 = V_S - I_1 Z_S$
- Terminating impedances heavily dictate the overall operational behavior of the network.



Input Impedance (Z_{in})

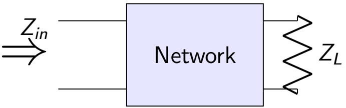
- Definition: The impedance seen looking into Port 1 when Port 2 is terminated with a load Z_L.
- $$Z_{in} = \frac{V_1}{I_1} \text{ (while } V_2 = -I_2 Z_L)$$
- Derived using Z-parameters:

$$V_1 = Z_{11} I_1 + Z_{12} I_2$$
$$V_2 = Z_{21} I_1 + Z_{22} I_2$$

- Substitute $I_2 = -V_2/Z_L$ into the second equation, solve for I_2, and plug into the first to get:

Input Impedance Formula

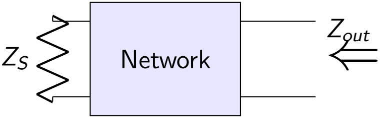
$$Z_{in} = Z_{11} - \frac{Z_{12} Z_{21}}{Z_{22} + Z_L}$$

- 

Output Impedance (Z_{out})

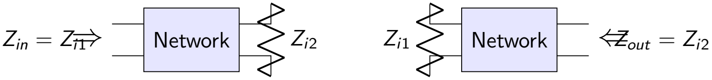
- Definition: The impedance seen looking back into Port 2 when Port 1 is terminated with the source impedance Z_S, and the independent source V_S is turned off ($V_S = 0$).
- $$Z_{out} = \frac{V_2}{I_2} \text{ (while } V_1 = -I_1 Z_S)$$
- Following a similar derivation using Z-parameters:
- Substitute $I_1 = -V_1/Z_S$ and $V_S = 0$ into the Z-parameter equations:
- **Output Impedance Formula**

  $$Z_{out} = Z_{22} - \frac{Z_{12}Z_{21}}{Z_{11} + Z_S}$$



Introduction to Image Impedance

- Concept: What if we want to terminate a network such that the impedance is perfectly 'matched' in both directions?
- An asymmetrical two-port network has two Image Impedances: Z_{i1} and Z_{i2}.
- Condition 1: If Port 2 is terminated with Z_{i2}, the input impedance at Port 1 is exactly Z_{i1}.
- Condition 2: If Port 1 is terminated with Z_{i1}, the input impedance looking into Port 2 is exactly Z_{i2}.
- When these conditions are met, the network is said to be *image matched*.



Deriving Image Impedance Formulas I

- Let's express Z_{i1} and Z_{i2} in terms of ABCD (Transmission) parameters.
- Input impedance with load Z_L is: $Z_{in} = \frac{AZ_L+B}{CZ_L+D}$
- Set $Z_L = Z_{i2}$, then $Z_{in} = Z_{i1}$:

Condition 1 Substitution

$$(1)\ Z_{i1} = \frac{AZ_{i2}+B}{CZ_{i2}+D}$$

- Looking backwards, output impedance with source $Z_S = Z_{i1}$ is Z_{i2}:

Condition 2 Substitution

$$(2)\ Z_{i2} = \frac{DZ_{i1}+B}{CZ_{i1}+A}$$

Deriving Image Impedance Formulas II

- Solving these simultaneous equations yields the formulas for Z_{i1} and Z_{i2}.

Image Impedance Equations

- Solving the previous equations gives:

$$Z_{i1} = \sqrt{\frac{AB}{CD}} \qquad \text{(Image impedance at Port 1)}$$

$$Z_{i2} = \sqrt{\frac{BD}{AC}} \qquad \text{(Image impedance at Port 2)}$$

- Alternatively, using Open-Circuit (Z_{oc}) and Short-Circuit (Z_{sc}) impedances:
  - $Z_{oc1} = A/C$ and $Z_{sc1} = B/D$
  - $Z_{oc2} = D/C$ and $Z_{sc2} = B/A$
- Therefore:

Practical Formula

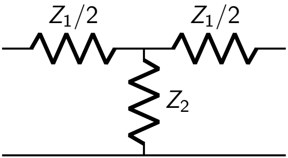
$$Z_{i1} = \sqrt{Z_{oc1}Z_{sc1}} \quad \text{and} \quad Z_{i2} = \sqrt{Z_{oc2}Z_{sc2}}$$

Characteristic vs Image Impedance

- **Symmetrical Networks**: A network where Port 1 and Port 2 are indistinguishable (e.g., a simple T-network with equal series arms).
- For a symmetrical network, $A = D$.
- Substitute $A = D$ into our Image Impedance formulas:

$$Z_{i1} = \sqrt{\frac{AB}{CA}} = \sqrt{\frac{B}{C}}$$
$$Z_{i2} = \sqrt{\frac{BB}{AC}} = \sqrt{\frac{B}{C}}$$

- Therefore, $Z_{i1} = Z_{i2} = Z_0$ (Characteristic Impedance).
- Characteristic impedance is a special case of image impedance for symmetrical networks.



Symmetrical T-Network

Calculation Example: Finding Z_{in}

- **Problem:** A two-port network has Z-parameters: $Z_{11} = 40\Omega$, $Z_{12} = Z_{21} = 20\Omega$, $Z_{22} = 30\Omega$. A load $Z_L = 10\Omega$ is connected to Port 2. Calculate the input impedance Z_{in}.
- **Formula:**

$$Z_{in} = Z_{11} - \frac{Z_{12}Z_{21}}{Z_{22} + Z_L}$$

- **Calculation:**

$$Z_{in} = 40 - \frac{(20)(20)}{30 + 10}$$
$$Z_{in} = 40 - \frac{400}{40}$$
$$Z_{in} = 40 - 10 = 30\Omega$$

- **Result:** The source sees a 30Ω impedance.

Quick Check

- **Question 1:** What condition is applied to Port 2 to measure the driving point impedance Z_{11}?

  (Answer: Port 2 is open-circuited, $I_2 = 0$)

- **Question 2:** For a fully symmetrical network, how do the two image impedances Z_{i1} and Z_{i2} relate?

  (Answer: They are perfectly equal, known as Characteristic Impedance Z_0)

- **Question 3:** In the image impedance formula $Z_i = \sqrt{Z_{oc} Z_{sc}}$, what does Z_{sc} stand for?

  (Answer: Short-Circuit input impedance)

Summary

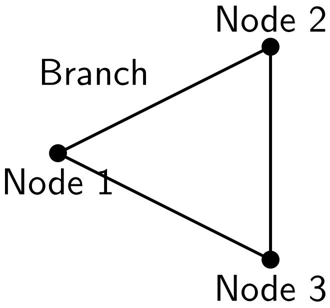
- **Driving Point Impedance:** Impedance looking into a port with the other port strictly open (Z_{11}, Z_{22}).
- **Transfer Impedance:** Relates voltage at one port to current at the other (Z_{12}, Z_{21}).
- **Input/Output Impedances:** The actual impedances seen when the network is loaded (Z_L) and driven by a source (Z_S).
- **Image Impedance:** Unique terminating impedances (Z_{i1}, Z_{i2}) that result in simultaneous impedance matching at both ports.
- **Characteristic Impedance:** The specific case of image impedance in a perfectly symmetrical network (Z_0).

Key Takeaways

- The impedance of a two-port network is dynamic; what you see at the input depends heavily on what is connected at the output.
- Image matching is crucial for cascading multiple filter sections or transmission lines without power loss due to reflections.
- Geometric mean of open and short circuit measurements ($Z_i = \sqrt{Z_{oc}Z_{sc}}$) is the most practical way to find image impedance in a laboratory setting.

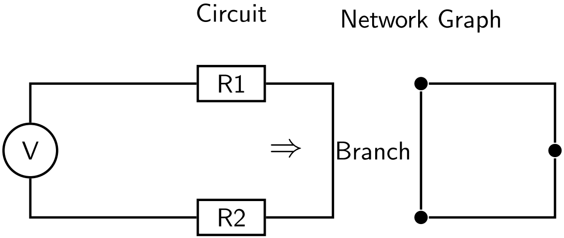
Next Lecture Preview

- **Next Topic:** Network Topology
- We will shift our focus to the graphical representation of electrical circuits.
- Key terms to cover:
  - Branch
  - Node
  - Loop
  - Mesh
- We will learn how to reduce complex circuits into topological graphs to analyze their fundamental structure.



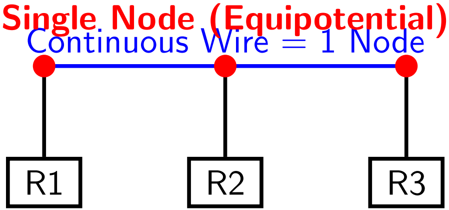
What is Network Topology?

- Topology refers to the interconnection pattern of network elements.
- It deals with the structural properties of a circuit rather than the types of elements (resistors, capacitors, etc.).
- By ignoring the electrical nature of components and replacing them with simple lines, we form a Network Graph.
- Helps in systematically applying Kirchhoff's Laws (KCL and KVL).



Defining a Node

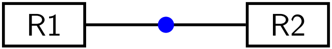
- A **Node** is a junction or a connection point where two or more circuit elements meet.
- Represented as a point or dot in a network graph.
- In a schematic, an entire continuous wire without any component is considered a single node.
- Nodes are points where we apply Kirchhoff's Current Law (KCL).



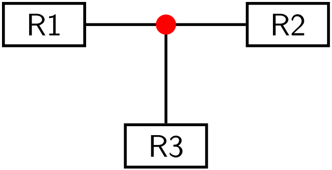
Simple vs. Principal Nodes

- **Simple Node**: A connection point where exactly *two* elements are connected in series.
- **Principal (or Essential) Node**: A connection point where *three or more* elements meet.
- Principal nodes are crucial because current divides at these points.
- KCL equations are typically written for Principal Nodes.

Simple Node



Principal Node (Current Splits)

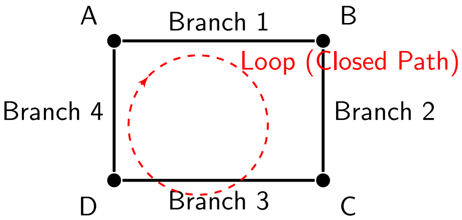


Defining a Branch

- A **Branch** represents a single path in a network, containing one simple element and connecting two nodes.
- It can be an active element (voltage/current source) or a passive element (R, L, C).
- The number of branches in a circuit equals the number of elements.
- Current through a single branch is continuous and identical at all points within the branch.

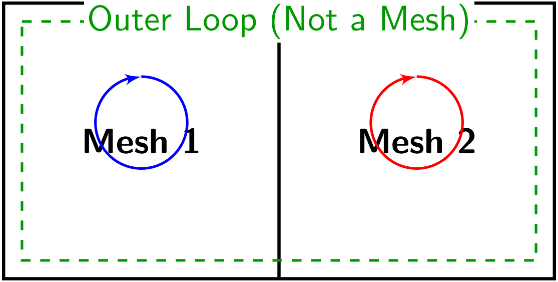
Defining a Path and a Loop

- **Path**: A continuous sequence of branches traversed without passing through any node more than once.
- **Loop**: A closed path in a circuit.
- To form a loop, start at a node, travel through a set of branches, and return to the starting node.
- No node should be crossed twice (except the starting/ending node).
- Kirchhoff's Voltage Law (KVL) is applied around loops.



Defining a Mesh

- A **Mesh** is a restricted, special kind of loop.
- It is a loop that does *not* contain any other loops within its interior.
- Think of meshes as the empty 'windows' of a circuit diagram.
- Meshes are only defined for planar circuits (circuits that can be drawn flat without crossing lines).

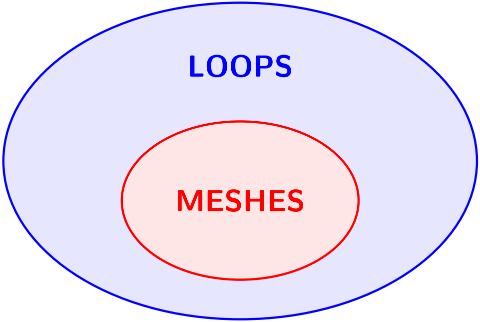


Loop vs. Mesh: Key Differences

Important Distinction

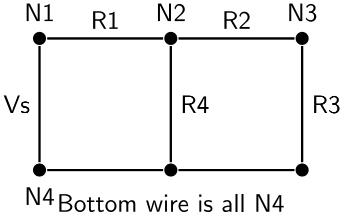
Rule of Thumb: All meshes are loops, but not all loops are meshes.

- **Mesh**: The smallest possible closed path. No internal loops.
- **Loop**: Any closed path. Can contain other loops inside.
- Mesh Analysis uses meshes to form independent KVL equations.



Example: Counting Nodes, Branches, and Meshes

- Consider a circuit with: 1 Voltage Source, 4 Resistors (forming a 2-window circuit).
- **Branches (b)**: 5
- **Nodes (n)**: 4
- **Meshes (m)**: Count the individual 'windows'.
- **Loops (l)**: Include meshes + outer loops.



Fundamental Topological Relationship

- In any planar network, the number of independent loops (or meshes) is related to the branches and nodes by:

Euler's Formula for Electrical Networks

$$m = b - n + 1$$

Where:
$m =$ number of meshes (independent loops)
$b =$ number of branches
$n =$ number of nodes

$$\mathbf{m = b - n + 1}$$

Verification of the Formula

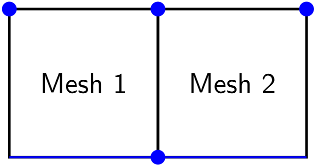
- Let's test the formula

$$m = b - n + 1$$

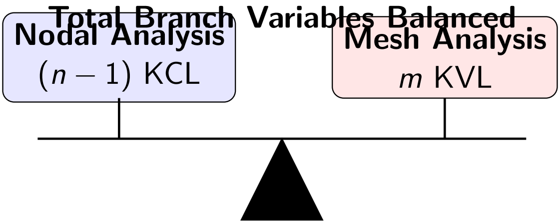
- **Given Circuit:** A 2-window circuit (e.g., T-network with a source)
- **Branches (b)** $= 5$ (1 source, 4 resistors)
- **Nodes (n)** $= 4$ (N1, N2, N3, N4)
- **Calculation**:

$$m = 5 - 4 + 1$$
$$m = 2$$

- The circuit indeed has 2 meshes!



- Why define these terms so strictly?
- **Nodal Analysis**: Requires exactly $(n-1)$ independent KCL equations.
- **Mesh Analysis**: Requires exactly $m = (b - n + 1)$ independent KVL equations.
- Ensures a solvable system of linear equations without redundant information.



Quick Check

- **Question 1**: Can a mesh contain another mesh inside it?

Quick Check

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  - **Answer:** No, by definition, a mesh cannot contain internal loops.
- **Question 2**: In a circuit with 6 branches and 4 nodes, how many independent meshes are there?

Quick Check

- **Question 1**: Can a mesh contain another mesh inside it?
  - **Answer:** No, by definition, a mesh cannot contain internal loops.

- **Question 2**: In a circuit with 6 branches and 4 nodes, how many independent meshes are there?
  - **Answer:** $m = b - n + 1$
  - $m = 6 - 4 + 1 = 3$ meshes.

Summary

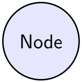
Term	Definition
Node	Junction of 2 or more elements (Principal = 3+).
Branch	A path connecting two nodes, containing an element.
Loop	Any closed path.
Mesh	A closed path with no other loops inside.
Key Formula	$m = b - n + 1$

- **Key Takeaways**:
  - A node is a junction point; principal nodes are where current divides.
  - All meshes are loops, but not all loops are meshes.
  - The topological formula dictates the number of independent KVL equations.

Preview of Next Lecture

Next Topic: Series/Parallel Resistors & Node Analysis

- We will apply our knowledge of nodes to simplify circuits.
- We will learn how to write KCL equations systematically using Nodal Analysis.
- Bring your calculators!



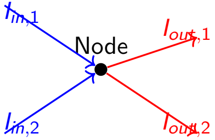
$$+\sum I = 0$$

Agenda for Today's Lecture

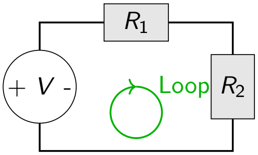
- Recap of KCL and KVL
- Resistors in Series and Parallel
- Voltage and Current Divider Rules
- Star-Delta / Delta-Star Transformations
- Introduction to Nodal Analysis
- Steps for Formulating Nodal Equations
- The Concept of Supernodes
- Summary and Next Lecture Preview

Recap: Kirchhoff's Laws (KCL & KVL)

- **Kirchhoff's Current Law (KCL):**
  - Algebraic sum of all currents entering and exiting a node is zero: $\sum_{n=1}^{N} I_n = 0$.
  - Based on the principle of conservation of electric charge.
- **Kirchhoff's Voltage Law (KVL):**
  - Algebraic sum of all voltages around any closed loop is zero: $\sum_{m=1}^{M} V_m = 0$.
  - Based on the principle of conservation of energy.



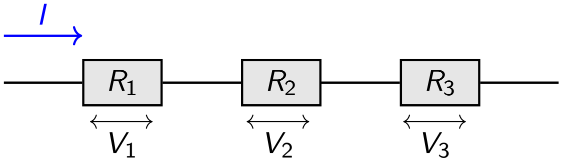
KCL: $\sum I_{in} = \sum I_{out}$



KVL: $\sum V_{drops} = \sum V_{rises}$

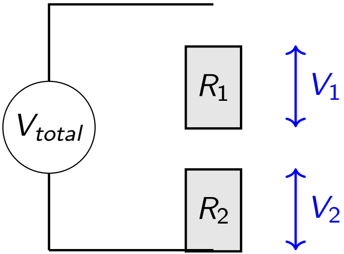
Resistors in Series

- **Definition:** Connected end-to-end, carrying the exact same current.
- **Equivalent Resistance (R_{eq}):** $R_{eq} = R_1 + R_2 + R_3 + \cdots + R_n$
- **Properties:**
  - Same current I flows through all elements.
  - Total voltage V_{total} is the sum of voltage drops (KVL).
  - Equivalent resistance is always greater than the largest individual resistor.



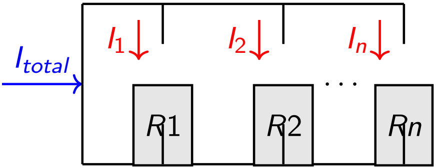
Voltage Divider Rule

- **Purpose:** Find voltage across a specific resistor in a series circuit without finding total current first.
- **General Formula:** $V_x = V_{total} \times \frac{R_x}{R_{eq}}$
- **Two Resistor Case:**
  - $V_1 = V_{total} \times \frac{R_1}{R_1+R_2}$
  - $V_2 = V_{total} \times \frac{R_2}{R_1+R_2}$
- **Application:** Potentiometers and sensor signal conditioning.



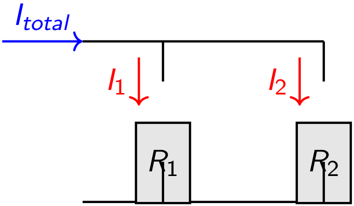
Resistors in Parallel

- **Definition:** Terminals are connected to the same two nodes (they share the same voltage).
- **Equivalent Resistance (R_{eq}):** $\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_n}$
- **Two Resistor Shortcut:** $R_{eq} = \frac{R_1 \times R_2}{R_1 + R_2}$
- **Properties:**
  - Voltage across all parallel elements is identical.
  - Total current $I_{total} = I_1 + I_2 + \cdots + I_n$ (KCL).
  - Equivalent resistance is strictly less than the smallest individual resistor.



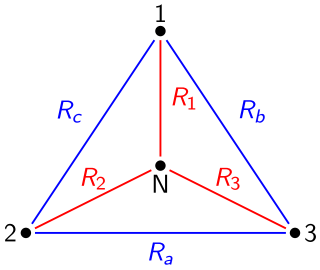
Current Divider Rule

- **Purpose:** Determines how a total current splits between parallel branches.
- **General Formula:** $I_x = I_{total} \times \frac{R_{eq}}{R_x}$
- **Two Resistor Case (Very Common):**
  - $I_1 = I_{total} \times \frac{R_2}{R_1 + R_2}$
  - $I_2 = I_{total} \times \frac{R_1}{R_1 + R_2}$
- **Note:** Current takes the path of least resistance. The larger resistor gets the smaller share of the current.



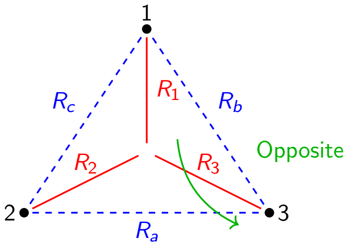
Star-Delta (Wye-Delta) Transformations

- **Problem:** Some networks cannot be simplified using just series/parallel rules (e.g., bridge circuits).
- **Solution:** Transform a Star (Y) network into an equivalent Delta (Δ) network, or vice versa.
- **Delta to Star ($\Delta \to Y$):**
  - $R_1 = \frac{R_b R_c}{R_a+R_b+R_c}$
  - $R_2 = \frac{R_c R_a}{R_a+R_b+R_c}$
  - $R_3 = \frac{R_a R_b}{R_a+R_b+R_c}$
- **Rule of Thumb:** Star resistor = (Product of adjacent Delta resistors) / (Sum of all Delta resistors).



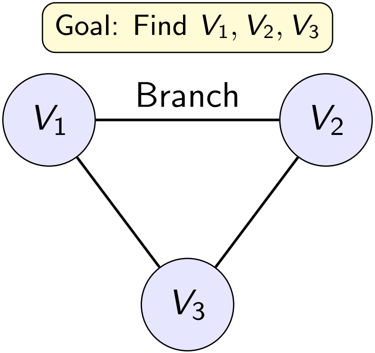
Star to Delta Transformation

- **Star to Delta ($Y \rightarrow \Delta$):**
  - $R_a = \frac{R_1R_2+R_2R_3+R_3R_1}{R_1}$
  - $R_b = \frac{R_1R_2+R_2R_3+R_3R_1}{R_2}$
  - $R_c = \frac{R_1R_2+R_2R_3+R_3R_1}{R_3}$
- **Rule of Thumb:** Delta resistor = (Sum of all possible Star resistor pairwise products) / (Opposite Star resistor).
- **Symmetrical Case:** If all resistors are equal (R), then $R_\Delta = 3R_Y$ and $R_Y = \frac{R_\Delta}{3}$.



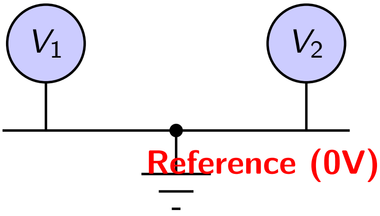
Introduction to Nodal Analysis

- **What is it?** Systematic method to determine voltage at every node in a circuit relative to a reference node.
- **Core Principle:** Based strictly on Kirchhoff's Current Law (KCL) and Ohm's Law.
- **Why use it?**
  - Highly structured and easily programmable (used by SPICE simulators).
  - Reduces number of equations compared to basic branch current methods.
  - Efficient for circuits with many parallel branches and current sources.



The Reference Node (Ground)

- Every nodal analysis begins by selecting a **Reference Node**.
- The reference node is assigned a voltage of **0 Volts** ($V = 0V$).
- **Selection Strategies:**
  - The node with the most branches connected to it.
  - The negative terminal of a voltage source.
  - The physical ground of the circuit, if specified.
- All other node voltages in the circuit are calculated relative to this reference node.



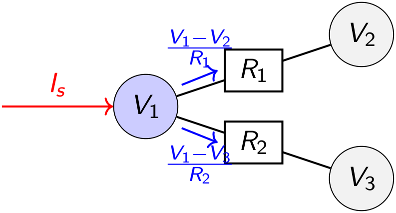
Steps for Nodal Analysis

1. **Identify Nodes:** Count total principal nodes (N).
2. **Assign Reference:** Choose one node as reference ($0V$).
3. **Assign Variables:** Label voltages at remaining $N-1$ nodes as $V_1, V_2, \dots, V_{N-1}$.
4. **Apply KCL:** At each non-reference node, apply KCL. Assume all unknown currents are leaving the node.
5. **Express Currents:** Use Ohm's Law to express branch currents: $I = \frac{V_{from} - V_{to}}{R}$.
6. **Solve:** Solve the resulting system of $N-1$ linear equations.

1. Identify N → 2. Ref (0V) → 3. Variables → 4. Apply KCL → 5. Ohm's Law → 6. Solve System

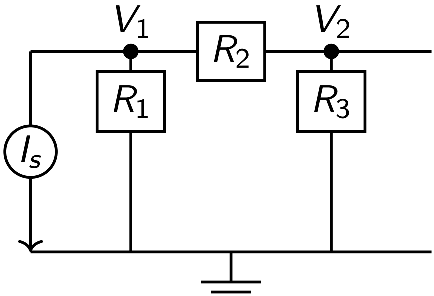
Formulating KCL Equations

- **For a generic Node n with voltage V_n:**
- **Currents leaving the node through resistors:**
  - $I_x = \frac{V_n - V_{adjacent}}{R_x}$
- **Currents from independent current sources:**
  - If entering the node: Subtract the value (or put on RHS as positive).
  - If leaving the node: Add the value (or put on RHS as negative).
- **General KCL Equation:**
  - $\sum \frac{V_n - V_{adjacent}}{R} = \sum I_{sources\ entering}$



Example: Nodal Analysis Formulation

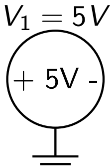
- **Circuit:** Node 1 (V_1), Node 2 (V_2), and Ground.
- R_1 between V_1 and Ground. R_2 between V_1 and V_2. R_3 between V_2 and Ground.
- Current source I_s entering Node 1.
- **Node 1 Equation:**
  - $\frac{V_1 - 0}{R_1} + \frac{V_1 - V_2}{R_2} = I_s$
- **Node 2 Equation:**
  - $\frac{V_2 - V_1}{R_2} + \frac{V_2 - 0}{R_3} = 0$



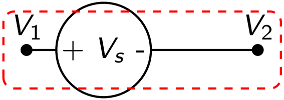
Handling Voltage Sources in Nodal Analysis

- **Case 1: Voltage source connected to Reference Node.**
  - This is the easiest case! The node connected to the positive terminal has a known voltage.
  - Example: A 5V source between Node 1 and Ground means $V_1 = 5V$.
- **Case 2: Voltage source connected between two non-reference nodes.**
  - This creates a **Supernode**.
  - We cannot write KCL for those two nodes individually because the current through the voltage source is unknown.

Case 1

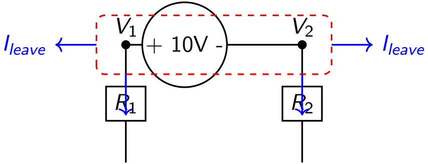


Case 2 (Supernode)



The Supernode Concept

- **Definition:** Formed by enclosing a voltage source (and any parallel elements) between two non-reference nodes.
- **Steps to solve:**
  - **KCL at the Supernode:** Apply KCL to the entire boundary as a single node.
  - **Voltage Constraint Equation:** Relate the two node voltages to the voltage source.
  - Example: $10V$ source from V_1 (+) to V_2 (-) $\rightarrow$ $V_1 - V_2 = 10V$.
- The KCL equation + constraint equation provides the system to solve.



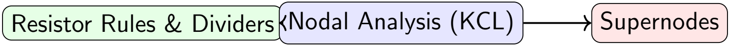
Advantages of Nodal Analysis

- **Fewer Equations:** If nodes $<$ meshes, Nodal Analysis requires a smaller matrix.
- **Direct Voltages:** Directly yields node voltages for trivial power/current calculations.
- **Non-Planar Circuits:** Applicable to non-planar circuits, whereas basic Mesh analysis is not.
- **Algorithmic:** Perfect for computer implementation. Nodal conductance matrix is symmetric and easy to automate.

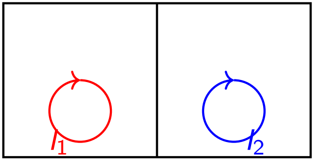
Feature	**Nodal Analysis**	**Mesh Analysis**
Basis Law	KCL (Currents)	KVL (Voltages)
Primary Variables	Node Voltages (V)	Mesh Currents (I)
Best for	Many parallel branches	Many series branches
Circuit Type	Planar & Non-Planar	Planar only (standard)

Summary

- Series resistors share current; Parallel resistors share voltage.
- Voltage and Current divider rules are rapid shortcuts for basic analysis.
- Star-Delta transformations help simplify complex topological networks.
- Nodal Analysis uses KCL to find unknown node voltages relative to a reference (ground).
- Assume currents leave the node when formulating equations: $\frac{V_n - V_{adj}}{R}$.
- Floating voltage sources between nodes require the Supernode technique.



- **Lecture 8: Mesh Analysis**
- Introduction to Mesh (Loop) Analysis based on KVL.
- Formulating mesh equations for planar circuits.
- Handling current sources in mesh analysis.
- The concept of the Supermesh.
- Comparing Nodal vs. Mesh Analysis: Which one to choose?



Agenda

- Recap of Nodal Analysis and KVL
- Loops vs. Meshes
- Introduction to Mesh Analysis
- Steps to Perform Mesh Analysis
- Mesh Equations Formulation
- Practical Examples (2-Mesh Circuits)
- Handling Current Sources: The Supermesh
- Summary and Q&A

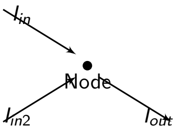
Node Analysis (Recap):

- Based on Kirchhoff's Current Law (KCL).
- Variables are node voltages.
- Solves for voltage at principal nodes.

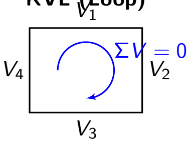
Kirchhoff's Voltage Law (KVL):

- The algebraic sum of all voltages around any closed loop is zero.
- $\Sigma V = 0$
- Mesh analysis heavily relies on KVL.

KCL (Node)

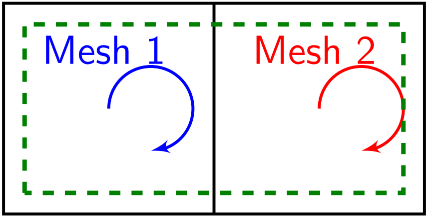


KVL (Loop)



Loops vs. Meshes

- **Loop:** Any closed path in a circuit. Can contain other loops inside it.
- **Mesh:** A specific type of loop that does not contain any other loops within it. The most elementary closed path.
- *Note:* Mesh analysis is only applicable to planar circuits (circuits that can be drawn on a 2D plane with no crossing branches).



Outer Loop (Not a Mesh)

Introduction to Mesh Analysis

- **What is Mesh Analysis?**
  - A method used to calculate unknown currents in a planar circuit.
  - Uses KVL equations.
  - Variables are mesh currents, not branch currents.
- **Why use Mesh Analysis?**
  - Reduces the number of equations needed compared to direct application of Kirchhoff's Laws.
  - Number of equations = Number of meshes (M).
  - **Formula:** $M = B - N + 1$ (where B = branches, N = nodes).

Steps to Perform Mesh Analysis

1. Identify all meshes in the planar circuit.

↓

2. Assign a clockwise mesh current ($i_1, i_2, \ldots$) to each mesh.

↓

3. Express voltage drops across resistors via Ohm's Law.

↓

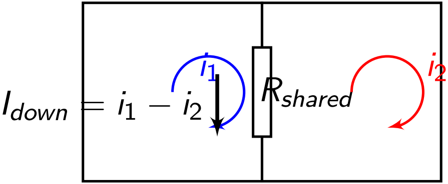
4. Apply KVL to each mesh to write equations.

↓

5. Solve the simultaneous equations for mesh currents.

Mesh Currents vs. Branch Currents

- **Mesh Current (i_m):** A fictitious current that circulates around a single mesh.
- **Branch Current (I_b):** The actual physical current flowing through a specific circuit element.
- **Relationship:**
  - If a branch belongs to only one mesh, $I_b = i_m$.
  - If a branch is shared between two meshes, actual current is the algebraic sum of mesh currents: e.g., $I_b = i_1 - i_2$.



Writing Mesh Equations (Standard Form)

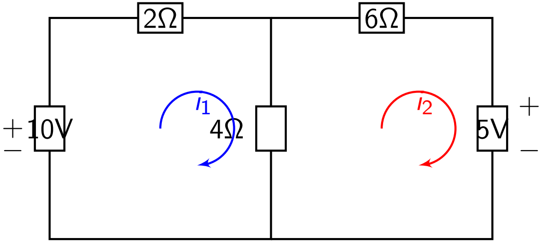
- For a circuit with resistive elements only, equations can be formulated by inspection:
- **Mesh 1:** $R_{11}i_1 - R_{12}i_2 - \cdots - R_{1n}i_n = V_1$
- **Mesh 2:** $-R_{21}i_1 + R_{22}i_2 - \cdots - R_{2n}i_n = V_2$
- **Where:**
  - R_{kk}: Self-resistance of mesh k (sum of all resistors in mesh k).
  - R_{kj}: Mutual resistance shared between mesh k and mesh j.
  - V_k: Algebraic sum of voltage sources driving current in mesh k.

$$\begin{bmatrix} R_{11} & -R_{12} \\ -R_{21} & R_{22} \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

Example 1: Simple 2-Mesh Circuit (Setup)

Problem: Find mesh currents i_1 and i_2.

- **Circuit Details:**
  - Two voltage sources: $V_1 = 10V$, $V_2 = 5V$ (opposing)
  - Resistors: $R_1 = 2\Omega$, $R_2 = 4\Omega$ (shared), $R_3 = 6\Omega$
- **Step 1 & 2:** Assign clockwise currents i_1 for Mesh 1 and i_2 for Mesh 2.



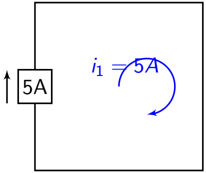
Example 1: Simple 2-Mesh Circuit (Solution)

- **Apply KVL to Mesh 1:**
  - $-10 + 2i_1 + 4(i_1 - i_2) = 0 \Rightarrow \mathbf{6i_1 - 4i_2 = 10}$
- **Apply KVL to Mesh 2:**
  - $4(i_2 - i_1) + 6i_2 + 5 = 0 \Rightarrow \mathbf{-4i_1 + 10i_2 = -5}$
- **Solving the System:**
  - From Eq 1: $i_1 = \frac{10+4i_2}{6}$
  - Substitute into Eq 2: $-4\left(\frac{10+4i_2}{6}\right) + 10i_2 = -5$
  - **Result:** $i_1 = 1.82A$, $i_2 = 0.23A$

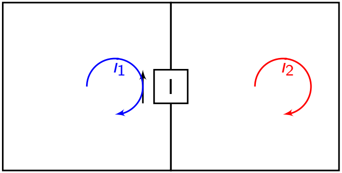
Handling Current Sources in Mesh Analysis

- **Case 1: Current source in an outer branch (unshared)**
  - This is the easiest case. The mesh current is directly determined by the source.
  - If an outer branch of Mesh 1 has a $5A$ source, then $i_1 = 5A$ (or $-5A$ depending on direction).
- **Case 2: Current source shared between two meshes**
  - This creates a **Supermesh**.
  - Standard KVL cannot be directly applied through the current source branch because its voltage is unknown.

Case 1: Outer Branch



Case 2: Shared Branch



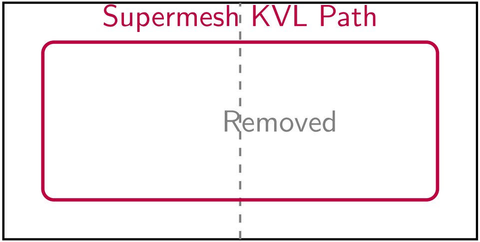
Concept of Supermesh

Supermesh

Formed when two meshes share a current source (dependent or independent). Created by temporarily removing the current source and any elements in series with it.

How to solve:

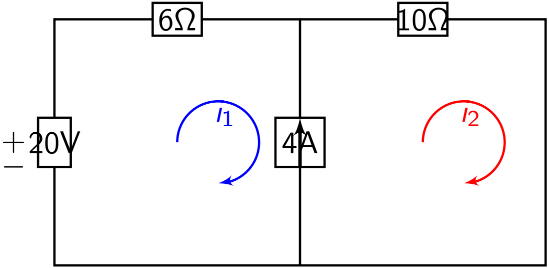
1. Write the KVL equation around the large Supermesh boundary.
2. Write a constraint equation using the current source and the two mesh currents it shares: $I_{source} = i_x - i_y$.



Example 2: Circuit with Supermesh (Setup)

Problem: Find i_1 and i_2.

- **Circuit Details:**
  - Voltage source $V_1 = 20V$ in Mesh 1.
  - Resistor $R_1 = 6\Omega$ in Mesh 1.
  - Resistor $R_2 = 10\Omega$ in Mesh 2.
  - Shared branch: A 4A current source is shared between Mesh 1 and Mesh 2.
- **Setup:** Define i_1 and i_2 clockwise.



Example 2: Circuit with Supermesh (Solution)

- **Step 1: Constraint Equation**
  - The 4A source flows upwards. In the shared branch, i_2 flows up and i_1 flows down.
  - Therefore: $i_2 - i_1 = 4 \Rightarrow \mathbf{i_2 = i_1 + 4}$
- **Step 2: Supermesh KVL Equation**
  - Ignore the shared branch and write KVL around the outer perimeter.
  - $\mathbf{-20 + 6i_1 + 10i_2 = 0}$
- **Step 3: Solve**
  - $-20 + 6i_1 + 10(i_1 + 4) = 0$
  - $16i_1 + 40 = 20 \Rightarrow 16i_1 = -20 \Rightarrow \mathbf{i_1 = -1.25A}$
  - $i_2 = -1.25 + 4 = \mathbf{2.75A}$

Quick Check I

Question 1

Mesh analysis is an application of which fundamental law?

A) KCL

B) KVL ✓

C) Ohm's Law only

Quick Check II

Question 2

In a circuit with 5 nodes and 7 branches, how many mesh equations are needed?

A) 2

B) 3 ✓

C) 4

Summary & Next Lecture Preview I

Summary:

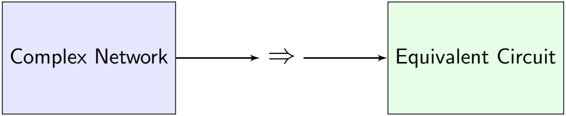
- Mesh analysis solves for unknown mesh currents using KVL.
- It is applicable only to planar circuits.
- For shared components, branch current is the algebraic sum of mesh currents.
- Supermeshes are required when current sources are shared between meshes.

Summary & Next Lecture Preview II

Next Lecture: Network Theorems

- Introduction to Network Theorems.
- Thevenin's and Norton's Theorems.
- Superposition Theorem.

- Electronics Circuit Network & Measurement (DI03011031)
- Unit 1: Network Elements and Network Analysis
- Lecture 9: Network Theorems

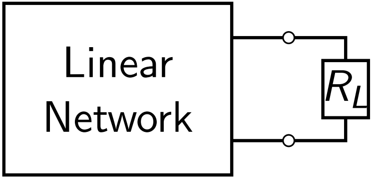


Agenda

- Introduction to Network Theorems
- Superposition Theorem
- Thevenin's Theorem
- Norton's Theorem
- Thevenin-Norton Equivalence (Source Transformation)
- Handling Dependent Sources
- Quick Check & Summary

Introduction to Network Theorems

- Why Network Theorems?
- **Simplification**: Reduce complex linear circuits into simpler equivalent models.
- **Variable Loads**: Allow for quick recalculations when a single load element changes, without re-solving the entire circuit.
- **Multiple Sources**: Easily handle networks containing several independent DC or AC sources.
- **Scope**: Applicable only to Linear, Bilateral networks (components like resistors, inductors, and capacitors).



Superposition Theorem: Statement

- **Superposition Principle**
- In any linear, bilateral network containing two or more independent sources, the response (voltage or current) in any element is the algebraic sum of the responses caused by each independent source acting alone.
- **Condition for Deactivation**
- While considering one source, all other independent sources must be deactivated:
- **Voltage Sources**: Replaced by a short circuit ($V = 0$).
- **Current Sources**: Replaced by an open circuit ($I = 0$).
- (Note: Internal impedances of the sources, if any, remain in the circuit.)

- **Step-by-Step Procedure**
- Select one independent source to remain active.
- Deactivate all other independent sources in the network.
- Calculate the voltage or current response in the desired branch due to this single source.
- Repeat Steps 1-3 for every independent source in the circuit.
- Algebraically add all the individual responses to find the total response.
- **Important**: Pay attention to the assumed direction of current or polarity of voltage for each partial response during the final algebraic summation.

Superposition Theorem: Limitation on Power

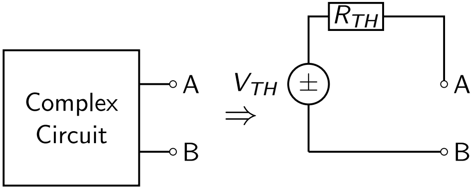
- Does Superposition Apply to Power?
- **No!** Superposition only applies to linear quantities (Voltage and Current).
- Power is a non-linear quantity since it is proportional to the square of voltage or current.
- $P = I^2 R$ or $P = V^2/R$
- Therefore, $P_{total} \neq P_1 + P_2$
- To find total power dissipated by an element, you must first find the total current or total voltage using superposition, and then calculate power.

Mathematical Emphasis

$(I_1 + I_2)^2 R \neq (I_1^2 R + I_2^2 R)$
Cross term $2 I_1 I_2 R$ is missing on the right side!

Thevenin's Theorem: Concept and Statement

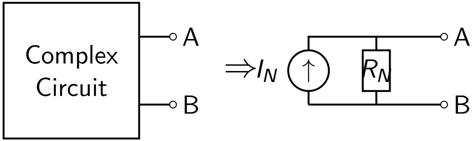
- **Thevenin's Theorem**
- Any linear, bilateral two-terminal network containing sources and impedances can be replaced by a simpler equivalent circuit consisting of a single voltage source (V_{TH}) in series with a single equivalent resistance (R_{TH}).
- V_{TH} **(Thevenin Voltage)**: The open-circuit voltage across the two terminals.
- R_{TH} **(Thevenin Resistance)**: The equivalent resistance looking into the two terminals with all independent sources deactivated.



Thevenin's Theorem: Steps for Analysis

- **Step-by-Step Procedure**
- **Isolate the Load**: Remove the load resistor (R_L) from the terminals (say, A and B).
- **Find V_{TH}**: Calculate the open-circuit voltage across terminals A and B using any method (Nodal, Mesh, etc.).
- **Find R_{TH}**: Deactivate all independent sources (short voltage sources, open current sources). Calculate the equivalent resistance looking back into terminals A and B.
- **Construct Equivalent**: Draw the Thevenin equivalent circuit with V_{TH} in series with R_{TH}.
- **Calculate Load Response**: Reconnect R_L. Find load current: $I_L = \frac{V_{TH}}{R_{TH}+R_L}$.

- **Norton's Theorem**
- Any linear, bilateral two-terminal network can be replaced by an equivalent circuit consisting of a single current source (I_N) in parallel with a single equivalent resistance (R_N).
- I_N **(Norton Current)**: The short-circuit current that flows through the two terminals when they are shorted together.
- R_N **(Norton Resistance)**: The equivalent resistance looking into the two terminals with all independent sources deactivated. (Note: $R_N = R_{TH}$)



Norton's Theorem: Steps for Analysis

- **Step-by-Step Procedure**
- **Short the Terminals**: Remove the load resistor (R_L) and place a short circuit across terminals A and B.
- **Find I_N**: Calculate the current flowing through this short circuit.
- **Find R_N**: Remove the short circuit. Deactivate all independent sources and calculate the equivalent resistance looking into terminals A and B. ($R_N = R_{TH}$)
- **Construct Equivalent**: Draw the Norton equivalent circuit with I_N in parallel with R_N.
- **Calculate Load Response**: Reconnect R_L. Find load current using current divider rule: $I_L = I_N \times \frac{R_N}{R_N+R_L}$.

Source Transformation & Thevenin-Norton Equivalence

- **Equivalence Relationship**
- Thevenin and Norton equivalent circuits are directly related through Source Transformation.
- **To convert Thevenin to Norton**:
- $I_N = \frac{V_{TH}}{R_{TH}}$ and $R_N = R_{TH}$
- **To convert Norton to Thevenin**:
- $V_{TH} = I_N \times R_N$ and $R_{TH} = R_N$
- This duality allows you to compute whichever parameter (V_{TH} or I_N) is easier for a given circuit, and then transform to the desired equivalent.

$$\begin{array}{c} \text{Thevenin} \\ V_{TH} \text{ \& series } R_{TH} \end{array} \xleftrightarrow[I_N = V_{TH}/R_{TH}]{V_{TH} = I_N R_N} \begin{array}{c} \text{Norton} \\ I_N \text{ \& parallel } R_N \end{array}$$

Handling Dependent Sources

- **Calculating R_{TH} with Dependent Sources**
- If a network contains dependent (controlled) sources, you cannot deactivate them when finding R_{TH}.
- **Method using an External Source**:
- Remove the load R_L.
- Deactivate all independent sources.
- Keep all dependent sources active.
- Apply a test voltage source (V_{test}, e.g., 1V) across terminals A and B.
- Calculate the resulting current (I_{test}) entering the network.
- $R_{TH} = \frac{V_{test}}{I_{test}}$.
- (Alternatively, apply a test current I_{test} and find V_{test})

Comparison of Network Theorems

- **Which Theorem to Use?**

Feature	**Superposition**	**Thevenin's**	**Norton's**
Best Used For	Finding individual contributions of multiple sources.	Analyzing circuits with a variable load component.	Same as Thevenin, often preferred for parallel networks.
Model Output	Total V or I response directly.	Equivalent Voltage Source + Series R.	Equivalent Current Source + Parallel R.
Constraint	Linear circuits only. Cannot calculate power directly.	Must identify a specific two-terminal port.	Must identify a specific two-terminal port.

Practical Application Example

- **Applying Thevenin's Theorem**
- Consider a bridge circuit where the center galvanometer acts as the load.
- Solving the entire bridge using mesh analysis requires 3 mesh equations.
- **Thevenin approach**:
- Remove the galvanometer.
- Calculate V_{TH} (voltage difference between the two bridge nodes).
- Calculate R_{TH} (parallel-series combination of bridge arms).
- Reconnect galvanometer to the simple V_{TH}-R_{TH} series circuit to find current.
- This turns a complex 3×3 matrix problem into basic voltage dividers and parallel resistors.

Quick Check

- **Question:**
- A two-terminal network is analyzed. It is found that the open-circuit voltage across the terminals is 24V. When the terminals are short-circuited, a current of 4A flows. What is the Thevenin equivalent resistance (R_{TH}) of the network?
- A) $6\,\Omega$
- B) $96\,\Omega$
- C) $0.16\,\Omega$
- D) Cannot be determined without circuit details.

Quick Check: Answer

- **Answer:** A) $6\,\Omega$
- **Explanation:**
- We know that the Thevenin voltage V_{TH} is the open-circuit voltage ($24V$).
- The Norton current I_N is the short-circuit current ($4A$).
- From source transformation principles, we know:
- $R_{TH} = \frac{V_{TH}}{I_N}$
- Therefore:
- $R_{TH} = \frac{24V}{4A} = 6\,\Omega$

Summary

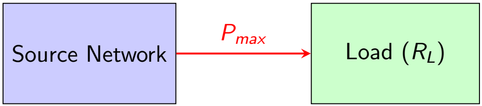
- **Superposition Theorem**: Total response in a linear circuit with multiple independent sources is the sum of responses to each source acting alone.
- **Thevenin's Theorem**: Replaces a complex linear network with a single voltage source (V_{TH}) in series with a resistance (R_{TH}).
- **Norton's Theorem**: Replaces a complex linear network with a single current source (I_N) in parallel with a resistance (R_N).
- **Equivalence**: Thevenin and Norton circuits are interchangeable via source transformation ($V_{TH} = I_N R_N$, $R_{TH} = R_N$).
- **Dependent Sources**: Must be left active when calculating equivalent resistance; a test source is required.

Next Lecture Preview

- **What's Next?**
- In our next lecture, we will conclude our study of Network Theorems by exploring:
- **Maximum Power Transfer Theorem**: How to choose a load to extract maximum power from a source.
- **Reciprocity Theorem**: The bidirectional nature of linear networks.
- **Principle of Duality**: Converting circuits between voltage/current and series/parallel domains.
- **Preparation**: Review the power calculation formulas ($P = V^2/R, P = I^2R$) and basic calculus (derivatives for maximization).

Network Theorems: Max Power Transfer, Reciprocity, Duality

- Course: Electronics Circuit Network & Measurement (DI03011031)
- Unit 1: Network Elements and Network Analysis
- Lecture 10: Network Theorems (Part 2)
- Topics:
- Maximum Power Transfer Theorem
- Reciprocity Theorem
- Principle of Duality

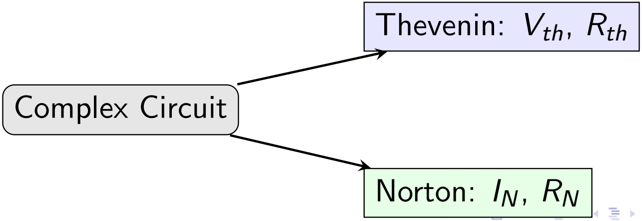


Agenda

- Recap of Previous Theorems
- Maximum Power Transfer Theorem
  - Statement & Proof
  - Efficiency Considerations
  - Solved Example
- Reciprocity Theorem
  - Statement & Conditions
  - Verification Process
- Principle of Duality
  - Dual Elements & Quantities
  - Constructing Dual Networks
- Summary and Quick Check

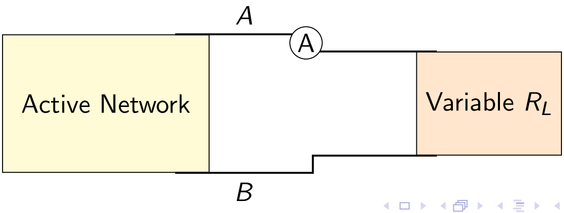
Recap: Thevenin, Norton, Superposition

- Thevenin's Theorem: Replaces a complex linear circuit with a single voltage source (V_{th}) in series with a resistance (R_{th}).
- Norton's Theorem: Replaces a circuit with a single current source (I_N) in parallel with a resistance (R_N). Note: $R_{th} = R_N$.
- Superposition Theorem: In a linear circuit with multiple independent sources, the response in any branch is the algebraic sum of the responses caused by each source acting alone.
- These theorems simplify the calculation of load current and voltage.



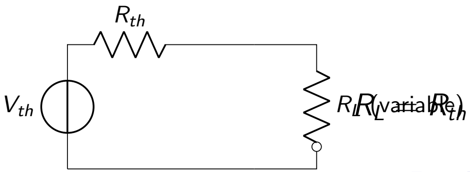
Introduction to Maximum Power Transfer Theorem

- Motivation: In electronic systems (like audio amplifiers or RF transmitters), it is often necessary to transfer maximum possible power to a load, rather than maximizing voltage or current.
- Concept: The power transferred from a source to a load depends on the resistance of the load relative to the internal resistance of the source.
- How do we find the optimum load resistance that draws maximum power from a given active network?



Maximum Power Transfer Theorem: Statement

- Statement:
- A resistive load connected to a DC network receives maximum power when the load resistance (R_L) is equal to the internal resistance (Thevenin resistance, R_{th}) of the source network as seen from the load terminals.
- Condition for Maximum Power:
- $R_L = R_{th}$
- The circuit is replaced by its Thevenin equivalent: V_{th} in series with R_{th}.



Maximum Power Transfer: Mathematical Proof

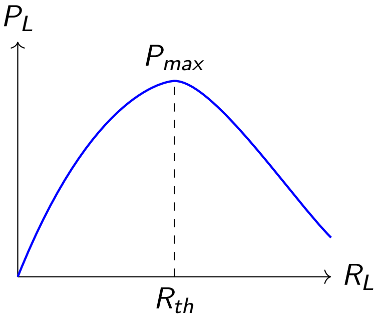
- Current in the circuit: $I = \frac{V_{th}}{R_{th}+R_L}$
- Power delivered to the load: $P_L = I^2 R_L = \left(\frac{V_{th}}{R_{th}+R_L}\right)^2 R_L$
- To find the maximum power, differentiate P_L with respect to R_L and equate to zero:
- $\frac{dP_L}{dR_L} = \frac{V_{th}^2[(R_{th}+R_L)^2 - R_L \cdot 2(R_{th}+R_L)]}{(R_{th}+R_L)^4} = 0$
- Simplifying:
  $(R_{th} + R_L) - 2R_L = 0 \implies R_{th} - R_L = 0 \implies R_L = R_{th}$

Key Result

Maximum power is transferred when $R_L = R_{th}$

Maximum Power Calculation and Efficiency

- When $R_L = R_{th}$:
- Max Power: $P_{max} = \left(\frac{V_{th}}{2R_{th}}\right)^2 R_{th} = \frac{\mathbf{V_{th}^2}}{\mathbf{4R_{th}}}$
- Efficiency (η) at Maximum Power Transfer:
- Power delivered to load: $P_L = I^2 R_L$
- Total power generated: $P_T = I^2(R_{th} + R_L) = I^2(2R_L) = 2P_L$
- Efficiency $\eta = \frac{P_L}{P_T} \times 100 = \frac{P_L}{2P_L} \times 100 = \mathbf{50}\%$
- Note: This theorem is used where power level is low (e.g., communications), not in power systems where 50% efficiency is unacceptable.



Solved Example: Maximum Power Transfer

- Problem: Find the value of R_L for maximum power transfer and the maximum power delivered.
- Given Circuit: A $10V$ source in series with a 4Ω resistor, parallel to a 6Ω resistor, followed by R_L.
- Solution:
- Find V_{th}: Open R_L. Voltage across 6Ω resistor.
- $V_{th} = 10 \times \frac{6}{4+6} = 6V$
- Find R_{th}: Short the $10V$ source.
- $R_{th} = 4\Omega \parallel 6\Omega = \frac{4\times6}{4+6} = 2.4\Omega$
- For max power, $R_L = R_{th} = 2.4\Omega$
- $P_{max} = \frac{V_{th}^2}{4R_{th}} = \frac{6^2}{4\times2.4} = \frac{36}{9.6} = \mathbf{3.75W}$

Introduction to Reciprocity Theorem

- Concept: Certain networks exhibit a symmetrical property where the positions of an excitation (source) and its corresponding response (current or voltage) can be interchanged without changing the ratio of response to excitation.
- Applicability: Applies only to linear, bilateral, and passive networks (contains resistors, inductors, capacitors, but NO dependent sources or active elements like transistors).
- Transfer Impedance: The ratio of excitation voltage to response current remains constant.

Reciprocity Theorem: Statement

- Statement:
- In any linear, bilateral network with a single voltage source V in branch 'A' producing a current I in branch 'B', if the voltage source V is moved to branch 'B', it will produce the exact same current I in branch 'A'.
- Conditions:
- The network must have only ONE independent source.
- Initial conditions must be zero.
- Dependent sources are not allowed.

Verification of Reciprocity Theorem

- Step-by-Step Verification:
- Insert a voltage source V in branch 1 and place a short-circuit ammeter in branch 2.
- Calculate or measure the current I in branch 2. Find the transfer resistance $R_T = \frac{V}{I}$.
- Remove the voltage source from branch 1 and replace it with a short circuit.
- Insert the identical voltage source V into branch 2 and place the ammeter in branch 1.
- Calculate or measure the new current I' in branch 1.
- If $I = I'$, the reciprocity theorem is verified.

Solved Example: Reciprocity Theorem

- Circuit: T-network with series arms $R_1 = 2\Omega$, $R_2 = 4\Omega$ and shunt arm $R_3 = 6\Omega$.
- Case 1: $10V$ source at input (port 1), output (port 2) shorted.
- Total resistance seen by source: $R_{eq1} = 2 + (6 \parallel 4) = 2 + 2.4 = 4.4\Omega$
- Source current: $I_S = 10/4.4 = 2.27A$
- Current at port 2: $I_2 = I_S \times \frac{6}{6+4} = 2.27 \times 0.6 = \mathbf{1.36A}$
- Case 2: $10V$ source at output (port 2), input (port 1) shorted.
- Total resistance seen by source: $R_{eq2} = 4 + (6 \parallel 2) = 4 + 1.5 = 5.5\Omega$
- Source current: $I_{S2} = 10/5.5 = 1.82A$
- Current at port 1: $I_1 = I_{S2} \times \frac{6}{6+2} = 1.82 \times 0.75 = \mathbf{1.36A}$
- Verified: $I_1 = I_2 = 1.36A$

Principle of Duality: Introduction

- Definition: Two electrical networks are said to be duals of each other if the mesh equations of one network have the identical mathematical form as the nodal equations of the other.
- Concept: In electrical engineering, many concepts come in pairs that share identical mathematical relationships (e.g., Ohm's law: $V = IR$ and $I = GV$).
- Utility: If you know the behavior of a circuit, you instantly know the behavior of its dual circuit by simply swapping dual quantities.

Dual Elements and Quantities

- To convert a network to its dual, replace every element/concept with its dual counterpart:

Original Quantity	**Dual Quantity**
Voltage (V)	Current (I)
Resistance (R)	Conductance (G)
Inductance (L)	Capacitance (C)
Reactance (X)	Susceptance (B)
Impedance (Z)	Admittance (Y)
Mesh / Loop	Node
Series Connection	Parallel Connection
Open Circuit	Short Circuit
KVL	KCL

Constructing a Dual Network

- Graphical Method for Drawing Dual Circuits:
- Place a node at the center of each mesh of the original circuit. Place one reference node outside the circuit.
- Draw lines connecting the nodes such that each line crosses exactly one element of the original circuit.
- Replace the crossed original element with its dual element in the new line.
- Orientation: A voltage source causing clockwise mesh current becomes a current source feeding into the corresponding node.

Example: Principle of Duality

- Original Circuit: A voltage source V in series with an inductor L and a resistor R.
- Equation (KVL): $V = L\frac{di}{dt} + Ri$
- Dual Circuit Construction:
- The single mesh implies one non-reference node and one reference node.
- Series connection becomes parallel connection.
- V becomes current source I.
- L becomes capacitor C.
- R becomes conductance G.
- Dual Circuit: A current source I in parallel with a capacitor C and a conductance G.
- Equation (KCL): $I = C\frac{dv}{dt} + Gv$

Quick Check

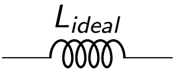
- Question 1: Under what condition is maximum power transferred to a load in a DC circuit?
- Answer: When $R_L = R_{th}$
- Question 2: What is the efficiency of power transfer when maximum power is delivered?
- Answer: 50%
- Question 3: Can the Reciprocity Theorem be applied to a circuit with dependent sources?
- Answer: No, only to linear, bilateral networks with independent sources.
- Question 4: What is the dual of a series inductor?
- Answer: A parallel capacitor.

Summary & Next Lecture Preview

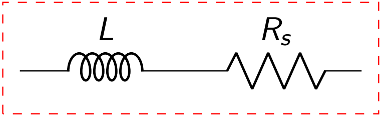
- Summary:
- Maximum Power Transfer: P_{max} achieved when $R_L = R_{th}$, resulting in 50% efficiency. Crucial for communication circuits.
- Reciprocity Theorem: Interchanging a single source and its response does not change the ratio (transfer impedance) in linear, bilateral networks.
- Principle of Duality: Allows transformation of circuits into dual forms (e.g., Series $\leftrightarrow$ Parallel, KVL $\leftrightarrow$ KCL).
- Next Lecture:
- Moving to Unit 2: Resonance Circuits and Passive Filters.
- Topic: Quality factor of a Coil and Capacitor.

Ideal vs. Practical Components

- Ideal Inductor: Pure inductance (L), zero resistance. Does not dissipate any active power.
- Practical Inductor (Coil): Has wire resistance (R_s) in series with inductance (L). Dissipates some power.
- Ideal Capacitor: Pure capacitance (C), infinite leakage resistance. No power loss.
- Practical Capacitor: Has some equivalent series resistance (ESR) or parallel leakage resistance. Exhibits minor power losses.



Ideal Inductor



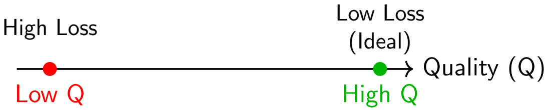
Practical Inductor

What is Quality Factor (Q)?

- Concept: Q-factor is a dimensionless parameter that describes how 'ideal' a reactive component is.
- It measures the ratio of energy stored to energy dissipated per cycle.
- General Formula:

$$Q = 2\pi \times \frac{\text{Maximum Energy Stored per Cycle}}{\text{Energy Dissipated per Cycle}}$$

- A higher Q indicates a component closer to ideal (lower energy loss).



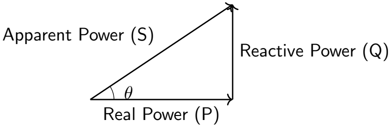
Q-Factor: Power Perspective

- Q-factor can also be defined using power ratios in an AC circuit.

$$Q = \frac{\text{Reactive Power}}{\text{Average (Active) Power}}$$

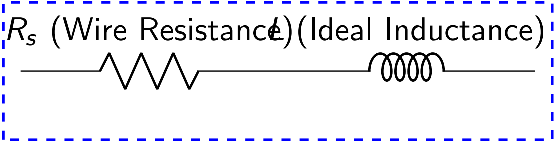
$$Q = \frac{I^2 X}{I^2 R} = \frac{X}{R}$$

- Where:
  - X = Reactance of the component (inductive or capacitive)
  - R = Equivalent series resistance of the component



Q-Factor of an Inductor (Coil) - Model

- A practical inductor is modeled as an ideal inductance L in series with a small resistance R_s.
- Impedance: $Z = R_s + jX_L$ where $X_L = \omega L = 2\pi fL$
- The resistance R_s represents the ohmic resistance of the copper wire.
- The goal in coil design is to make R_s as small as possible relative to X_L.



Practical Inductor (Coil) Model

Deriving Q for a Series Coil

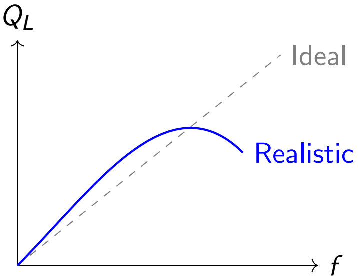
- From the power definition:

$$Q_L = \frac{X_L}{R_s}$$

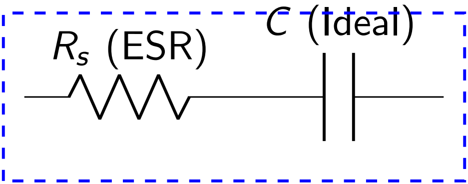
- Substitute $X_L = \omega L$:

$$Q_L = \frac{\omega L}{R_s} = \frac{2\pi f L}{R_s}$$

- Key Takeaways:
  - Q_L is directly proportional to frequency f (assuming R_s is constant).
  - At higher frequencies, skin effect increases R_s, causing Q to eventually peak and drop.



- A practical capacitor can be modeled with an Equivalent Series Resistance (ESR), denoted R_s.
- Impedance: $Z = R_s - jX_C$ where $X_C = \frac{1}{\omega C}$
- ESR represents losses in the dielectric and the resistance of the leads.
- Good capacitors (like ceramic or film) have very low ESR.



Practical Capacitor Model

Deriving Q for a Series Capacitor

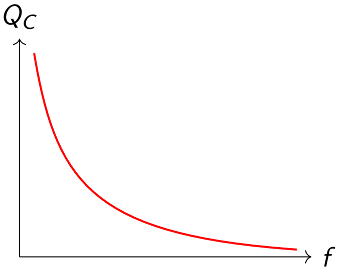
- Using the general formula $Q = \frac{X}{R}$:

$$Q_C = \frac{X_C}{R_s}$$

- Substitute $X_C = \frac{1}{\omega C}$:

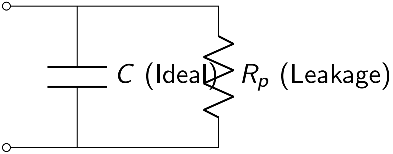
$$Q_C = \frac{1}{\omega C R_s} = \frac{1}{2\pi f C R_s}$$

- Observation: For a series model, Q_C is inversely proportional to frequency f.



Q-Factor of a Capacitor - Parallel Model

- Capacitor losses are sometimes better represented by a large Parallel Resistance (R_p).
- Here, R_p represents the leakage current passing *through* the dielectric material.
- Admittance: $Y = \frac{1}{R_p} + j\omega C$
- An ideal capacitor would have $R_p = \infty$ (no leakage).



Deriving Q for a Parallel Capacitor

- For a parallel circuit, Q is the ratio of Susceptance (B_C) to Conductance (G):

$$Q_C = \frac{B_C}{G}$$

- Substitute $B_C = \omega C$ and $G = \frac{1}{R_p}$:

$$Q_C = \frac{\omega C}{1/R_p} = \omega C R_p = 2\pi f C R_p$$

- Observation: For a parallel model, Q_C is directly proportional to frequency.

Comparison

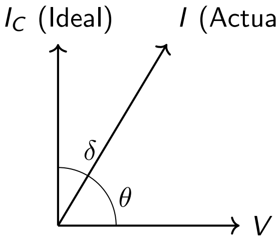
Series Model: $Q_C = \frac{1}{\omega C R_s}$ (Inversely proportional to f)
Parallel Model: $Q_C = \omega C R_p$ (Directly proportional to f)

Dissipation Factor (D)

- Capacitor quality is often specified by its Dissipation Factor (D) rather than Q.
- The Dissipation Factor is simply the reciprocal of the Quality Factor.

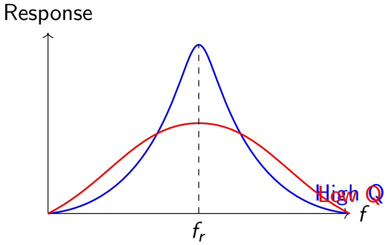
$$D = \frac{1}{Q}$$

- For a series model: $D = \omega C R_s = \frac{R_s}{X_C}$
- Also equal to the tangent of the loss angle ($\tan \delta$).
- Ideal Component: $Q = \infty$, $D = 0$.



Significance of High vs. Low Q

- High Q Components (e.g., Q ¿ 100):
  - Very low energy loss per cycle.
  - Essential for sharp, highly selective RF filters and tuned circuits.
  - High voltage/current magnification at resonance.
- Low Q Components (e.g., Q ¡ 10):
  - Higher energy dissipation (more heating).
  - Result in broad, less selective frequency responses.
  - Can be useful for damping unwanted oscillations (snubber circuits).



Quick Check

- **Question 1:** If the series resistance of a coil is doubled, what happens to its Q-factor? (Assuming frequency is constant)

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- **Question 3:** What is the ideal Q-factor for a pure capacitor?

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- *Answer: They are inversely proportional,* $D = \frac{1}{Q}$.
- **Question 3:** What is the ideal Q-factor for a pure capacitor?
- *Answer: Infinity (∞), because its resistance is zero.*

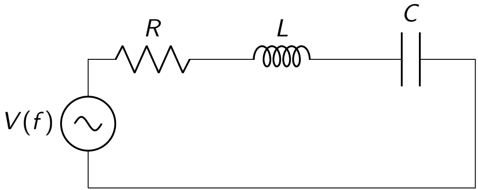
Summary

- Quality Factor (Q) measures energy storage vs. energy dissipation.
- Higher Q indicates a more ideal component, vital for sharp resonant circuits.

Parameter	**Coil / Inductor**	**Capacitor**
Series Model Q	$Q_L = \frac{\omega L}{R_s}$	$Q_C = \frac{1}{\omega C R_s}$
Parallel Model Q	-	$Q_C = \omega C R_p$
Dissipation Factor (D)	$D = \frac{1}{Q_L}$	$D = \frac{1}{Q_C}$

Next Lecture Preview

- Topic: Series Resonant Circuits & Frequency Response
- Key Concepts to Cover:
  - Connecting R, L, and C in series.
  - What happens at the resonant frequency (f_r).
  - How component Q affects the overall circuit Q.
  - Plotting and analyzing the frequency response curve.



Series Resonant Circuits & Frequency Response

- Electronics Circuit Network & Measurement
- Unit 2: Resonance Circuits and Passive Filters
- **Topics Covered:**
    - Introduction to Resonance
    - Series RLC Circuit Analysis
    - Resonant Frequency Derivation
    - Frequency Response Curves
    - Bandwidth and Selectivity

Agenda

- Recap: Quality Factor
- Introduction to Electrical Resonance
- Series RLC Circuit Configuration
- Condition for Series Resonance
- Variation of Circuit Parameters with Frequency
- Frequency Response and Bandwidth
- Voltage Magnification
- Summary & Quick Check

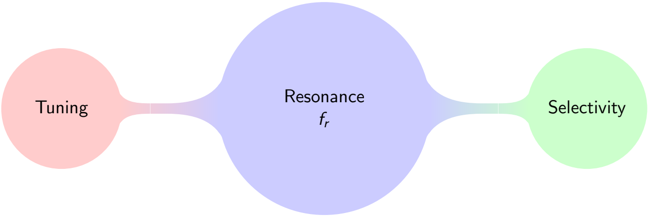
Recap: Quality Factor

- **Quality Factor (Q):**
- Measures the efficiency or 'goodness' of an inductor or capacitor.
- Inductor: $Q_L = \frac{X_L}{R_{series}} = \frac{\omega L}{R_{series}}$
- Capacitor: $Q_C = \frac{X_C}{R_{series}} = \frac{1}{\omega C R_{series}}$
- High Q implies low energy dissipation (low resistance) relative to the energy stored.

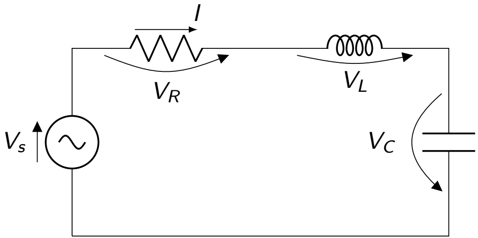
$Q_L = \frac{\omega L}{R_{series}}$ $\quad$ $Q_C = \frac{1}{\omega C R_{series}}$

Introduction to Electrical Resonance

- **What is Resonance?**
- Resonance occurs in an AC circuit when the effects of inductance and capacitance cancel each other out at a specific frequency.
- At this specific frequency, called the **Resonant Frequency (f_r)**, the circuit behaves purely resistively.
- **Applications:** Radio and TV tuning (selecting a specific station frequency), filters, and oscillators.

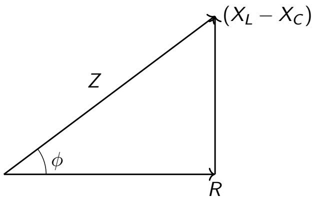


- **Circuit Configuration:**
- A resistor (R), an inductor (L), and a capacitor (C) connected in series with an AC voltage source.
- The same current I flows through all three elements.
- **Voltages:**
  - $V_R = I \cdot R$ (In phase with I)
  - $V_L = I \cdot X_L$ (Leads I by 90°)
  - $V_C = I \cdot X_C$ (Lags I by 90°)



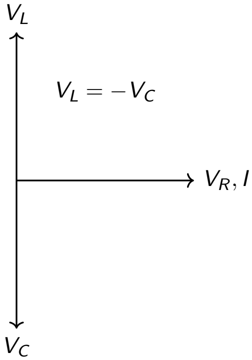
Impedance of a Series RLC Circuit

- **Total Impedance (Z):**
- The total impedance is the vector sum of resistance and net reactance.
- $Z = R + j(X_L - X_C)$
- **Magnitude:** $|Z| = \sqrt{R^2 + (X_L - X_C)^2}$
- **Phase Angle:** $\phi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right)$
- The circuit can be inductive ($X_L > X_C$), capacitive ($X_C > X_L$), or resistive ($X_L = X_C$).



Condition for Series Resonance

- **The Resonant Condition:**
- Resonance occurs when the net reactance is zero.
- $X_L = X_C \implies X_L - X_C = 0$
- **Implications at Resonance:**
  - Impedance is minimum: $Z = R$
  - Current is maximum: $I = \frac{V}{R}$
  - Phase angle is zero ($\phi = 0$): Voltage and current are in phase.
  - Power factor is unity (PF $= 1$).



Derivation of Resonant Frequency (f_r)

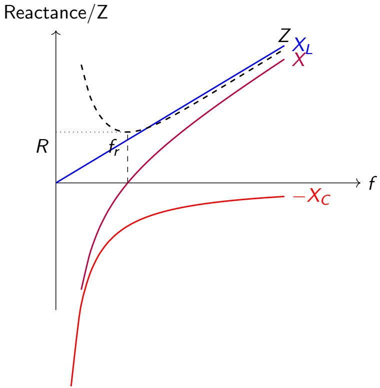
- **Finding the specific frequency:**
- At resonance: $X_L = X_C$
- Substitute the formulas: $2\pi f_r L = \frac{1}{2\pi f_r C}$
- Rearrange to solve for f_r:
- $(2\pi f_r)^2 = \frac{1}{LC} \implies 2\pi f_r = \frac{1}{\sqrt{LC}}$
- **Resonant Frequency:**

Formulas

$$f_r = \frac{1}{2\pi\sqrt{LC}} \text{ (in Hertz)}$$
$$\omega_r = \frac{1}{\sqrt{LC}} \text{ (in rad/s)}$$

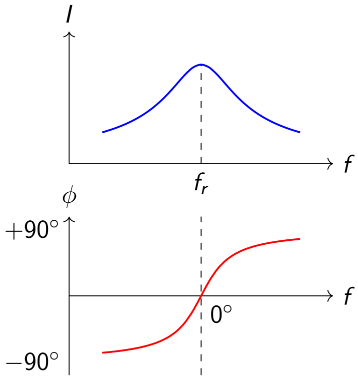
Variation of Reactance & Impedance

- **How parameters change as frequency increases:**
- $X_L = 2\pi fL$: Increases linearly with frequency.
- $X_C = \frac{1}{2\pi fC}$: Decreases non-linearly with frequency.
- Net Reactance ($X = X_L - X_C$): Goes from negative (capacitive) to zero at f_r, to positive (inductive).
- Impedance (Z): Very high at low frequencies, dips to minimum (R) at f_r, and rises again at high frequencies.



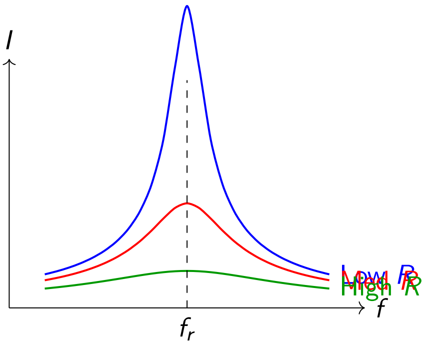
Current and Phase Angle vs. Frequency

- **Current ($I = \frac{V}{Z}$):**
- Very low at low and high frequencies due to high Z.
- Maximum at f_r ($I_{max} = \frac{V}{R}$).
- **Phase Angle (ϕ):**
  - $f < f_r$: Capacitive. Current leads voltage (negative ϕ).
  - $f = f_r$: Resistive. In phase ($\phi = 0°$).
  - $f > f_r$: Inductive. Current lags voltage (positive ϕ).



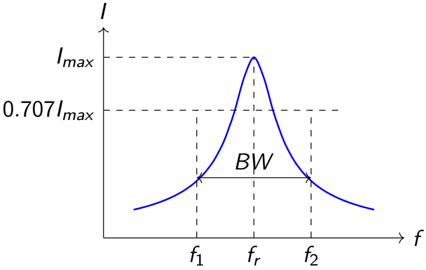
The Frequency Response Curve

- **The Current Resonance Curve:**
- Plots Current (I) vs. Frequency (f).
- Forms a bell-shaped curve peaking at f_r.
- The steepness or 'sharpness' of this curve determines the circuit's selectivity.
- A smaller resistance R results in a higher peak current and a sharper curve.
- A larger resistance R results in a lower, broader curve.



Half-Power Frequencies & Bandwidth

- **Bandwidth (BW or Δf):**
- The range of frequencies over which the power dissipated is at least half of the maximum power.
- **Half-Power Frequencies (f_1, f_2):** Frequencies where current is $\frac{I_{max}}{\sqrt{2}}$ (or 70.7% of max).
- f_1 = Lower cut-off frequency. f_2 = Upper cut-off frequency.
- Bandwidth Equation: $BW = f_2 - f_1$
- Power at f_1, f_2 is $P = I^2R = (\frac{I_{max}}{\sqrt{2}})^2R = \frac{1}{2}P_{max}$



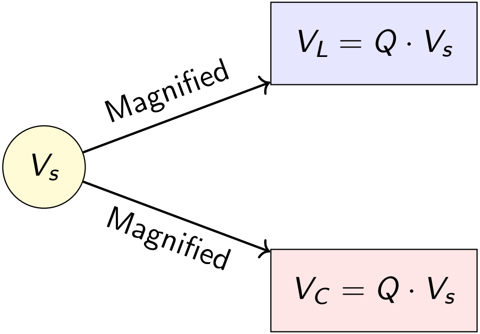
Quality Factor (Q) in Series Resonance

- **Q-Factor as Selectivity:**
- In a series circuit, Q defines the sharpness of the resonance curve.
- $Q = \frac{\text{Resonant Frequency}}{\text{Bandwidth}} = \frac{f_r}{BW}$
- Higher Q = Narrower Bandwidth = Higher Selectivity.
- **Q in terms of circuit components:**
- $Q = \frac{X_L}{R} = \frac{2\pi f_r L}{R}$
- Substituting $f_r = \frac{1}{2\pi\sqrt{LC}}$, we get:
- $Q = \frac{1}{R}\sqrt{\frac{L}{C}}$

High Q	Low Q
Sharp Peak, Narrow BW	Flat Peak, Wide BW

Voltage Magnification Effect

- **A unique property of series resonance:**
- At resonance, the voltage across the inductor (V_L) and capacitor (V_C) can be much higher than the supply voltage (V_s)!
- $V_L = I \cdot X_L = (\frac{V_s}{R}) \cdot X_L = V_s \cdot (\frac{X_L}{R}) = V_s \cdot Q$
- $V_C = I \cdot X_C = (\frac{V_s}{R}) \cdot X_C = V_s \cdot (\frac{X_C}{R}) = V_s \cdot Q$
- $V_L = V_C = Q \cdot V_s$
- Because Q is typically ¿ 1, the circuit acts as a Voltage Multiplier.



Summary of Series Resonance

- **Key Characteristics at $f = f_r$:**
- Inductive and Capacitive Reactances are equal: $X_L = X_C$
- Total Impedance is minimum and purely resistive: $Z = R$
- Circuit Current is maximum: $I_{max} = \frac{V}{R}$
- Power Factor is unity: $PF = 1$ (Phase angle $= 0$)
- Resonant Frequency: $f_r = \frac{1}{2\pi\sqrt{LC}}$
- Voltage Magnification: $V_L = V_C = Q \cdot V_{source}$

Parameter	$f < f_r$ **(Low)**	$f = f_r$ **(Resonance)**	$f > f_r$ **(High)**
Net Reactance	Capacitive ($X_C > X_L$)	Zero ($X_L = X_C$)	Inductive ($X_L > X_C$)
Impedance (Z)	High	Minimum ($Z = R$)	High
Current (I)	Low	Maximum (V/R)	Low
Phase Angle (ϕ)	Negative (I leads V)	Zero (In phase)	Positive (I lags V)
Power Factor	Leading	Unity (1)	Lagging

Quick Check

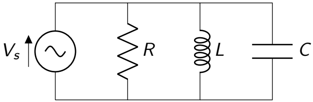
- **Question:** A series RLC circuit has $R = 10\,\Omega$, $L = 100$ mH, and $C = 10\,\mu F$. What happens to the bandwidth if the resistance is increased to $20\,\Omega$?
- A) Bandwidth decreases
- B) **Bandwidth increases** ✓
- C) Bandwidth remains unchanged
- D) Resonant frequency shifts

Key Takeaways

- Resonance happens when reactive components cancel ($X_L = X_C$).
- Series Resonance provides minimum impedance and maximum current, acting as a highly selective band-pass filter.
- The Q-factor dictates the sharpness of the frequency response; higher Q means a narrower bandwidth.
- Beware of Voltage Magnification across the inductor and capacitor at resonance.

Next Lecture Preview

- **Parallel Resonant Circuits & Frequency Response**
- Configuration of Parallel RLC circuits (Tank Circuits).
- Condition for Parallel Resonance (Anti-resonance).
- Dynamic Impedance and Current Magnification.
- Comparison between Series and Parallel Resonance.

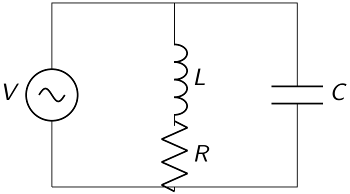


Agenda for Today's Lecture

- Introduction to Parallel Resonant Circuits
- Practical Tank Circuit Model
- Derivation of Parallel Resonant Frequency (f_r)
- Dynamic Impedance at Resonance
- Current at Resonance and Current Magnification
- Frequency Response Curves (Impedance & Current)
- Quality Factor (Q) and Bandwidth
- Comparison: Series vs. Parallel Resonance

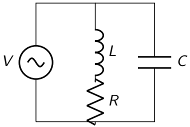
Introduction to Parallel Resonant Circuits

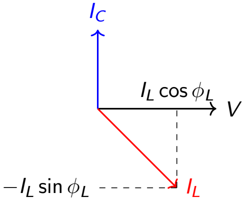
- **Definition:** A parallel circuit consisting of an inductor (with internal resistance) and a capacitor is in resonance when the total line current is perfectly in phase with the applied voltage.
- **Power Factor:** At resonance, the circuit has a unity power factor (p.f. = 1).
- **Alternative Name:** Often referred to as an anti-resonant circuit or tank circuit.
- **Primary Characteristic:** Unlike a series resonant circuit (which has minimum impedance), a parallel resonant circuit exhibits maximum impedance at the resonant frequency.



Practical Parallel LC Circuit Model (Tank Circuit)

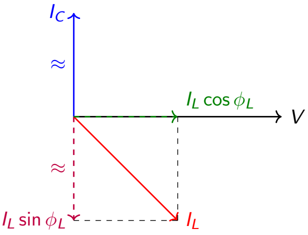
- **Ideal vs. Practical:** An ideal parallel LC circuit has no resistance. In practice, the inductor coil always possesses some resistance (R).
- **Circuit Configuration:**
  - Branch 1: A coil with inductance L and resistance R in series.
  - Branch 2: A capacitor C (assumed ideal, zero resistance).
- **Phasor Approach:**
  - Voltage V is common across both branches.
  - Current I_L lags V by an angle ϕ_L (less than 90° due to R).
  - Current I_C leads V by exactly 90°.





Condition for Parallel Resonance

- Total Line Current (I): The vector sum of I_L and I_C. $I = \vec{I_L} + \vec{I_C}$
- Resolving I_L into components:
  - Active (in-phase) component: $I_L \cos \phi_L$
  - Reactive (quadrature) component: $I_L \sin \phi_L$
- **Resonance Condition:** Occurs when the net reactive current is zero.
- Therefore, the reactive component of the inductive branch must perfectly cancel the capacitive current.
- **Equation:** $I_C = I_L \sin \phi_L$



Derivation of Parallel Resonant Frequency (f_r)

- From the resonance condition: $I_C = I_L \sin\phi_L$
- We know:
  - $I_C = \frac{V}{X_C} = \frac{V}{1/\omega C} = V\omega C$
  - $I_L = \frac{V}{Z_L}$
  - $\sin\phi_L = \frac{X_L}{Z_L} = \frac{\omega L}{Z_L}$
- Substituting these into the condition:

$$V\omega C = \left(\frac{V}{Z_L}\right)\left(\frac{\omega L}{Z_L}\right)$$

$$C = \frac{L}{Z_L^2}$$

$$Z_L^2 = \frac{L}{C}$$

Calculating the Resonant Frequency

- We found: $Z_L^2 = \frac{L}{C}$
- Since $Z_L^2 = R^2 + X_L^2 = R^2 + (2\pi f_r L)^2$
- Therefore: $R^2 + (2\pi f_r L)^2 = \frac{L}{C}$
- Solving for f_r:

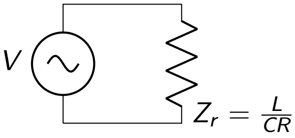
$$(2\pi f_r L)^2 = \frac{L}{C} - R^2$$

$$f_r = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \left(\frac{R}{L}\right)^2}$$

$$f_r = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \left(\frac{R}{L}\right)^2}$$

Dynamic Impedance at Resonance (Z_r)

- At resonance, the net reactive current is zero. The total current I is purely resistive.
- $I = I_L \cos \phi_L$
- We know $I_L = \frac{V}{Z_L}$ and $\cos \phi_L = \frac{R}{Z_L}$
- $I = \frac{V}{Z_L} \cdot \frac{R}{Z_L} = \frac{VR}{Z_L^2}$
- Substitute $Z_L^2 = \frac{L}{C}$:
- $I = \frac{VR}{L/C} = V\left(\frac{CR}{L}\right)$
- Therefore, the Dynamic Impedance (Z_r) is:
- $Z_r = \frac{V}{I} = \frac{L}{CR}$

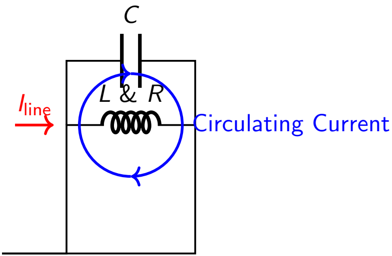


Characteristics at Parallel Resonance

- **Maximum Impedance:** $Z_r = \frac{L}{CR}$ is a very high value.
- **Minimum Current:** Because impedance is maximum, the line current drawn from the source is at its absolute minimum: $I_{\min} = \frac{V}{Z_r}$.
- **Resistive Nature:** The circuit acts as a pure resistor (unity power factor).
- **Rejector Circuit:** Due to the high impedance, it 'rejects' the line current at the resonant frequency, making it useful in radio frequency tuning and filter circuits.

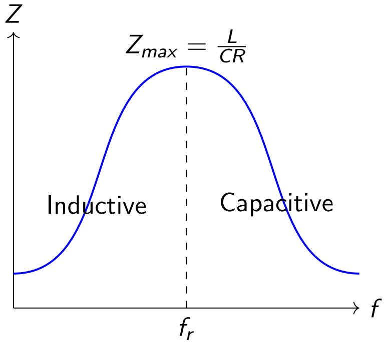
Current Magnification

- In a series circuit, we have voltage magnification. In a parallel circuit, we have current magnification.
- The line current I is very small.
- However, the current circulating internally between the inductor and capacitor (I_C and I_L) can be very large.
- **Circulating Current:** The capacitor discharges into the inductor, and the magnetic field of the inductor collapses to charge the capacitor. Energy oscillates back and forth.
- Magnification $= \frac{\text{Circulating Current}}{\text{Line Current}} \approx Q$



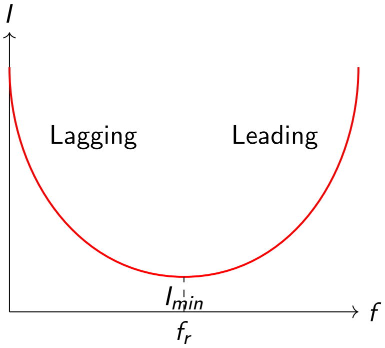
Frequency Response: Impedance vs Frequency

- Let's look at how impedance (Z) changes with frequency (f).
- **Below f_r:** X_L is small, X_C is large. Inductive branch draws more current. The circuit is Inductive.
- **At f_r:** X_L and X_C effects balance out. Impedance is purely resistive and at its Maximum ($Z = L/CR$).
- **Above f_r:** X_L is large, X_C is small. Capacitive branch draws more current. The circuit is Capacitive.



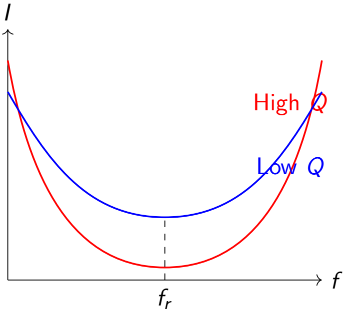
Frequency Response: Current vs Frequency

- The line current (I) is inversely proportional to impedance.
- **Below f_r:** Impedance is low, so line current is high (lagging).
- **At f_r:** Impedance is maximum, so line current is at its Minimum (in-phase).
- **Above f_r:** Impedance drops again, so line current increases (leading).
- The current curve is a 'dip' or 'notch' curve.



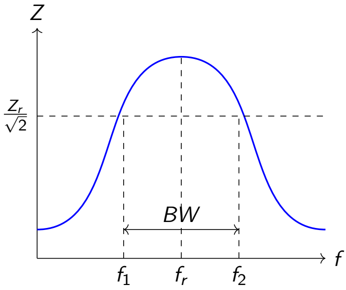
Quality Factor (Q) in Parallel Resonance

- The Quality Factor (Q) defines the sharpness of resonance.
- Defined as the ratio of circulating current to line current: $Q = \frac{I_C}{I}$
- Alternatively, defined via power: $Q = 2\pi \frac{\text{Max energy stored}}{\text{Energy dissipated/cycle}}$
- For a practical parallel circuit (where R is small compared to X_L):
  - $Q \approx \frac{1}{R}\sqrt{\frac{L}{C}}$
  - $Q = \frac{2\pi f_r L}{R} = \frac{\omega_r L}{R}$



Bandwidth in Parallel Circuits

- **Bandwidth (BW):** The range of frequencies between the half-power points (f_1 and f_2) on the impedance curve.
- Since power is proportional to I^2 or V^2/Z, the half-power points occur where the impedance drops to $\frac{Z_r}{\sqrt{2}}$.
- $BW = f_2 - f_1 = \frac{f_r}{Q}$
- Same relationship as in series circuits:
  - Higher Q $\implies$ Narrower Bandwidth (Higher Selectivity)
  - Lower Q $\implies$ Wider Bandwidth (Lower Selectivity)

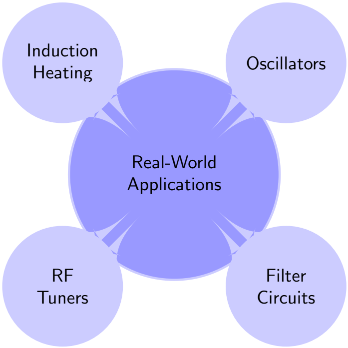


Comparison: Series vs. Parallel Resonance

Feature	**Series Resonance (Acceptor)**	**Parallel Resonance (Rejector)**
Impedance at f_r	Minimum ($Z = R$)	Maximum ($Z = L/CR$)
Current at f_r	Maximum (V/R)	Minimum (V/Z_r)
Magnification	Voltage Magnification	Current Magnification
Power Factor	Unity (1)	Unity (1)
Below f_r	Capacitive (Leading I)	Inductive (Lagging I)
Above f_r	Inductive (Lagging I)	Capacitive (Leading I)

Real-World Applications

- **Radio Frequency (RF) Tuners:** Tank circuits are used in receivers to tune into a specific frequency while rejecting others.
- **Filter Circuits:** Band-stop filters (notch filters) block a specific band of frequencies.
- **Oscillators:** LC tank circuits define the frequency of oscillation in many RF transmitters.
- **Induction Heating:** Parallel resonance is used to generate large circulating currents to heat metals.



Quick Check

Question 1: In a parallel resonant circuit, what happens to the line current at the resonant frequency?
(a) It becomes maximum
(b) It becomes zero
(c) It becomes minimum
(d) It becomes infinite

Question 2: The dynamic impedance of a practical parallel resonant circuit is given by:
(a) CR/L
(b) L/CR
(c) LC/R
(d) R/LC

Summary & Next Class

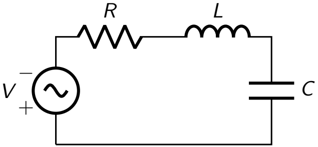
- **Summary:**
  - Parallel resonance occurs when the reactive component of line current is zero (unity p.f.).
  - The resonant frequency is $f_r = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \left(\frac{R}{L}\right)^2}$.
  - At resonance, the circuit exhibits maximum dynamic impedance $(Z_r = L/CR)$ and minimum line current.
  - The circuit acts as a current magnifier and a frequency rejector.
- **Next Class (Lecture 14):**
  - Numerical examples and calculation of resonance parameters for both series and parallel circuits.

Agenda

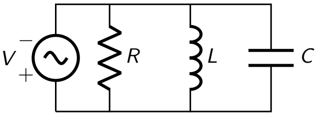
- Recap of Series and Parallel Resonance
- Essential Resonance Parameters (f_r, Q, BW)
- Parameter Formulas for Series Resonance
- Calculation Example: Series Circuit
- Parameter Formulas for Parallel Resonance
- Dynamic Impedance (Z_d)
- Calculation Example: Parallel Circuit
- Selectivity and the Impact of Resistance
- Quick Check & Summary

Recap: Resonance in AC Circuits

- Resonance occurs when inductive reactance equals capacitive reactance ($X_L = X_C$).
- **Series Resonance:** Minimum impedance ($Z = R$), Maximum current, Known as an Acceptor Circuit.
- **Parallel Resonance:** Maximum impedance, Minimum line current, Known as a Rejector Circuit.
- In both cases, the circuit exhibits purely resistive behavior at resonance (power factor = 1).



Series RLC

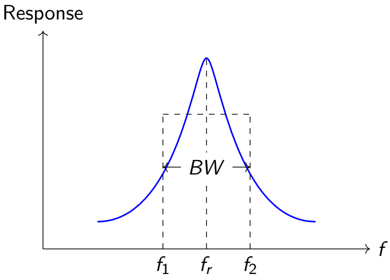


Parallel RLC

Essential Resonance Parameters

When analyzing resonant circuits, we must calculate several vital parameters:

- **Resonance Frequency (f_r):** The precise frequency where $X_L = X_C$.
- **Quality Factor (Q):** A dimensionless measure of the sharpness of resonance (ratio of energy stored to energy dissipated per cycle).
- **Bandwidth (BW):** The frequency range $(f_2 - f_1)$ over which the circuit's power is at least half of its maximum value.
- **Half-Power Frequencies (f_1, f_2):** The lower and upper cutoff frequencies defining the bandwidth boundary.



Formulas: Series Resonance (f_r and Q)

- **Resonance Frequency:**

$$f_r = \frac{1}{2\pi\sqrt{LC}}$$

- **Quality Factor (Q_s):**

$$Q_s = \frac{\omega_r L}{R} = \frac{1}{R}\sqrt{\frac{L}{C}}$$

(A higher Q means a narrower, sharper tuning curve and less energy loss in the resistor.)

- **Impedance at resonance:**

$$Z_{min} = R$$

Formulas: Bandwidth and Cutoff Frequencies

- **Bandwidth (BW):**

$$BW = f_2 - f_1 = \frac{f_r}{Q_s} = \frac{R}{2\pi L}$$

- **Half-Power (Cutoff) Frequencies:**
  Assuming a reasonably high Q ($Q \geq 10$), the curve is symmetric around f_r:

$$f_1 = f_r - \frac{BW}{2}$$

$$f_2 = f_r + \frac{BW}{2}$$

  (Note: For low Q circuits, precise quadratic formulas must be used instead of this symmetric approximation.)

Example Calculation: Series Resonance

Problem: A series circuit has $R = 10\,\Omega$, $L = 0.1\,\text{H}$, and $C = 10\,\mu\text{F}$. Calculate f_r, Q, and BW.

Solution:

- **Find f_r:**

$$f_r = \frac{1}{2\pi\sqrt{0.1 \times 10 \times 10^{-6}}} = \frac{1}{2\pi\sqrt{10^{-6}}} \approx \mathbf{159.15}\,\text{Hz}$$

- **Find Q_s:**

$$Q_s = \frac{1}{10}\sqrt{\frac{0.1}{10 \times 10^{-6}}} = 0.1 \times \sqrt{10000} = \mathbf{10}$$

- **Find BW:**

$$BW = \frac{f_r}{Q_s} = \frac{159.15}{10} = \mathbf{15.91}\,\text{Hz}$$

Formulas: Practical Parallel Resonance

For a practical tank circuit (a coil with R and L in parallel with a capacitor C):

- **Resonance Frequency:**

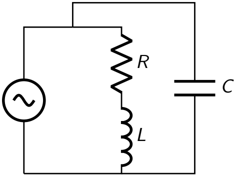
$$f_r = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$$

  (If the coil resistance R is very small, $\frac{R^2}{L^2}$ is negligible, and it simplifies to the series formula $\frac{1}{2\pi\sqrt{LC}}$.)

- **Quality Factor (Q_p):**

$$Q_p = \frac{\omega_r L}{R}$$

  (Similar to series if R is in the inductive branch).



In a parallel resonant circuit, the impedance is purely resistive and reaches its maximum value at resonance. This peak impedance is called **Dynamic Impedance (Z_d)**.

Dynamic Impedance Formula

$$Z_d = \frac{L}{CR}$$

Significance:

- A smaller internal resistance R in the coil leads to a much higher dynamic impedance.
- This effectively blocks the resonant frequency signal, justifying the term "rejector circuit."

Example Calculation: Parallel Resonance

Problem: A tank circuit has $C = 100\,\text{pF}$, and a coil with $L = 100\,\mu\text{H}$ and $R = 20\,\Omega$. Find the approximate f_r and the dynamic impedance Z_d.

Solution:

- **Check simplification:** $\frac{R^2}{L^2} = 4 \times 10^{10}$, which is much smaller than $\frac{1}{LC} = 10^{14}$. We can use the simplified formula.
- **Find f_r:**

$$f_r \approx \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{10^{-14}}} \approx \mathbf{1.59}\,\text{MHz}$$

- **Find Z_d:**

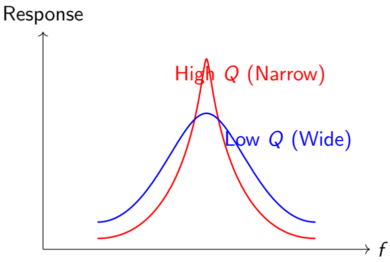
$$Z_d = \frac{100 \times 10^{-6}}{100 \times 10^{-12} \times 20} = \frac{100\mu}{2\text{n}} = 50,000\,\Omega = \mathbf{50}\,\text{k}\boldsymbol{\Omega}$$

Selectivity and Bandwidth Relationship

- **Selectivity:** The ability of a circuit to select a desired frequency band while rejecting others.
- **Relationship:** Selectivity is directly proportional to Q and inversely proportional to BW.

$$\text{Selectivity} \propto Q \propto \frac{1}{BW}$$

- **Implication:** A narrow bandwidth means high selectivity. This is essential for applications like radio receivers, where tuning into a specific station without interference from adjacent channels is required.



Impact of Resistance (R)

Parameter	Series Resonance Effect	Parallel Resonance Effect
Impedance at f_r	Increases ($Z = R$)	Decreases ($Z_d = L/CR$)
Current/Response	Decreases	Decreases
Quality Factor (Q)	Decreases	Decreases
Bandwidth (BW)	Widens	Widens

- **Effect on Series Resonance:** Increasing R increases total impedance at resonance, decreases peak current, decreases Q, and widens bandwidth.
- **Effect on Parallel Resonance:** Increasing the coil's R slightly lowers exact resonance frequency, significantly decreases dynamic impedance, decreases Q, and widens bandwidth.

Practical Application: Tuning a Radio

- When you tune a radio, you are adjusting a variable capacitor in an LC resonant circuit.
- **Changing C:** Alters the resonance frequency f_r to match the incoming broadcast frequency.
- **Antenna Signal:** The circuit provides maximum voltage (series) or maximum impedance (parallel) specifically at this f_r.
- **Required Q:** The Q must be precisely designed to provide a specific bandwidth (e.g., 10 kHz for AM radio). Too narrow, and you lose audio fidelity; too wide, and you hear multiple stations.

Quick Check

- **Question 1:** If the capacitance in a series resonant circuit is increased by a factor of 4, how does the resonance frequency change?
  A) Doubles
  B) **Halves (Correct)**
  C) Increases by 4
  D) Decreases by 4

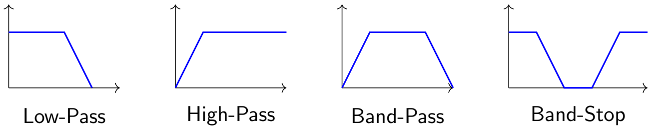
- **Question 2:** The dynamic impedance of a parallel resonant circuit is given by:
  A) R/LC
  B) L/CR **(Correct)**
  C) C/LR
  D) $1/(2\pi\sqrt{LC})$

Summary

- **Resonance Frequency (f_r):** Primarily determined by L and C ($f_r = \frac{1}{2\pi\sqrt{LC}}$).
- **Quality Factor (Q):** Determines the sharpness of the peak; inversely proportional to R.
- **Bandwidth (BW):** Defined as $BW = \frac{f_r}{Q}$.
- **Dynamic Impedance (Z_d):** Maximum impedance at parallel resonance, $Z_d = \frac{L}{CR}$.
- Calculating these parameters is essential for designing filters and selective tuning circuits.

Next Lecture Preview

- **Next Topic:** Classification of Passive Filter Circuits
- What are electrical filters?
- Differences between Low-Pass, High-Pass, Band-Pass, and Band-Stop filters.
- Applications of passive filters in signal processing and communication systems.

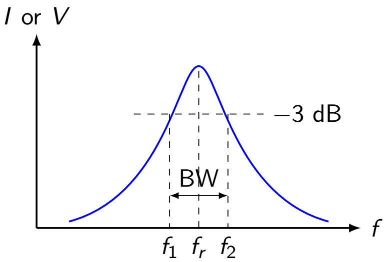


Agenda

- Recap of Resonance Parameters
- Introduction to Electrical Filters
- Key Filter Terminology
- Passive vs. Active Filters
- Classification of Filters by Frequency Response
- Detailed Look: LPF, HPF, BPF, and BSF
- Ideal vs. Practical Characteristics
- Summary & Next Steps

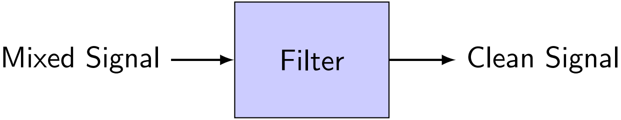
Recap: Resonance Parameters

- Resonance Frequency (f_r): The frequency at which the inductive reactance equals the capacitive reactance ($X_L = X_C$).
- Bandwidth (BW): The range of frequencies over which the circuit's power is at least half of its maximum value ($f_2 - f_1$).
- Quality Factor (Q): A measure of the sharpness of the resonance. $Q = \frac{f_r}{BW}$.
- Higher Q indicates a narrower bandwidth, providing higher selectivity.
- These principles of frequency selectivity naturally lead us to filter circuits.



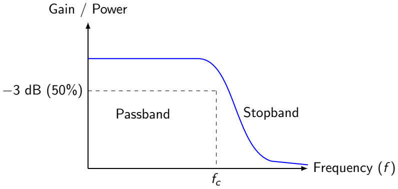
What is an Electrical Filter?

- **Definition**: An electrical filter is a frequency-selective circuit that passes a specified band of frequencies and attenuates (blocks) signals of frequencies outside this band.
- **Purpose**: Used to separate desired signals from unwanted signals, such as noise or interference.
- **Core Principle**: Filters utilize frequency-dependent reactive components, namely Inductors (L) and Capacitors (C).
- **Remember**:
  - Capacitor Reactance: $X_C = \frac{1}{2\pi fC}$ (Decreases with frequency)
  - Inductor Reactance: $X_L = 2\pi fL$ (Increases with frequency)



Key Filter Terminology

- **Passband**: The range of frequencies that are allowed to pass through the filter with minimal attenuation.
- **Stopband** (Attenuation Band): The range of frequencies that are heavily attenuated or blocked by the filter.
- **Cutoff Frequency** (f_c): The boundary frequency that separates the passband from the stopband.
- At f_c, the output power drops to 50% (or the voltage drops to 70.7%, i.e., -3 dB point) of its maximum passband value.
- **Attenuation** (α): The reduction in signal strength, usually measured in decibels (dB).



Passive vs. Active Filters

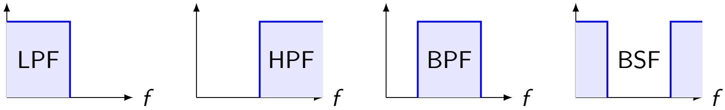
Feature	**Passive Filters**	**Active Filters**
Components	Resistors (R), Inductors (L), Capacitors (C)	Op-Amps, Resistors (R), Capacitors (C)
Power Supply	Not required	External DC power supply required
Signal Gain	≤ 1 (Cannot amplify, only attenuate)	> 1 (Can amplify the signal)
Frequency Suitability	Suitable for high frequencies	Great for low frequencies (avoids bulky L)

- Passive filters are built using only passive components and do not require external power.
- Active filters use active components (like Op-Amps) and can amplify signals but are typically limited at very high frequencies.

Classification Based on Frequency Response

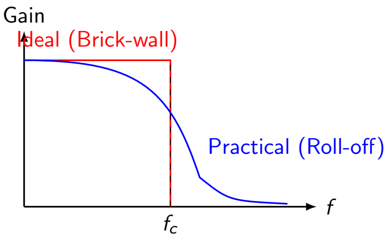
Passive filters are primarily classified into four standard types based on which frequencies they pass:

- **Low Pass Filter (LPF)**: Passes low frequencies, blocks high frequencies.
- **High Pass Filter (HPF)**: Passes high frequencies, blocks low frequencies.
- **Band Pass Filter (BPF)**: Passes a specific middle band of frequencies, blocks both lower and higher frequencies.
- **Band Stop / Reject Filter (BSF)**: Blocks a specific band of frequencies, passes all others.



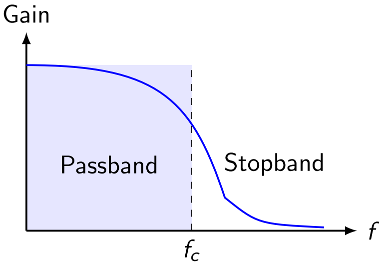
Ideal vs. Practical Filter Characteristics

- **Ideal Filters**:
  - Have a "brick-wall" response.
  - Transition from passband to stopband is instantaneous at f_c.
  - Zero attenuation in the passband, infinite in the stopband.
  - Physically impossible to realize.
- **Practical Filters**:
  - Roll-off is gradual, not vertical.
  - There is a transition region between the passband and stopband.
  - Have some small insertion loss in the passband.



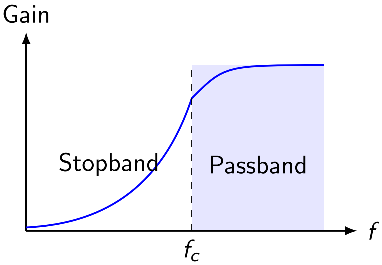
1. Low Pass Filter (LPF)

- **Function**: Allows frequencies from 0 Hz (DC) up to the cutoff frequency (f_c) to pass.
- Attenuates all frequencies above f_c.
- **Passband**: $0 \leq f < f_c$
- **Stopband**: $f > f_c$
- **Common usage**: Removing high-frequency noise from audio signals, smoothing DC power supply outputs.



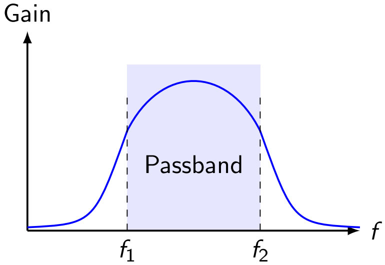
2. High Pass Filter (HPF)

- **Function**: Allows frequencies above the cutoff frequency (f_c) to pass.
- Attenuates all frequencies below f_c, including DC (0 Hz).
- **Passband**: $f > f_c$
- **Stopband**: $0 \leq f < f_c$
- **Common usage**: Blocking DC offsets in amplifier stages (AC coupling), sending high frequencies to tweeters in speakers.



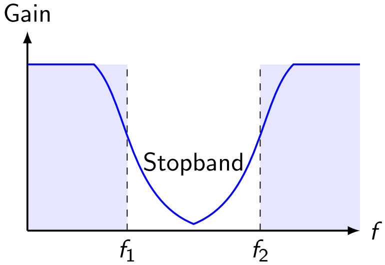
3. Band Pass Filter (BPF)

- **Function**: Allows a specific range of frequencies to pass while attenuating frequencies both above and below this range.
- Has two cutoff frequencies: Lower cutoff (f_1) and Upper cutoff (f_2).
- **Passband**: $f_1 < f < f_2$
- **Stopband**: $f < f_1$ AND $f > f_2$
- **Bandwidth**: $BW = f_2 - f_1$
- **Common usage**: Radio receivers to tune into a specific station's frequency while blocking others.



4. Band Stop / Band Reject Filter (BSF)

- **Function**: Attenuates a specific range of frequencies while passing all frequencies outside this range.
- Also known as a Notch Filter (if the rejection band is very narrow).
- Has two cutoff frequencies: Lower (f_1) and Upper (f_2).
- **Stopband**: $f_1 < f < f_2$
- **Passband**: $f < f_1$ AND $f > f_2$
- **Common usage**: Eliminating a specific noise frequency, such as 50 Hz or 60 Hz power line interference in instruments.

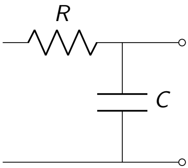


Implementing Passive Filters (Preview)

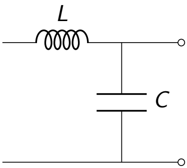
How do we actually build these?

- **RC Filters**: Resistor and Capacitor networks. Used mainly for low power and lower frequency applications.
- **RL Filters**: Resistor and Inductor networks. Less common due to inductor size and losses.
- **LC Filters**: Inductor and Capacitor networks. Provide sharper roll-off and are widely used in RF (Radio Frequency) applications where high Q is needed.

We will explore the mathematical design of Constant-k LC filters in upcoming lectures.



RC Low Pass



LC Low Pass

Quick Check

Question 1: Which filter type would you use to remove a constant DC voltage offset (0 Hz) from an AC signal?

- A) Low Pass Filter
- B) High Pass Filter **(Correct)**
- C) Band Pass Filter

Question 2: A filter has a passband between 10 kHz and 20 kHz. What type of filter is this?

- A) Band Stop Filter
- B) High Pass Filter
- C) Band Pass Filter **(Correct)**

Summary & Key Takeaways

- Filters are frequency-selective circuits used to pass or block specific frequency bands.
- Passive filters use R, L, and C and do not amplify the signal.
- Four main classifications based on frequency response:
  - **LPF**: Passes below f_c
  - **HPF**: Passes above f_c
  - **BPF**: Passes between f_1 and f_2
  - **BSF**: Blocks between f_1 and f_2
- Practical filters have a gradual transition (roll-off), unlike ideal brick-wall filters.

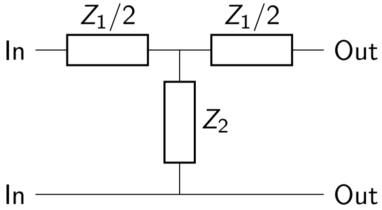
Next Lecture Preview

Next Topic: Introduction to Constant-k Type Filters

- Symmetrical T and π networks for filters.
- Understanding the 'Constant-k' classification.
- Deriving the design equations for Constant-k Low Pass and High Pass filters.
- **Preparation**: Review the Z and Y parameters for two-port networks (from Unit 1), as we will use them to analyze filter sections.

Introduction to Constant-k Type Filters

- Welcome to Lecture 16! Today we delve into the core principles of constant-k prototype filters, forming the foundation for passive filter design.



Agenda

- What is a Constant-k Filter?
- The 'Constant-k' Principle and Equation
- T-Section and Pi-Section Networks
- Constant-k Low Pass Filter (LPF) Architecture
- Constant-k High Pass Filter (HPF) Architecture
- Limitations of Constant-k Filters
- Quick Knowledge Check
- Summary

Recap: Passive Filter Fundamentals

- Passive Filters: Built using only passive components (Resistors, Inductors, Capacitors). No external power supply.
- Passband: Range of frequencies that are transmitted with minimum attenuation.
- Stopband (Attenuation Band): Range of frequencies that are highly attenuated.
- Cut-off Frequency (f_c): The boundary frequency between passband and stopband.

What is a Constant-k Filter?

- Also known as prototype filters.
- They are the original and simplest type of passive image-parameter filters.
- Designed so that the product of the series impedance (Z_1) and the shunt impedance (Z_2) is a constant, independent of frequency.
- They form the basic building blocks from which more complex filters (like m-derived filters) are developed.

Constant-k Section

The Constant-k Principle

Fundamental Equation

$$Z_1 \times Z_2 = k^2 = R_0^2$$

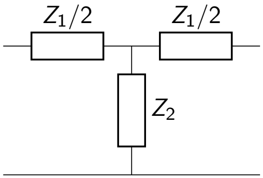
- Z_1: Total series arm impedance.
- Z_2: Total shunt arm impedance.
- k or R_0: A real, purely resistive constant known as the Design Impedance or nominal characteristic impedance.
- Since R_0^2 is real and positive, Z_1 and Z_2 must be pure reactances of opposite signs (one inductive, one capacitive).

Deriving Design Impedance (R_0)

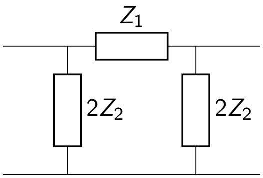
- Let series arm be purely inductive: $Z_1 = j\omega L$
- Let shunt arm be purely capacitive: $Z_2 = \frac{1}{j\omega C}$
- Product: $Z_1 Z_2 = (j\omega L) \times \left(\frac{1}{j\omega C}\right) = \frac{L}{C}$
- Therefore: $R_0^2 = \frac{L}{C} \implies R_0 = \sqrt{\frac{L}{C}}$
- R_0 is completely independent of frequency (ω canceled out).

T-Section and Pi-Section Networks

- Any filter section can be arranged as a symmetrical T-network or a symmetrical π-network.
- **T-Section:** Two series arms ($Z_1/2$), one shunt arm (Z_2).
- π-**Section:** One series arm (Z_1), two shunt arms ($2Z_2$).



T-Section



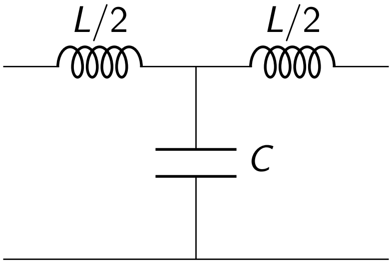
π-Section

Introduction to Constant-k LPF

- **Objective:** Pass low frequencies ($f < f_c$) and attenuate high frequencies ($f > f_c$).
- **Component Selection:**
  - **Series Element (Z_1):** Must block high frequencies $\rightarrow$ Inductor (L). Reactance $X_L = 2\pi fL$ increases with frequency.
  - **Shunt Element (Z_2):** Must short high frequencies to ground $\rightarrow$ Capacitor (C). Reactance $X_C = 1/(2\pi fC)$ decreases with frequency.

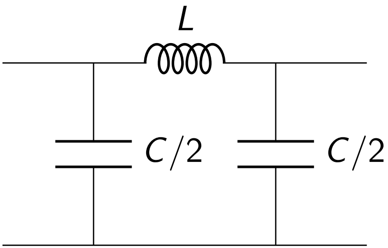
Constant-k LPF: T-Section

- Applying the LPF components to a symmetrical T-network:
- The total series inductance is L. For the T-section, this is split into two series inductors: $L/2$ each.
- The shunt capacitance remains a single capacitor: C.
- This forms an inductor-capacitor-inductor 'T' shape.



Constant-k LPF: Pi-Section

- Applying the LPF components to a symmetrical π-network:
- The series inductance remains a single inductor: L.
- The total shunt capacitance is C. Since we need $2Z_2$ for the shunt arms, the impedance is doubled. For a capacitor, doubling impedance means halving the capacitance: $C/2$ each.
- This forms a capacitor-inductor-capacitor 'π' shape.

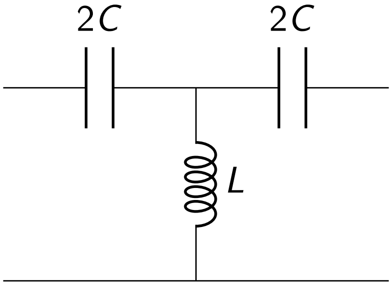


Introduction to Constant-k HPF

- **Objective:** Pass high frequencies ($f > f_c$) and attenuate low frequencies ($f < f_c$).
- **Component Selection:**
  - **Series Element (Z_1):** Must block low frequencies/DC $\rightarrow$ Capacitor (C).
  - **Shunt Element (Z_2):** Must short low frequencies to ground $\rightarrow$ Inductor (L).

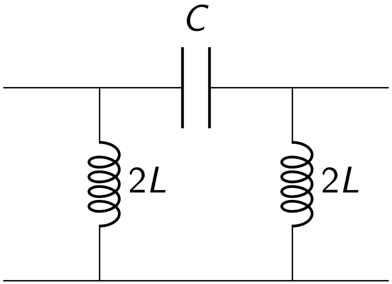
Constant-k HPF: T-Section

- Applying the HPF components to a symmetrical T-network:
- The total series capacitance is C. Splitting it to half impedance ($Z_1/2$) means doubling the capacitance value: $2C$ each.
- The shunt inductance remains a single inductor: L.



Constant-k HPF: Pi-Section

- Applying the HPF components to a symmetrical π-network:
- The series capacitance remains a single capacitor: C.
- The shunt inductance is L. For the π-section, we need $2Z_2$, which means the inductance value is doubled: $2L$ each.



Limitations of Constant-k Filters

- Despite their simplicity, prototype constant-k filters have significant drawbacks in practical applications:
- **Non-constant Characteristic Impedance:** The characteristic impedance (Z_0) varies drastically across the passband and approaches zero or infinity at the cut-off frequency. This causes severe impedance mismatch and reflections.
- **Slow Roll-off Rate:** The transition from the passband to the stopband is gradual. It does not provide sharp, immediate attenuation at f_c.

Quick Check

Question:

If the series element Z_1 of a constant-k filter is an inductor, what must the shunt element Z_2 be to satisfy the condition $Z_1 Z_2 = R_0^2$?

Options:

- A) A Resistor
- B) An Inductor
- C) A Capacitor
- D) A Transformer

Summary

- Constant-k filters are prototype filters defined by $Z_1 Z_2 = R_0^2$.
- Design Impedance (R_0): $R_0 = \sqrt{L/C}$.
- LPF: Series L, Shunt C. Passes low frequencies.
- HPF: Series C, Shunt L. Passes high frequencies.
- Structures: Can be implemented as T or π symmetrical sections.
- Limitations: Varying impedance in the passband and slow attenuation roll-off.

Next Lecture Preview

- **Topic: Design of Constant-k Low Pass Filter**
- In the next lecture, we will:
  - Derive the mathematical design equations for determining the exact values of L and C.
  - Use the cut-off frequency (f_c) and design impedance (R_0) to calculate component values.
  - Solve numerical examples for designing Constant-k LPF T and π networks.

Design of Constant-k Low Pass Filter

- Course Code: DI03011031
- Course: Electronics Circuit Network & Measurement
- Unit 2: Resonance Circuits and Passive Filters
- Lecture 17

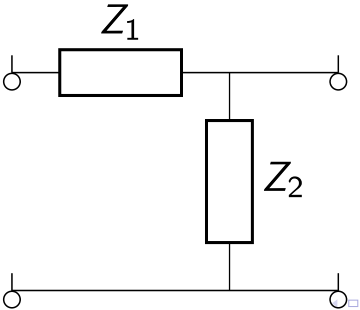
Constant-k
Low Pass Filter

Agenda for Today

- Concept of Constant-k Filters
- Characteristics of an Ideal vs. Practical Low Pass Filter
- T-Section and π-Section Networks for LPF
- Derivation of Design Equations
- Attenuation and Phase Characteristics
- Step-by-Step Design Procedure
- Numerical Example
- Summary & Next Steps

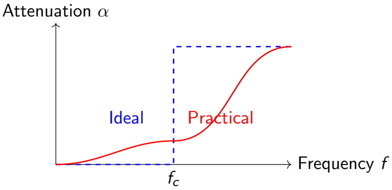
What is a Constant-k Filter?

- Also known as the Zobel network or prototype filter section.
- The fundamental property: The product of series impedance (Z_1) and shunt impedance (Z_2) is a constant real number.
- $$Z_1 \times Z_2 = k^2 = R_0^2$$
- Where:
  - Z_1 = Series arm impedance
  - Z_2 = Shunt arm impedance
  - k or R_0 = Nominal or Design Impedance (purely resistive and constant over the passband)



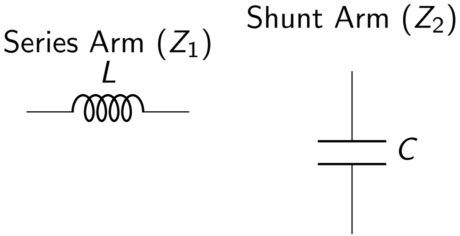
Low Pass Filter (LPF) Characteristics

- Function: Passes frequencies below a critical cut-off frequency (f_c) and attenuates frequencies above it.
- Passband: $0 \leq f < f_c$
- Stopband: $f > f_c$
- Ideal LPF: Zero attenuation in passband, infinite attenuation in stopband.
- Practical Constant-k LPF: Attenuation gradually increases after f_c.



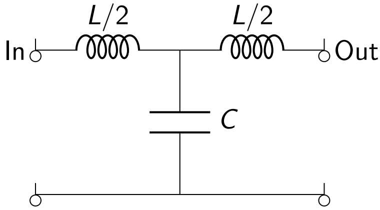
Components of a Constant-k LPF

- To pass low frequencies, the series arm must have low impedance at low frequencies and high impedance at high frequencies.
- $\rightarrow$ Series Arm (Z_1) uses an Inductor (L). ($X_L = j\omega L$)
- To shunt (bypass) high frequencies, the shunt arm must have high impedance at low frequencies and low impedance at high frequencies.
- $\rightarrow$ Shunt Arm (Z_2) uses a Capacitor (C). ($X_C = 1/j\omega C$)
- Checking the Constant-k condition:
- $$Z_1 Z_2 = (j\omega L)\left(\frac{1}{j\omega C}\right) = \frac{L}{C} = R_0^2$$



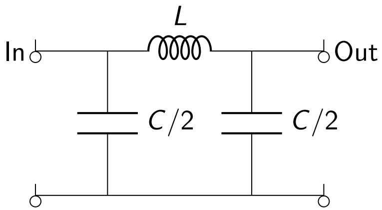
Constant-k LPF: T-Section

- A symmetrical T-section is formed by splitting the series impedance.
- Two series arms, each with impedance $Z_1/2$
- One shunt arm with impedance Z_2
- For an LPF:
  - Series elements: Inductors of value $L/2$
  - Shunt element: Capacitor of value C



Constant-k LPF: π-Section

- A symmetrical π-section is formed by splitting the shunt admittance (or doubling the shunt impedance).
- One series arm with impedance Z_1
- Two shunt arms, each with impedance $2Z_2$
- For an LPF:
  - Series element: Inductor of value L
  - Shunt elements: Capacitors of value $C/2$



Cut-off Frequency Derivation (Part 1)

- For a symmetrical network, the passband condition is:
-
$$-1 \leq \frac{Z_1}{4Z_2} \leq 0$$
- Substitute $Z_1 = j\omega L$ and $Z_2 = \frac{1}{j\omega C}$:
-
$$\frac{Z_1}{4Z_2} = \frac{j\omega L}{4(1/j\omega C)} = \frac{-\omega^2 LC}{4}$$
- Therefore:
-
$$-1 \leq \frac{-\omega^2 LC}{4} \leq 0$$

Cut-off Frequency Derivation (Part 2)

- The upper limit corresponds to the cut-off frequency ω_c:
- $$\frac{-\omega_c^2 LC}{4} = -1$$
- $$\omega_c^2 = \frac{4}{LC} \implies \omega_c = \frac{2}{\sqrt{LC}}$$
- Since $\omega_c = 2\pi f_c$:
- $$2\pi f_c = \frac{2}{\sqrt{LC}} \implies f_c = \frac{1}{\pi\sqrt{LC}}$$
- This is the cut-off frequency of the Constant-k LPF.

Design Equations for L and C

- We have two key equations:
- $R_0 = \sqrt{\frac{L}{C}}$
- $f_c = \frac{1}{\pi\sqrt{LC}}$
- Multiplying both equations:
- $$R_0 \times f_c = \sqrt{\frac{L}{C}} \times \frac{1}{\pi\sqrt{LC}} = \frac{1}{\pi C} \implies C = \frac{1}{\pi f_c R_0}$$
- Dividing (1) by (2):
- $$\frac{R_0}{f_c} = \pi L \implies L = \frac{R_0}{\pi f_c}$$

Summary of Design Formulas

Formulas for L and C

To design a Constant-k LPF given f_c and R_0:

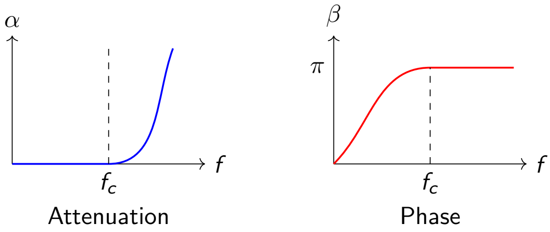
$$L = \frac{R_0}{\pi f_c} \quad \text{(Henries)}$$

$$C = \frac{1}{\pi f_c R_0} \quad \text{(Farads)}$$

- For T-Section: Use $L/2$, $L/2$, and C.
- For π-Section: Use $C/2$, $C/2$, and L.

Attenuation and Phase Characteristics

- Passband ($f < f_c$): Attenuation $\alpha = 0$ (ideally). Phase shift β increases from 0 to π radians.
- Stopband ($f > f_c$): Attenuation α increases rapidly. Phase shift $\beta = \pi$ (constant).
- Limitation: The attenuation near cut-off is not very sharp. This is a primary disadvantage of the Constant-k filter.



Numerical Example: Problem Statement

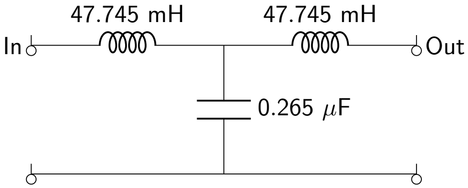
- Design a Constant-k T-section Low Pass Filter with the following specifications:
- Cut-off frequency, $f_c = 2$ kHz
- Design impedance, $R_0 = 600\ \Omega$
- Tasks:
  - Calculate the values of L and C.
  - Determine the component values for the T-section.
  - Draw the final circuit diagram.

Numerical Example: Solution (Step 1)

- Given: $f_c = 2000$ Hz, $R_0 = 600\ \Omega$
- Calculate Inductance (L):
- $$L = \frac{R_0}{\pi f_c} = \frac{600}{\pi \times 2000}$$
- $$L = 0.09549 \text{ H} = 95.49 \text{ mH}$$
- Calculate Capacitance (C):
- $$C = \frac{1}{\pi f_c R_0} = \frac{1}{\pi \times 2000 \times 600}$$
- $$C = 0.2652 \times 10^{-6} \text{ F} = 0.265\ \mu\text{F}$$

Numerical Example: Solution (Step 2)

- Determine T-Section Component Values:
- For a T-section, the series arms are $L/2$ and the shunt arm is C.
- $$L_1 = L_2 = \frac{L}{2} = \frac{95.49\ \text{mH}}{2} = 47.745\ \text{mH}$$
- $$C_1 = C = 0.265\ \mu\text{F}$$



Quick Check

- **Question 1:** What is the relationship between Z_1 and Z_2 in a constant-k filter?
- **Question 2:** In a constant-k π-section LPF, what is the value of the shunt capacitors if total capacitance is C?

Quick Check

- **Question 1:** What is the relationship between Z_1 and Z_2 in a constant-k filter?
- **Question 2:** In a constant-k π-section LPF, what is the value of the shunt capacitors if total capacitance is C?

- **Answers:**
- $Z_1 \times Z_2 = R_0^2$ (a constant)
- $C/2$ in each shunt branch.

Summary

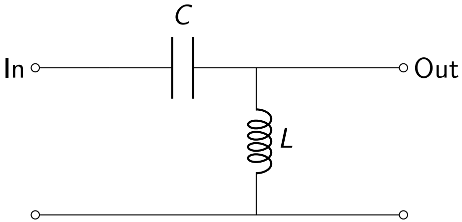
- Constant-k LPF: Uses an inductor in series and a capacitor in shunt.
- Designed for a specific cut-off frequency (f_c) and design impedance (R_0).
- Formulas: $L = R_0/(\pi f_c)$ and $C = 1/(\pi f_c R_0)$.
- Can be implemented as a T-section (split L) or π-section (split C).
- Suffers from a gradual attenuation roll-off rather than a sharp cut-off.

Preview of Next Lecture

- Next Topic: Design of Constant-k High Pass Filter
- Reversing the roles of inductors and capacitors.
- Derivation of HPF cut-off frequency and design equations.
- T and π configurations for High Pass Filters.

Design of Constant-k High Pass Filter

- Welcome to Lecture 18! Today we will focus on designing Constant-k High Pass Filters, covering their T and Pi sections, and understanding their mathematical design principles.

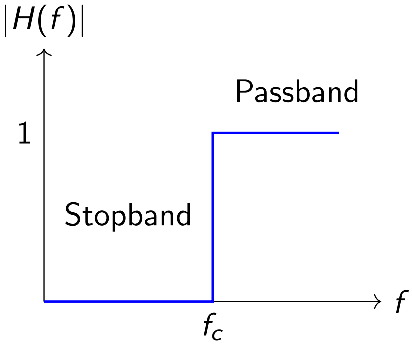


Agenda

- Introduction to High Pass Filters
- Constant-k HPF Fundamentals
- T-Section HPF Configuration
- Pi-Section (π-Section) HPF Configuration
- Characteristic Impedance (Z_0)
- Cut-off Frequency (f_c) Derivation
- Design Equations for L and C
- Attenuation and Phase Characteristics
- Design Example & Problem Solving
- Summary & Quick Quiz

Introduction to High Pass Filters

- A High Pass Filter (HPF) allows signals with frequencies higher than a certain threshold (cutoff frequency, f_c) to pass through and attenuates frequencies lower than f_c.
- Ideal HPF Characteristics:
  - Passband: $f > f_c$ (Zero attenuation)
  - Stopband: $f < f_c$ (Infinite attenuation)
- In a passive HPF, the series components must have low impedance at high frequencies, and shunt components must have low impedance at low frequencies.



Constant-k Filter Concept (Review)

- In a constant-k filter, the product of the series arm impedance (Z_1) and the shunt arm impedance (Z_2) is a real constant independent of frequency.
- Condition:

$$Z_1 \cdot Z_2 = R_0^2 = k^2$$

- Where:
  - R_0 is the characteristic or design impedance.
- For a High Pass Filter, the series arm is capacitive (Z_1) and the shunt arm is inductive (Z_2).

$$Z_1 = \frac{1}{j\omega C}, \quad Z_2 = j\omega L \implies Z_1 \cdot Z_2 = \frac{L}{C} = R_0^2$$

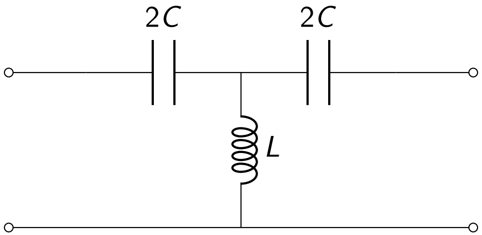
Impedances in Constant-k HPF

- For a constant-k High Pass Filter:
  - Series Arm Impedance (Z_1): A capacitor C is used.
  - $Z_1 = \frac{1}{j\omega C} = -j\frac{1}{\omega C}$
  - Shunt Arm Impedance (Z_2): An inductor L is used.
  - $Z_2 = j\omega L$
- Constant-k Verification:

$$Z_1 \cdot Z_2 = \left(\frac{1}{j\omega C}\right) \cdot (j\omega L) = \frac{L}{C} = R_0^2$$

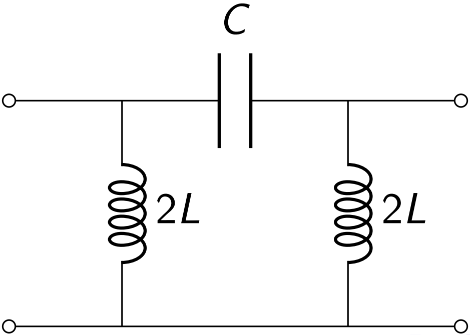
T-Section Configuration for HPF

- In a T-section network, the series arm is split into two halves, each with impedance $\frac{Z_1}{2}$, and the full shunt arm Z_2 is in the middle.
- Component Values:
  - Since Z_1 is capacitive (C), $\frac{Z_1}{2} = \frac{1}{j\omega(2C)}$.
  - Therefore, the series capacitors are $2C$ each.
  - The shunt inductor remains L.



Pi-Section (π-Section) Configuration for HPF

- In a Pi-section network, the full series arm Z_1 is kept intact, and the shunt arm is split into two parallel branches, each with impedance $2Z_2$.
- Component Values:
  - The series capacitor remains C.
  - Since Z_2 is inductive (L), $2Z_2 = j\omega(2L)$.
  - Therefore, the two shunt inductors are $2L$ each.



Cut-off Frequency (f_c) of HPF

- The cutoff frequency separates the passband from the stopband. It occurs when the reactance ratio reaches a critical point.
- For an HPF, the passband condition is:

$$-1 \leq \frac{Z_1}{4Z_2} \leq 0$$

- At the cutoff frequency (f_c):

$$\frac{Z_1}{4Z_2} = -1$$

- Substituting $Z_1 = \frac{1}{j\omega C}$ and $Z_2 = j\omega L$:

$$\frac{1}{j\omega_c C \cdot 4(j\omega_c L)} = -1 \implies \frac{1}{-4\omega_c^2 LC} = -1$$

Cut-off Frequency (f_c) Derivation (Cont.)

- Solving the equation from the previous slide:

$$4\omega_c^2 LC = 1 \implies \omega_c = \frac{1}{2\sqrt{LC}}$$

- Since $\omega_c = 2\pi f_c$, the cut-off frequency is:

$$f_c = \frac{1}{4\pi\sqrt{LC}}$$

- Note how this differs from the Low Pass Filter where $f_c = \frac{1}{\pi\sqrt{LC}}$.

$$f_c = \frac{1}{4\pi\sqrt{LC}}$$

Design Equations for L and C

- To design a filter, we are usually given the cut-off frequency (f_c) and the design impedance (R_0). We need to find L and C.
- We know two key relations:

$$R_0 = \sqrt{\frac{L}{C}}, \quad f_c = \frac{1}{4\pi\sqrt{LC}}$$

- Solving these simultaneously yields:

$$L = \frac{R_0}{4\pi f_c}$$

$$C = \frac{1}{4\pi f_c R_0}$$

Frequency Response Characteristics

- Attenuation (α):
  - Passband ($f > f_c$): Attenuation is zero (ideal). $\alpha = 0$.
  - Stopband ($f < f_c$): Attenuation increases as frequency decreases towards zero.
- Phase Shift (β):
  - Passband ($f > f_c$): Phase varies from $-\pi$ at f_c to 0 at infinite frequency.
  - Stopband ($f < f_c$): Phase shift is fixed at $-\pi$.

Design Example: Problem Statement

- Problem:
  - Design a constant-k High Pass Filter (both T and Pi sections) with a cut-off frequency of 10 kHz and a design impedance of $600\,\Omega$.
- Given:
  - $f_c = 10$ kHz $= 10,000$ Hz
  - $R_0 = 600\,\Omega$
- Objective:
  - Calculate L and C, then determine component values for T and Pi networks.

Design Example: Calculating L and C

- Calculate Inductance (L):

$$L = \frac{R_0}{4\pi f_c} = \frac{600}{4 \cdot \pi \cdot 10,000}$$

$$L \approx 4.77 \text{ mH}$$

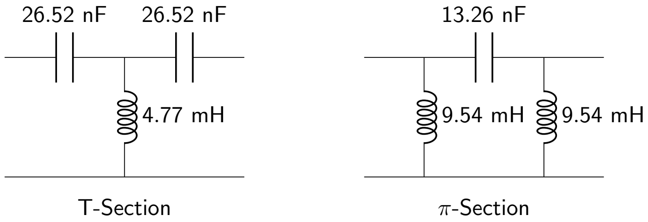
- Calculate Capacitance (C):

$$C = \frac{1}{4\pi f_c R_0} = \frac{1}{4 \cdot \pi \cdot 10,000 \cdot 600}$$

$$C \approx 13.26 \text{ nF}$$

Design Example: T and Pi Section Values

- T-Section Component Values:
  - Series Capacitors: $2C = 2 \times 13.26$ nF $= 26.52$ nF (Two required)
  - Shunt Inductor: $L = 4.77$ mH (One required)
- Pi-Section Component Values:
  - Series Capacitor: $C = 13.26$ nF (One required)
  - Shunt Inductors: $2L = 2 \times 4.77$ mH $= 9.54$ mH (Two required)



Quick Check

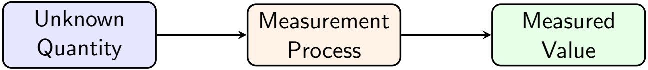
- Question 1: In a Pi-section HPF, what is the value of the shunt inductors if the base inductance is L?
  a) L/2
  b) L
  c) 2L
- Question 2: What is the formula for the cutoff frequency of a constant-k HPF?
  a) $1/(2\pi\sqrt{LC})$
  b) $1/(4\pi\sqrt{LC})$
  c) $1/(\pi\sqrt{LC})$

Summary & Next Lecture Preview

- Summary:
  - High pass filters attenuate low frequencies and pass high frequencies.
  - Constant-k HPF has series capacitors and shunt inductors.
  - Design formulas: $L = \frac{R_0}{4\pi f_c}$ and $C = \frac{1}{4\pi f_c R_0}$.
  - T-section uses $2C, 2C$ and L. Pi-section uses C and $2L, 2L$.
- Next Lecture:
  - We will begin Unit 3, moving away from filters to measurement characteristics, starting with "Characteristics of Instruments: Accuracy & Precision".

Introduction to Measurement

- Measurement is the process of determining the value of a physical variable.
- It involves comparing an unknown quantity with an accepted standard.
- The measured value is inherently an approximation of the true value.
- To evaluate measurement quality, we define specific static characteristics.



What is an Instrument?

Definition

An instrument is a device or mechanism used to determine the present value of a quantity under observation.

- Instruments translate a physical variable (like voltage, current, or temperature) into a human-readable or machine-processable format.
- **Examples in Electronics:**
  - Voltmeter (measures potential difference)
  - Ammeter (measures electric current)
  - Oscilloscope (displays signal waveforms)

The Concept of Error

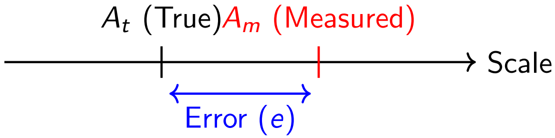
Error Definition

Error is the algebraic difference between the measured value and the true (or expected) value of the quantity.

Formula: $e = A_m - A_t$

- e: Absolute Error
- A_m: Measured Value
- A_t: True Value

- Error indicates the deviation of the reading from the actual value.
- No measurement is completely free of error.



Absolute vs. Relative Error

- **Absolute Error (e):** The direct difference, measured in the same units as the quantity (e.g., ± 0.1 V).
- **Relative Error (ϵ_r):** The ratio of the absolute error to the true value.

Formulas

$$\epsilon_r = \frac{|A_m - A_t|}{A_t}$$
$$\%\text{Error} = \epsilon_r \times 100\%$$

Case 1: Small True Value	**Case 2: Large True Value**
1V error on 10V	1V error on 1000V
$\epsilon_r = 1/10 = 10\%$	$\epsilon_r = 1/1000 = 0.1\%$

Accuracy

Definition

Accuracy is the degree of closeness or conformity of a measured value to the true (or accepted standard) value.

- It indicates how correct a measurement is.
- High accuracy means the measured value is very close to the true value (low error).
- Accuracy is often determined by the calibration of the instrument.

Expressing Accuracy

- Accuracy can be specified in several ways:
  - **Point Accuracy:** Accuracy at a specific point on the scale.
  - **Accuracy as % of True Value:** e.g., $\pm 0.5\%$ of true value.
  - **Accuracy as % of Full-Scale Deflection (FSD):** e.g., $\pm 1\%$ of FSD.
- **Note:** Error calculated based on FSD can lead to large relative errors when measuring small values on that scale.

Precision

Definition

Precision refers to the degree of agreement within a group of measurements or instruments.

- It is a measure of the consistency or repeatability of measurements.
- High precision means that repeated measurements of the same quantity yield very similar results.
- **Precision does not guarantee accuracy.**

Significant Figures in Precision

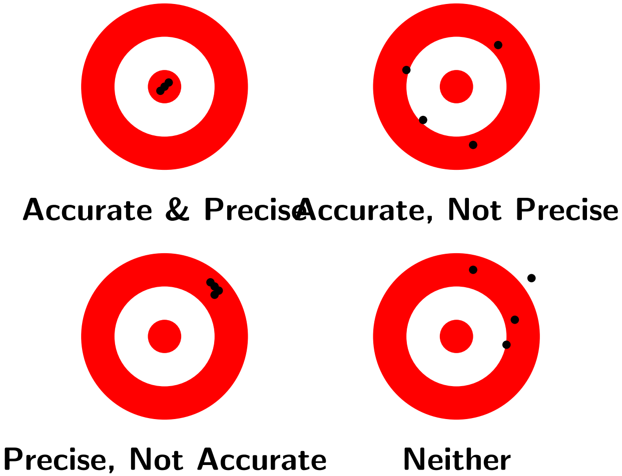
- Precision is often mathematically related to the number of significant figures in a measurement.
- More significant figures imply higher precision.

Example

- Reading 1: 10.1 V (3 significant figures)
- Reading 2: 10.134 V (5 significant figures)

- Reading 2 is more precise, but its accuracy still depends on calibration against a true standard.

Accuracy vs. Precision (Target Analogy)



Repeatability

Definition

The closeness of agreement among repeated measurements of the same variable under the same operating conditions.

- **Conditions for Repeatability:**
  - Same measurement procedure
  - Same observer
  - Same measuring instrument used under the same conditions
  - Same location
  - Repetition over a short period of time

Reproducibility

Definition

The closeness of agreement among repeated measurements of the same variable under changed operating conditions.

- **Conditions for Reproducibility:**
  - Different measurement principles or methods
  - Different observers
  - Different measuring instruments
  - Different locations or environmental conditions
  - Measurements taken over a long period of time

Repeatability vs. Reproducibility

Factor	**Repeatability**	**Reproducibility**
Time Span	Short-term	Long-term / Global
User / Observer	Same	Different
Equipment	Same	Different
Environment	Same	Different
Key Question	"Does it drift right now?"	"Can it be verified independently?"

Question: An instrument consistently reads exactly 2.5V higher than the actual voltage. This instrument is:

A) Accurate but not precise

B) **Precise but not accurate** ✓

C) Both accurate and precise

D) Neither accurate nor precise

Summary & Key Takeaways

- **Instruments** quantify physical variables for observation.
- **Error** is the difference between measured and true values.
- **Accuracy** indicates conformity to the true value (lack of error).
- **Precision** indicates consistency of repeated measurements (lack of scatter).
- **Repeatability** is consistency under identical, short-term conditions.
- **Reproducibility** is consistency under changed or long-term conditions.

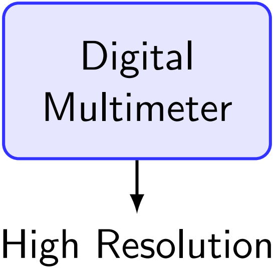
Preview for Next Lecture

Next Topic: Characteristics of Instruments: Sensitivity & Resolution

- What we will cover:
  - Sensitivity and scale factor
  - Resolution and threshold
  - Linearity and Response time
- **Preparation:** Review the concepts of input and output relationships in mathematical functions, as this will aid in understanding linearity.

Welcome to Lecture 20

- Course: Electronics Circuit Network & Measurement (DI03011031)
- Unit 3: Characteristic of Measurement and Bridges
- Topic: Characteristics of Instruments: Sensitivity & Resolution
- Instructor: GTU Professor

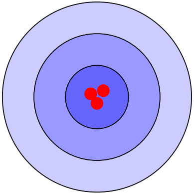


Agenda for Today

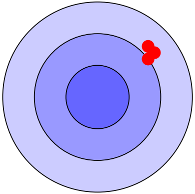
- Recap of Previous Lecture
- Concept of Sensitivity
- Deflection Factor (Inverse Sensitivity)
- Mathematical Representation of Sensitivity
- Concept of Resolution (Discrimination)
- Threshold vs. Resolution
- Relationship Between Sensitivity and Resolution
- Real-world Examples and Practical Implications
- Summary and Q&A

Recap: Accuracy vs. Precision

- Accuracy: How close a measured value is to the true or accepted value.
- Precision: How close repeated measurements are to each other (repeatability/reproducibility).
- An instrument can be precise but inaccurate, or accurate but imprecise.
- Today, we introduce two more crucial metrics that define an instrument's capability: Sensitivity and Resolution.



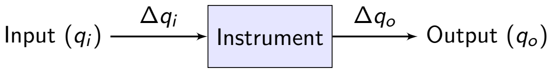
Accurate & Precise



Precise, Not Accurate

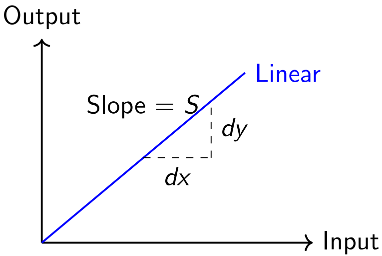
Introduction to Sensitivity

- Definition: Sensitivity is the ratio of the change in the output of an instrument to a change in the value of the quantity being measured.
- It denotes the smallest change in the measured variable to which the instrument responds.
- Formula: $Sensitivity(K) = \frac{\Delta q_o}{\Delta q_i}$
- Where Δq_o = Change in output signal, Δq_i = Change in input signal.



Static Sensitivity Graph

- For a linear instrument, sensitivity is constant and represents the slope of the calibration curve.
- Slope $S = \frac{dy}{dx}$
- For non-linear instruments, sensitivity varies depending on the input level.
- Higher slope = Higher Sensitivity.

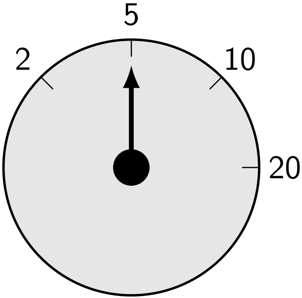


Units of Sensitivity

- The units of sensitivity depend entirely on the input and output quantities.
- Example 1: A galvanometer's sensitivity might be expressed in millimeters per microampere ($mm/\mu A$).
- Example 2: A temperature sensor (thermocouple) might have sensitivity in millivolts per degree Celsius ($mV/^\circ C$).
- Example 3: A Bourdon tube pressure gauge: degrees of rotation per Pascal ($^\circ/Pa$).

Instrument	**Input**	**Output**	**Sensitivity Unit**
Galvanometer	Current (μA)	Deflection (mm)	$mm/\mu A$
Thermocouple	Temperature ($^\circ C$)	Voltage (mV)	$mV/^\circ C$
Bourdon Gauge	Pressure (Pa)	Rotation ($^\circ$)	$^\circ/Pa$

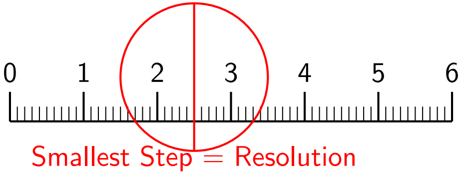
- The inverse of sensitivity is called the Deflection Factor or Scale Factor.
- Definition: It is the ratio of the change in input to the corresponding change in output.
- Formula: $DeflectionFactor = \frac{1}{Sensitivity} = \frac{\Delta q_i}{\Delta q_o}$
- Often used in oscilloscopes (e.g., Volts/div).



Volts/div Knob

Understanding Resolution

- Definition: Resolution (or Discrimination) is the smallest measurable input change that can be detected and indicated by the instrument.
- It is the smallest increment of the quantity being measured that causes a noticeable change in the output.
- Expressed as an absolute value or a percentage of full-scale deflection (FSD).

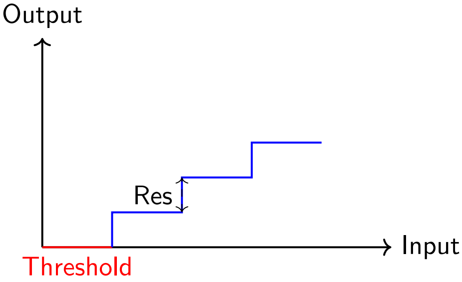


Resolution in Digital Instruments

- In digital instruments, resolution is strictly determined by the number of digits in the display.
- A 3 1/2 digit multimeter has a maximum display of 1999.
- If the full scale is 200V, the smallest step is $200V/2000 = 0.1V$.
- Resolution = Smallest significant digit change.

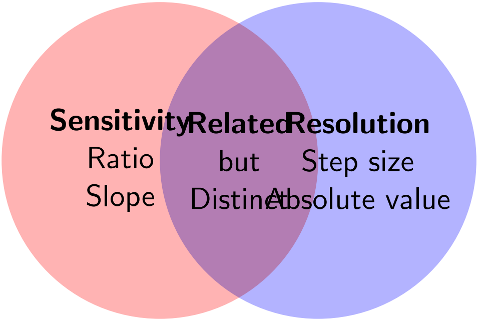
Resolution vs. Threshold

- Threshold: The minimum input value required to cause a detectable initial change in the output from a zero state.
- Resolution: The smallest measurable change *anywhere* along the scale, not just starting from zero.
- If an instrument starts from zero, the first noticeable movement defines its threshold. Subsequent smallest noticeable steps define resolution.



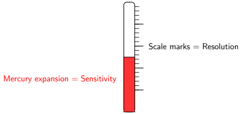
Sensitivity vs. Resolution: The Difference

- Sensitivity is a parameter representing a *relationship* (slope). High sensitivity means a small input gives a large output.
- Resolution is a *magnitude* (an absolute value). High resolution means a very small change in input can be detected.
- An instrument can be highly sensitive but have poor resolution if the indicating scale is coarse or noisy.



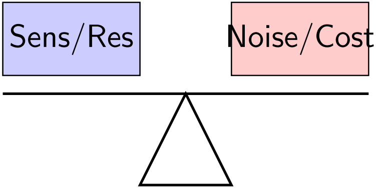
Practical Example: Thermometer

- Consider a mercury thermometer.
- Sensitivity: Determined by the capillary tube diameter and bulb volume. (How far the mercury travels per degree).
- Resolution: Determined by the printed scale marks. If marked every 0.1°C, resolution is 0.1°C.
- To improve resolution, we can make the markings finer, but only if the sensitivity (mercury travel) is high enough to differentiate.



Impact on Measurement Quality

- High sensitivity is desirable, but too much can lead to instability (noise amplification).
- High resolution is necessary for precision measurements, but implies higher cost and complexity.
- System design requires balancing sensitivity, resolution, range, and noise immunity based on application requirements.



Quick Check

Self-Assessment

- **Question 1:** If a voltmeter output changes by 2V for a 10mV input change, what is its sensitivity?
- **Question 2:** True or False? The deflection factor is the exact same concept as sensitivity.
- **Question 3:** A digital scale reads 100.00 grams. What is its likely resolution?

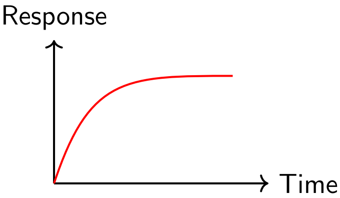
Summary

Key Takeaways

- **Sensitivity:** Ratio of change in output to change in input ($\Delta q_o / \Delta q_i$).
- **Deflection Factor:** Inverse of sensitivity.
- **Resolution:** The smallest input increment that produces a measurable output change.
- **Threshold:** The minimum input to start reading from zero.
- Both are fundamental parameters for selecting the right measuring instrument.

Next Lecture Preview

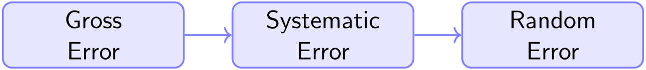
- Topic: Linearity and Response Time
- We will explore what it means for an instrument to be linear.
- We will analyze dynamic characteristics: how fast an instrument responds to changing inputs (Response Time, Lag, and Dynamic Error).
- Please review the concepts of static vs. dynamic systems.



Dynamic Characteristics Preview

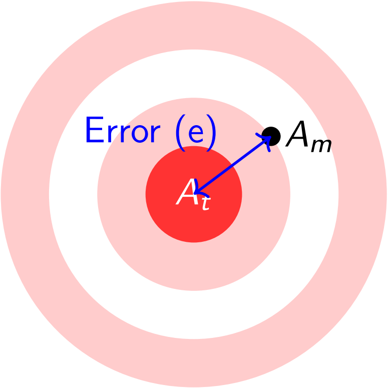
Agenda

- Introduction to Measurement Errors
- Broad Classification of Errors
- Gross Errors and Their Prevention
- Systematic Errors: Instrumental, Environmental, Observational
- Random (Residual) Errors
- Summary and Q&A



What is an Error in Measurement?

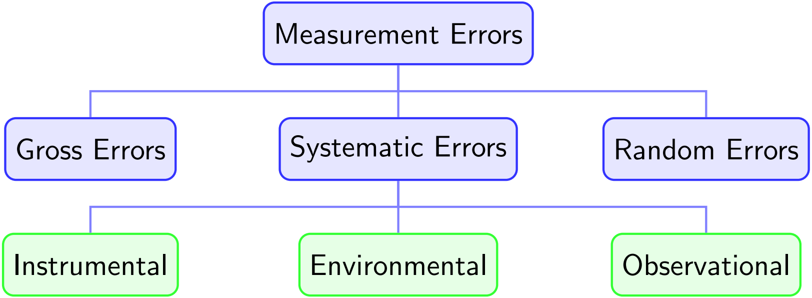
- **Error Definition:** The deviation of the measured value from the true (or expected) value of the quantity being measured.
- Mathematically: Error (e) $= A_m - A_t$
- Where A_m is the measured value and A_t is the true value.
- No measurement is perfect; some degree of error is always present.
- The goal of measurement science is to identify, quantify, and minimize these errors.



Broad Classification of Errors

Measurement errors are generally classified into three major categories:

- **Gross Errors:** Caused by human mistakes in reading or using instruments.
- **Systematic Errors:** Occur due to instrument shortcomings, environmental changes, or flawed observation methods.
- **Random Errors:** Unpredictable fluctuations that occur even when gross and systematic errors are removed.



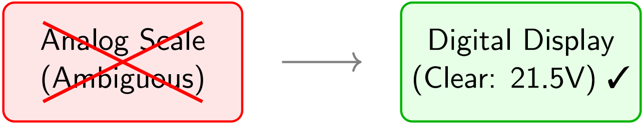
Gross Errors

- **Nature:** Strictly human-induced mistakes.
- **Common Causes:**
  - Misreading the instrument scale (e.g., reading 31.5 instead of 21.5).
  - Incorrect recording of data.
  - Improper use of instruments (e.g., applying wrong voltage range).
  - Incorrect wiring or setup of the measurement circuit.
- **Characteristics:** They cannot be treated mathematically or statistically because they are unpredictable in magnitude and sign.

Prevention of Gross Errors

Since gross errors are human mistakes, they can be avoided by exercising care.

- **Preventative Measures:**
  - Take multiple readings instead of relying on a single measurement.
  - Have a second person verify the readings.
  - Use digital instruments to eliminate scale reading ambiguities.
  - Follow standard operating procedures (SOPs) strictly.
  - Ensure proper training of the personnel taking measurements.

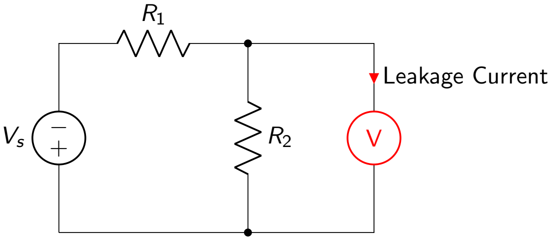


Systematic Errors

- **Definition:** Errors that remain constant or change according to a specific law on repeated measurements.
- Unlike gross errors, these are reproducible and predictable.
- They skew the measurement consistently in one direction (either always positive or always negative).
- **Three Sub-categories:**
  - Instrumental Errors
  - Environmental Errors
  - Observational Errors

Systematic Errors: Instrumental Errors

- Causes inherent to the measuring instrument itself:
- **Inherent shortcomings:** Friction in bearings of moving coil instruments, irregular spring tension.
- **Misuse of instrument:** Using an instrument beyond its specified bandwidth or overloading it.
- **Loading Effect:** When the instrument alters the circuit conditions (e.g., a voltmeter with low internal resistance drawing significant current).



Minimization of Instrumental Errors

Instrumental errors can be significantly reduced by:

- **Calibration:** Regularly comparing the instrument against a standard reference.
- **Applying Correction Factors:** Using known error curves supplied by the manufacturer to correct readings mathematically.
- **Proper Selection:** Choosing the right instrument for the job (e.g., using a high-impedance voltmeter to avoid loading effects).
- **Maintenance:** Routine servicing to reduce mechanical friction or wear.

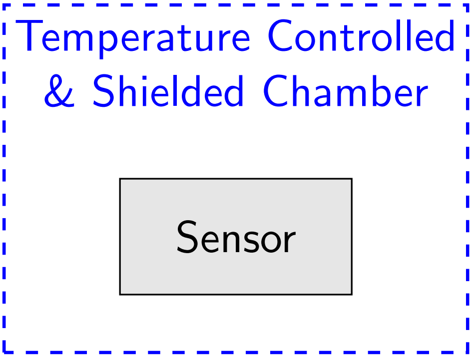
Systematic Errors: Environmental Errors

- **Definition:** Errors caused by changes in the external conditions surrounding the instrument.
- **Influencing Factors:**
  - **Temperature:** Alters resistance of coils, affects spring stiffness.
  - **Humidity:** Can cause leakage paths or affect capacitance.
  - **Pressure & Altitude:** Impacts certain transducers.
  - **External Magnetic/Electrostatic Fields:** Can induce stray voltages or deflect moving coils.

Minimization of Environmental Errors

To mitigate environmental effects:

- **Air Conditioning / Enclosures:** Maintain constant temperature and humidity.
- **Magnetic Shielding:** Use high-permeability materials (like mu-metal) to shield sensitive components from stray fields.
- **Compensation Circuits:** Use thermistors or RTDs in the circuit to automatically counteract temperature drift.
- **Zero Setting:** Re-zeroing the instrument in the actual operating environment before taking measurements.



Systematic Errors: Observational Errors

- **Definition:** Errors introduced by the observer's viewing angle or habits.
- **Parallax Error:**
  - Occurs when the pointer is not viewed exactly perpendicular to the scale.
  - Extremely common in analog instruments with a gap between the pointer and the scale.
- **Mitigation:**
  - Use meters with anti-parallax mirrors (align pointer with its reflection).
  - Shift to digital readouts to completely eliminate parallax.

Random (Residual) Errors

- **Definition:** Errors that remain even after gross and systematic errors have been completely eliminated.
- **Causes:** Unknown, fluctuating, and unpredictable small variations in the measurement system or environment (e.g., thermal noise in electronics).
- **Characteristics:**
  - They occur randomly in both positive and negative directions.
  - They follow the laws of probability (Gaussian/Normal distribution).
- **Handling:** Best minimized by taking a large number of readings and calculating the statistical mean (average).

Quick Check

- **Question 1:** An operator reads 10.5V instead of 10.05V on a meter. Which type of error is this?
  (Answer: Gross Error)

- **Question 2:** A measuring instrument constantly reads 0.2A higher than the actual current due to a stiff spring. Which error category does this fall under?
  (Answer: Systematic - Instrumental Error)

- **Question 3:** How can we reduce random errors?
  (Answer: By taking multiple readings and calculating their statistical mean.)

Summary

Error Type	Primary Cause	Mitigation Strategy
Gross Errors	Human mistakes (misreading, wrong connections)	Care, verification, training, digital meters
Systematic (Inst.)	Flaws in tool or loading effects	Calibration, correct instrument choice
Systematic (Env.)	Changes in temp, humidity, magnetic fields	Shielding, compensation, climate control
Systematic (Obs.)	Viewing angle (Parallax error)	Anti-parallax mirrors, digital readouts
Random Errors	Unpredictable fluctuations/noise	Statistical averaging (taking mean of multiple readings)

Next Lecture Preview

- **Topic: Limiting Errors & Calculation**
- **What we will cover:**
  - What are Guarantee Errors (Limiting Errors)?
  - How manufacturers specify the accuracy of instruments.
  - Calculating the limiting error for combinations of quantities (Addition, Subtraction, Multiplication).
  - Practical numerical examples on error calculation.

Electronics Circuit Network & Measurement

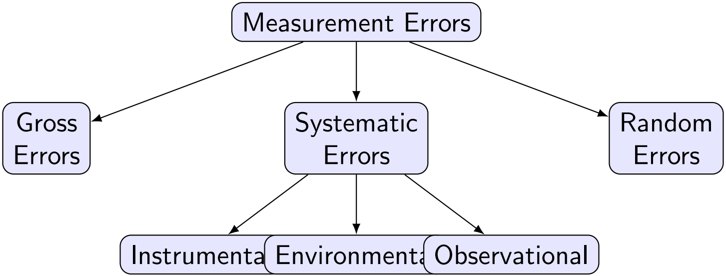
Unit 3: Characteristic of Measurement and Bridges

Lecture 22: Limiting Errors & Calculation

Agenda

- Recap of Measurement Errors
- Concept of Limiting Errors (Guarantee Errors)
- Absolute vs. Relative Limiting Errors
- Impact of Scale Reading on Accuracy
- Propagation of Errors in Composite Calculations
- Mathematical Formulations for Error Propagation
- Numerical Examples and Problem Solving

Recap: Types of Measurement Errors



- **Gross Errors:** Human mistakes in reading, recording, or calculating.
- **Systematic Errors:** Predictable and consistent errors (e.g., friction, temperature).
- **Random Errors:** Unpredictable fluctuations remaining after removing other errors.

Introduction to Limiting Errors

- Manufacturers specify instrument accuracy within a certain percentage of the full-scale reading.
- These defined limits of deviation from the specified value are called **Limiting Errors** or **Guarantee Errors**.
- The true value of a quantity is guaranteed to be within these limits.

$$\text{Actual Value } (A_a) = \text{Measured Value } (A_s) \pm \text{Limiting Error } (\delta A)$$

- **Example:** A resistor specified as $100\,\Omega \pm 5\%$ has a guaranteed value between $95\,\Omega$ and $105\,\Omega$.

Absolute vs. Relative Limiting Error

Absolute Limiting Error (δA)

The actual magnitude of the error in units of measurement.

$$\delta A = A_a - A_s$$

Relative Limiting Error (ε_r)

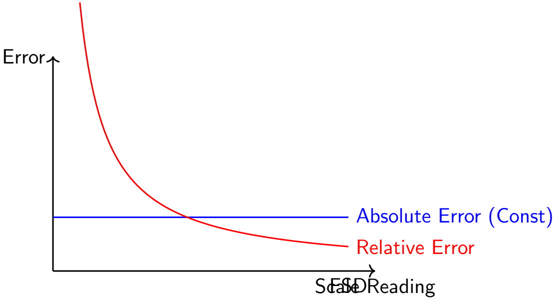
The ratio of absolute error to the specified nominal value.

$$\varepsilon_r = \frac{\delta A}{A_s}$$

- **Percentage Limiting Error ($\%\varepsilon_r$):** $\%\varepsilon_r = \left(\frac{\delta A}{A_s}\right) \times 100$
- Thus, Absolute Error $\delta A = \varepsilon_r \times A_s$.

Impact of Scale Reading on Relative Error

- Accuracy is usually guaranteed as a percentage of Full Scale Deflection (FSD).
- Absolute error remains constant throughout the scale.
- Consequently, the **Relative Error increases** as the measured value decreases from full scale.



Conclusion: Always select an instrument range such that the reading is as close to the full scale as possible.

Problem: A 0-150 V voltmeter has a guaranteed accuracy of 1% of full scale reading. Calculate the percentage limiting error when measuring: 1) 150 V 2) 75 V

Solution:

- Absolute error $(\delta A) = 1\%$ of 150 V $= 0.01 \times 150 = 1.5$ V

At 150 V Reading

$$\% \text{ Error} = \left(\frac{1.5}{150}\right) \times 100$$
$$= 1\%$$

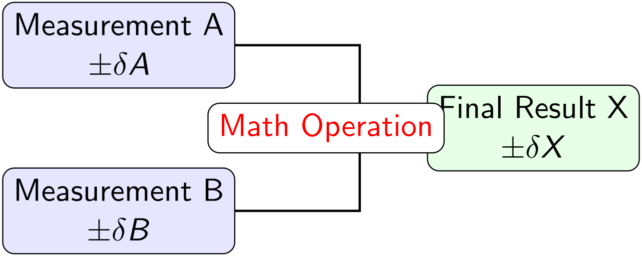
At 75 V Reading

$$\% \text{ Error} = \left(\frac{1.5}{75}\right) \times 100$$
$$= 2\%$$

Observation: The percentage error doubles at half scale.

Propagation of Errors (Composite Errors)

- Often, a final result is computed from multiple independent measurements.
- Each measurement has its own limiting error, which propagates to the final result.
- We evaluate the **worst-case scenario** to find the limiting error of the computed result.



- **Operations to consider:** Addition, Subtraction, Multiplication, Division, Powers.

Error Propagation: Addition and Subtraction

For Addition

$X = x_1 + x_2 + \cdots + x_n$

Absolute Error:
$\delta X = \pm(\delta x_1 + \delta x_2 + \cdots + \delta x_n)$

For Subtraction

$X = x_1 - x_2$

Absolute Error:
$\delta X = \pm(\delta x_1 + \delta x_2)$

Key Rule

In addition and subtraction, the **absolute errors** of the individual quantities are directly **added** to yield the total absolute limiting error. (Even for subtraction, errors add up for worst-case limits).

Error Propagation: Multiplication

- **For Multiplication:** $X = x_1 \times x_2$
- Let x_1 have relative error ε_{r1}, and x_2 have relative error ε_{r2}.

Relative Error of X: $\varepsilon_{rX} = \pm(\varepsilon_{r1} + \varepsilon_{r2})$

Percentage Error: $\%\varepsilon_{rX} = \pm(\%\varepsilon_{r1} + \%\varepsilon_{r2})$

Key Rule

In multiplication, the **relative (or percentage) errors** of the individual quantities are **added** to yield the relative limiting error of the result.

Error Propagation: Division

- **For Division:** $X = x_1/x_2$
- Similar to multiplication, we consider the worst-case relative errors.

Multiplication

% Error: $\varepsilon_{rX} = \pm(\varepsilon_{r1} + \varepsilon_{r2})$

Division

% Error: $\varepsilon_{rX} = \pm(\varepsilon_{r1} + \varepsilon_{r2})$

Key Rule

Just like multiplication, in division, the **percentage limiting errors** of the quantities are **added** to determine the final percentage error.

Error Propagation: Powers and Roots

- For a quantity raised to a power: $X = x^n$

$$\%\varepsilon_{rX} = \pm(n \times \%\varepsilon_{rx})$$

- For a root: $X = x^{1/n}$

$$\%\varepsilon_{rX} = \pm\left(\frac{1}{n} \times \%\varepsilon_{rx}\right)$$

Key Rule

The percentage error of a quantity raised to a power is **multiplied** by that power. This emphasizes why high-power terms are highly sensitive to errors.

Numerical Example: Series Resistors

Problem: Three resistors are connected in series: $R_1 = 50\,\Omega \pm 5\%$, $R_2 = 100\,\Omega \pm 2\%$, $R_3 = 150\,\Omega \pm 1\%$. Find total resistance and its limiting error.

Solution: Convert % errors to absolute errors:

- $\delta R_1 = 5\%$ of $50 = 2.5\,\Omega$
- $\delta R_2 = 2\%$ of $100 = 2.0\,\Omega$
- $\delta R_3 = 1\%$ of $150 = 1.5\,\Omega$

Total Resistance $R = 50 + 100 + 150 = \mathbf{300\,\Omega}$
Total Absolute Error $\delta R = \pm(2.5 + 2.0 + 1.5) = \pm\mathbf{6.0\,\Omega}$

- **Total Percentage Error** $= \left(\frac{6.0}{300}\right) \times 100 = \pm\mathbf{2\%}$

Numerical Example: Power Calculation

Problem: Power $P = I^2 R$. Measured Current $I = 10\,\text{A} \pm 2\%$, Measured Resistance $R = 50\,\Omega \pm 5\%$. Find percentage limiting error in Power.

Solution: Using propagation rules for powers and multiplication:

$$\%\varepsilon_{rP} = \pm\left[(2 \times \%\varepsilon_{rI}) + \%\varepsilon_{rR}\right]$$
$$\%\varepsilon_{rP} = \pm\left[(2 \times 2\%) + 5\%\right]$$
$$\%\varepsilon_{rP} = \pm\left[4\% + 5\%\right] = \pm\mathbf{9}\%$$

Power $P = 10^2 \times 50 = 5000\,\text{W}$. Error $= \pm 9\%$

Minimizing Error Propagation in Practice

- Choose high-accuracy components for terms raised to powers.
- Measure as close to full-scale as possible to reduce relative errors.
- **Avoid subtraction** of quantities that are nearly equal, as this drastically increases relative error.

✗ **Example:** If $X = 100 \pm 1$ and $Y = 98 \pm 1$,
$Z = X - Y = 2 \pm 2$.
The relative error is $\pm 100\%$!

Quick Knowledge Check

Question 1

If a 100V range voltmeter has 1% FSD error, what is the absolute error at 20V reading?

A) 0.2V B) 1V C) 5V D) 20V

Question 2

When calculating $X = (A \times B)/C$, how do we find the total percentage error?

A) Multiply errors

B) Add absolute errors

C) Add percentage errors of A, B, and C

Summary

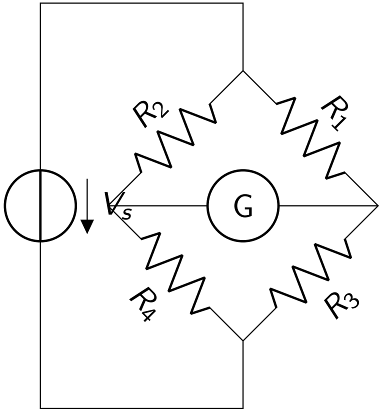
- **Limiting Errors:** Maximum bounds of deviation specified by manufacturers.
- **Scale Reading:** Absolute error is constant; relative error increases at lower readings.

Mathematical Operation	Error Type to Add
Addition & Subtraction	Absolute Errors (δx)
Multiplication & Division	Percentage (Relative) Errors ($\%\varepsilon$)
Powers (x^n)	Multiply percentage error by n

Proper instrument range selection is critical for accurate measurements.

Next Lecture Preview

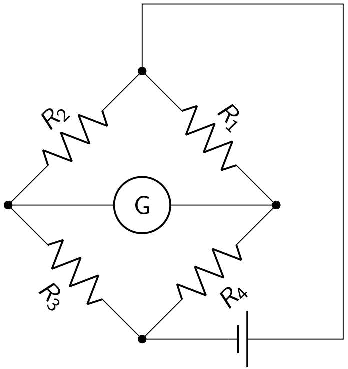
- **Topic:** Intro to AC and DC Bridges
- We will transition from general instrument characteristics to specific measurement circuits.
- Working principles of Bridges.
- Applications of Bridges in precise electrical parameter measurement.



Intro to AC and DC Bridges

Principles, Working, and Applications

- Welcome to Lecture 23. Today we begin our exploration of bridge circuits, which are foundational tools for precise electrical measurements.
- We will cover the basic topology of both DC and AC bridges.
- We will derive their fundamental balance equations.
- We will examine their primary applications in measuring resistance, inductance, and capacitance.

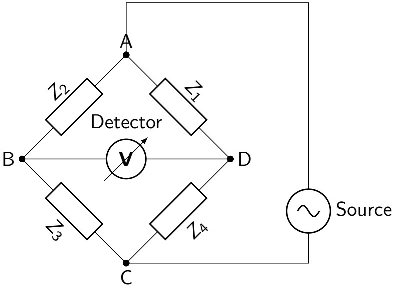


Agenda

- Introduction to Bridge Circuits
- The DC Bridge: General Form and Balance Condition
- Detectors and Sources in DC Bridges
- Applications of DC Bridges
- The AC Bridge: General Form
- AC Bridge Balance: Magnitude and Phase Criteria
- Detectors and Sources in AC Bridges
- Applications of AC Bridges
- Summary and Quick Check

Introduction to Bridge Circuits

- A Bridge Circuit is a topology consisting of four branches (arms) connected in a quadrilateral fashion.
- **Nodes:** Four nodes where arms meet.
- **Excitation:** A source (voltage or current) is applied across one pair of opposite nodes.
- **Detection:** A null detector (like a galvanometer) is connected across the other pair of opposite nodes.
- **Purpose:** Used primarily for the precise measurement of component values (R, L, C) by achieving a 'balanced' state where the detector reads zero.

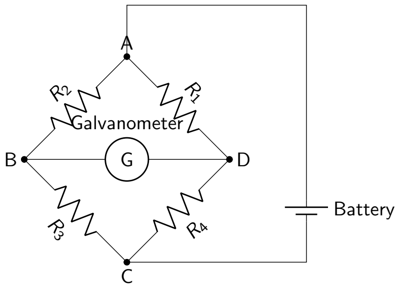


Why Use Bridge Circuits?

- Why not just use an ohmmeter or LCR meter?
- **High Accuracy:** Null-point measurement is independent of the detector's calibration. The accuracy depends only on the standard components in the bridge arms.
- **Sensitivity:** Bridge circuits can detect minute changes in impedance, making them ideal for sensor interfacing (e.g., strain gauges, RTDs).
- **Immunity to Source Variations:** At the balance point, fluctuations in the supply voltage do not affect the balance equation or the measured value.

The DC Bridge: General Form

- The simplest bridge is the DC bridge, consisting of four purely resistive arms (R_1, R_2, R_3, R_4).
- **Source:** A DC power supply (battery) is connected between nodes A and C.
- **Detector:** A sensitive Galvanometer (G) is connected between nodes B and D.
- **Operation:** The values of the known resistors are adjusted until the potential at node B equals the potential at node D ($V_B = V_D$).
- When $V_B = V_D$, no current flows through the galvanometer ($I_g = 0$). This is the Null Point.



DC Bridge: Balance Condition Derivation

- At balance, $I_g = 0$, meaning $V_B = V_D$. Therefore, the voltage drop from A to B equals the drop from A to D: $V_{AB} = V_{AD}$.
- Let I_1 flow through R_1 and I_2 flow through R_2.
- Since $I_g = 0$, I_1 also flows through R_3, and I_2 flows through R_4.
- $V_{AB} = I_1 R_1$ and $V_{AD} = I_2 R_2 \implies I_1 R_1 = I_2 R_2$ — (Eq 1)
- Also, $V_{BC} = V_{DC} \implies I_1 R_3 = I_2 R_4$ — (Eq 2)
- Dividing Eq 1 by Eq 2:

$$\frac{I_1 R_1}{I_1 R_3} = \frac{I_2 R_2}{I_2 R_4} \implies \frac{R_1}{R_3} = \frac{R_2}{R_4}$$

Sources and Detectors in DC Bridges

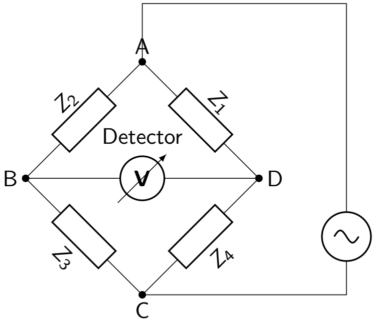
- **Sources:**
  - Typically highly stable DC batteries or regulated DC power supplies.
  - Voltage must be sufficient to provide good sensitivity but low enough to avoid self-heating of the resistors (which changes their resistance).
- **Detectors:**
  - D'Arsonval Galvanometer: Most common. Highly sensitive to small DC currents.
  - Digital Multimeters (DMMs): In microvolt mode, can be used in modern setups.
  - Electronic Null Detectors: Use chopper-stabilized amplifiers for extreme sensitivity.

Applications of DC Bridges

- DC Bridges are primarily used for measuring purely resistive components.
- **Measurement of Unknown Resistance:** Using standard decades to find an unknown R_x.
- **Cable Fault Location:** Murray loop and Varley loop tests use bridge principles to find shorts or grounds in underground cables.
- **Sensor Interfacing:**
  - **Strain Gauges:** Measure minute resistance changes due to mechanical stress.
  - **RTDs (Resistance Temperature Detectors):** Measure temperature variations as resistance changes.
  - Bridge configuration converts these small ΔR changes into measurable voltage variations.

The AC Bridge: General Form

- AC Bridges extend the DC bridge concept to measure complex impedances ($Z = R + jX$).
- The four arms consist of impedances Z_1, Z_2, Z_3, Z_4, which can be combinations of R, L, and C.
- **Source:** An AC voltage source (usually a sine wave oscillator) applied across opposite nodes.
- **Detector:** An AC null detector applied across the other nodes.
- **Operation:** Just like the DC bridge, the goal is to achieve zero potential difference across the detector nodes.



AC Bridge: Balance Equations

- Using the same logic as the DC bridge, at balance, the voltage drops must be equal.
- $V_{AB} = V_{AD}$ and $V_{BC} = V_{DC}$
- $I_1 Z_1 = I_2 Z_2$ and $I_1 Z_3 = I_2 Z_4$
- Dividing the equations yields the general balance condition for an AC bridge:

$$Z_1 Z_4 = Z_2 Z_3$$

- In polar form, $Z = |Z|\angle\theta$. Substituting this into the balance equation:

$$(|Z_1|\angle\theta_1)(|Z_4|\angle\theta_4) = (|Z_2|\angle\theta_2)(|Z_3|\angle\theta_3)$$

Magnitude and Phase Balance Criteria

- From the polar equation, we can equate the magnitudes and the phase angles separately. An AC bridge requires TWO conditions to be met for balance:

Conditions for AC Bridge Balance

Magnitude Condition:

$$|Z_1| \cdot |Z_4| = |Z_2| \cdot |Z_3|$$

Phase Angle Condition:

$$\theta_1 + \theta_4 = \theta_2 + \theta_3$$

- **Crucial Note:** Achieving balance requires alternating adjustments of at least two variable components (typically one for resistive balance, one for reactive balance).

Detectors in AC Bridges

- Because the excitation is AC, DC galvanometers cannot be used. Common AC detectors include:
- **Headphones:** Used at audio frequencies (250 Hz to 4 kHz). The human ear is very sensitive to the null point (silence).
- **Vibration Galvanometers:** Tuned mechanical devices for low frequencies (power line freq, 50/60 Hz up to 1 kHz).
- **Tunable Amplifier Detectors:** Electronic amplifiers tuned to the source frequency, rejecting noise and harmonics. Provide visual meter deflection.
- **Cathode Ray Oscilloscope (CRO):** Excellent for balancing both magnitude and phase simultaneously (using Lissajous figures).

Sources in AC Bridges

- The choice of source depends on the frequency required for the measurement.
- **Power Line (50 Hz):** Used for low-frequency measurements, especially for large inductors or capacitors used in power systems. Often requires a step-down transformer.
- **Audio Frequency Oscillators:** Generate stable sine waves in the 1 kHz to 10 kHz range. These are the most common sources for general-purpose L and C bridges.
- **Radio Frequency (RF) Oscillators:** Used for high-frequency bridges. Require special shielding and grounding to prevent parasitic capacitances from ruining the measurement.

Applications of AC Bridges

- AC Bridges are versatile and have numerous applications:

Measurement Parameter	**Specific Bridge Examples**
Inductance (L)	Maxwell's, Hay's, Anderson Bridge
Capacitance (C)	Schering, De Sauty Bridge
Frequency (f)	Wien Bridge
Quality / Dissipation Factor	Derived from R, L, C measurements

- **Quality Factor (Q) & Dissipation Factor (D):** Bridges naturally yield the resistive losses in inductors and capacitors, allowing calculation of Q and D.

Quick Check

- **Question 1:** In a DC bridge, the balance equation is derived by setting the detector current to what value?
- **Question 2:** Why must an AC bridge satisfy two balance conditions instead of one?
- **Question 3:** Name one common detector used in an AC bridge operating at 1 kHz.

Summary

- Bridges offer high accuracy through null-deflection measurement.
- DC Bridges consist of pure resistors and are balanced when opposite arm products are equal: $R_1 R_4 = R_2 R_3$.
- DC bridges are used for measuring resistance and interfacing sensors like strain gauges.
- AC Bridges measure complex impedances.
- Balance in an AC bridge requires matching both magnitude ($|Z_1||Z_4| = |Z_2||Z_3|$) and phase ($\theta_1 + \theta_4 = \theta_2 + \theta_3$).
- AC bridges require oscillators for sources and AC-sensitive detectors (like CROs or tuned amps).

Next Lecture Preview

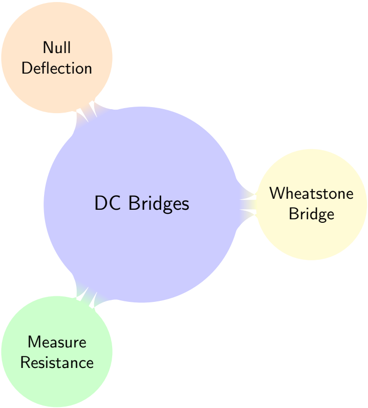
- In our next lecture, we will dive deep into a specific and highly important DC bridge:
- **The Wheatstone Bridge**
  - Detailed working principle.
  - Derivation of sensitivity.
  - Limitations of the Wheatstone bridge for very low and very high resistances.
  - Practical configurations.

Agenda

- Introduction to the Wheatstone Bridge
- Circuit Construction and Components
- The Principle of Null Deflection
- Mathematical Derivation of the Balance Condition
- Measurement of Unknown Resistance
- Bridge Sensitivity and Unbalanced State

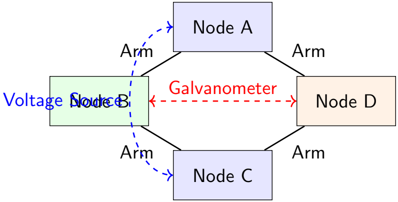
Introduction to DC Bridges

- DC bridges are primarily used to measure unknown electrical resistance.
- They operate on the principle of comparing an unknown value with a known standard.
- The Wheatstone Bridge is the most common and accurate DC bridge for measuring medium resistances (1 Ω to 100 kΩ).
- Invented by Samuel Hunter Christie in 1833, but improved and popularized by Sir Charles Wheatstone in 1843.



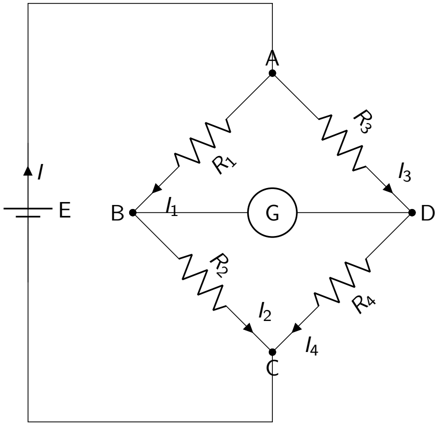
What is a Wheatstone Bridge?

- A circuit consisting of four resistive arms joined to form a quadrilateral.
- A DC voltage source is connected across one pair of opposite junctions.
- A null detector (usually a sensitive Galvanometer) is connected across the other pair of opposite junctions.
- It determines the unknown resistance by balancing the two legs of the bridge circuit.



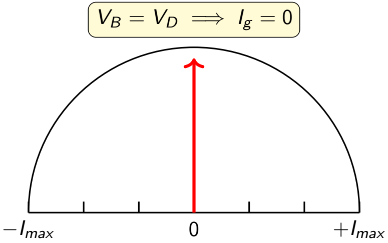
Circuit Diagram of Wheatstone Bridge

- Four resistors: R_1, R_2, R_3, R_4 forming arms AB, BC, AD, and DC.
- Battery (E) connected between nodes A and C.
- Galvanometer (G) connected between nodes B and D.
- Current I splits at node A into I_1 (through R_1) and I_3 (through R_3).



Principle of Operation: Null Deflection

- The bridge works on the principle of 'Null Deflection'.
- When the potential (voltage) at Node B equals the potential at Node D ($V_B = V_D$), the bridge is said to be 'balanced'.
- At balance, the potential difference across the galvanometer is zero ($V_B - V_D = 0$).
- Therefore, no current flows through the galvanometer ($I_g = 0$).



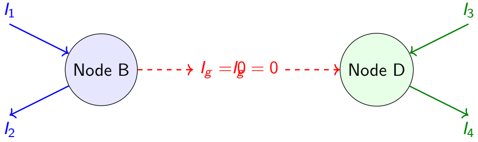
Derivation of Balance Condition - Step 1

- Assume the bridge is balanced ($I_g = 0$).
- Applying Kirchhoff's Current Law (KCL) at Node B:

$$I_1 = I_2 + I_g \implies I_1 = I_2 \text{ (since } I_g = 0)$$

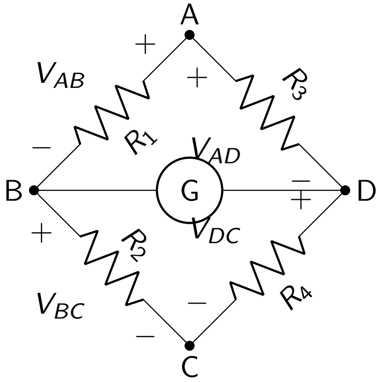
- Applying KCL at Node D:

$$I_3 = I_4 - I_g \implies I_3 = I_4 \text{ (since } I_g = 0)$$



Derivation of Balance Condition - Step 2

- Since $V_B = V_D$, the voltage drop across R_1 equals the voltage drop across R_3:
  $V_{AB} = V_{AD} \implies I_1 R_1 = I_3 R_3$ — (Equation 1)
- Similarly, the voltage drop across R_2 equals the voltage drop across R_4:
  $V_{BC} = V_{DC} \implies I_2 R_2 = I_4 R_4$ — (Equation 2)



Derivation of Balance Condition - Final

- Divide Equation 1 by Equation 2:
  $\frac{I_1 R_1}{I_2 R_2} = \frac{I_3 R_3}{I_4 R_4}$
- Since $I_1 = I_2$ and $I_3 = I_4$, the currents cancel out:
  $\frac{R_1}{R_2} = \frac{R_3}{R_4}$ or $R_1 R_4 = R_2 R_3$
- This is the fundamental balance equation of the Wheatstone Bridge.

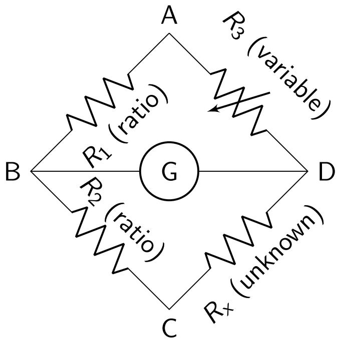
$$\frac{R_1}{R_2} = \frac{R_3}{R_4}$$

Balance Condition

Measurement of Unknown Resistance

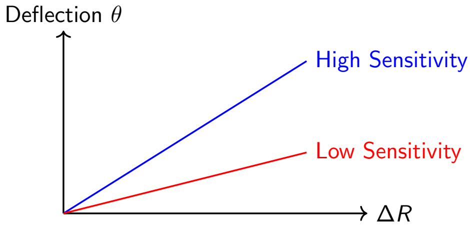
- Let R_4 be the unknown resistance (R_x).
- Let R_3 be a standard variable resistance.
- R_1 and R_2 are known as the 'ratio arms'.
- Rearranging the balance equation:

$$R_x = R_3 \times \left(\frac{R_2}{R_1}\right)$$



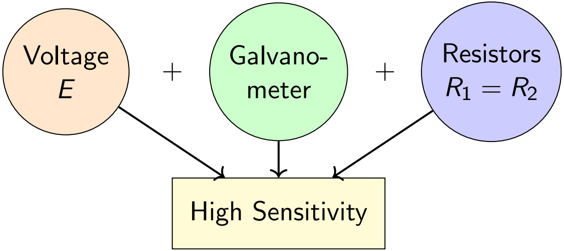
Sensitivity of the Wheatstone Bridge

- Sensitivity indicates how much the galvanometer deflects for a small unbalance in resistance.
- Voltage Sensitivity (S_v) = Deflection (θ) / Voltage (V)
- Bridge Sensitivity $(S_B) = \frac{\Delta\theta}{\Delta R/R}$
- Higher sensitivity allows for more precise measurements.



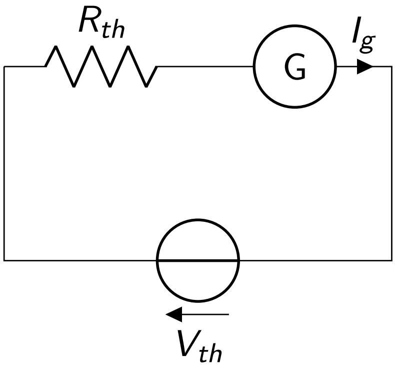
Factors Affecting Sensitivity

- Magnitude of the excitation voltage (Battery E): Higher voltage = higher sensitivity.
- Sensitivity of the galvanometer itself.
- The ratio of the bridge arms (R_1/R_2): Maximum sensitivity occurs when all four arms have equal resistance ($R_1 = R_2 = R_3 = R_4$).



Unbalanced Wheatstone Bridge

- If $R_1/R_2 \neq R_3/R_4$, current flows through the galvanometer.
- The current can be calculated using Thevenin's Theorem.
- The bridge is viewed from the galvanometer terminals (B and D).
- $V_{th} = E\left(\frac{R_1}{R_1+R_2} - \frac{R_3}{R_3+R_4}\right)$
- $R_{th} = (R_1||R_2) + (R_3||R_4)$



Quick Check

- **Question:** In a balanced Wheatstone bridge, $R_1 = 100\,\Omega$, $R_2 = 1000\,\Omega$, and the standard arm $R_3 = 250\,\Omega$. What is the unknown resistance R_x (R_4)?
  - A) $25\,\Omega$
  - B) $2500\,\Omega$
  - C) $250\,\Omega$
  - D) $400\,\Omega$

Quick Check

- **Question:** In a balanced Wheatstone bridge, $R_1 = 100\,\Omega$, $R_2 = 1000\,\Omega$, and the standard arm $R_3 = 250\,\Omega$. What is the unknown resistance R_x (R_4)?
  - A) $25\,\Omega$
  - B) $2500\,\Omega$
  - C) $250\,\Omega$
  - D) $400\,\Omega$

Answer

Correct Answer: B

$$R_x = R_3 \times \left(\frac{R_2}{R_1}\right) = 250 \times \left(\frac{1000}{100}\right) = 2500\,\Omega$$

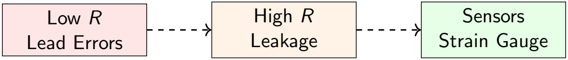
Summary

- The Wheatstone Bridge is a four-arm DC circuit used to measure medium resistances.
- It operates on the principle of null deflection ($V_B = V_D$, $I_g = 0$).
- The balance condition is $\mathbf{\frac{R_1}{R_2} = \frac{R_3}{R_4}}$.
- Sensitivity is maximum when all four resistive arms are equal.

$$\text{Balance: } \frac{R_1}{R_2} = \frac{R_3}{R_4}$$

Next Lecture Preview

- Topic: Wheatstone Bridge: Limitations & Applications
- Why can't we use it for very low resistances? (Contact and lead resistance)
- Why can't we use it for very high resistances? (Leakage currents)
- Practical applications in industry and sensors.

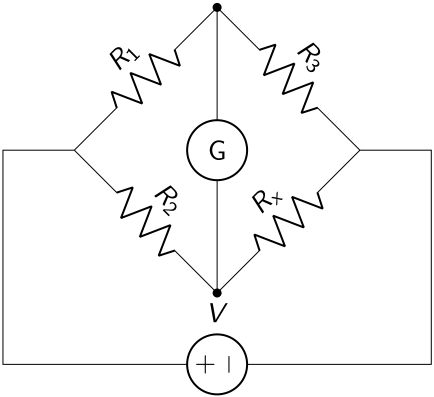


Agenda

- Recap of Wheatstone Bridge Basics
- Range of Resistance Measurement
- Limitation: Measurement of Low Resistance
- Limitation: Measurement of High Resistance
- Limitation: Heating Effect & Thermoelectric EMFs
- Application: Measurement of Medium Resistances
- Application: Strain Gauges
- Application: Temperature Sensors (RTDs)
- Application: Cable Fault Location
- Quick Check
- Summary

Recap: The Balanced Wheatstone Bridge

- Circuit Layout: Four arms with resistors R_1, R_2, R_3, and R_x.
- Galvanometer: Connected between the midpoints to detect zero current.
- Balance Condition: Occurs when the voltage potential at both midpoints is equal.
- Equation: $\frac{R_1}{R_2} = \frac{R_3}{R_x} \implies R_x = R_3 \times \frac{R_2}{R_1}$
- Primary Use: Measuring unknown resistance R_x accurately.



Classification of Resistances

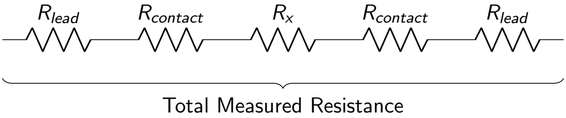
To understand limitations, we must classify resistance values:

- **Low Resistance:** Less than 1 Ω (e.g., armature windings, cables).
- **Medium Resistance:** 1 Ω to 100 $k\Omega$ (e.g., electronic circuit resistors, heater elements).
- **High Resistance:** Greater than 100 $k\Omega$ (e.g., insulation resistance of cables, dielectric leakage).
- **Key Takeaway:** Wheatstone bridge is primarily suited for Medium Resistances.

Low ($< 1\ \Omega$)	Medium ($1\ \Omega - 100\ k\Omega$)	High ($> 100\ k\Omega$)

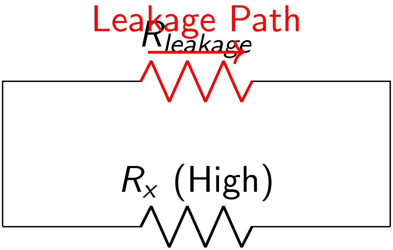
Limitation 1: Measuring Low Resistance ($< 1\ \Omega$)

- **Contact Resistance:** The resistance at the binding posts and switches becomes significant.
- **Lead Resistance:** The connecting wires have their own resistance (often in milli-ohms).
- **Impact:** When measuring an R_x of 0.1 Ω, a lead resistance of 0.05 Ω introduces a 50% error.
- **Solution:** Kelvin Double Bridge is used instead to eliminate lead resistance errors.



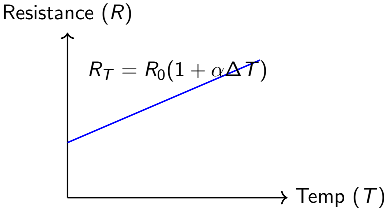
Limitation 2: Measuring High Resistance ($> 100\ k\Omega$)

- **Leakage Currents:** At high resistances, unwanted leakage paths (e.g., across the surface of insulators or test terminals) appear in parallel with the unknown resistor.
- **Sensitivity Issues:** High resistance limits the current flowing through the bridge arms.
- **Impact:** The galvanometer deflection becomes too small to detect the null point accurately.
- **Solution:** Megger or specialized Megohm bridges with guard circuits are used.



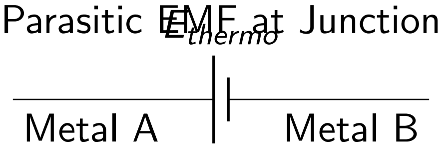
Limitation 3: Heating Effect (I^2R Loss)

- **Power Dissipation:** Current flowing through the bridge arms causes I^2R heating.
- **Temperature Coefficient:** Resistance changes with temperature ($R_T = R_0[1 + \alpha \Delta T]$).
- **Impact:** If excessive voltage is applied to increase sensitivity, the resistors heat up, changing their value and causing drift in the balance point.
- **Precaution:** Use minimal excitation voltage and limit the time the bridge is powered.



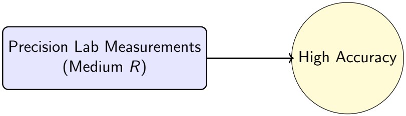
Limitation 4: Thermoelectric EMFs

- **Seebeck Effect:** Junctions of dissimilar metals (e.g., copper wires connected to brass terminals) generate a small voltage when there is a temperature gradient.
- **Impact:** These small parasitic DC voltages add to or subtract from the galvanometer reading, creating a false null point.
- **Mitigation:**
  - Use a more sensitive galvanometer.
  - Take measurements, then reverse the battery polarity and average the results.



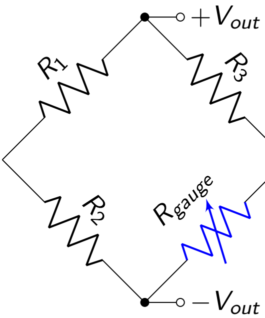
Application 1: Measurement of Medium Resistances

- **Standard Use:** The primary laboratory application of the Wheatstone bridge.
- **Accuracy:** Very high accuracy (typically 0.1% to 1%) for measuring resistors in the 1 Ω to 100 $k\Omega$ range.
- **Usage:** Calibration of instruments, quality control of manufactured resistors, and precision lab work.



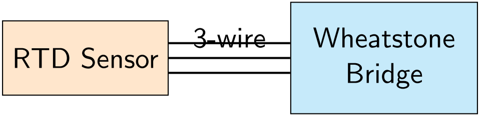
Application 2: Strain Gauges (Sensor Integration)

- **Strain Gauge:** A sensor whose resistance changes when it is stretched or compressed.
- **Quarter-Bridge Configuration:** One arm of the Wheatstone bridge is the strain gauge; the other three are fixed resistors.
- **Working:**
  - Bridge is balanced at rest.
  - Mechanical strain changes R_{gauge}.
  - Bridge becomes unbalanced, producing an output voltage V_{out} directly proportional to the strain.



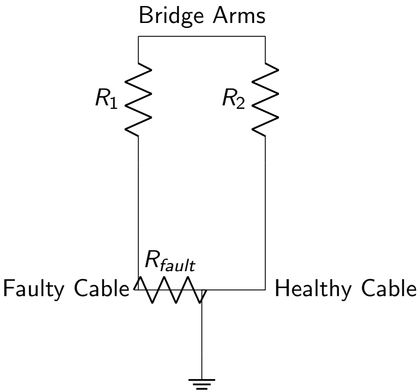
Application 3: Temperature Measurement (RTDs)

- **RTD (Resistance Temperature Detector):** A sensor (often Platinum) that changes resistance predictably with temperature.
- **Integration:** The RTD acts as the unknown arm R_x.
- **Continuous Monitoring:** Instead of manually balancing the bridge, the output unbalance voltage is measured continuously and calibrated to display temperature.
- **Lead Compensation:** 3-wire and 4-wire bridge setups are used to cancel out lead resistance errors.



Application 4: Fault Location in Cables

- **Murray Loop Test:** A modified Wheatstone bridge used to locate ground faults or short circuits in underground telephone or power cables.
- **Principle:** The loop of the faulty cable and a sound return cable forms two arms of the bridge.
- **Advantage:** By balancing the bridge, the exact distance to the fault can be calculated based on the proportional resistance of the cable length.



Quick Check

Question 1

Why is a Wheatstone bridge not suitable for measuring a 0.01 Ω resistor?

A) Leakage currents are too high
B) Lead and contact resistances cause massive errors
C) The power dissipation is zero

Question 2

Which sensor is commonly placed in a Wheatstone bridge to measure mechanical stress?

A) Thermocouple
B) Capacitor
C) Strain Gauge

Summary

- **Ideal Range:** Wheatstone bridge is best for medium resistances ($1\ \Omega - 100\ k\Omega$).
- **Low Resistance Limits:** Plagued by contact and lead resistance (use Kelvin bridge instead).
- **High Resistance Limits:** Plagued by leakage currents and low sensitivity (use Megger instead).
- **Other Errors:** Thermoelectric EMFs and I^2R heating must be managed.
- **Applications:** Crucial for reading resistive sensors like Strain Gauges, Light Dependent Resistors (LDRs), and RTDs.

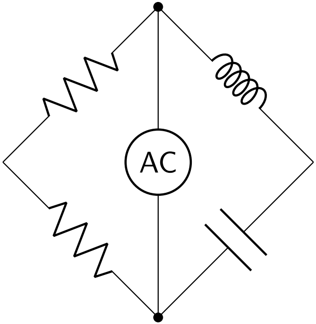
Range	**Limitation**	**Alternative**
Low ($< 1\ \Omega$)	Lead/Contact Resistance	Kelvin Double Bridge
High ($> 100\ k\Omega$)	Leakage & Low Sensitivity	Megger
Medium	Best fit (0.1% – 1% accuracy)	N/A

Preview for Next Lecture

Next Topic: Maxwell's Bridge

What to Expect:

- Transitioning from DC to AC Bridges.
- How to measure unknown Inductance using standard capacitance.
- Phasor diagrams and AC balance conditions.
- **Preparation:** Review the concepts of AC impedance ($Z = R + jX_L$).



Maxwell's Bridge

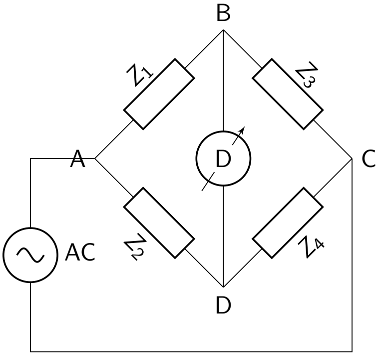
- Course: Electronics Circuit Network & Measurement (DI03011031)
- Unit 3: Characteristic of Measurement and Bridges
- Topic: Maxwell's Bridge
- Lecture: 26

Agenda

- Introduction to AC Bridges for Inductance
- Maxwell's Inductance Bridge (Circuit & Derivation)
- Maxwell's Inductance-Capacitance (L-C) Bridge
- Derivations and Balance Equations
- Phasor Diagrams
- Advantages and Limitations

Introduction to AC Bridges

- AC bridges are used to measure unknown impedances (inductance, capacitance).
- They operate on AC supply and use an AC detector (e.g., headphones, vibration galvanometers).
- The general balance condition for a four-arm AC bridge is $Z_1 Z_4 = Z_2 Z_3$.
- Balance requires equalizing both magnitude and phase angle:
  - $|Z_1||Z_4| = |Z_2||Z_3|$
  - $\theta_1 + \theta_4 = \theta_2 + \theta_3$



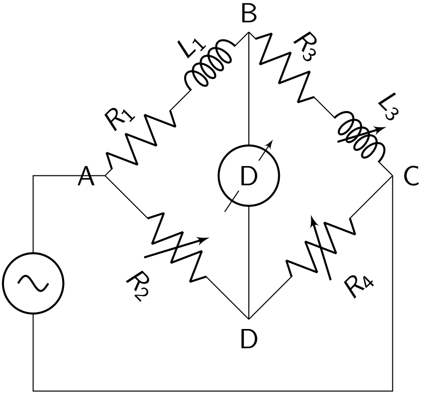
Measuring Inductance: Maxwell's Bridge

- Maxwell's bridge is specifically designed to measure an unknown inductance.
- It comes in two primary configurations:
  - **Maxwell's Inductance Bridge:** Compares unknown inductance with a known standard inductance.
  - **Maxwell's Inductance-Capacitance (L-C) Bridge:** Measures unknown inductance using a standard known capacitance.
- The L-C bridge is much more widely used due to the superior characteristics of standard capacitors.

Inductance Bridge	**L-C Bridge**
Uses standard inductor	Uses standard capacitor
Bulky, high interference	Compact, zero magnetic field
Less practical	Highly preferred in industry

Maxwell's Inductance Bridge: Circuit Diagram

- Arm AB (Z_1): Unknown inductance L_1 with effective resistance R_1.
- Arm AD (Z_2): Known variable resistance R_2.
- Arm BC (Z_3): Known standard variable inductance L_3 of resistance R_3.
- Arm CD (Z_4): Known variable resistance R_4.
- Detector: Connected between points B and D.
- Source: AC voltage connected between points A and C.



Maxwell's Inductance Bridge: Derivation (Part 1)

- The complex impedances of the four arms are:
  - $Z_1 = R_1 + j\omega L_1$
  - $Z_2 = R_2$
  - $Z_3 = R_3 + j\omega L_3$
  - $Z_4 = R_4$
- At balance, the condition is:

Balance Condition

$Z_1 Z_4 = Z_2 Z_3$

- Substituting the impedances:

$$(R_1 + j\omega L_1)R_4 = R_2(R_3 + j\omega L_3)$$

Maxwell's Inductance Bridge: Derivation (Part 2)

- Expanding the equation:

$$R_1 R_4 + j\omega L_1 R_4 = R_2 R_3 + j\omega L_3 R_2$$

- Equating the real parts for resistive balance:

$$R_1 R_4 = R_2 R_3 \implies \mathbf{R_1 = \frac{R_2 R_3}{R_4}}$$

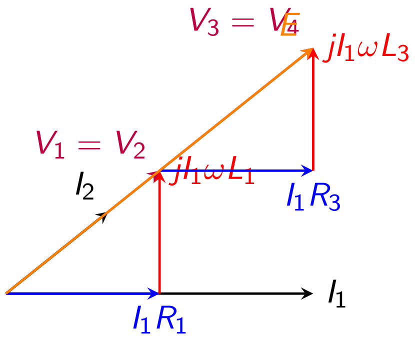
- Equating the imaginary parts for reactive balance:

$$\omega L_1 R_4 = \omega L_3 R_2 \implies \mathbf{L_1 = \frac{R_2 L_3}{R_4}}$$

- Note that ω cancels out during the derivation.

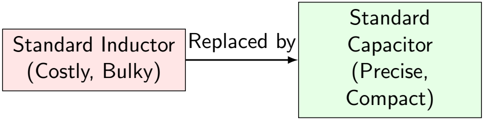
Phasor Diagram: Inductance Bridge

- Reference Phasor: Current I_1 (flows through arms AB and BC at balance).
- Branch Current: Current I_2 flows through arms AD and CD.
- Voltage drops:
  - $V_1 = I_1 R_1 + jI_1 \omega L_1$ (Leads I_1)
  - $V_2 = I_2 R_2$ (In phase with I_2)
  - $V_3 = I_1 R_3 + jI_1 \omega L_3$ (Leads I_1)
  - $V_4 = I_2 R_4$ (In phase with I_2)
- At balance, node voltages are equal: $V_1 = V_2$ and $V_3 = V_4$.



Limitations of Maxwell's Inductance Bridge

- Requires a standard variable inductance, which is extremely expensive and difficult to calibrate accurately.
- Standard inductors suffer from stray magnetic fields and inherent winding resistance.
- Mutual inductance between the unknown coil and the standard coil can introduce significant measurement errors.
- **Solution:** Use capacitance as the standard instead of inductance.

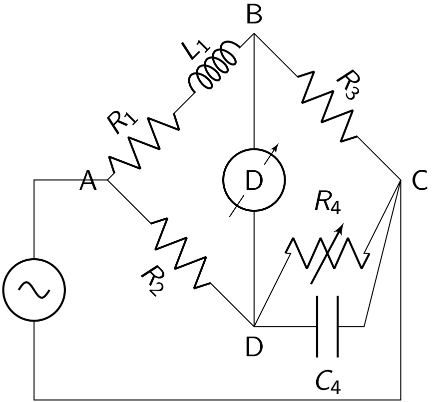


Maxwell's Inductance-Capacitance (L-C) Bridge

- Often referred to as the **Maxwell-Wien bridge**.
- Specifically measures unknown inductance in terms of a known standard capacitor.
- Standard capacitors are overwhelmingly preferred because they are:
  - Nearly ideal (extremely low dielectric loss).
  - Immune to stray magnetic fields.
  - Highly compact and cost-effective.
- A standard variable capacitor or standard resistors are adjusted to achieve null balance.

Maxwell's L-C Bridge: Circuit Diagram

- Arm AB (Z_1): Unknown inductance L_1 and resistance R_1 in series.
- Arm AD (Z_2): Known non-inductive resistance R_2.
- Arm BC (Z_3): Known non-inductive resistance R_3.
- Arm CD (Z_4): Standard capacitor C_4 in parallel with variable resistance R_4.
- Detector and Source: Placed conventionally across the opposite junctions.



Maxwell's L-C Bridge: Derivation (Part 1)

- Complex Impedances & Admittance:
  - $Z_1 = R_1 + j\omega L_1$
  - $Z_2 = R_2$
  - $Z_3 = R_3$
  - $Y_4 = \frac{1}{R_4} + j\omega C_4$ (Using admittance for parallel arm CD)
- Modified balance equation:

$$Z_1 = Z_2 Z_3 Y_4$$

- Substitute values:

$$R_1 + j\omega L_1 = R_2 R_3 \left(\frac{1}{R_4} + j\omega C_4 \right)$$

Maxwell's L-C Bridge: Derivation (Part 2)

- Expanding the right-hand side:

$$R_1 + j\omega L_1 = \frac{R_2 R_3}{R_4} + j\omega R_2 R_3 C_4$$

- Equating real parts:

$$\mathbf{R_1 = \frac{R_2 R_3}{R_4}}$$

- Equating imaginary parts:

$$\omega L_1 = \omega R_2 R_3 C_4 \implies \mathbf{L_1 = R_2 R_3 C_4}$$

- The unknown variables L_1 and R_1 are found **without frequency dependence**.

Quality Factor (Q) Measurement

- The Quality Factor (Q) of the unknown coil is defined as $Q = \frac{\omega L_1}{R_1}$.
- Substituting the derived values for L_1 and R_1:

$$Q = \frac{\omega(R_2 R_3 C_4)}{\frac{R_2 R_3}{R_4}}$$

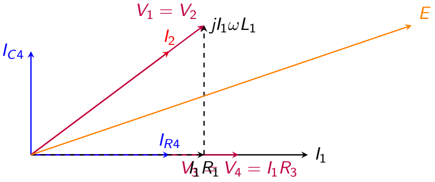
Quality Factor

$$Q = \omega C_4 R_4$$

- Maxwell's L-C bridge is most suitable for measuring **medium Q coils** ($1 < Q < 10$).

Phasor Diagram: L-C Bridge

- $V_1 = V_2$ and $V_3 = V_4$ at balance condition.
- Current I_1 flows through arms AB and BC; current I_2 flows through AD and CD.
- Arm CD has parallel elements, so current I_2 splits into I_{R4} (resistive) and I_{C4} (capacitive).
- I_{C4} leads the voltage V_4 by precisely 90°.
- The vector sum of I_{R4} and I_{C4} equals the total branch current I_2.



Advantages and Limitations of L-C Bridge

Advantages:

- Balance equations are entirely independent of supply frequency.
- Simple and straightforward algebraic expressions for L_1 and R_1.
- Uses standard capacitors which are highly accurate, stable, and virtually loss-free.

Limitations:

- Requires a variable standard capacitor, which can be mechanically complex and expensive.
- Strictly limited to measuring medium Q coils ($1 < Q < 10$).
- For high Q coils ($Q > 10$), R_4 becomes impractically large to achieve balance.

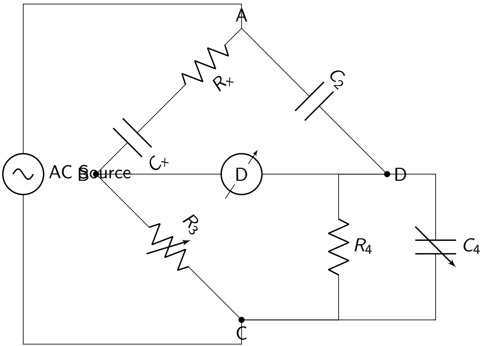
Quick Check

- **Q1:** What is the primary operational difference between Maxwell's Inductance Bridge and Maxwell's L-C Bridge?
- **Q2:** Why are standard capacitors overwhelmingly preferred over standard inductors in precision bridge circuits?
- **Q3:** For what specific range of Q-factor is Maxwell's L-C bridge considered suitable and accurate?

Summary & Next Lecture Preview

- **Summary:**
  - Maxwell's bridge is the standard for inductance measurement.
  - The Inductance-Capacitance bridge avoids errors inherent to standard inductors.
  - Balance is independent of frequency: $L_1 = R_2 R_3 C_4$ and $R_1 = \frac{R_2 R_3}{R_4}$.
  - Most suitable for medium Q coils ($1 < Q < 10$).
- **Next Lecture Preview:**
  - We will introduce the Schering Bridge, widely used for measuring unknown capacitance, relative permittivity, and dielectric loss.

- Electronics Circuit Network & Measurement
- Unit 3: Characteristic of Measurement and Bridges
- Lecture 27: Schering Bridge



Agenda

- Introduction to Schering Bridge
- Circuit Diagram and Components
- AC Bridge Balance Condition
- Derivation of C_x and R_x
- Dissipation Factor (Dielectric Loss)
- Phasor Diagram
- High Voltage Schering Bridge
- Advantages, Limitations, and Applications
- Quick Check & Summary

Introduction to Schering Bridge

What is a Schering Bridge?

An AC bridge circuit used to measure unknown capacitance, dielectric loss, and relative permittivity.

- Widely used in the measurement of the insulation properties of cables, equipment, and capacitor testing.
- Named after Harald Schering, it compares the unknown capacitor with a standard, high-quality, loss-free capacitor.
- Highly accurate for evaluating the quality of a dielectric material.

Schering Bridge Circuit Diagram

- **Basic Circuit Configuration**
- The bridge consists of four arms (AB, BC, CD, DA), an AC source, and a null detector.
- **Arm AB (Z_1):** Unknown capacitor C_x in series with its internal resistance R_x.
- **Arm AD (Z_2):** Standard, loss-free capacitor C_2.
- **Arm BC (Z_3):** Non-inductive standard resistor R_3.
- **Arm CD (Z_4):** Non-inductive resistor R_4 in parallel with a variable standard capacitor C_4.

Circuit Components & Parameters

Component	**Description & Physical Significance**
C_x	Unknown capacitance to be measured.
R_x	Series resistance representing the dielectric loss in C_x.
C_2	Standard capacitor (negligible loss, $R = 0$).
R_3	Non-inductive variable resistor.
R_4	Non-inductive variable resistor.
C_4	Variable standard capacitor.

- At least two components (typically R_3 and C_4) are varied to achieve a null response in the detector.

AC Bridge Balance Condition

General Balance Equation

For any four-arm AC bridge with an AC source across nodes A-C and a detector across nodes B-D, the balance condition is achieved when the current through the detector is zero.

- This occurs when the voltage drop across Arm AB equals the voltage drop across Arm AD.
- **General Equation:**

$$Z_1 \cdot Z_4 = Z_2 \cdot Z_3$$

- Where Z_1, Z_2, Z_3, Z_4 are the complex impedances of the respective arms.

Impedance Equations of Arms

Formulating the Impedances

Based on the circuit diagram, we define the impedance for each arm:

- **Arm 1 (Z_1):**
  $Z_1 = R_x + \frac{1}{j\omega C_x}$
- **Arm 2 (Z_2):**
  $Z_2 = \frac{1}{j\omega C_2}$
- **Arm 3 (Z_3):**
  $Z_3 = R_3$
- **Arm 4 (Z_4):** $R_4 \parallel C_4$
  $Z_4 = \frac{R_4 \cdot \frac{1}{j\omega C_4}}{R_4 + \frac{1}{j\omega C_4}} = \frac{R_4}{1 + j\omega C_4 R_4}$

Deriving the Balance Equation

Substituting Impedances

- Substitute the impedances into the balance condition ($Z_1 Z_4 = Z_2 Z_3$):

$$\left(R_x + \frac{1}{j\omega C_x}\right) \cdot \left(\frac{R_4}{1 + j\omega C_4 R_4}\right) = \left(\frac{1}{j\omega C_2}\right) \cdot R_3$$

- Rearranging to isolate the terms:

$$\left(R_x + \frac{1}{j\omega C_x}\right) \cdot R_4 = \frac{R_3}{j\omega C_2} \cdot (1 + j\omega C_4 R_4)$$

Expanding the Equation:

$$R_x R_4 + \frac{R_4}{j\omega C_x} = \frac{R_3}{j\omega C_2} + \frac{R_3(j\omega C_4 R_4)}{j\omega C_2}$$

Simplifying the right side:

$$R_x R_4 - j\frac{R_4}{\omega C_x} = -j\frac{R_3}{\omega C_2} + \frac{R_3 C_4 R_4}{C_2}$$

Equating Real Parts:

$$R_x R_4 = \frac{R_3 C_4 R_4}{C_2} \implies R_x = R_3 \frac{C_4}{C_2}$$

Equating Imaginary Parts:

$$\frac{R_4}{\omega C_x} = \frac{R_3}{\omega C_2} \implies C_x = C_2 \frac{R_4}{R_3}$$

Final Expressions for C_x and R_x

Measurement Results

- From the balance conditions, the unknown parameters are calculated as:

Unknown Capacitance:

$$C_x = C_2 \frac{R_4}{R_3}$$

Equivalent Series Resistance:

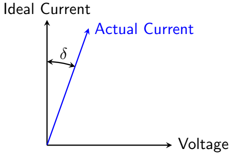
$$R_x = R_3 \frac{C_4}{C_2}$$

- Notice that both equations are independent of frequency (ω).
- The bridge balance is achieved by varying C_4 and R_3.

Dissipation Factor (Dielectric Loss)

- **What is Dissipation Factor?**
- Real capacitors have some power loss due to dielectric imperfections.
- This loss is quantified by the Dissipation Factor (D), also known as $\tan\delta$.
- δ is the loss angle (the angle by which the current falls short of a perfect 90° lead over voltage).

$$D = \tan\delta = \omega C_x R_x$$



Calculating D

- Substitute the derived values of C_x and R_x into the Dissipation Factor formula:

$$D = \omega C_x R_x$$

$$D = \omega \left(C_2 \frac{R_4}{R_3} \right) \left(R_3 \frac{C_4}{C_2} \right)$$

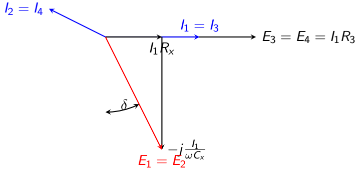
- Simplifying the terms (C_2 and R_3 cancel out):

$$D = \omega C_4 R_4$$

- The dissipation factor depends on the frequency ω, and the values of C_4 and R_4.
- Usually, R_4 is kept fixed, and C_4 is calibrated directly in terms of dissipation factor.

Phasor Diagram at Balance

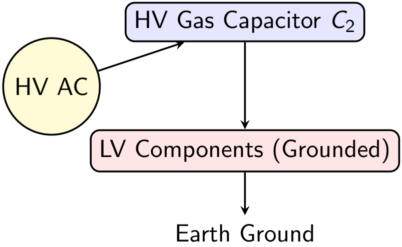
- At balance ($I_{detector} = 0$):
- $I_1 = I_3$ and $I_2 = I_4$
- Voltage drops: $E_1 = E_2$ and $E_3 = E_4$
- Take E_3 (across R_3) as reference.
- I_1 is in phase with E_3.
- E_1 has a resistive drop ($I_1 R_x$) and capacitive drop (lagging I_1 by 90°).
- I_2 leads E_2 (purely capacitive) by exactly 90°.



High Voltage Schering Bridge

Testing Under Operational Stress

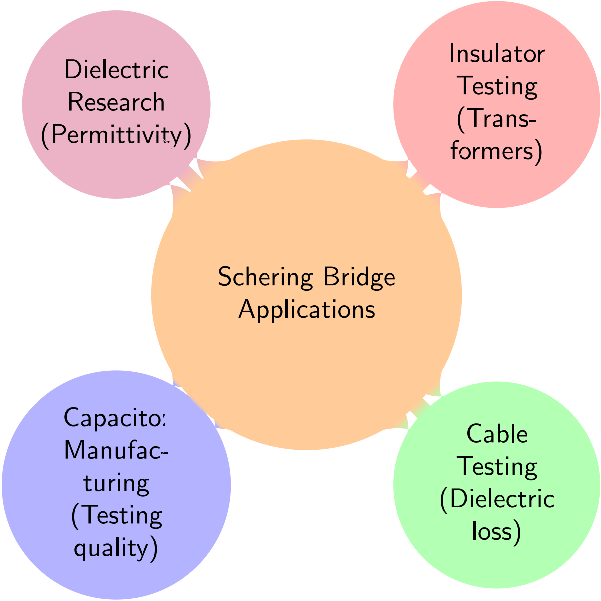
- Used to test insulation (cables, insulators) at high working voltages.
- **Modifications:**
  - High voltage AC source is applied.
  - The standard capacitor C_2 is a high-voltage, compressed gas (SF_6) capacitor.
  - The low-voltage components (R_3, R_4, C_4, and the detector) are grounded to protect the operator.
  - Earth screens and guard rings are used to eliminate stray capacitance errors.



Advantages and Limitations

✓ **Advantages**	✗ **Limitations**
Balance equations are independent of frequency.	Cannot be used to measure inductances.
Can measure small capacitances accurately.	At high frequencies, stray capacitances can cause significant errors.
Dial of capacitor C_4 can be directly calibrated to read the dissipation factor.	Requires proper shielding (Wagner earth connection) for HF measurements.
Highly safe for operators when configured as a High Voltage bridge (grounded low-voltage arms).	

Applications of Schering Bridge



Quick Check

Assessment Questions

1. In a Schering bridge, what does the component R_x physically represent?
2. True or False: The balance condition of a Schering bridge depends on the supply frequency.
3. What is the formula for the dissipation factor (D) obtained from the Schering bridge?
4. Why is the standard capacitor C_2 chosen to be a gas-filled capacitor in high voltage applications?

Summary & Key Takeaways

- **Lecture Summary**
- The Schering Bridge is widely used for measuring unknown capacitance and dielectric loss.
- **The balance equations are:**
  - $C_x = C_2 \frac{R_4}{R_3}$
  - $R_x = R_3 \frac{C_4}{C_2}$
- The Dissipation factor ($\tan \delta$) indicates insulation quality and is given by $\omega C_4 R_4$.
- High voltage Schering bridges ground the variable arms to ensure operator safety and use standard gas capacitors.

Next Lecture Preview

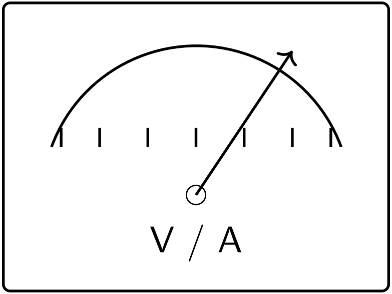
Up Next: Unit 4

Moving Iron vs Moving Coil Instruments

- We will begin Unit 4: Basic Electrical Parameter Measurement.
- Study the construction and working principles of PMMC (Permanent Magnet Moving Coil) instruments.
- Understand Moving Iron (MI) instruments.
- Compare the advantages and applications of both types of analog meters.

Lecture 28: Moving Iron vs Moving Coil Instruments

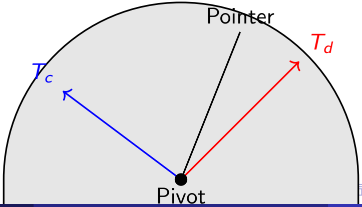
- Course: Electronics Circuit Network & Measurement (DI03011031)
- Unit 4: Basic Electrical Parameter measurement
- Topic: Differentiate between moving iron and moving coil type instruments



Agenda

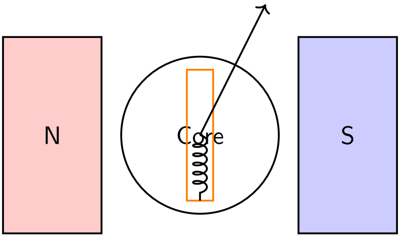
- Introduction to Analog Instruments
- Essential Torques (Deflecting, Controlling, Damping)
- Moving Coil (PMMC) Instruments: Construction & Working
- Moving Iron (MI) Instruments: Attraction & Repulsion Types
- Torque Equations
- Comprehensive Comparison (PMMC vs. MI)
- Summary

- Indicating Instruments: Instruments that indicate the value of the electrical quantity being measured at the time of measurement.
- Examples: Ammeters, Voltmeters, Wattmeters.
- For any indicating instrument to work properly, three torques are essential:
- Deflecting Torque (T_d): Moves the pointer from zero.
- Controlling Torque (T_c): Opposes the deflecting torque; stops pointer at correct reading & returns it to zero.
- Damping Torque ($T_{damping}$): Kills the oscillations so pointer settles quickly.



Moving Coil Instruments (PMMC)

- PMMC: Permanent Magnet Moving Coil.
- Principle: When a current-carrying coil is placed in a magnetic field, it experiences a force proportional to the current.
- Core Components:
  - U-shaped Permanent Magnet (provides stable magnetic field).
  - Rectangular aluminum former wound with fine copper wire (the moving coil).
  - Spring (provides controlling torque and carries current).
- Only works on DC. If AC is applied, torque reverses rapidly, average torque is zero.



PMMC: Torque Equation

- Deflecting Torque (T_d):
- $T_d = B \cdot I \cdot N \cdot A$
- Where:
  - B = Flux density (Wb/m^2)
  - I = Current in coil (Amps)
  - N = Number of turns
  - A = Area of coil (m^2)
- Controlling Torque (T_c): Provided by springs, $T_c = K \cdot \theta$ (where θ is deflection angle).
- At steady state: $T_d = T_c \implies BINA = K\theta \implies \theta \propto I$.
- Result: Uniform (linear) scale.

PMMC: Advantages & Disadvantages

Advantages:

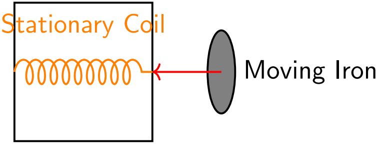
- Uniform (linear) scale.
- High sensitivity and accuracy.
- Low power consumption (typically 25 to 200 μW).
- High torque-to-weight ratio (less frictional error).
- Excellent eddy current damping (due to aluminum former).

Disadvantages:

- Cannot be used for AC measurements.
- More expensive than Moving Iron types.
- Errors due to aging of magnets and springs.

Moving Iron (MI) Instruments

- Principle: A piece of soft iron is placed in the magnetic field of a stationary electromagnet (the coil). The iron gets magnetized and experiences a force.
- Works on both AC and DC: Since the soft iron is always attracted (or repelled if two pieces are magnetized similarly) regardless of current direction, average torque is non-zero for AC.
- Two Types:
  - Attraction Type
  - Repulsion Type



MI: Attraction Type

- Construction: Contains a stationary hollow cylindrical coil.
- A flat eccentric soft iron disc is pivoted eccentrically near the coil.
- Working:
  - When current flows, a magnetic field is produced inside the coil.
  - The soft iron disc is drawn inward towards the strongest part of the magnetic field.
  - This movement deflects the pointer.
- Damping: Air friction damping is used (a piston in an air chamber).

MI: Repulsion Type

- Construction: Contains two iron vanes inside a stationary coil.
- Fixed Vane: Attached to the coil frame.
- Moving Vane: Attached to the moving spindle.
- Working:
  - Current passing through the coil magnetizes both vanes in the *same* direction.
  - Similar poles repel each other, pushing the moving vane away.
  - The repulsion force turns the spindle and pointer.
- Also uses air friction damping.

MI: Torque Equation

- Deflecting Torque (T_d) in MI instruments:
- $T_d = \frac{1}{2} I^2 \frac{dL}{d\theta}$
- Where:
  - I = RMS current
  - L = Inductance of the coil
  - θ = Deflection angle
- Controlling Torque: Spring control, $T_c = K\theta$.
- At steady state: $K\theta = \frac{1}{2} I^2 \frac{dL}{d\theta} \implies \theta \propto I^2$.
- Result: Non-uniform (cramped at the beginning) scale.

MI: Advantages & Disadvantages

Advantages:

- Can be used for both AC and DC circuits.
- Cheap, robust, and simple construction (no permanent magnet).
- Capable of handling momentary overloads well.

Disadvantages:

- Non-uniform scale.
- Lower accuracy compared to PMMC.
- Errors introduced by hysteresis, stray magnetic fields, and frequency variations (in AC).
- Higher power consumption.

Difference: Principle & Usage

Parameter	**Moving Coil (PMMC)**	**Moving Iron (MI)**
Working Principle	Force on a current-carrying coil in a magnetic field	Attraction/Repulsion of soft iron in a magnetic field
Usage	DC Only	Both AC and DC
Magnetic Field	Permanent Magnet	Electromagnet (Coil)
Moving Part	Coil	Soft Iron piece

Difference: Scale & Damping

Parameter	Moving Coil (PMMC)	Moving Iron (MI)
Scale	Uniform (Linear)	Non-uniform (cramped at ends)
Damping	Eddy current damping	Air friction damping
Controlling Torque	Spring control	Spring or Gravity control
Deflecting Torque	$T_d \propto I$	$T_d \propto I^2$

Difference: Performance & Cost

Parameter	Moving Coil (PMMC)	Moving Iron (MI)
Sensitivity & Accuracy	Very High	Lower than PMMC
Power Consumption	Very Low	Higher
Cost	More Expensive	Cheaper
Hysteresis Error	Absent	Present (due to iron core)

Quick Check

- **Question 1:** Why does a PMMC meter not measure AC?
- **Answer:** The permanent magnetic field is fixed. Alternating current creates alternating torque, and average torque over a cycle is zero, so the pointer just vibrates.
- **Question 2:** Why is the scale of an MI instrument non-uniform?
- **Answer:** Because its deflecting torque is proportional to the square of the current ($T_d \propto I^2$).

Application Context

- **Where to use PMMC?**
  - DC ammeters and voltmeters in electronic circuits.
  - Galvanometers (null detection in bridges).
  - Because of high sensitivity, they are used as the base mechanism in analog multimeters.
- **Where to use MI?**
  - AC mains switchboards and panels (voltage and current monitoring).
  - Motor starters and industrial AC measurement.
  - Cheap, rugged applications where 1-2% error is acceptable.

Summary

- **PMMC Instruments:**
  - Linear scale, high accuracy, DC only.
  - Works on Lorentz force ($T_d \propto I$).
  - Uses Eddy current damping.
- **Moving Iron Instruments:**
  - Non-linear scale, rugged, works on AC & DC.
  - Works on attraction/repulsion of magnetized iron ($T_d \propto I^2$).
  - Uses Air friction damping.
- Selection depends on precision needs, budget, and whether the signal is AC or DC.

Next Lecture Preview

- Topic: Electronic Multimeter and its advantages
- We will move from basic electromechanical movements to electronic instruments.
- How active components (amplifiers) increase the sensitivity of measurements.
- Reading assignment: Review basic operational amplifier concepts.

Lecture 29: Electronic Multimeter: Working & Advantages

- **Course Code:** DI03011031
- **Course Title:** Electronics Circuit Network & Measurement
- **Unit 4:** Basic Electrical Parameter Measurement
- **Topics Covered:** Working principles, block diagram, and advantages of Electronic Multimeters.

Electronic Multimeter (EMM)

Agenda for Today's Lecture

- Recap of Previous Lecture
- Introduction to Electronic Multimeters (EMM)
- Block Diagram and Architecture
- Measuring DC & AC Voltage
- Measuring Current & Resistance
- Concept of High Input Impedance
- Advantages of Electronic Multimeters
- Quick Check & Summary

Recap: Moving Iron vs Moving Coil Instruments

Permanent Magnet Moving Coil (PMMC)	Moving Iron (MI) Instruments
Works only on DC. Linear scale, highly accurate and sensitive.	Works on both AC and DC. Non-linear scale, less accurate than PMMC.

Transition to Electronics

Conventional meters load the circuit under test due to low input impedance. Electronic instruments solve this issue.

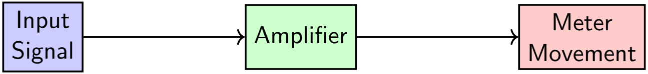
Introduction to Multimeters

- A Multimeter (or VOM - Volt-Ohm-Milliammeter) is an instrument used to measure voltage, current, and resistance.
- **Traditional Analog Multimeter:**
  - Uses a PMMC movement directly connected to the circuit.
  - Suffers from loading effect (low input impedance, e.g., 20 kΩ/V).
- **Electronic Multimeter (EMM):**
  - Incorporates active electronic components (amplifiers, FETs, Op-Amps).
  - Amplifies the input signal before feeding it to the display.

What is an Electronic Multimeter?

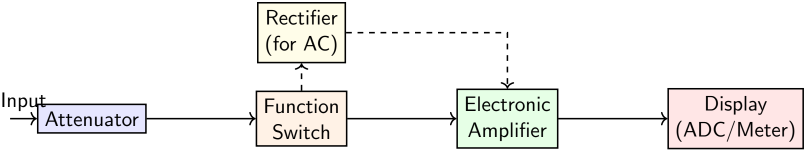
An Electronic Multimeter uses active solid-state components (like transistors or operational amplifiers) to enhance measurement capabilities.

- High input impedance (typically 10 MΩ or higher).
- Capable of measuring very small signals in millivolts or microamperes.
- Internal power supply (batteries) powers the electronic circuitry, not just the ohmmeter section.
- Available in analog (e.g., Solid-State Voltmeter) and digital (DMM) forms.



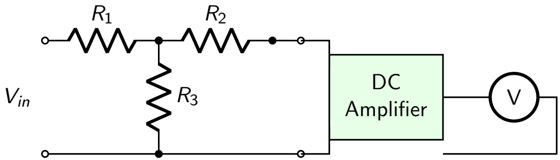
Block Diagram of an Electronic Multimeter

- **Input Probe / Attenuator:** Scales down high voltages to a safe level.
- **Function Switch:** Selects between AC V, DC V, Current, or Resistance.
- **Electronic Amplifier:** Provides high input impedance and amplifies signals.
- **Rectifier (for AC):** Converts AC signals to DC.
- **Meter Movement / ADC:** Displays the measured value.



Measuring DC Voltage

- The DC voltage is applied across the input probes.
- An **Attenuator** (Voltage Divider) scales down large voltages.
- A **DC Amplifier** (FET input stage) isolates the circuit.
- The amplified signal drives a PMMC meter or ADC.
- **Advantage:** FET input ensures near-zero current is drawn.

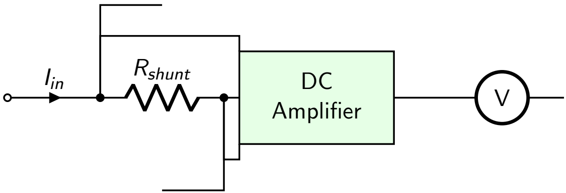


Measuring AC Voltage

- AC voltage enters the attenuator network.
- The signal is passed through a **Rectifier Circuit** (half-wave or full-wave).
- The rectified pulsating DC is filtered to obtain an average or RMS value.
- The DC signal is then amplified and sent to the meter.
- **Note:** Most basic EMMs are average-responding but calibrated to display RMS values for sine waves.

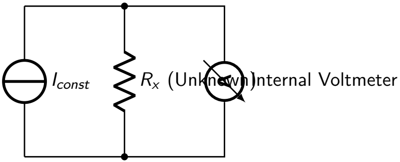
Measuring Current

- The multimeter is connected in series with the circuit.
- The current flows through a known internal **Shunt Resistor**.
- The voltage drop across the shunt is directly proportional to the current ($V = I \times R$).
- This small voltage drop is amplified by the internal DC amplifier and displayed.
- **Advantage:** Electronic amplification allows the use of very small shunt resistors, minimizing the 'burden voltage'.



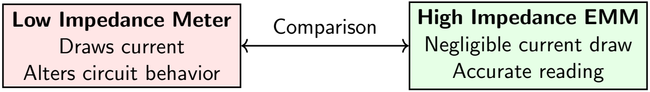
Measuring Resistance

- The meter uses its internal battery to supply a **Constant Current Source** to the unknown resistor.
- According to Ohm's Law ($V = I \times R$), the voltage developed across the resistor is directly proportional to its resistance.
- This voltage is measured by the internal voltmeter circuit.
- The scale is calibrated to read directly in Ohms (Ω).
- **Advantage:** Provides linear scales in digital versions, unlike non-linear scales of passive analog ohmmeters.



The Concept of High Input Impedance

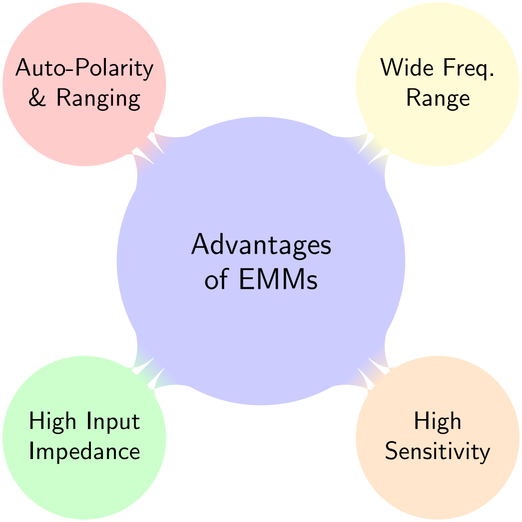
- **Input Impedance:** The effective resistance the measuring instrument presents to the circuit.
- **Loading Effect:** A low impedance meter draws current, lowering the voltage it is trying to measure.
- High impedance (e.g., 10 MΩ) draws negligible current.
- **EMM Advantage:** EMMs use FETs (Field Effect Transistors) at the input stage. This provides an input impedance that remains constantly high across all voltage ranges.



Analog vs. Digital Electronic Multimeters

Analog Electronic Multimeter	**Digital Multimeter (DMM)**
Uses electronic amplifiers but displays the result on a PMMC pointer scale.	Uses electronic signal conditioning followed by an Analog-to-Digital Converter (ADC).
Good for observing fluctuating signals.	Displays results on an LCD/LED screen.
Subject to parallax errors.	Eliminates parallax error and offers higher resolution.

Advantages of Electronic Multimeters



Additional Advantages & Features

- **Overload Protection:** Electronic circuits can include fast-acting protection mechanisms against overvoltage.
- **Multi-functionality:** Modern EMMs include capacitance, frequency, temperature, and transistor hFE testing.
- **No Parallax Error:** In digital variants, the exact number is displayed.
- **Improved Accuracy:** Reduced frictional errors (in DMMs) and temperature compensation ensure reliable readings.

Capacitance | Frequency | Temperature | Transistor hFE

Applications of Electronic Multimeters

- **Electronics Repair & Diagnostics:** Troubleshooting PCBs, testing transistors and diodes.
- **Automotive Electronics:** Checking battery health, sensor outputs, and alternator ripples.
- **Research & Laboratory:** Precision measurements requiring high accuracy and minimal circuit disturbance.
- **Industrial Maintenance:** Checking power supplies, control circuits, and motor drives.

Quick Check

Question 1

What is the primary advantage of using an FET at the input stage of an electronic multimeter?

- A) To decrease measurement time
- B) To provide high input impedance
- C) To increase power consumption
- D) To convert DC to AC

Question 2

How does an electronic multimeter measure resistance?

- A) By passing a constant current and measuring the voltage drop
- B) By measuring the frequency of the resistor

Summary & Next Lecture Preview

Summary:

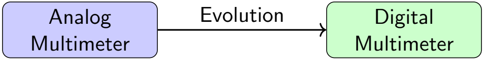
- Electronic multimeters use active amplifiers to enhance measurement.
- High input impedance prevents the 'loading effect'.
- Capable of precise measurement of V, I, and R with high sensitivity.
- EMMs offer distinct advantages like overload protection and multi-functionality over traditional passive VOMs.

Next Lecture Preview:

- **Digital Panel Meters:** Principles for measuring voltage, current, and frequency digitally.

Recap: Electronic Multimeters

- High Input Impedance: Reduces loading effect on the circuit being tested, typically $> 10\ M\Omega$.
- Active Components: Uses op-amps, FETs, or BJTs for signal amplification and conditioning.
- Versatility: Can measure AC/DC voltage, current, and resistance accurately.
- Limitations: While versatile, analog electronic multimeters still require reading a scale, introducing human reading errors (parallax).



Agenda

- ➢ Introduction to Digital Panel Meters (DPM)
- ➢ Basic Block Diagram of a DPM
- ➢ The Heart of DPM: Dual-Slope A/D Converter
- ➢ Voltage Measurement with DPMs
- ➢ Current Measurement with DPMs
- ➢ DPM Specifications (Resolution and Digits)
- ➢ Advantages of Digital Panel Meters

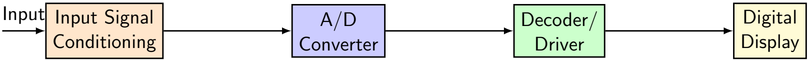
Introduction to Digital Panel Meters (DPMs)

- Definition: A Digital Panel Meter (DPM) is an electronic instrument that measures an electrical parameter (like voltage or current) and displays the value digitally on a screen.
- Form Factor: Designed to be mounted on instrumentation panels, control boards, or equipment chassis.
- Core Difference: Replaces the moving coil mechanism of analog meters with solid-state analog-to-digital conversion and digital displays (LED/LCD).
- Primary Purpose: Accurate, easily readable, single-function monitoring.

Block Diagram of a Basic DPM

The generic architecture of a DPM consists of:

- Input Signal Conditioning: Attenuates (scales down) or amplifies the input signal to a standard range (e.g., 0 to 2V DC).
- Analog-to-Digital Converter (ADC): Converts the conditioned analog DC voltage into a proportional digital signal.
- Decoder / Display Driver: Translates the digital code (often BCD) into signals that can light up specific segments.
- Digital Display: 7-segment LED or LCD displaying the numerical value.



The Role of the A/D Converter

- The ADC is the most critical component of any digital instrument.
- Function: It translates a continuous analog voltage into a discrete digital number.
- Common Types in DPMs:
  - Dual-Slope Integrating ADC: Most widely used due to its excellent noise rejection and high accuracy at low speeds.
  - Successive Approximation ADC: Used where higher speed is required, but less common in standard DPMs.
- The accuracy and resolution of the DPM depend entirely on the ADC.

Dual-Slope Integration ADC Principle

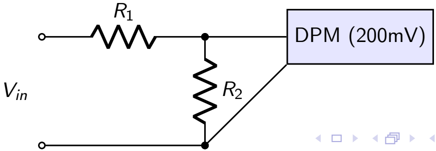
How it works (in two phases):

- Run-up (Integration) Phase: The unknown input voltage (V_{in}) is integrated for a fixed, predetermined time (T_1). The capacitor charges to a level proportional to V_{in}.
- Run-down (De-integration) Phase: The input is switched to a known internal reference voltage ($-V_{ref}$). The capacitor discharges back to zero.
- Measurement: The time taken to discharge (T_2) is measured using a digital counter.

$$V_{in} = V_{ref} \times \left(\frac{T_2}{T_1} \right)$$

Voltage Measurement using DPM (Digital Voltmeter)

- Direct DC Voltage Measurement: If measuring a low DC voltage (e.g., $0 - 200$ mV), it can often be fed directly to the ADC.
- High DC Voltage: Requires a Voltage Divider (Attenuator) network.
- Example: To measure 200V with a 200mV ADC, the input must be attenuated by a factor of 1000.
- AC Voltage Measurement: Requires an internal Precision Rectifier (AC-to-DC converter) before the signal reaches the ADC.
- The DPM is usually calibrated to display the RMS value of a sine wave.



Current Measurement using DPM

- A standard DPM ADC only responds to voltage.
- Ohm's Law Application: To measure current, the DPM measures the voltage drop across a known, precision resistor called a Shunt.

$$V_{drop} = I_{unknown} \times R_{shunt}$$

- Internal vs. External Shunts:
  - Low Currents (mA range): The shunt resistor is internal to the DPM.
  - High Currents (Amps range): An external heavy-duty shunt is used. The DPM measures the millivolt drop across this external shunt.

Understanding Shunt Resistors

- Material: Made from materials like Manganin, which has a very low temperature coefficient of resistance (resistance doesn't change with heat).
- Value: Extremely low resistance (e.g., 0.01 Ω or 0.001 Ω) to minimize the voltage drop in the actual circuit (burden voltage).
- Example: A 50A/50mV external shunt means when 50A flows through it, exactly 50mV drops across it.
- The DPM is set to the 200mV range but its display is calibrated to show "50.0" when it receives 50mV.

DPM Specifications: Digits

- DPMs are classified by the number of digits they can display.
- Full Digit: Can display any number from 0 to 9.
- Half Digit (1/2 digit): The most significant digit (leftmost) can only display a '1' or be blank (or sometimes '0').
- Example: $3\frac{1}{2}$ Digit DPM
  - Three full digits $(0 - 9)$.
  - One half digit (blank or 1).
  - Maximum display value: 1999.
- If measuring a 20V range, the max reading is 19.99V.

DPM Specifications: Resolution and Accuracy

- Resolution: The smallest change in the input signal that produces a change in the display.
- For a $3\frac{1}{2}$ digit meter on a 200mV range, max reading is 199.9mV.
- Resolution $=$ 0.1mV (or 100 μV).
- Accuracy: How close the displayed value is to the true value.
- Specified as: $\pm$(% of reading $+$ number of counts).
- Example: $\pm(0.5\% + 1$ digit).

Range	**Max Display**	**Resolution**
200 mV	199.9 mV	0.1 mV
2 V	1.999 V	1 mV
20 V	19.99 V	10 mV

Advantages of Digital Panel Meters

Compared to Analog Panel Meters:

- No Parallax Error: Digits are read directly, eliminating observation errors from different angles.
- Higher Accuracy & Resolution: Easily achieves accuracies of 0.1% or better, far exceeding analog meters.
- Faster Reading: Immediate numerical value without waiting for a pointer to settle (no mechanical inertia).
- Data Processing: Digital outputs (like BCD or RS-232/RS-485) can be sent directly to computers or PLCs for logging and control.
- Durability: No delicate moving coils or springs to break under vibration.

Question

A DPM is rated as a $4\frac{1}{2}$ digit meter. What is the maximum numerical value it can display?

A) 4999

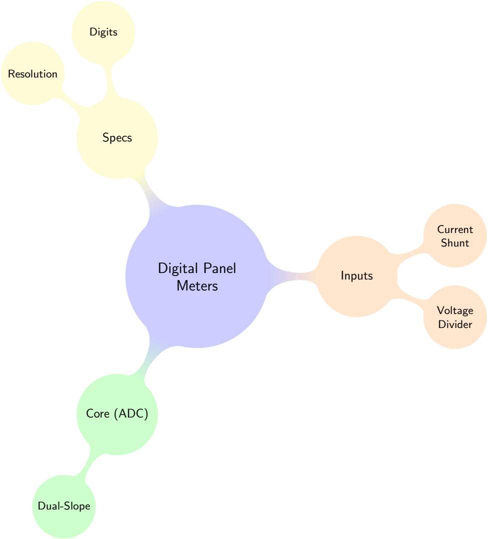
B) 9999

C) 19999

D) 40000

Correct Answer: C) 19999

Summary



Digits
Resolution
Specs
Digital Panel Meters
Inputs
Current Shunt
Voltage Divider
Core (ADC)
Dual-Slope

Next Lecture Preview

Topic: Digital Panel Meters - Frequency Measurement

In the next lecture, we will complete our study of DPMs by exploring how digital meters measure frequency.

We will cover:

- The block diagram of a Digital Frequency Meter.
- Time bases and gating principles.
- Signal counting mechanisms.

Digital Panel Meters: Frequency Measurement

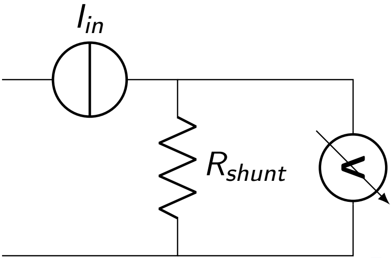
- Course: Electronics Circuit Network & Measurement (DI03011031)
- Unit 4: Basic Electrical Parameter measurement
- Lecture 31: Digital Panel Meters: Frequency Measurement

Agenda

- Recap of Voltage/Current DPMs
- Introduction to Digital Frequency Measurement
- Basic Principle & Formula
- Block Diagram of a Digital Frequency Meter
- Subsystems (Schmitt Trigger, Time Base, Main Gate)
- Timing Diagram
- Low Frequency Measurement (Period Measurement)
- Sources of Error

Recap: Digital Voltage & Current Meters

- **Function:** Convert continuous analog voltage/current into discrete digital numbers.
- **Core Component:** Analog-to-Digital Converter (ADC).
- **Dual-Slope ADC:** The most common ADC for DPMs due to its excellent noise rejection and accuracy.
- **Current Measurement:** Achieved by passing the unknown current through a known precision shunt resistor and measuring the resulting voltage drop.



Introduction to Digital Frequency Measurement

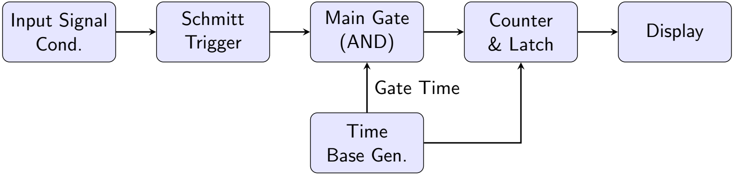
- **Frequency (f):** The number of cycles of a periodic waveform that occur in one second. Measured in Hertz (Hz).
- **Digital Approach:** Instead of using analog tuned circuits or bridges, digital meters literally count the number of cycles over a precise, known time interval.
- **Advantages:**
  - Extremely high accuracy and resolution.
  - Direct numerical readout (no parallax error).
  - Wide frequency range (from fractions of a Hz to GHz).

Basic Principle of Frequency Measurement

- **The Core Equation:** $f = \frac{N}{t}$
- $f =$ Frequency of the unknown signal.
- $N =$ Number of pulses (cycles) counted.
- $t =$ Known time interval (gate time).
- **Example:** If we keep a gate open for precisely $t = 1$ second, and we count $N = 5000$ pulses, then the frequency is exactly 5000 Hz.
- If $t = 0.1$ second and $N = 500$, $f = 5000$ Hz.

Block Diagram of a Digital Frequency Meter

- The Digital Frequency Meter (DFM) consists of the following key blocks:

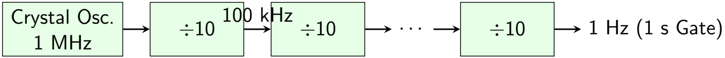


Signal Conditioning & Schmitt Trigger

- **Signal Conditioning:** The unknown input signal (f_x) may have a very low amplitude or be too high. Amplifiers or attenuators normalize the voltage level.
- **Schmitt Trigger:**
  - Digital counters only understand distinct ON (1) and OFF (0) pulses.
  - The Schmitt trigger converts any periodic analog signal (sine wave, triangle wave, complex wave) into a sharp square wave.
  - The frequency of the output square wave is identical to the input frequency.

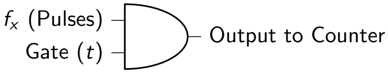
The Time Base Generator

- **Purpose:** To generate a highly accurate and stable time interval (t) to control the Main Gate.
- **Components:**
  - **Crystal Oscillator:** Usually generates a high frequency (e.g., 1 MHz) with excellent temperature and aging stability.
  - **Decade Dividers:** A series of divide-by-10 flip-flop circuits.
- **Operation:** A 1 MHz signal has a period of $1\mu s$. Passing it through six decade dividers (10^6) produces a signal with a 1 second period.



The Main Gate (AND Logic)

- **Function:** Acts as a switch that only lets the input signal pass through for the precise duration of the time base pulse.
- **Implementation:** Usually a simple 2-input AND gate.
- **Inputs:**
  - The train of pulses from the Schmitt trigger (Unknown frequency f_x).
  - The gating pulse from the Time Base (Known time t).
- **Output:** Pulses only appear at the output when the Gating pulse is HIGH (Logic 1).

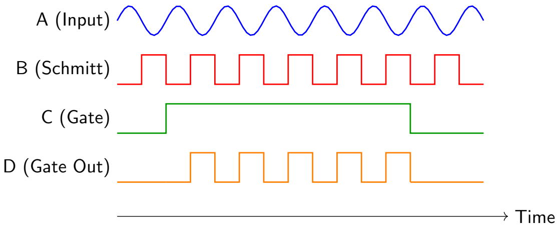


Decade Counter, Latch, and Display

- **Decade Counter:** A series of cascaded BCD (Binary Coded Decimal) counters. They count the pulses emerging from the Main Gate.
- **Latch (Memory):**
  - Captures the final count at the exact moment the Main Gate closes.
  - Holds this value stable for the display while the counter is reset to zero to start the next measurement.
- **Decoder/Display:** Converts the BCD data from the latch into signals that light up the 7-segment LED/LCD display.

Timing Diagram of a DFM

- **Waveform A (Input):** Analog sine wave (f_x).
- **Waveform B (Schmitt):** Continuous square wave of freq f_x.
- **Waveform C (Gate):** A precise rectangular pulse of duration t (e.g., 1 s).
- **Waveform D (Gate Out):** A discrete burst of square waves. The number of cycles equals the frequency if $t = 1$ s.
- **Waveform E (Latch/Reset):** Pulses to latch data and reset counter.



Challenge: Low Frequency Measurement

- **The Problem:** If the unknown frequency is very low (e.g., 2 Hz) and we use a 1 second gate time, the counter will only count 2 pulses.
- The resolution is terrible (2 ± 1 count error is a 50% error!).
- **The Solution: Period Measurement Mode.**
- Instead of counting unknown pulses for a known time, we count *known time base pulses* for the duration of *one unknown cycle*.
- We reverse the roles of the inputs at the Main Gate.

Sources of Error in Digital Frequency Meters

- Despite high accuracy, DFMs have three primary sources of error:
- ± 1 **Count Error (Gating Error):** The gating pulse is completely asynchronous (not synchronized) with the unknown input signal. The gate may open or close halfway through an input pulse, leading to a ± 1 uncertainty in the final count.
- **Time Base Error:** Inaccuracies in the crystal oscillator due to temperature variations, aging, or initial calibration errors.
- **Trigger Level Error:** Noise on the input signal can cause the Schmitt trigger to fire falsely, adding extra spurious counts.

Minimizing ±1 Count Error

- **Impact of ±1 count:**
- Percentage Error $= \frac{\pm 1}{N} \times 100\%$
- If N is small (low frequency), the error is huge.
- If N is large (high frequency), the error is negligible.
- **Mitigation:**
  - For high frequencies, use longer gate times (e.g., 10 s instead of 1 s) to increase N.
  - For low frequencies, use the Period Measurement technique.

Count (N)	**Error (±1)**	**Percentage Error**
10	1	10.0%
1000	1	0.1%
1000000	1	0.0001%

Quick Check

- **Question 1:** Which component in the DFM converts a sine wave into a square wave?
- **Question 2:** If the time base gate is set to 0.1 seconds, and the counter reads 1500, what is the frequency?
- **Question 3:** Why is Period Measurement preferred for low frequencies?

Summary

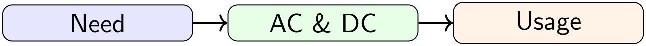
- Digital Frequency Meters measure frequency by counting cycles over a precise time interval ($f = N/t$).
- A Schmitt trigger prepares the analog signal by squaring it.
- A Crystal Oscillator and dividers provide a highly accurate time base (gate time).
- The Main Gate (AND gate) combines the input signal and the time base.
- ± 1 Count Error is inherent due to the asynchronous nature of the gate and input signals.

Next Lecture Preview

- **Topic: Clamp-on Ammeter, Electronic Wattmeter & Energy Meter**
- How to measure current without breaking the circuit.
- Working principle of the Clamp-on Ammeter (current transformer).
- Basics of Electronic Wattmeters for measuring power.
- Introduction to Energy Meters.

Agenda

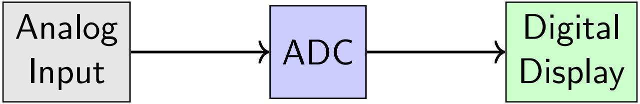
- Limitations of Traditional Ammeters
- Introduction to the Clamp-on Ammeter
- Working Principle: AC Current Measurement
- Working Principle: DC Current Measurement
- Step-by-Step Usage Guide
- Important Precautions
- Advantages and Applications



Previous Lecture Recap

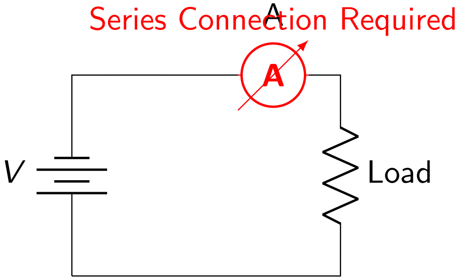
Digital Panel Meters (DPM)

- Convert analog inputs to digital form using ADCs (like Dual-Slope integration).
- Display readings numerically on LEDs or LCDs.
- High accuracy and lack of observational parallax errors.
- Used for voltage, current, and frequency monitoring on instrument panels.



Limitations of Traditional Current Measurement

- **Standard Ammeter Connection:** Must be connected in series with the load.
- **Circuit Interruption:** Requires breaking the circuit (cutting wires or disconnecting terminals) to insert the meter.
- **Safety Risks:** Exposes the user to live, potentially high-voltage conductors.
- **Downtime:** Requires shutting down the system before measurement.
- **Impractical for High Current:** Standard internal shunts cannot handle massive industrial currents.



What is a Clamp-on Ammeter?

Definition

An electrical test tool that combines a basic digital multimeter with a current sensor (the "clamp").

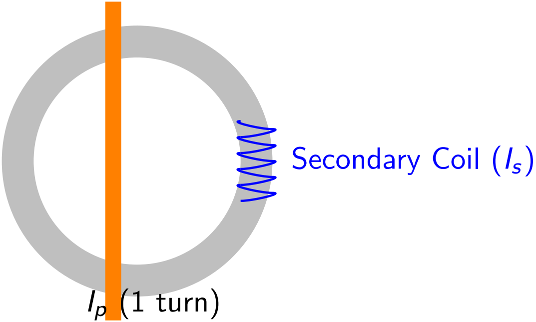
Key Feature: Non-intrusive Measurement

- Measures current without physical contact with live parts.
- No need to break the circuit or power down equipment.
- The hinged "jaw" opens and clamps around a single conductor.
- Primarily detects the magnetic field generated by the flowing current.

Working Principle: AC Clamp Meter

Based on Current Transformer (CT) Principle:

- Used exclusively for Alternating Current (AC).
- The conductor being measured acts as the **Primary Winding** ($N_1 = 1$ turn).
- The iron core jaws contain a **Secondary Winding** ($N_2 =$ multiple turns) wrapped around them.
- AC current in the conductor creates a varying magnetic flux.
- This flux induces an alternating current in the secondary winding.
- The induced current is proportional to the primary current.



The Current Transformer Equation

Transformation Ratio

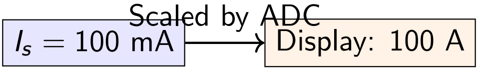
$$I_p \times N_p = I_s \times N_s$$

- I_p: Primary Current (current in the conductor)
- N_p: Primary Turns (always 1 for a clamped wire)
- I_s: Secondary Current (induced inside the meter)
- N_s: Secondary Turns (usually high, e.g., 1000)

Calculation:

$$I_s = \frac{I_p}{N_s}$$

Example: If $I_p = 100$ A and $N_s = 1000$, then $I_s = 0.1$ A (100 mA).



Why CT Clamp Meters Cannot Measure DC

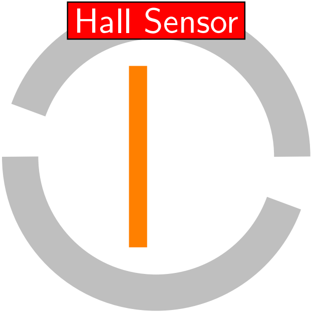
The Problem with DC:

- Transformers require a *changing* magnetic flux ($d\Phi/dt$) to induce a voltage/current.
- Direct Current (DC) flows continuously in one direction at a constant magnitude.
- A constant DC current produces a **static magnetic field**.
- Since $d\Phi/dt = 0$, no current is induced in the secondary winding.
- Therefore, a standard AC clamp meter will read **0.00 A** when clamped around a DC wire.

Working Principle: DC Clamp Meter

Based on the Hall Effect Principle:

- Capable of measuring **both AC and DC** current.
- The jaws are made of solid iron but contain a small air gap.
- A semiconductor **Hall Effect Sensor** is placed in this gap.
- The conductor's current (DC or AC) generates a magnetic field that concentrates in the iron core and passes through the sensor.
- The sensor generates a voltage proportional to the magnetic field strength.



Understanding the Hall Effect

How it works:

- A constant control current is passed through the thin semiconductor (Hall chip).
- When a magnetic field (from the measured wire) intersects the chip perpendicularly, it deflects the charge carriers.
- This charge accumulation creates a potential difference across the transverse sides of the chip.
- This is the **Hall Voltage** (V_H).

Hall Voltage Equation

$$V_H \propto B \times I_c$$

- B: Magnetic field strength (proportional to wire current)
- I_c: Control current (constant)

Step-by-Step: How to Use a Clamp Meter

1. **Select the Function:** Turn the dial to the appropriate current setting (AC or DC).
2. **Zero the Meter (DC only):** If measuring DC, press the "Zero" or "Rel" button to remove Earth's magnetic field interference.
3. **Open Jaws:** Press the trigger to open the clamp.
4. **Enclose ONE Conductor:** Place exactly **one** wire inside the jaws and release the trigger to close them tightly.
5. **Read Value:** View the current reading on the LCD display.
6. **Hold Feature:** Press "Hold" if working in a tight spot to freeze the reading.

Crucial Precaution: Clamp Only ONE Wire

The Most Common Mistake: Clamping a whole power cable (containing both Phase/Live and Neutral wires).

What happens?

- Current flows out through the Live wire and returns through the Neutral wire.
- Their magnetic fields are **equal in magnitude but opposite in direction**.
- The fields **cancel each other out**.
- The meter will read **0.00 Amps**.

Solution

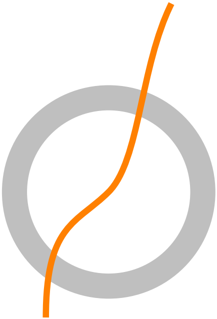
Use a line splitter if you cannot separate the individual wires of a cord.

Additional Measurement Techniques

Measuring Low Currents (Multiplier Technique):

- If a current is too small for the meter to read accurately (e.g., 0.5A on a 1000A scale).
- **Solution:** Loop the wire through the clamp jaws multiple times (e.g., 10 loops).
- **Result:** The meter reads 10 times the actual current (reads 5.0A).
- **Final Calculation:** Divide the reading by the number of loops ($5.0A/10 = 0.5A$).

Safety: Always wear appropriate PPE (Personal Protective Equipment) when working near live panels.



Looped Wire inside Clamp

Advantages & Applications

Advantages:

- **Safety:** Non-contact measurement isolates user from high voltages.
- **Convenience:** No need to shut down equipment or break circuits.
- **High Range:** Can easily measure hundreds or thousands of Amperes.

Applications:

- Troubleshooting industrial motors and compressors (inrush current).
- HVAC system diagnostics.
- Electrical panel load balancing.
- Automotive diagnostics (DC clamps for alternators).

Quick Check

Question 1: Why does an AC-only clamp meter read zero when placed on a DC circuit?

A) DC current produces no magnetic field.

B) Transformers require a changing magnetic flux ($d\Phi/dt$) to induce current.

C) DC voltage is too low.

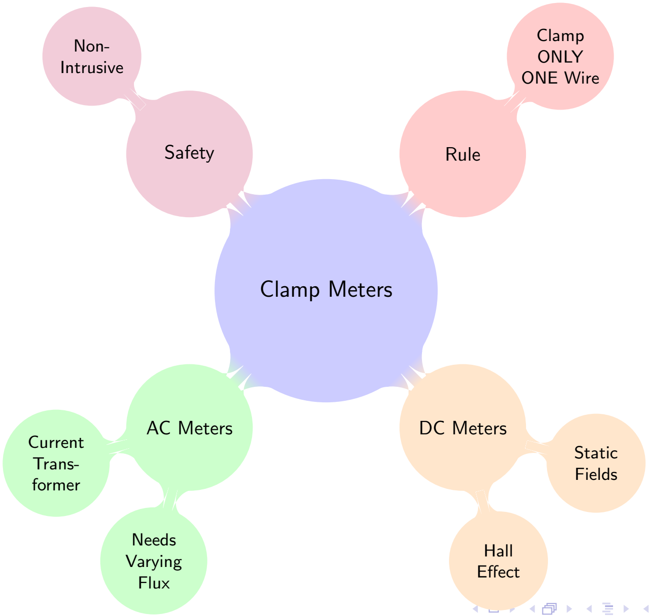
Question 2: What sensor is used to measure DC current in a modern clamp meter?

A) Current Transformer

B) Strain Gauge

C) Hall Effect Sensor

Summary



Non-
Intrusive
Safety
Clamp
ONLY
ONE Wire
Rule
Clamp Meters
AC Meters
Current
Trans-
former
Needs
Varying
Flux
DC Meters
Static
Fields
Hall
Effect

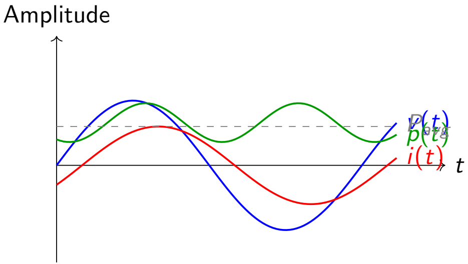
Preview for Next Lecture

Lecture 33: Electronic Wattmeter

- Measuring electrical power (Watts) rather than just current or voltage.
- Working principles of analog and electronic wattmeters.
- Electrodynamometer instruments.
- Application in measuring power consumption.

Concept of Power in AC Circuits

- Instantaneous power $p(t)$ is the product of instantaneous voltage and current: $p(t) = v(t) \times i(t)$.
- For sinusoidal steady-state, Average Power $P = V_{rms} \times I_{rms} \times \cos(\phi)$.
- $\cos(\phi)$ is the power factor, and ϕ is the phase angle between voltage and current.
- Accurate power measurement requires simultaneous sensing of both voltage and current and calculating their exact product.



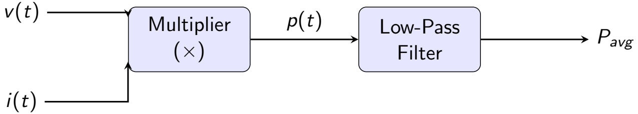
Need for Electronic Wattmeters

- Traditional Electrodynamometer Wattmeters have moving parts and heavy coils.
- They have a limited frequency response (mostly suitable for 50/60 Hz power frequencies).
- They consume relatively high power from the circuit being measured (loading effect).
- Electronic Wattmeters are required for high-frequency measurements, non-sinusoidal waveforms, and low power factor loads.
- They offer high input impedance for voltage sensing and very low resistance for current sensing.

Electrodynamometer	**Electronic Wattmeter**
Mechanical moving parts	Solid-state components
Narrow bandwidth (50 Hz)	Broad bandwidth (up to kHz/MHz)
High loading effect	Negligible loading
Poor at very low PF	Excellent at low PF

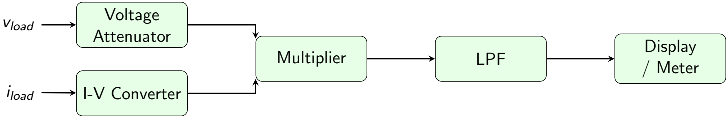
Basic Principle of Electronic Wattmeter

- The fundamental principle is analog multiplication of two signals representing load voltage $v(t)$ and load current $i(t)$.
- The product $p(t) = v(t) \times i(t)$ represents instantaneous power.
- Passing $p(t)$ through a Low-Pass Filter (LPF) or integrator yields the time-averaged DC value.
- This DC value is directly proportional to the Average Real Power P and is displayed on a meter.



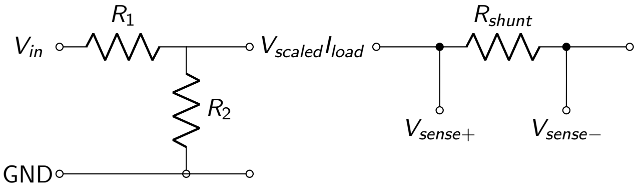
Block Diagram of Electronic Wattmeter

- **Input Attenuator:** Scales down the load voltage to a safe, measurable level (e.g., $\pm$5V).
- **I-V Converter:** Uses a shunt resistor to convert load current into a proportional voltage signal.
- **Multiplier Module:** Multiplies the scaled voltage and current signals.
- **Low-Pass Filter (LPF):** Extracts the DC component (average power) from the multiplier output.
- **Amplifier and Display:** Amplifies the DC signal to drive a Digital Panel Meter or analog gauge.



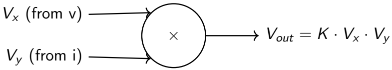
Input Scaling & Sensing

- **Voltage Sensing:** Typically a resistive voltage divider network. It must be frequency compensated to avoid phase shift errors.
- **Current Sensing:** For low currents, a precision shunt resistor is used ($V_i = I_{load} \times R_{shunt}$).
- For isolation or high currents, a wideband current transformer (CT) or Hall effect sensor is used.
- Crucial requirement: The sensing circuits must not introduce any differential phase shift between the voltage and current signals.



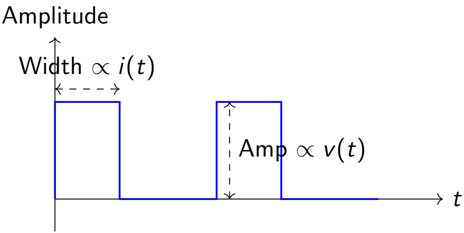
Analog Multiplier Circuit

- The Multiplier is the heart of the analog electronic wattmeter.
- Produces an output voltage $V_{out} = K \times V_x \times V_y$, where K is a scaling constant.
- Common techniques:
  - Quarter-Square Multiplier (using squaring circuits).
  - Variable Transconductance Multiplier (Gilbert Cell).
  - Time-Division Multiplier (TDM/PWM).



Time-Division Multiplier (TDM) Principle

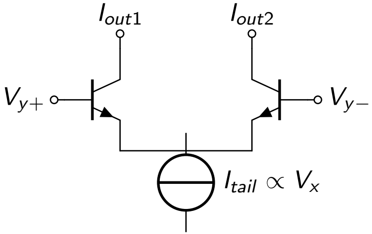
- TDM is widely used in high-precision electronic wattmeters.
- Generates a rectangular pulse train.
- The amplitude of the pulse is proportional to one input (e.g., Voltage $v(t)$).
- The duty cycle of the pulse is proportional to the other input (e.g., Current $i(t)$).
- The average value of this pulse train is exactly proportional to the product $v(t) \times i(t)$.



Variable Transconductance Multiplier

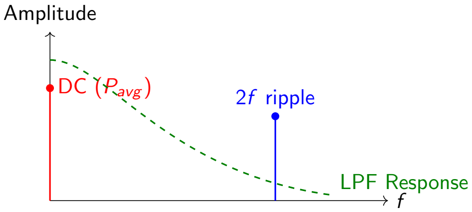
- Utilizes the principle that the transconductance (g_m) of a differential BJT pair is proportional to its emitter tail current.
- Input voltage V_x controls the tail current.
- Input V_y is applied to the differential base inputs.
- The output differential current is proportional to $V_x \times V_y$.
- Often implemented as a Gilbert Cell, offering very wide bandwidth (MHz range).

$$\Delta I_{out} \propto V_x \cdot V_y$$

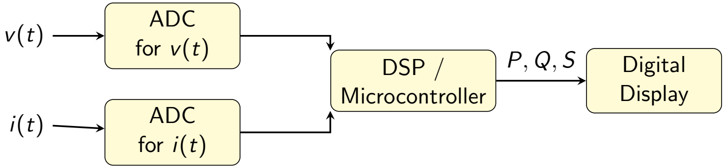


The Averaging Circuit (LPF)

- The output of the multiplier contains a DC component (Real Power) and an AC component at twice the line frequency.
- A Low-Pass Filter (active RC filter with an Op-Amp) extracts the DC component.
- The cutoff frequency of the LPF must be well below the fundamental frequency (e.g., < 10 Hz for a 50 Hz system).
- This ensures a stable, ripple-free reading on the display.

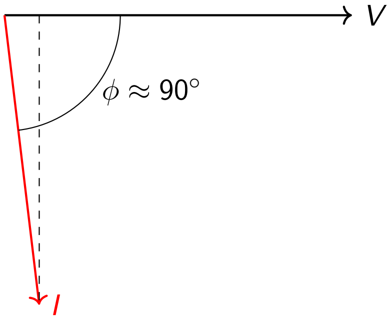


- Modern wattmeters use digital techniques instead of analog multipliers.
- Fast Analog-to-Digital Converters (ADCs) sample $v(t)$ and $i(t)$ simultaneously at high rates.
- A microcontroller or DSP multiplies the sampled pairs mathematically.
- Calculation: $P = \frac{1}{N}\sum_{k=1}^{N}(v_k \times i_k)$.
- Can also calculate Reactive Power (Q), Apparent Power (S), and Harmonics.



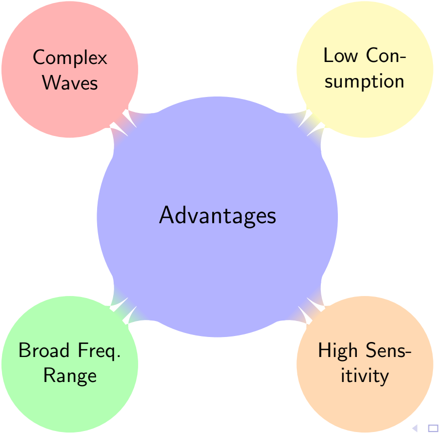
Measuring Low Power Factor

- Electrodynamometer wattmeters struggle at low power factors (highly inductive/capacitive loads) because $\cos(\phi)$ is near zero.
- Small internal phase errors cause massive percentage errors in reading.
- Electronic wattmeters feature precision phase-compensated sensing and accurate multipliers.
- They provide much higher accuracy even at power factors below 0.1.



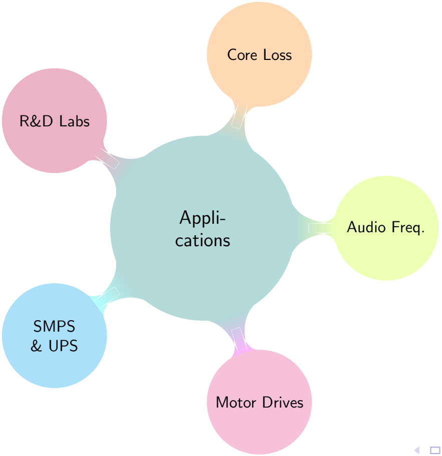
Advantages of Electronic Wattmeter

- **Broad Frequency Range:** Can measure power from DC up to hundreds of kHz.
- **High Sensitivity:** Capable of measuring very low power levels (milliwatts or microwatts).
- **Low Power Consumption:** Consumes negligible power from the circuit under test.
- **Complex Waveforms:** Accurately measures power of non-sinusoidal waveforms with high harmonic content.



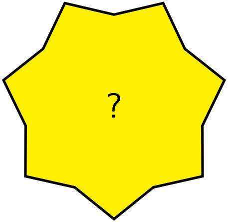
Applications

- Power measurement in Switched-Mode Power Supplies (SMPS) and UPS systems.
- Analyzing efficiency of Variable Frequency Drives (VFDs) and motor controls.
- Audio frequency power measurement.
- Core loss testing in magnetic materials and transformers at various frequencies.
- General-purpose precision power measurement in R&D labs.



Questions

1. What is the primary function of the multiplier in an electronic wattmeter?
2. Why is a Low-Pass Filter necessary after the multiplier stage?

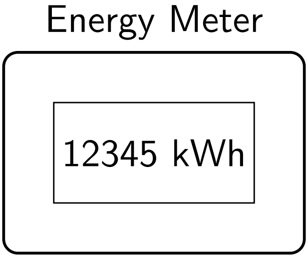


Summary

- Electronic wattmeters solve the frequency and loading limitations of mechanical wattmeters.
- They rely on an analog or digital multiplier to find the instantaneous power $v(t) \times i(t)$.
- A Low-Pass Filter is used to average the signal to find Active Power P.
- Common multiplier technologies include Time-Division and Transconductance (Gilbert cell).
- They are essential for analyzing high-frequency, non-sinusoidal, and low power factor circuits.

Next Lecture Preview

- Introduction to Electrical Energy (kWh)
- Energy Meter Construction
- Working Principle of Electromechanical Induction Energy Meter
- Electronic/Digital Energy Meters



Lecture Agenda

- Power vs. Energy Concept
- Induction Type Energy Meter: Introduction
- Constructional Details (Driving, Moving, Braking systems)
- Working Principle and Torque Equations
- Errors in Energy Meters and Compensations
- Creeping and Its Prevention
- Digital / Electronic Energy Meters

Electrical Power vs. Electrical Energy

Electrical Power (P)	**Electrical Energy (E)**
Rate at which energy is transferred	Total amount of work done over time
Measured in Watts (W) or Kilowatts (kW)	Measured in Watt-hours (Wh) or Kilowatt-hours (kWh)
$P = V \times I \times \cos(\phi)$	$E = \int (V \times I \times \cos(\phi))\, dt$

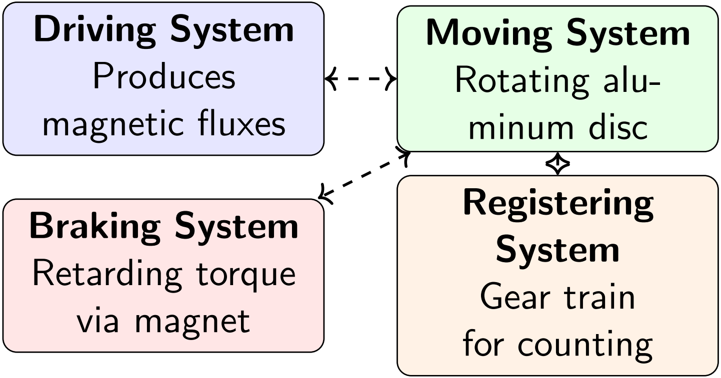
Key Conversion

1 Unit of Electricity = 1 kWh = 1000 Watts consumed for 1 hour.

Introduction to Induction Type Energy Meter

- Most common type of AC energy measuring instrument used in domestic and industrial applications (legacy).
- Works on the principle of electromagnetic induction.
- Continuously measures instantaneous voltage and current, multiplies them, and integrates them with respect to time.
- **Key Component**: An aluminum disc placed between two electromagnets.
- Not suitable for DC measurement (induction requires alternating flux).

The induction type single-phase energy meter consists of four main systems:



The Driving System

Shunt Magnet (Voltage Coil):

- Connected in parallel.
- Many turns, fine wire.
- Highly inductive; I lags V by $\approx 90^\circ$.
- Flux $\Phi_{sh} \propto V$.

Series Magnet (Current Coil):

- Connected in series.
- Few turns, thick wire.
- Flux $\Phi_{se} \propto I$.

Copper Shading Bands

Used on the central limb of the shunt magnet to adjust the phase angle exactly to 90°.

Moving System:

- Consists of a light aluminum disc mounted on a vertical spindle.
- Positioned in the air gap between series and shunt magnets.
- Eddy currents induced by Φ_{sh} and Φ_{se} produce driving torque.

Braking System:

- Uses a permanent magnet positioned near the edge of the disc.
- Disc cutting permanent flux induces eddy currents.
- Creates braking torque proportional to speed (Lenz's Law).
- Moving magnet inward/outward adjusts torque.

Registering (Counting) System

- Translates the mechanical rotation of the disc into electrical energy units (kWh).
- Spindle has a worm gear that engages with a pinion on the gear train.
- Gear ratios designed such that specific revolutions = 1 kWh.

Meter Constant (K)

$K = \frac{\text{Number of revolutions}}{\text{Energy in kWh}}$

Typically expressed as rev/kWh (e.g., 600 rev/kWh).

Working Principle

- Alternating fluxes Φ_{sh} and Φ_{se} pass through the aluminum disc.
- Fluxes induce alternating eddy currents I_{esh} and I_{ese} in the disc.
- Interaction (Φ_{sh} with I_{ese}, and Φ_{se} with I_{esh}) produces net driving torque (T_d).
- Steady conditions: $T_d = T_b$ (Braking torque).
- Disc speed (N) is directly proportional to active power (P).
- Total revolutions over time t equal total energy consumed.

Torque Equations

Driving Torque (T_d):

- $T_d \propto \Phi_1 \Phi_2 \sin(\beta) \cos(\alpha)$
- With $\Delta = 90^\circ$, $T_d \propto VI \cos(\phi)$
- $T_d = K_1 \times P$

Braking Torque (T_b):

- $T_b = K_2 \times N$

Steady State:

- $T_d = T_b \Rightarrow K_1 P = K_2 N \Rightarrow N \propto P$
- $\int N\,dt \propto \int P\,dt \Rightarrow \text{Total Rotations} \propto \text{Energy}$

Errors in Induction Energy Meters

Error Type	Cause & Compensation
Phase Error	Shunt flux must lag V by exactly 90°. Fixed by adjusting copper shading bands.
Speed Error	Variation in brake magnet strength. Fixed by adjusting brake magnet position.
Friction Error	Mechanical friction causes under-registration at light loads. Compensated by shading loop on central limb.
Temperature Error	Resistance changes in coils/magnets. Compensated by temperature-sensitive magnetic shunts.

What is Creeping?

- Slow, continuous rotation of the disc when there is no load current, but voltage is applied.

Causes:

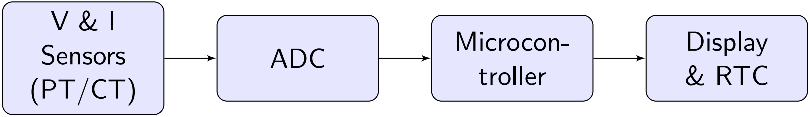
- Over-compensation for friction.
- Stray magnetic fields or high supply voltage.

Prevention (Anti-Creep Mechanisms):

- **Opposite Holes**: Two diametrically opposite holes drilled in the disc. Distorts flux and creates a locking torque.
- **Small Iron Piece**: Attached to disc edge, attracted to brake magnet.

Digital / Electronic Energy Meter

- Replaces mechanical parts with solid-state electronics. High accuracy, no creeping.



- **Sensors**: Scale down V and I.
- **ADC**: Converts to digital signals.
- **Microcontroller**: Multiplies and integrates to get energy.

Advantages of Digital Energy Meters

- **High Accuracy**: Typically Class 1.0 or better; unaffected by wear or friction.
- **Tamper Detection**: Can detect reverse current, magnetic interference, or bypassed neutral wires.
- **Multi-parameter Display**: Shows V, I, kW, kVA, PF, and maximum demand alongside kWh.
- **Time of Use (TOU) Billing**: RTC allows different tariff rates for peak/off-peak hours.
- **Communication**: Supports optical ports, RS485, RF, or GSM for Automatic Meter Reading (AMR/AMI).

Quick Check / Review Questions

- What is the fundamental difference between what a wattmeter measures and an energy meter measures?
- Which part of the induction meter provides the 90° phase shift for the shunt magnet?
- How is the speed of the aluminum disc controlled in an induction energy meter?
- What is ‘creeping’ and how is it prevented?
- Name two advantages of an electronic energy meter over an induction type.

Summary & Next Lecture Preview

Summary:

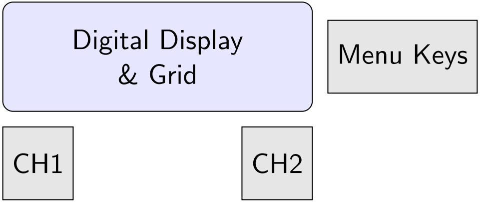
- Energy is the time integration of electrical power ($E = \int P\,dt$).
- Induction meters use driving, moving, braking, and registering systems.
- Creeping and friction errors are resolved mechanically.
- Digital meters use ADCs and DSP for high accuracy and anti-tamper.

Next Lecture: Digital Storage Oscilloscope (DSO)

- Focus on its construction, block diagram, working principle, and advantages.

Digital Storage Oscilloscope: Construction & Block Diagram

- Basic Electrical Parameter Measurement
- Topic: DSO Construction & Block Diagram
- Course: Electronics Circuit Network & Measurement (DI03011031)
- Focus:
  - Transitioning from Analog to Digital Measurement.
  - Fundamental Architecture of DSO.
  - Functional blocks and internal signal flow.



Agenda

- Review: Limitations of Analog Oscilloscopes (CRO)
- Introduction: What is a Digital Storage Oscilloscope (DSO)?
- Core Concept: Sampling and Quantization
- Deep Dive: DSO Block Diagram
- Sub-System Analysis:
    - Input Signal Conditioning
    - Analog-to-Digital Converter (ADC)
    - Acquisition Memory and Processing
    - Display System (DAC & Screen)
- Quick Check & Summary

Limitations of Analog Oscilloscopes (CRO)

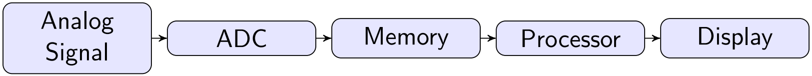
Before exploring DSOs, we must understand the shortcomings of Cathode Ray Oscilloscopes:

- **Transient Signal Capture:** Cannot capture or hold one-shot transient events effectively; the phosphor glow fades quickly.
- **Pre-trigger Viewing:** Impossible to view events that occurred *before* the trigger point.
- **Storage Constraints:** Relies on special storage CRTs (analog storage) which are expensive, fragile, and have limited retention time.
- **Signal Processing:** Lacks ability to perform mathematical operations (FFT, integration, differentiation) on the captured waveforms.
- **Data Transfer:** Cannot interface easily with computers for data logging or analysis.

Introduction to Digital Storage Oscilloscopes (DSO)

A Digital Storage Oscilloscope (DSO) represents an analog signal by sampling it and converting the voltage levels into digital information.

- **Digital Conversion:** Uses an Analog-to-Digital Converter (ADC) to digitize the input waveform.
- **Digital Memory:** Stores the digitized waveform data in solid-state memory (RAM).
- **Reconstruction:** Uses a microprocessor and a DAC or direct digital rendering to reconstruct the waveform on a display.
- **Infinite Storage:** The signal remains in memory as long as power is applied or until overwritten.



Core Concepts: Sampling and Nyquist Criterion

To accurately represent an analog signal digitally, the DSO must sample it correctly.

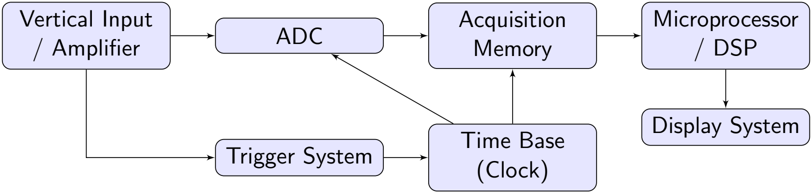
- **Sampling:** Taking periodic 'snapshots' of the continuous analog signal.
- **Nyquist-Shannon Sampling Theorem:** The sampling frequency (f_s) must be at least twice the highest frequency component (f_{max}) of the signal.

$$f_s \geq 2f_{max}$$

- **Aliasing:** If $f_s < 2f_{max}$, the reconstructed signal will appear as a false, lower-frequency wave (alias).
- **Practical Application:** Modern DSOs sample at 5 to 10 times the signal frequency for accurate shape reconstruction (e.g., 100 MHz scope may have 1 GSa/s sampling rate).

DSO Architecture: The Complete Block Diagram

The architecture of a DSO consists of several interacting blocks:



Analog Front-End (AFE)

- **Function:** Conditions the incoming analog signal to a level suitable for the ADC.
- **Attenuator:** Reduces high-voltage signals (e.g., switching from 10V/div to 50V/div).
- **Amplifier:** Boosts weak signals to utilize the full dynamic range of the ADC.
- **Impedance Matching:** Typically provides 1 MΩ or 50 Ω input impedance to prevent circuit loading.
- **Bandwidth Limit:** Often includes a low-pass filter to remove high-frequency noise and prevent aliasing before the signal reaches the ADC.

Block 2: Analog-to-Digital Converter (ADC)

The Core of the DSO

- **Function:** Converts continuous analog voltage into discrete digital values (binary numbers) at intervals dictated by the clock.
- **Resolution:** Defined by the number of bits (typically 8-bit, 10-bit, or 12-bit).
  - An 8-bit ADC divides the vertical range into $2^8 = 256$ discrete levels.
  - A 12-bit ADC provides $2^{12} = 4096$ levels, offering significantly higher vertical precision.
- **Type:** DSOs primarily use Flash ADCs due to their extremely high speed, capable of operating at Giga-samples per second (GSa/s).

Storing the Waveform

- **Function:** A high-speed digital memory (RAM) that continuously stores the binary data stream from the ADC.
- **Memory Depth:** Represents the maximum number of sample points the DSO can store in a single acquisition. Example: 1 Mpts.
- **Relationship:**

$$\text{Capture Time} = \frac{\text{Memory Depth}}{\text{Sample Rate}}$$

- **Circular Buffer:** Memory operates as a circular buffer. Data is continuously written, and the oldest data is overwritten until a trigger event stops the acquisition.

Block 4: Trigger and Control Logic

Synchronizing the Capture

- **Trigger Circuit:** Monitors the incoming analog signal (or digitized data) to identify a specific event (e.g., rising edge crossing a 2V threshold).
- **Pre-trigger viewing:** Because data is constantly flowing into the circular memory buffer, when the trigger occurs, the memory already holds data from *before* the event.
- **Time Base (Clock):** Provides the precision timing pulses to the ADC and memory. It dictates the horizontal scale (Time/Div) on the screen.

The Digital Brain

- **Function:** Coordinates all sub-systems, manages the user interface, and processes the raw digital data.
- **Signal Processing (DSP):**
  - Waveform math: addition, subtraction, multiplication.
  - Fast Fourier Transform (FFT) for frequency domain analysis.
  - Digital filtering (Low-pass, High-pass).
- **Measurements:** Automatically calculates parameters like Peak-to-Peak voltage, Frequency, Period, Rise Time, and Duty Cycle directly from the digital arrays.

Reconstructing the Signal

- **Modern Displays:** Use high-resolution LCD or OLED screens.
- **Rendering:** The microprocessor maps the memory array to pixel coordinates on the screen.
- **Interpolation:** If there are few sample points between pixels, the processor uses algorithms (like $\sin(x)/x$ interpolation or linear interpolation) to draw smooth lines connecting the dots.
- **User Interface:** Overlays grids (graticules), measurement texts, menus, and multiple color-coded channels on the same screen.

DSO Construction: Physical Front Panel

The physical construction of a DSO front panel is organized logically into functional groups:

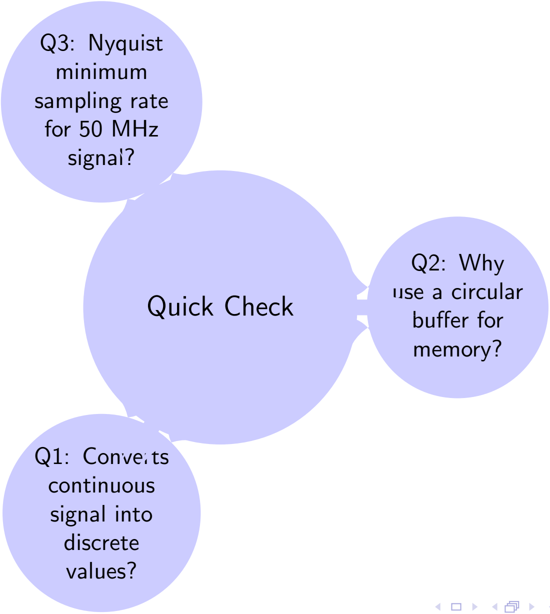
- **Vertical Controls:** Independent Volts/Div knobs and position controls for each channel.
- **Horizontal Controls:** A shared Time/Div knob and horizontal position control.
- **Trigger Controls:** Trigger level knob, edge selection, and source selection.
- **Menu & Math Keys:** Buttons to access FFT, cursor measurements, automated measurements, and display settings.
- **Interfaces:** USB and LAN ports for PC connectivity, firmware upgrades, and saving data.

Sampling Techniques in DSO

DSOs employ different sampling methods based on the signal type:

- **Real-Time Sampling:**
  - The entire waveform is captured in a single trigger event.
  - Required for single-shot, transient, or non-repetitive signals.
  - Limited by the maximum sampling rate of the ADC.
- **Equivalent-Time Sampling:**
  - Reconstructs a waveform by taking a few samples from multiple repetitive cycles.
  - Achieves an artificially high 'effective' sample rate (e.g., 100 GSa/s).
  - Strict Requirement: The signal must be perfectly periodic and stable.

Quick Check


Q3: Nyquist minimum sampling rate for 50 MHz signal?
Quick Check
Q2: Why use a circular buffer for memory?
Q1: Converts continuous signal into discrete values?

Summary

- DSOs digitize analog signals using ADCs and store them in memory for analysis and display.
- The core block diagram consists of: Analog Input $\rightarrow$ ADC $\rightarrow$ Memory $\rightarrow$ DSP $\rightarrow$ Display.
- ADC resolution dictates vertical precision; Sampling Rate dictates horizontal (timing) precision and bandwidth.
- Memory depth determines how long a signal can be captured at a high sample rate.
- DSOs offer massive advantages in transient capture, pre-trigger viewing, and digital signal processing (FFT, automated measurements) compared to analog CROs.

Preview for Next Lecture

Next Topic: DSO Working & Advantages

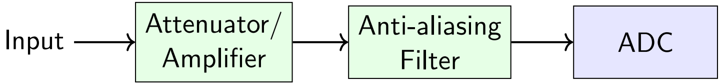
In the upcoming lecture, we will build upon this architectural knowledge and discuss:

- Detailed operational workflow of the DSO.
- Comprehensive advantages of DSOs over analog CROs.
- Practical applications and parameter measurements (Voltage, Time, Frequency) using DSOs.

- Welcome to Lecture 36.
- Today's focus will be on the operational dynamics and the significant advantages of the Digital Storage Oscilloscope (DSO) over conventional analog instruments.

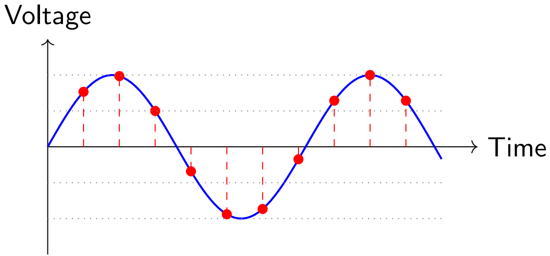
Working Principle: Signal Acquisition
The Analog Front End

- **Step 1: Signal Conditioning**
- The incoming analog signal is first fed into the vertical amplifier.
- **Purpose:** Attenuation or amplification to map the signal amplitude optimally into the Analog-to-Digital Converter's (ADC) input range.
- **Filtering:** Anti-aliasing filters limit the bandwidth to satisfy the Nyquist criterion ($f_s \geq 2f_{max}$).
- **Impedance Matching:** Typically provides 1 MΩ in parallel with a small capacitance (∼15 pF) to avoid loading the circuit under test.

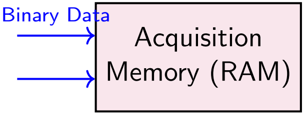


The Digitization Process
Analog-to-Digital Conversion (ADC)

- **Step 2: Sampling and Quantization**
- **Sampling:** The continuous-time signal is converted to discrete-time samples. The sample rate (F_s) dictates temporal resolution (e.g., 1 GSa/s = 1 sample every nanosecond).
- **Quantization:** The ADC assigns a digital value to each sample's amplitude.
- **Resolution:** Defined by the ADC bit depth (typically 8-bit, 10-bit, or 12-bit). An 8-bit ADC provides $2^8 = 256$ discrete voltage levels.
- **Flash ADCs:** Commonly used in high-speed DSOs to achieve gigasample per second rates.



- **Step 3: Storing the Waveform**
- The digital values from the ADC are written into Acquisition Memory (RAM).
- **Memory Depth (Record Length):** Defines how many samples can be stored in a single acquisition (e.g., 10k points, 1M points).
- **Relationship:** Acquisition Time $= \frac{\text{Memory Depth}}{\text{Sample Rate}}$
- Deep memory allows a DSO to maintain a high sampling rate over a longer time base setting, preventing aliasing and preserving signal detail.

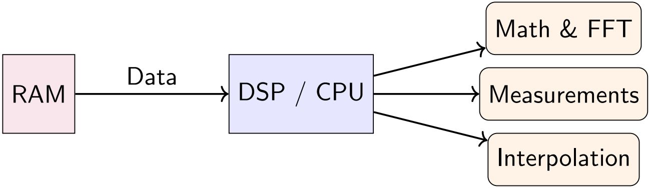


Memory Depth $= N$ samples

Signal Processing and CPU

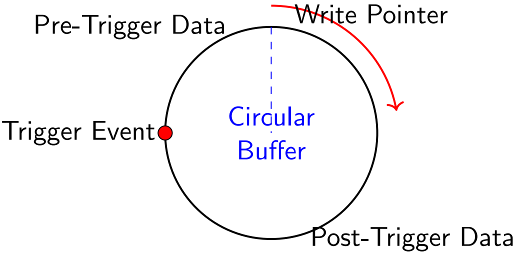
The Brains of the Operation

- **Step 4: Data Manipulation**
- The onboard microprocessor or DSP (Digital Signal Processor) accesses the stored data.
- **Math Operations:** Can perform addition, subtraction, multiplication, or FFT (Fast Fourier Transform) on the waveform data.
- **Measurements:** Automatically calculates parameters like V_{pp}, V_{rms}, frequency, rise time, and duty cycle based on the stored digital array.
- **Interpolation:** Algorithms (like Sine(x)/x or linear) are applied to reconstruct the visual waveform between actual sample points.



- **Step 5: Visualization**
- Unlike analog CRT scopes, modern DSOs use LCD or TFT flat-panel displays.
- **Display Memory:** Processed data is mapped to pixel coordinates in a frame buffer.
- **Intensity Grading:** Advanced DSOs emulate analog persistence by changing pixel color or brightness based on how often a specific voltage-time point is hit, highlighting signal anomalies or jitter.
- Earlier DSOs required a DAC to drive raster CRT displays, but modern all-digital displays map memory directly to pixel matrices.

- Digital triggering operates on the digitized data stream rather than the analog input, providing unparalleled flexibility:
- **Pre-triggering:** The ability to view events that occurred *before* the trigger point. The ADC continuously fills memory in a circular buffer; when the trigger occurs, the scope stops acquisition after capturing the required post-trigger samples.
- **Complex Triggers:**
  - Pulse Width Trigger (captures glitches)
  - Logic/Pattern Trigger (AND/OR logic across multiple channels)
  - Runt Trigger (captures signals that fail to reach a threshold)



Operating Modes of DSO

How the Display Updates

- **Normal / Auto Mode:** Continually acquires and updates the display. Auto forces a trigger if none is found; Normal waits for a valid trigger.
- **Single Shot Mode:** Captures a single event and freezes it on the screen. Perfect for transient analysis (e.g., power-on inrush current).
- **Roll Mode:** At slow timebase settings, the waveform scrolls continuously from right to left across the screen, similar to a strip chart recorder. Useful for monitoring slow physical phenomena.

Advantage 1: Storage and Persistence

Infinite Retention

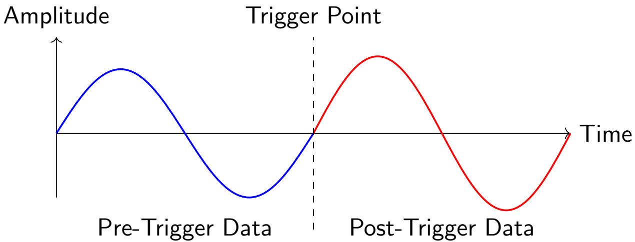
- **Infinite Storage:** Once a waveform is digitized, it can be stored indefinitely in memory or saved to a USB drive as a CSV or image file.
- **No Fading:** Analog CRTs rely on phosphor persistence; the image fades quickly. DSOs display the stored waveform continuously without degradation.
- **Reference Traces:** A "golden" waveform can be saved and overlaid on the screen to compare with live signals (e.g., checking manufacturing tolerances).

- **Automated Measurements:** Eliminates human error in reading graticules. Instantly displays Rise Time, Fall Time, Period, Frequency, Peak-to-Peak, Mean, and RMS values.
- **Cursors:** On-screen cursors provide precise ΔV and Δt measurements.
- **Advanced Math:**
  - Signal addition/subtraction (e.g., differential measurements).
  - Fast Fourier Transform (FFT) allows viewing the frequency domain spectrum of a time-domain signal.

Advantage 3: Pre-Trigger Viewing

Looking Back in Time

- **The Problem with Analog:** An analog scope only starts drawing the trace *after* the trigger occurs, meaning the leading edge or the cause of the trigger is often lost.
- **The DSO Solution:** Because the ADC writes to a continuous circular buffer, the scope holds history. When the trigger condition is met, the user can configure the scope to display 50% pre-trigger and 50% post-trigger data.
- **Benefit:** Crucial for troubleshooting microcontrollers or determining the root cause of a voltage spike.



Advantage 4: Connectivity and Portability

Modern Integration

- **Interfacing:** Equipped with USB, LAN (Ethernet), and GPIB ports.
- **Remote Control:** Can be fully controlled via PC software (e.g., SCPI commands) for automated test setups in manufacturing.
- **Form Factor:** Digital circuitry is highly integrated. Modern DSOs are lighter, thinner, and consume less power than bulky CRT-based analog oscilloscopes.
- **Hardcopy:** Easy to print screens or save data for documentation.

Feature	Analog Oscilloscope	Digital Storage Oscilloscope (DSO)
Storage	Very limited (phosphor decay)	Infinite (digital memory)
Pre-trigger	Impossible	Standard feature
Measurements	Manual (visual estimation)	Automated and precise
Signal Math	Basic (add/invert)	Advanced (FFT, integration, etc.)
Display	CRT (Bulky, fragile)	LCD (Thin, robust)
Data Transfer	Not possible	USB, LAN, PC Connectivity

Quick Knowledge Check

Test Your Understanding

- **Question 1:** What defines the vertical resolution of a DSO?
  - (A) Memory Depth
  - (B) Sampling Rate
  - (C) ADC Bit Depth
  - (D) Display Size

- **Question 2:** Which feature allows a user to see the waveform data immediately before a trigger event occurs?
  - (A) Roll Mode
  - (B) Pre-triggering
  - (C) FFT
  - (D) Attenuation

Summary

Key Takeaways from Lecture 36

- **Working Principle:** DSOs acquire analog signals, digitize them using an ADC, and store them in RAM for processing and display.
- **Crucial Specs:** Sample rate determines time resolution; ADC bit depth determines voltage resolution; Memory depth determines how long a high sample rate can be sustained.
- **Triggering:** Digital buffering allows for advanced triggering and viewing pre-trigger history.
- **Advantages:** DSOs provide permanent storage, automated measurements, FFT analysis, and easy PC connectivity.

Next Lecture Preview

Moving to Transducers

- In our next session, we will begin Unit 5: Transducers and Sensors.
- **Topics for Lecture 37:**
  - Introduction to Transducers
  - Classification of Transducers (Active, Passive, Primary, Secondary)
  - Resistive, Capacitive, and Inductive Transducers
- **Preparation:** Review basic concepts of resistance, capacitance, and inductance.

Classification of Transducers

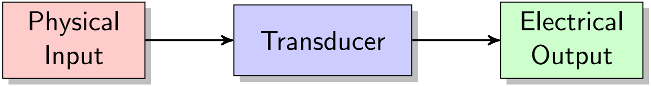
- Course: Electronics Circuit Network & Measurement (DI03011031)
- Unit 5: Transducers and Sensors
- Lecture 37: Introduction and Classification of Transducers
- Focus: Active, Passive, Primary, Secondary, Resistive, Capacitive, and Inductive types.

Agenda

- Definition and Role of Transducers
- Elements of a Transducer System
- Broad Classification Methods
- Active vs. Passive Transducers
- Primary vs. Secondary Transducers
- Classification by Transduction Principle
    - Resistive
    - Capacitive
    - Inductive
- Selection Criteria

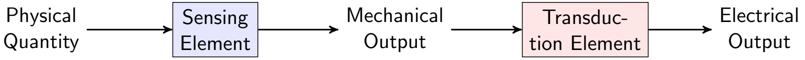
What is a Transducer?

- **Definition:** A device that converts one form of energy into another.
- **In Measurement Systems:** Specifically refers to devices that convert a physical, non-electrical quantity (temperature, pressure, force, displacement) into a measurable electrical signal (voltage, current, resistance).
- **Key Function:** Acts as the interface between the physical process and the electronic measuring instrument.
- **Sensor vs. Transducer:** A sensor responds to a physical stimulus, while a transducer encompasses both the sensor and the signal conditioning to provide an electrical output.



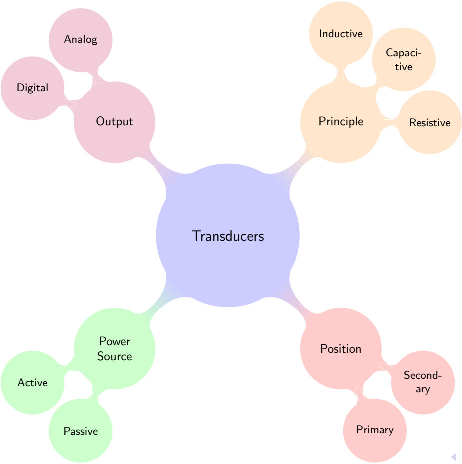
Elements of a Transducer

- **Sensing Element (Detector):** The part of the transducer that responds directly to the physical phenomenon.
- **Transduction Element:** The part that converts the output of the sensing element into an electrical signal.
- **Example - Bourdon Tube with LVDT:**
  - Physical Input: Pressure
  - Sensing Element: Bourdon tube (converts pressure to displacement)
  - Transduction Element: LVDT (converts displacement to voltage)



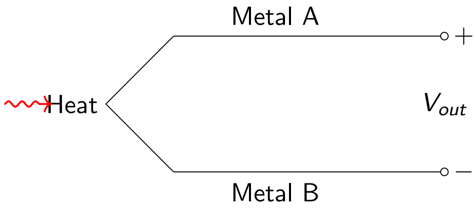
Broad Classification of Transducers

- Transducers can be classified based on several criteria:
  - **Based on Power Source:** Active and Passive.
  - **Based on Position/Function:** Primary and Secondary.
  - **Based on Electrical Principle:** Resistive, Capacitive, Inductive.
  - **Based on Output Format:** Analog and Digital.
  - **Transducers and Inverse Transducers:** Converting physical to electrical vs. electrical to physical.



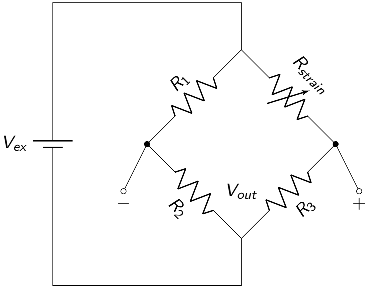
Active Transducers

- **Definition:** Self-generating transducers that do not require an external power source to produce an electrical output.
- **Operating Principle:** They generate voltage or current inherently by extracting energy from the physical quantity being measured.
- **Characteristics:**
  - Output is typically very small (mV or μA).
  - Require high-gain amplifiers for signal processing.
- **Common Examples:**
  - Thermocouple (Seebeck effect)
  - Piezoelectric crystal (Piezoelectric effect)
  - Photovoltaic cell



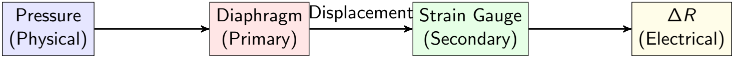
Passive Transducers

- **Definition:** Transducers that require an external power source (excitation voltage) to operate.
- **Operating Principle:** The physical quantity changes a passive electrical parameter (Resistance, Capacitance, or Inductance). The external circuit converts this parameter change into a measurable voltage or current.
- **Characteristics:**
  - Can provide higher output levels depending on the excitation voltage.
  - External power source is mandatory.
- **Common Examples:** Strain Gauge (Change in R), Thermistor (Change in R), LVDT (Change in L)



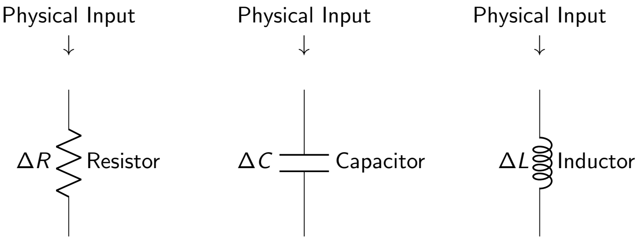
Primary vs. Secondary Transducers

- **Primary Transducer:** The first device in the measurement chain that directly responds to the physical variable and converts it, often into a mechanical variable (like displacement).
  - *Example:* A diaphragm converting pressure into displacement.
- **Secondary Transducer:** Receives the output of the primary transducer (e.g., displacement) and converts it into an electrical signal.
  - *Example:* A strain gauge attached to the diaphragm converting the displacement into a change in resistance.



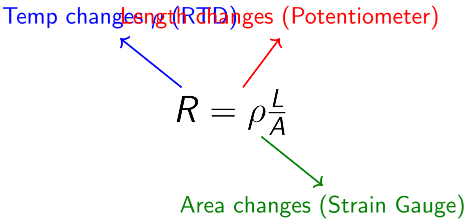
Classification by Electrical Principle

- Passive transducers are further categorized by the electrical parameter that changes in response to the physical variable:
  - **Resistive Transducers:** Change in electrical resistance (ΔR).
  - **Capacitive Transducers:** Change in electrical capacitance (ΔC).
  - **Inductive Transducers:** Change in electrical inductance (ΔL).
- These changes are typically converted to a voltage signal using a Wheatstone bridge or an AC bridge circuit.



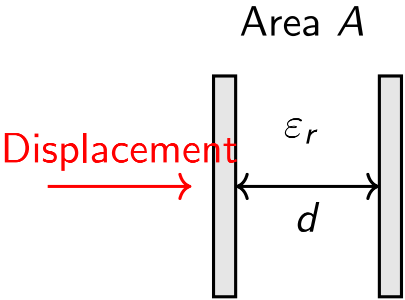
Resistive Transducers

- **Principle:** Resistance of a conductor is given by $R = \rho \frac{L}{A}$
  - ρ = Resistivity (temperature dependent)
  - L = Length of conductor
  - A = Cross-sectional area
- **Mechanism:** A physical quantity can change ρ, L, or A, thereby changing R.
- **Examples:**
  - **Potentiometer:** Displacement changes effective length (L).
  - **Strain Gauge:** Force changes length (L) and area (A).
  - **RTD / Thermistor:** Temperature changes resistivity (ρ).



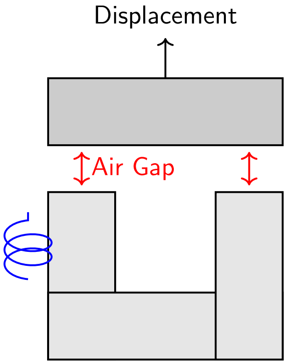
Capacitive Transducers

- **Principle:** Capacitance of a parallel plate capacitor is $C = \frac{\varepsilon_0 \varepsilon_r A}{d}$
  - $\varepsilon_0 \varepsilon_r$ = Permittivity of the dielectric
  - A = Overlapping area of plates
  - d = Distance between plates
- **Mechanism:** Physical variables can change A, d, or the dielectric constant ε_r.
- **Applications:**
  - **Change in d:** Used for measuring minute displacements, pressure (capacitor microphone).
  - **Change in A:** Used for larger angular or linear displacements.
  - **Change in ε_r:** Used for measuring liquid levels or humidity.



Inductive Transducers

- **Principle:** Inductance of a coil depends on the number of turns, the magnetic core material, and the magnetic circuit geometry.
- Formulas: $L = \frac{N^2 \mu A}{l_c}$ or based on reluctance $L = \frac{N^2}{\Re}$
- **Mechanism:** Physical variables change the self-inductance, mutual inductance, or reluctance of the magnetic circuit.
- **Examples:**
  - **Variable Reluctance:** Moving a magnetic core changes the air gap, altering reluctance and thus self-inductance.
  - **LVDT:** Displacement of a core changes the mutual inductance between primary and secondary coils.

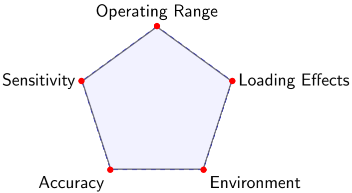


Comparing Passive Transducer Principles

Parameter	**Governing Formula**	**Key Variable Changed by Physical Input**	**Common Example**
Resistive	$R = \rho L/A$	Length, Area, Resistivity	Strain Gauge, RTD
Capacitive	$C = \varepsilon A/d$	Distance, Area, Dielectric	Condenser Mic, Level Sensor
Inductive	$L = N^2/\Re$	Permeability, Reluctance	Variable Reluctance Pick-up

Selection Criteria for Transducers

- Choosing the right transducer depends on several factors:
  - **Operating Range:** Must cover the expected range of the physical variable.
  - **Sensitivity:** Should provide sufficient output per unit of input.
  - **Accuracy and Linearity:** Requires high precision and ideally a linear input-output relationship.
  - **Environmental Compatibility:** Must withstand operating temperature, humidity, and corrosive elements.
  - **Loading Effects:** The transducer should not alter the physical phenomenon it is measuring.



Quick Check

- Does a thermocouple require an external DC power supply to measure temperature? Why or why not?
- In a capacitive pressure sensor, which parameter in the formula $C = \varepsilon A/d$ is most commonly varied by the applied pressure?
- Is a strain gauge considered an active or a passive transducer?

Answers

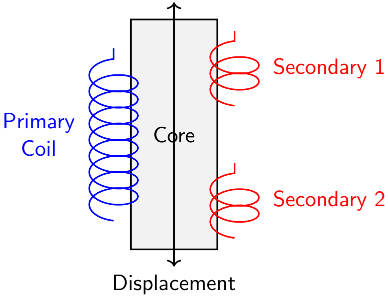
1. No, it is active (generates own output).
2. Distance d.
3. Passive (needs external power).

Summary

- Transducers convert physical variables into measurable electrical signals.
- Active Transducers generate their own electrical output (e.g., Thermocouples).
- Passive Transducers require an external power source and operate by changing R, L, or C.
- Transducers can be classified into Primary (physical to mechanical) and Secondary (mechanical to electrical).
- R, L, and C Transducers operate by physically altering the dimensional parameters that define resistance, inductance, and capacitance.

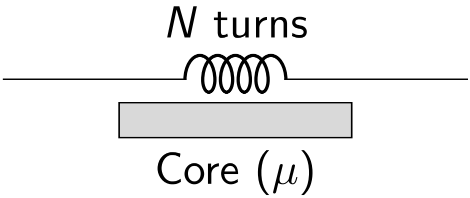
Next Lecture Preview

- **Topic:** Working of LVDT and Inductive Transducers
- **Key Concepts:**
  - Construction of Linear Variable Differential Transformer (LVDT)
  - Principle of mutual inductance
  - LVDT transfer characteristics and phase shift
  - Advantages and applications of LVDT in displacement measurement



Introduction to Inductive Transducers

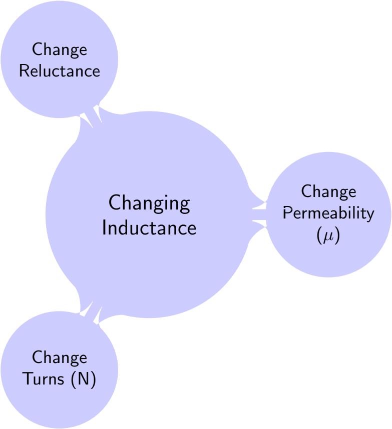
- **What are Inductive Transducers?**
- Passive transducers that require an external AC power source.
- Work on the principle of change in inductance to measure a physical quantity.
- **Governing Equation for Inductance (L):**
- $L = \frac{N^2 \mu A}{l}$
    - N = Number of turns in the coil
    - μ = Effective permeability of the medium
    - A = Cross-sectional area
    - l = Length of the magnetic path



Methods of Changing Inductance

To measure physical displacement, inductive transducers vary one of the following parameters:

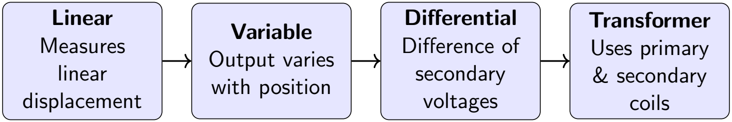
- **Change of Number of Turns (N):** Displacement changes the effective number of turns.
- **Change of Permeability (μ):** Moving a ferromagnetic core in and out of the coil changes the medium's permeability.
- **Change of Reluctance:** Changing the air gap between the magnetic core and the coil changes the magnetic path's reluctance (similar to changing length l).



Self-Inductance (Single Coil)	Mutual Inductance (Multiple Coils)
The displacement changes the inductance of a single coil. Example: Variable reluctance transducer where an air gap varies.	Involves a primary coil and one or more secondary coils. Displacement of a common magnetic core changes the magnetic coupling (flux linkage) between primary and secondary. This changes the induced voltage in the secondary coils. Prime Example: LVDT

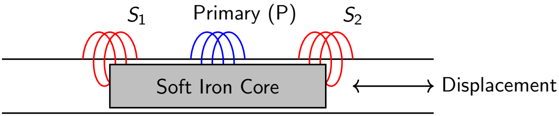
Linear Variable Differential Transformer (LVDT)

- **What is an LVDT?**
- An electromechanical transducer used to measure linear displacement.
- It translates linear motion into an alternating electrical signal.
- Works on the principle of mutual induction.



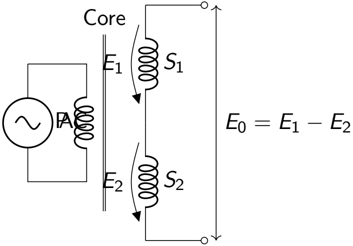
Construction of LVDT

- **Primary Winding (P):** Located at the center of the former. Excited by an AC voltage source (typically 1-10V, 50Hz - 20kHz).
- **Secondary Windings (S_1 & S_2):** Placed on either side of the primary, equidistant from the center. Connected in series opposition.
- **Magnetic Core:** A soft iron or nickel-iron alloy core. Free to move linearly inside the hollow former. Attached to the moving object whose displacement is to be measured.



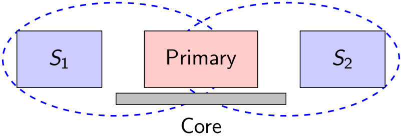
LVDT Circuit Diagram

- **Series Opposition Connection:** The two secondary windings S_1 and S_2 are connected such that their induced voltages oppose each other.
- Let E_1 be the voltage induced in S_1.
- Let E_2 be the voltage induced in S_2.
- **Differential Output Voltage (E_0):** $E_0 = E_1 - E_2$ (or $E_2 - E_1$, depending on connection)
- The phase of the output voltage indicates the direction of displacement.



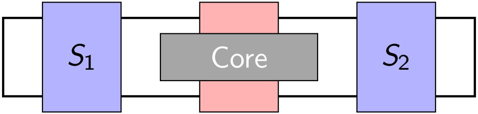
Working Principle of LVDT

- When an AC excitation voltage is applied to the primary, an alternating magnetic field is produced.
- This magnetic flux links with both secondary coils (S_1 and S_2) through the magnetic core.
- According to Faraday's Law, this varying flux induces an AC voltage in both secondaries (E_1 and E_2).
- The magnitude of E_1 and E_2 depends strictly on the position of the core.
- The core acts as a path for the magnetic flux. The coil closest to the core gets maximum flux linkage.



Working of LVDT - Case 1: Core at Null Position

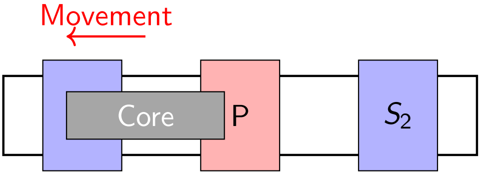
- **Null Position (Center):**
- The core is exactly at the center, symmetric to both S_1 and S_2.
- The magnetic flux linking with S_1 equals the flux linking with S_2.
- Therefore, induced voltage $E_1 = E_2$.
- **Output Voltage:**
- $E_0 = E_1 - E_2 = 0$ Volts.
- In reality, a very small residual voltage exists due to stray magnetic fields or minor imbalances, but ideally, it is zero.



$$E_1 = E_2 \implies E_0 = 0$$

Working of LVDT - Case 2: Core Moves to Left (S_1)

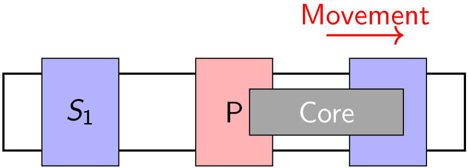
- **Core Moves Left (Towards S_1):**
- More core volume is inside the S_1 coil.
- The flux linking with S_1 increases, while flux linking with S_2 decreases.
- Therefore, induced voltage $E_1 > E_2$.
- **Output Voltage:**
- $E_0 = E_1 - E_2$
- The output voltage E_0 is positive and in-phase with the primary voltage.



$$E_1 > E_2 \implies E_0 > 0$$

Working of LVDT - Case 3: Core Moves to Right (S_2)

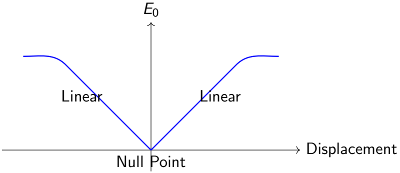
- **Core Moves Right (Towards S_2):**
- More core volume is inside the S_2 coil.
- The flux linking with S_2 increases, while flux linking with S_1 decreases.
- Therefore, induced voltage $E_2 > E_1$.
- **Output Voltage:**
- $E_0 = E_2 - E_1$ (magnitude is negative in algebraic sense relative to case 2).
- The output voltage E_0 is 180 degrees out-of-phase with the primary voltage.



$$E_2 > E_1 \implies E_0 < 0 \textbf{ (Phase Shift)}$$

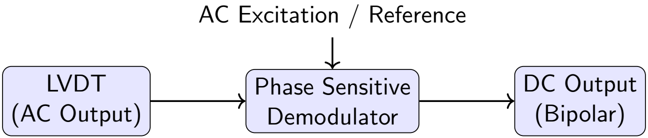
LVDT Transfer Characteristics

- **Displacement vs. Output Voltage Curve:**
- The graph of Output Voltage (E_0) versus Core Displacement is linear over a certain range.
- **Null Point:** At zero displacement, voltage is ideally zero (residual voltage exists practically).
- **Linear Range:** The curve is a straight line for small displacements from the null point.
- **Saturation Range:** If the core moves too far, the curve flattens out (becomes non-linear) because the core is leaving the coil area entirely.



Phase Sensitive Demodulation

- **Problem:** An AC voltmeter connected to the output will just show increasing voltage whether the core moves left or right. It cannot indicate direction.
- **Solution: Phase Sensitive Demodulator (PSD)**
- A circuit (like a synchronous detector) that compares the phase of the output signal (E_0) with the primary excitation signal.
- If in-phase: Outputs positive DC voltage.
- If 180° out-of-phase: Outputs negative DC voltage.
- Converts the AC output into a bipolar DC signal proportional to displacement.



Advantages of LVDT

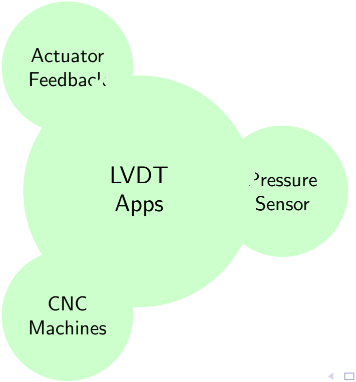
- **High Range:** Can measure displacements from fractions of a millimeter to several centimeters.
- **Frictionless Operation:** Since the core does not mechanically rub against the former, there is zero friction. Highly accurate and durable.
- **High Resolution:** Theoretically infinite resolution, limited only by the signal conditioning electronics.
- **High Sensitivity:** Typically around 40V/mm.
- **Ruggedness:** Can tolerate a high degree of shock and vibration.
- **Low Hysteresis:** Because there is no mechanical friction, hysteresis is almost negligible.

Disadvantages of LVDT

- **Stray Magnetic Fields:** Since it works on magnetic induction, external magnetic fields can cause interference. Magnetic shielding is required.
- **Temperature Sensitivity:** Changes in temperature affect coil resistance and core permeability.
- **AC Power Required:** Requires an external AC source (oscillator) and a demodulator circuit, adding complexity compared to a simple potentiometer.
- **Dynamic Response:** The excitation frequency limits the dynamic response; the measured motion must be significantly slower than the excitation frequency.

Applications of LVDT

- **Direct Measurement of Displacement:**
  - Used in CNC machines, robotics, and industrial automation to measure linear position.
  - Used in aerospace for flight control actuator position feedback.
- **Secondary Transducer:** Can be combined with primary sensing elements (like Bourdon tubes or proving rings) to measure:
  - **Force & Weight:** Using a spring/load cell setup.
  - **Pressure:** Translating bellows or diaphragm expansion into LVDT displacement.
  - **Tension:** In textile and paper mills.



Summary & Quick Check

Summary:

- Inductive transducers measure by varying inductance (self or mutual).
- LVDT is a mutual inductance transducer with 1 primary and 2 secondary coils.
- The secondary coils are in series opposition.
- Output magnitude indicates displacement amount; phase indicates direction.

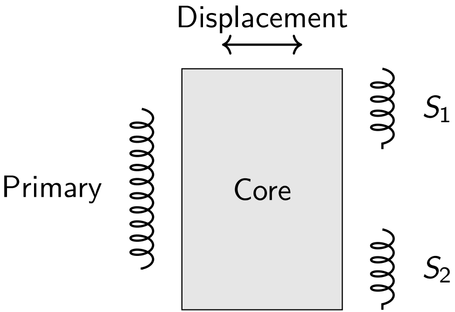
Quick Question:

If the core of an LVDT is exactly in the center, what is the theoretical output voltage?

(Answer: Zero, known as the null point)

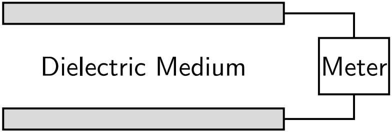
Recap: Inductive Transducers (LVDT)

- **Key Takeaways from Lecture 38:**
- Inductive Transducers: Operate on the principle of variations in self-inductance or mutual inductance.
- LVDT: Consists of one primary and two secondary windings (S_1 and S_2).
- Working: Core movement alters magnetic coupling.
- Core at Null: $V_1 = V_2 \rightarrow V_{out} = 0$
- Core moving right/left induces differential voltage proportional to displacement.
- Widely used for accurate linear displacement.



Introduction to Capacitive Transducers

- **Definition**: A passive transducer that converts a physical quantity (displacement, pressure, etc.) into a change in capacitance.
- **Basic Concept**: Consists of two parallel metal plates separated by a dielectric medium.
- **Transduction Principle**: Any physical change that alters the area of the plates, distance between them, or the dielectric material will result in a measurable change in capacitance.
- Mostly used for measuring extremely small displacements (e.g., micrometers).



Mathematical Principle of Operation

Capacitance Equation

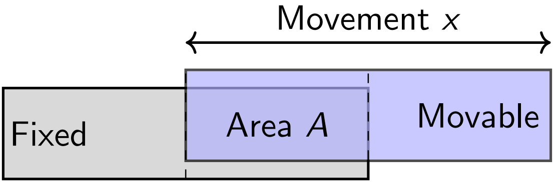
The capacitance C of a parallel plate capacitor is given by:

$$C = \frac{\varepsilon_0 \varepsilon_r A}{d}$$

- C = Capacitance in Farads (F)
- ε_0 = Permittivity of free space (8.854×10^{-12} F/m)
- ε_r = Relative permittivity (dielectric constant) of the medium
- A = Overlapping area of plates (m^2)
- d = Distance between the plates (m)

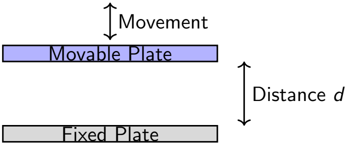
Principle 1: Varying Plate Area (A)

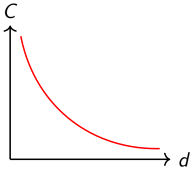
- **Concept**: One plate is fixed, and the other plate moves laterally (parallel to the fixed plate).
- **Effect**: The effective overlapping area A changes.
- **Relationship**: Capacitance C varies linearly with displacement x.
- $C \propto A$
- **Application**: Used for measuring relatively large linear displacements (1 cm to 10 cm). Can also be implemented in a cylindrical configuration for angular displacement measurement.



Principle 2: Varying Distance (d)

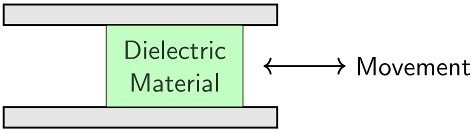
- **Concept**: One plate is fixed, and the other moves closer to or further away from it.
- **Effect**: The distance d between the plates changes.
- **Relationship**: Capacitance C varies non-linearly (hyperbolically) with distance d.
- $C \propto \frac{1}{d}$
- **Application**: Ideal for measuring extremely small displacements (e.g., 10^{-6} to 10^{-9} meters) due to high sensitivity. Frequently used in pressure, force, and microphone sensors.





Principle 3: Varying Dielectric Constant (ε_r)

- **Concept**: Both plates are fixed in area and distance. A dielectric material is moved in or out of the gap.
- **Effect**: The effective relative permittivity ε_r changes.
- **Relationship**: Capacitance changes linearly with the movement of the dielectric.
- **Application**:
  - Liquid level measurement (e.g., fuel tanks where the liquid acts as the changing dielectric).
  - Measuring thickness of materials.



Advantages & Disadvantages

Advantages

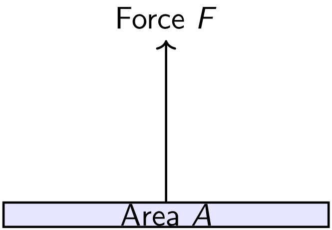
- **High Sensitivity**: Can detect minute displacements.
- **Low Power**: Operate with very minimal power.
- **Freq Response**: Light moving parts allow measuring high-frequency changes.
- **High Impedance**: Minimal loading effect.
- **Stability**: Good temperature stability.

Disadvantages

- **Non-linearity**: Distance method is non-linear over large ranges.
- **Stray Capacitance**: Cables add capacitance (needs shielded cables).
- **Edge Effects**: Electric field fringing causes errors.
- **Environment**: Dust/moisture alters dielectric constant.

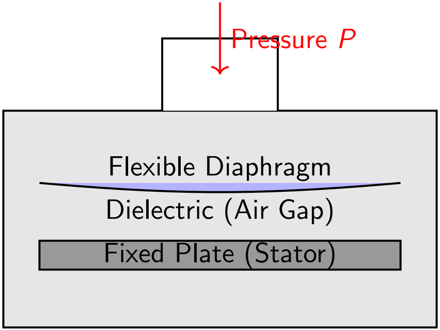
Pressure Measurement: Introduction

- Pressure is defined as force per unit area ($P = F/A$).
- A capacitive pressure transducer translates physical pressure into a capacitance change.
- **Core Principle**: Uses the varying distance (d) method.
- Converts fluid or gas pressure into physical displacement of a flexible diaphragm.



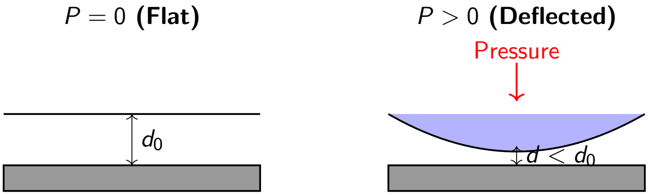
$$P = \frac{F}{A}$$

- **Components**:
  - **Fixed Plate**: A rigid metal electrode (stator).
  - **Movable Plate**: A thin, flexible metal diaphragm.
  - **Cavity**: A sealed gap acting as the dielectric.
  - **Pressure Port**: Inlet for fluid/gas.
- When $P = 0$, the diaphragm is flat. Distance is d_0.
- When $P > 0$ is applied, the diaphragm deflects.



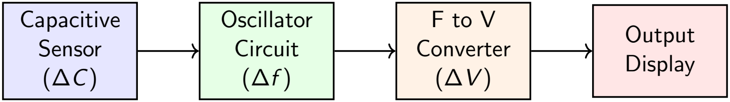
Working Principle of Pressure Sensor

- **Step 1**: External pressure P applies force to diaphragm.
- **Step 2**: The diaphragm bends inward, reducing the distance d.
- **Step 3**: Since $C \propto 1/d$, reducing distance causes an increase in total capacitance C.
- **Step 4**: This change in capacitance (ΔC) is measured.



Signal Conditioning for Capacitive Sensors

- Changes in capacitance are very small (pF range); standard multimeters cannot read them.
- **AC Bridge Circuit**: Capacitance forms one arm. Unbalance creates a voltage output.
- **Oscillator Circuit**: The variable capacitor is in an LC or RC oscillator tank circuit. Changing C changes frequency ($f = \frac{1}{2\pi\sqrt{LC}}$).
- **Charge Amplifier**: Converts small capacitive charge into a proportional voltage.



Question

In a capacitive pressure transducer, what is the primary parameter that changes to vary the capacitance?

A) The overlapping area of the plates
B) The dielectric constant of the fluid
C) The distance between the fixed plate and the diaphragm
D) The temperature of the plates

Answer on next slide.

Summary & Next Lecture Preview

Summary

- Capacitive transducers convert physical changes into capacitance variation ($C = \varepsilon A/d$).
- Three modes: Changing Area (linear), Changing Distance (highly sensitive, non-linear), Changing Dielectric.
- Pressure sensors utilize a flexible diaphragm to change distance d, altering capacitance.
- Key advantage: High sensitivity and frequency response. Key challenge: Stray capacitance.

Next Lecture Preview

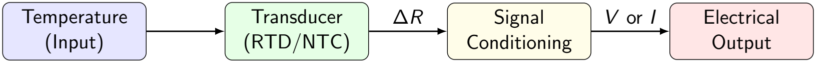
- We will shift focus from pressure to Temperature Transducers, starting with RTDs and Thermistors.

Agenda

- Introduction to Electrical Temperature Measurement
- Resistance Temperature Detectors (RTD) Principle
- RTD Materials and Construction
- RTD Signal Conditioning (Bridge Circuits)
- Thermistors: Principle and Types (NTC/PTC)
- Thermistor Characteristics and Equations
- Comparative Analysis: RTD vs. Thermistor
- Summary and Q&A

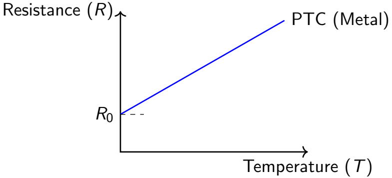
Introduction to Temperature Measurement

- Definition: Temperature transducers convert thermal energy into a proportional electrical signal (voltage, current, or resistance change).
- Resistive Temperature Sensors: Rely on the property of materials changing their electrical resistance with temperature variations.
- Key Requirements:
    - Predictable and repeatable relationship.
    - Sufficient sensitivity.
    - Stability over time.
    - Appropriate dynamic response.



RTD: Working Principle

- Resistance Temperature Detector (RTD): A passive transducer that operates on the principle of Positive Temperature Coefficient (PTC) of pure metals.
- As temperature increases, thermal vibrations of atoms increase, scattering electrons and increasing electrical resistance.
- Linear Approximation: $R_T = R_0[1 + \alpha(T - T_0)]$
  - R_T: Resistance at temperature T
  - R_0: Resistance at reference temperature T_0 (usually 0°C)
  - α: Temperature coefficient of resistance (/°C)



RTD: Mathematical Model

- For higher accuracy over wider ranges, the relationship is non-linear and described by the Callendar-Van Dusen equation:
- For $T > 0°C$:

$$R_T = R_0(1 + AT + BT^2)$$

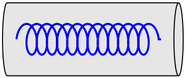
- For $T < 0°C$:

$$R_T = R_0[1 + AT + BT^2 + C(T - 100)T^3]$$

- Constants for Platinum (IEC 60751):
  - $A = 3.9083 \times 10^{-3} {}^\circ C^{-1}$
  - $B = -5.775 \times 10^{-7} {}^\circ C^{-2}$

RTD: Materials and Construction

- Common Materials:
  - **Platinum (Pt):** Best stability, wide range (-200 to 850°C), linear. Industry standard (e.g., Pt100, $R_0 = 100\Omega$).
  - **Copper (Cu):** Very linear, cheap, but oxidizes above 120°C.
  - **Nickel (Ni):** High sensitivity, highly non-linear, range up to 300°C.
- Construction Types:
  - **Wire-wound:** Pure wire wound on a ceramic or glass core.
  - **Thin-film:** Platinum deposited on a ceramic substrate, offering faster response and smaller size.

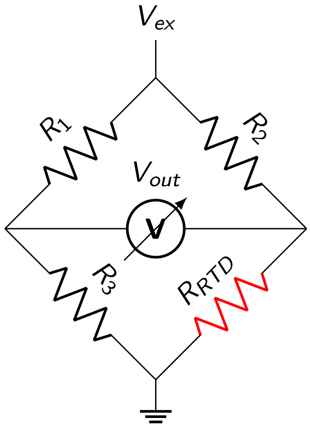


Wire-Wound

Thin-Film

RTD: Signal Conditioning

- RTDs are passive sensors; they require an excitation current.
- **Wheatstone Bridge Circuits:** Convert resistance change into a measurable voltage.
- **2-Wire Bridge:** Simple, but lead wire resistance introduces errors.
- **3-Wire Bridge:** Most common; compensates for lead wire resistance assuming equal lead resistances.
- **4-Wire Bridge:** Highest accuracy; completely eliminates lead wire errors (Kelvin sensing).



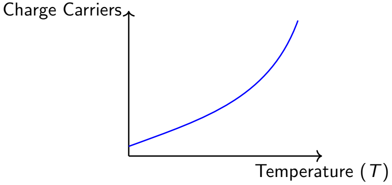
2-Wire RTD Bridge

RTD: Key Considerations

- **Self-Heating Error:**
  - The excitation current (I) passing through the RTD generates power ($P = I^2 R$).
  - This power heats the sensor slightly above the ambient temperature, causing measurement errors.
  - **Solution:** Keep excitation current minimal (typically 1 mA or less).
- **Advantages:** Exceptional stability, high accuracy, repeatable.
- **Disadvantages:** Lower sensitivity compared to thermistors, higher cost, slower response time (in wire-wound types).

Thermistors: Introduction and Principle

- **Thermistor (Thermal Resistor):** A semiconductor device made of ceramic-like mixtures of metal oxides (manganese, nickel, cobalt).
- **Working Principle:** Unlike pure metals, semiconductors have a Negative Temperature Coefficient (NTC).
- As temperature increases, more charge carriers are freed, significantly increasing conductivity and drastically reducing resistance.
- **Characteristics:** Extremely sensitive; a small temperature change causes a huge resistance change.



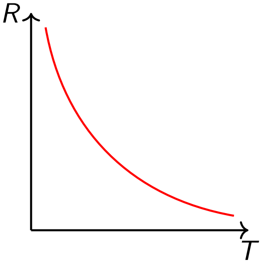
Thermistors: Types (NTC vs PTC)

NTC (Negative Temp. Coeff.):

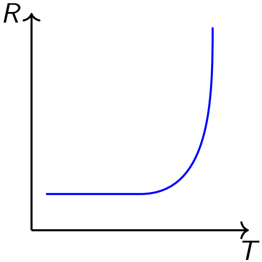
- Resistance decreases exponentially as temperature rises.
- Most commonly used for precision temperature measurement and control.

PTC (Positive Temp. Coeff.):

- Resistance increases with temperature, often abruptly at a 'switching' temperature.
- Used as resettable fuses or self-regulating heaters.



NTC Characteristic



PTC Characteristic

Thermistor: Mathematical Model (Beta Equation)

- The resistance of an NTC thermistor is highly non-linear and is approximately modeled by the **Beta parameter equation**:

$$R_T = R_0 \cdot e^{\beta\left(\frac{1}{T} - \frac{1}{T_0}\right)}$$

- Where:
  - R_T: Resistance at absolute temperature T (Kelvin)
  - R_0: Resistance at reference temperature T_0 (usually 298.15 K or 25°C)
  - β: Material constant (Beta value), typically between 3000 K and 5000 K.

Thermistor: Steinhart-Hart Equation

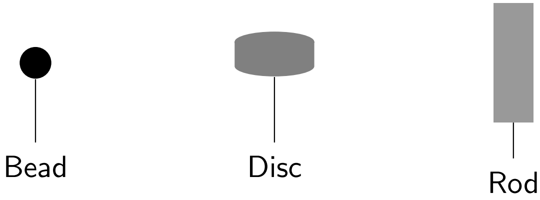
- For highly accurate measurements over a broader range, the **Steinhart-Hart equation** is used:

$$\frac{1}{T} = A + B(\ln R) + C(\ln R)^3$$

- Where:
  - T is temperature in Kelvin.
  - R is resistance in ohms.
  - A, B, C are the Steinhart-Hart coefficients derived from calibration at three different temperatures.
- Modern microcontrollers easily handle the logarithmic math required for this linearization.

Thermistor: Construction and Types

- Thermistors come in various physical forms to suit different applications:
  - **Bead Type:** Very small, fast response time, encased in glass for high stability.
  - **Disc Type:** Larger, slower response, handles higher power, good for general applications.
  - **Rod/Probe Type:** Rugged, encapsulated in metal for harsh environments.
  - **Chip/SMD:** Used for surface-mount printed circuit board temperature compensation.



Thermistor: Advantages & Limitations

- **Advantages:**
  - Very high sensitivity (changes up to 5% per °C).
  - Fast response time (especially bead types).
  - High base resistance (1kΩ to 100kΩ) minimizes lead wire resistance errors.
  - Low cost and small size.
- **Limitations:**
  - Highly non-linear characteristic requires complex linearization.
  - Limited operating temperature range (-50°C to $\sim$ 300°C).
  - Prone to self-heating errors due to smaller size.

Comparative Analysis: RTD vs. Thermistor

Feature	**RTD (e.g., Pt100)**	**NTC Thermistor**
Material	Metal (Platinum)	Semiconductor (Metal Oxides)
Temp Range	Very Wide (-200°C to 850°C)	Narrow/Moderate (-50°C to 300°C)
Linearity	Excellent (mostly linear)	Poor (exponential)
Sensitivity	Low (%/°C)	Very High (up to 5%/°C)
Base Resistance	Low (100 Ω, 1000 Ω)	High (10 kΩ, 100 kΩ)
Cost	High	Low

Quick Check

- **Question 1:** Which RTD configuration provides the most accurate measurement by eliminating lead resistance?
  - A) 2-wire bridge
  - B) 3-wire bridge
  - C) 4-wire bridge
  - D) 1-wire bridge
- **Question 2:** For an NTC thermistor, what happens to the resistance as temperature increases?
  - A) Increases linearly
  - B) Decreases exponentially
  - C) Remains constant
  - D) Increases exponentially

Summary

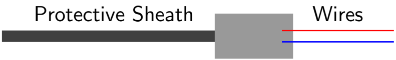
- RTDs utilize the positive temperature coefficient of metals (like Platinum) to offer highly stable, linear, and wide-range temperature measurement.
- RTDs require 3-wire or 4-wire configurations to eliminate lead resistance errors.
- Thermistors utilize semiconductor materials exhibiting a massive negative temperature coefficient, offering high sensitivity over a narrower range.
- Thermistors are governed by non-linear relationships, requiring equations like Steinhart-Hart for accurate calibration.

Next Lecture Preview

- **Topic:** Temperature Transducers: Thermocouple
- **Key Concepts to be Covered:**
  - Seebeck, Peltier, and Thomson effects
  - Working principle of Thermocouples
  - Cold Junction Compensation (CJC)
  - Types of industrial thermocouples (Type K, J, T, etc.)

Temperature Transducers: Thermocouple

- Course: Electronics Circuit Network & Measurement
- Unit 5: Transducers and Sensors
- Topic: Working and Principle of Thermocouples



Agenda

- Introduction to Thermoelectric Effects
- The Seebeck Effect
- Working Principle of Thermocouple
- Construction and Common Types (J, K, T, etc.)
- Measurement Junctions (Hot and Cold)
- Cold Junction Compensation
- Advantages and Disadvantages
- Practical Applications

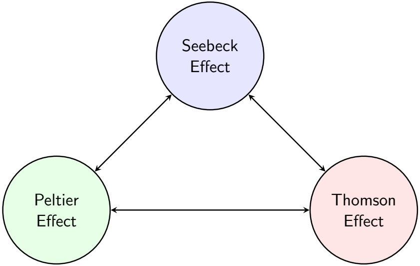
Introduction to Thermoelectricity

Thermoelectricity

Thermoelectricity is the direct conversion of temperature differences to electric voltage and vice versa.

Key Effects:

- **Seebeck Effect:** Generating voltage from a temperature gradient.
- **Peltier Effect:** Heating or cooling at a junction when current is forced through.
- **Thomson Effect:** Reversible heating/cooling in a single conductor with a temperature gradient and current flow.
- Thermocouples primarily rely on the **Seebeck Effect**.



- **Principle:** Discovered by Thomas Johann Seebeck in 1821.
- When two dissimilar metal conductors are joined at two points, and these junctions are kept at different temperatures, an electromotive force (EMF) is generated.

Equation

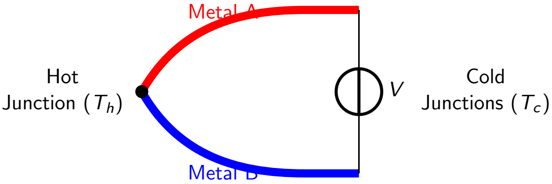
$$V = \alpha \cdot \Delta T$$

Where:

- V = Generated Seebeck voltage
- α = Seebeck coefficient (depends on the materials)
- ΔT = Temperature difference between the two junctions $(T_h - T_c)$

Working Principle of a Thermocouple

- A thermocouple forms a closed thermoelectric circuit.
- **Hot Junction** (Measuring Junction): Placed in the environment whose temperature is to be measured.
- **Cold Junction** (Reference Junction): Kept at a known, constant temperature (traditionally 0°C).
- The open-circuit voltage measured across the break in the circuit is directly proportional to the temperature difference.



Thermocouple Construction

- **Sensing Element:** Two bare wires of dissimilar metals welded together at the tip.
- **Insulation:** Wires must be electrically insulated from each other along their length (using ceramic beads, fiberglass, or mineral insulation).
- **Sheath/Protection Tube:** For industrial use, the element is enclosed in a stainless steel or Inconel sheath to protect against corrosive environments and mechanical damage.
- **Junction Types:**
  - **Exposed:** Fastest response, no protection.
  - **Grounded:** Welded to sheath, good heat transfer.
  - **Ungrounded:** Isolated from sheath, electrical noise immunity.

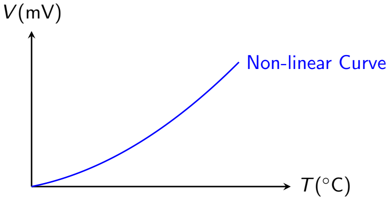
Standard Thermocouple Types

Different material pairs provide different temperature ranges and sensitivities.

Type	Material Pair	Range	Category
K	Chromel - Alumel	$-200°C$ to $+1260°C$	Base Metal (General purpose)
J	Iron - Constantan	$-40°C$ to $+750°C$	Base Metal (High sensitivity)
T	Copper - Constantan	$-200°C$ to $+350°C$	Base Metal (Low temps)
S, R, B	Platinum / Rhodium Alloys	Up to 1700°C	Noble Metal (High cost)

Thermocouple Voltage Characteristics

- Thermocouple output is generally in the millivolt (mV) range.
- **Sensitivity:** Varies by type. E.g., Type K is about $41\mu V/^{\circ}C$.
- **Non-linearity:**
  - The relationship between voltage and temperature is not perfectly linear over wide ranges.
  - Described by high-order polynomial equations defined by NIST.
  - Modern digital instruments use look-up tables or microprocessors to linearize the reading.

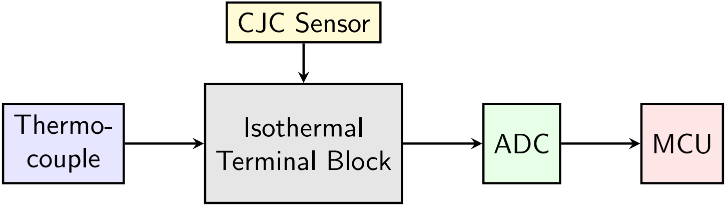


The Reference Junction Problem

- A voltmeter connected to the thermocouple wires creates new, unintended junctions at the instrument terminals.
- If the instrument is at room temperature (T_{ref}), the voltage measured is proportional to ($T_{\text{measure}} - T_{\text{ref}}$).
- To find the true T_{measure}, we must know exactly what T_{ref} is.
- Historically, the reference junction was physically submerged in an ice bath (0°C) to make the math easy.

Cold Junction Compensation (CJC)

- Ice baths are impractical for real-world applications.
- **Hardware CJC:**
  - An electronic temperature sensor (like a thermistor, RTD, or IC sensor) is placed directly on the terminal block where the thermocouple connects.
  - This sensor measures the local ambient temperature (T_{ref}).
  - The system calculates the equivalent thermocouple voltage for T_{ref} and adds it to the measured voltage.
  - This digitally simulates a 0°C reference junction.



Extension and Compensating Cables

- Thermocouple wires are often expensive or physically unsuitable for long runs to the control room.
- **Extension Grade Wires:** Made of the exact same material as the thermocouple but with lower grade/purity. Used for shorter distances.
- **Compensating Cables:** Made of cheaper alloys that have the same thermoelectric properties as the actual thermocouple over a limited ambient temperature range (e.g., 0 to 80°C).

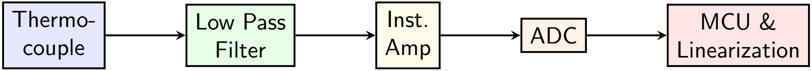
Important Rule

You **CANNOT** use standard copper wire to extend a thermocouple without shifting the cold junction location.

Signal Conditioning for Thermocouples

Due to the very small signal (microvolts), significant conditioning is required:

- **Amplification:** High-gain, low-noise instrumentation amplifiers are essential.
- **Filtering:** Susceptible to EMI and 50/60Hz power line noise; requires low-pass filtering.
- **Cold Junction Compensation:** Adding the reference temperature offset.
- **Linearization:** Converting the amplified voltage to a temperature using polynomial math.
- **Open Thermocouple Detection:** Injecting a tiny current to detect if the wire has broken.



Advantages and Limitations

Advantages:

- **Wide Range:** From -270°C to $> 2000^\circ$C.
- **Rugged & Robust:** Withstands severe vibration and harsh environments.
- **Fast Response Time:** Especially exposed junction types.
- **Self-Powered:** Requires no external excitation power.

Limitations:

- **Lower Accuracy:** Generally less accurate than RTDs or thermistors.
- **Complex Conditioning:** Requires CJC and microvolt amplification.
- **Non-linear:** Requires complex software/hardware for linearization.

Applications of Thermocouples

- **Industrial Processes:** Steel making, glass manufacturing, and foundries (high-temp monitoring).
- **Automotive:** Exhaust gas temperature (EGT) sensors, engine testing.
- **Home Appliances:** Gas furnaces, ovens, and water heaters (acting as a safety pilot light sensor).
- **Aerospace:** Jet engine turbine temperatures.
- **Cryogenics:** Monitoring extremely low temperatures using Type T.

Industrial Furnaces

Automotive EGT

Aerospace Turbines

Home Appliances

Quick Check

Question 1: Which thermoelectric effect is the primary principle behind the operation of a thermocouple?

- A) Peltier Effect
- B) Thomson Effect
- C) **Seebeck Effect**
- D) Hall Effect

Question 2: Why is Cold Junction Compensation (CJC) necessary?

- A) To amplify the microvolt signal
- B) **To account for the temperature at the instrument terminals**
- C) To linearize the output curve
- D) To prevent the wires from melting

Summary

- **Principle:** Thermocouples generate an EMF proportional to the temperature difference between hot and cold junctions (Seebeck Effect).
- **Construction:** Made from two dissimilar metals (e.g., Type K: Chromel/Alumel).
- **CJC:** Essential for accurate measurement, compensating for the ambient temperature of the measuring instrument.
- **Signal:** Output is in non-linear millivolts, requiring amplification, filtering, and linearization.
- **Usage:** Highly rugged, capable of measuring extreme temperatures, but less accurate than RTDs.

Next Lecture Preview

Topic: Modern Sensors: LM35, MQ2, DHT-11

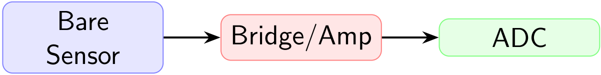
What we will cover:

- Integrated Circuit (IC) temperature sensors (LM35).
- How the LM35 provides a linear analog output (10mV/$^{\circ}$C).
- Introduction to Gas Sensors (MQ2) and Humidity Sensors (DHT-11).
- Integration of modern sensors with microcontrollers.

Evolution to Integrated Sensors

- Traditional Transducers vs. Integrated Sensors:
- Traditional: Require external signal conditioning (Wheatstone bridges, amplifiers, cold-junction compensation).
- Integrated (Modern): Combine sensing element and signal conditioning circuitry on a single chip.
- Advantages:
- Calibrated output directly proportional to the physical quantity.
- Higher noise immunity and reliability.
- Simplified interfacing with ADCs and microcontrollers (e.g., Arduino, Raspberry Pi).

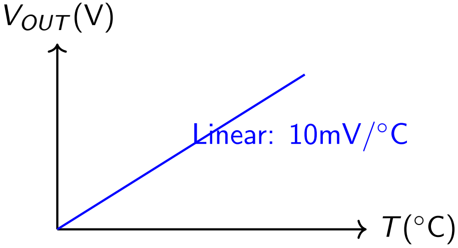
Traditional Setup



Modern Setup

LM35 Precision Temperature Sensor

- What is the LM35?
- An integrated circuit (IC) temperature sensor.
- Output voltage is linearly proportional to the Centigrade (Celsius) temperature.
- Key Specifications:
- Scale Factor: 10 mV/°C.
- Accuracy: ±0.5°C at room temperature.
- Temperature Range: −55°C to +150°C.
- Operating Voltage: 4V to 30V.

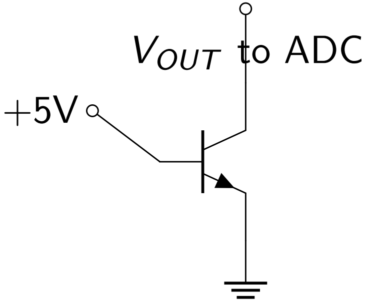


LM35: Working Principle

- Bandgap Temperature Sensor:
- Relies on the predictable temperature coefficient of the base-emitter voltage (V_{BE}) of transistors.
- Uses a pair of bipolar junction transistors (BJTs) with different emitter areas.
- The difference in their V_{BE} is directly proportional to absolute temperature (PTAT).
- Internal trimming resistors scale this PTAT voltage to output exactly 10 mV per degree Celsius.
- Does not require external calibration or trimming.

LM35: Circuit Connection & Calculation

- Basic Connection:
- Pin 1 to Supply (V_{CC}), Pin 3 to Ground, Pin 2 is Output (V_{OUT}).
- Formula for Temperature:
- $T(^{\circ}\text{C}) = \frac{V_{OUT}(\text{in Volts})}{0.010}$
- Example: If $V_{OUT} = 0.250$ V, then $T = \frac{0.250}{0.010} = 25^{\circ}$C.
- Negative Temperatures:
- Requires a pull-down resistor to a negative supply to allow V_{OUT} to sink current below 0V.



Quick Check 1

Question

An LM35 temperature sensor is connected to a voltmeter, and the reading is 375 mV. What is the current temperature measured by the sensor?

- A) 3.75°C

¡2-¿ **B)** 37.5°**C** ✓

- C) 375°C
- D) 3750°C

MQ2 Gas Sensor: Overview

- What is the MQ2?
- A metal oxide semiconductor (MOS) gas sensor.
- Sensitive to LPG, Propane, Methane, Hydrogen, Alcohol, and Smoke.
- Key Components:
- Sensing Element: Tin Dioxide (SnO_2).
- Heater Coil: Built-in heater to raise the SnO_2 layer to its operating temperature (approx. $200 - 300^{\circ}C$).
- Mesh Cover: Stainless steel net (anti-explosion) to allow gas in while preventing sparks from igniting external gas.

MQ2: Working Principle

- How does it detect gas?
- Clean Air: Oxygen molecules adsorb on the SnO_2 surface, trapping electrons. This creates a high resistance (low conductivity).
- Gas Presence: Combustible gases react with the adsorbed oxygen.
- Electron Release: This reaction releases the trapped electrons back into the SnO_2 conduction band.
- Resistance Drop: The electrical resistance of the sensor decreases dramatically as gas concentration increases.
- Output voltage increases proportionally in a voltage divider setup.

MQ2: Module Circuitry and Calibration

- Module Outputs:
- Analog Output (A0): Provides a continuous voltage corresponding to gas concentration.
- Digital Output (D0): Uses an onboard comparator (e.g., LM393) to output HIGH/LOW based on a threshold.
- Calibration:
- Sensor resistance in clean air (R_0) vs. resistance in target gas (R_S).
- Concentration in parts per million (ppm) is calculated using log-log sensitivity curves provided in the datasheet.
- Requires a 'burn-in' or pre-heating time of 24-48 hours for stable readings initially.

DHT-11 Digital Humidity and Temperature Sensor

- What is the DHT-11?
- A basic, ultra-low-cost digital temperature and humidity sensor.
- Specifications:
- Humidity: $20 - 80\%$ RH (Relative Humidity), $\pm 5\%$ accuracy.
- Temperature: $0 - 50°C$, $\pm 2°C$ accuracy.
- Output: Single-wire digital interface.
- Internal Components:
- Capacitive humidity sensing element.
- NTC (Negative Temperature Coefficient) thermistor.
- An 8-bit microcontroller to process analog signals into digital data.

DHT-11: Working Principle - Humidity

- Capacitive Humidity Sensor:
- Consists of two electrodes with a moisture-absorbing dielectric substrate (usually a polymer) between them.
- As humidity increases, the polymer absorbs water molecules from the air.
- The dielectric constant of the polymer changes, which alters the capacitance.
- The internal IC measures this capacitance change and converts it to a Relative Humidity (RH) value.

DHT-11: Working Principle - Temperature & Data

- Temperature Measurement:
- Uses an NTC Thermistor.
- Resistance decreases non-linearly as temperature increases.
- Internal IC applies Steinhart-Hart equations (or lookup tables) to determine temperature.
- Digital Data Format:
- Sends a 40-bit data stream serially over a single wire.
- Format: 8-bit integral RH data + 8-bit decimal RH data + 8-bit integral T data + 8-bit decimal T data + 8-bit checksum.
- Checksum ensures data integrity during transmission.

Quick Check 2

Question

Which physical parameter changes in the DHT-11 to measure Relative Humidity?

- A) Resistance of an NTC thermistor

¡2-¿ **B) Capacitance of a polymer dielectric** ✓

- C) The base-emitter voltage of a BJT
- D) The conductivity of a Tin Dioxide (SnO_2) layer

Comparative Analysis & Applications

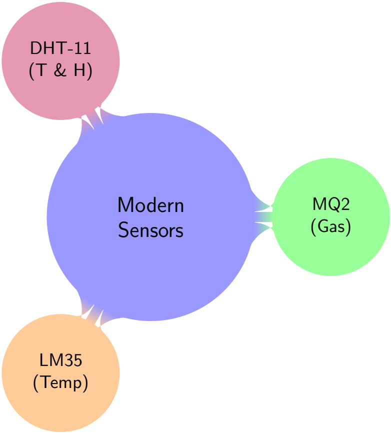
Sensor	Output Type	Applications
LM35 (Temp)	Analog (Voltage)	HVAC, electronics, battery monitoring
MQ2 (Gas)	Analog & Digital	Gas leak detectors, smoke alarms
DHT-11 (T+H)	Digital (Single-wire)	Weather stations, greenhouse, smart home

Interfacing with Microcontrollers

- General Steps for Interfacing:
- Power Supply: Ensure the sensor voltage matches the logic level of the MCU (e.g., 5V for Arduino Uno).
- Analog Sensors (LM35, MQ2 Analog Pin):
- Connect to ADC pins (e.g., A0).
- Convert raw ADC value $(0 - 1023)$ to voltage, then to physical unit using sensor formulas.
- Digital Sensors (DHT-11):
- Connect to any digital I/O pin.
- Use specific libraries (e.g., Adafruit DHT library) to handle the microsecond timing required to read the 40-bit stream.

Summary

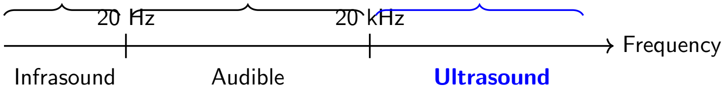
- LM35: Precision IC temperature sensor outputting a linear 10mV/°C analog voltage based on bandgap principles.
- MQ2: Gas sensor utilizing SnO_2 chemo-resistance, requiring a heating element, used to detect combustible gases.
- DHT-11: Integrated digital sensor combining a capacitive humidity element and an NTC thermistor, providing data via a 40-bit payload.
- Modern Sensors: Dramatically simplify instrumentation design by internalizing the complex physics and signal conditioning.



Next Lecture Preview

- Topic: Ultrasonic Sensors
- Working principle of Ultrasonic Transducers (Piezoelectric effect).
- Time of Flight (ToF) measurement methodology.
- Calculation of distance using the speed of sound.
- Common modules: HC-SR04.

- **Sound Spectrum:**
  - Infrasound: Below 20 Hz (inaudible to humans)
  - Acoustic/Audible: 20 Hz to $20,000$ Hz (human hearing range)
  - Ultrasound: Frequencies above 20 kHz ($20,000$ Hz)
- **Properties of Ultrasonic Waves:**
  - High frequency implies shorter wavelengths.
  - Highly directional, making them ideal for targeting specific areas.
  - Require a physical medium (air, liquid, or solid) to propagate; cannot travel in a vacuum.



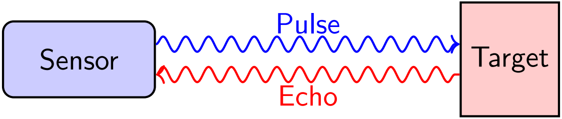
What is an Ultrasonic Sensor?

Definition

An electronic device that measures the distance to a target object by emitting ultrasonic sound waves and converting the reflected sound into an electrical signal.

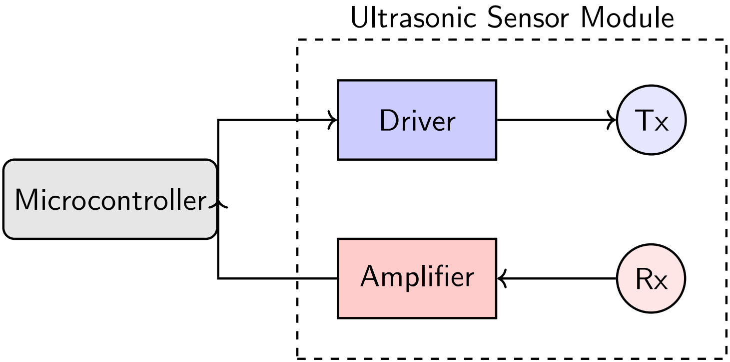
Key Characteristics:

- Provides non-contact distance measurement.
- Typically operates at frequencies like 40 kHz.
- Uses the echolocation principle, similar to how bats and dolphins navigate.



Core Components

- Ultrasonic sensors consist of three main parts:
  - **Transmitter (Tx):** Contains a piezoelectric crystal. Converts electrical oscillating signals into high-frequency sound waves (e.g., 40 kHz).
  - **Receiver (Rx):** Also contains a piezoelectric crystal. Captures the returning echo and converts the mechanical sound wave back into an electrical signal.
  - **Control Circuit (Transceiver Logic):** Generates the driving signal. Processes the received signal and calculates the time delay.



Working Principle: Time of Flight (ToF)

The Echolocation Process:

1. **Emission:** The transmitter sends out a short burst of ultrasonic waves.
2. **Propagation:** The waves travel through the air at the speed of sound.
3. **Reflection:** When the waves hit an object, they bounce back (echo).
4. **Reception:** The receiver detects the reflected waves.

Time of Flight (ToF)

The sensor measures the exact time interval between sending the pulse and receiving the echo. This time is directly proportional to the distance.

Distance Calculation Mathematics

Formula

$$Distance(D) = \frac{\text{Speed of Sound} \times \text{Time of Flight}}{2}$$

- **Constants & Variables:**
  - Speed of Sound (v): ≈ 343 m/s (or 0.0343 cm/μs) at 20°C in dry air.
  - Time (t): Total time taken for the wave to go and return.
  - Division by 2: Required because the measured time is for a round trip.
- **Example Calculation:**
  - If a sensor receives an echo after 500 μs:
  - $Distance = \frac{0.0343 \text{ cm}/\mu\text{s} \times 500\ \mu\text{s}}{2}$
  - $Distance = \frac{17.15 \text{ cm}}{2} = 8.575$ cm

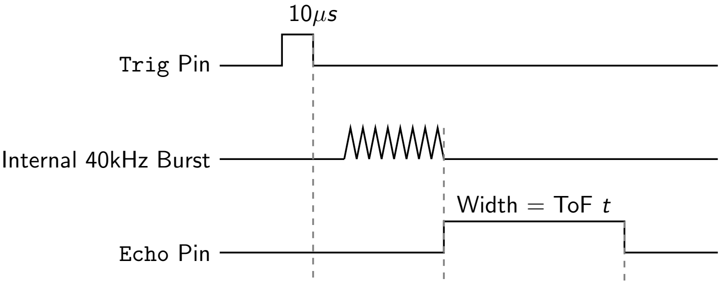
Practical Example: HC-SR04 Module

- The HC-SR04 is a highly popular, low-cost ultrasonic sensor module.
- **Specifications:**
  - Operating Voltage: 5V DC
  - Operating Frequency: 40 kHz
  - Measuring Range: 2 cm to 400 cm (4 meters)
  - Accuracy: ≈ 3 mm
- **Pin Configuration (4 Pins):**
  - `VCC`: 5V Power supply
  - `Trig` (Trigger): Input pin to initiate the measurement
  - `Echo`: Output pin that outputs a pulse proportional to the distance
  - `GND`: Ground

HC-SR04 Timing Diagram

Step-by-step Operation:

- **Trigger Pulse:** A 10 μs HIGH pulse is sent to the `Trig` pin.
- **Burst Emission:** The module sends eight 40 kHz ultrasonic pulses.
- **Echo Pulse:** The `Echo` pin goes HIGH immediately after the burst.
- **Measurement:** `Echo` stays HIGH for the Time of Flight (t).



- **Temperature and Humidity:**
  - The speed of sound varies with temperature: $v \approx 331.4 + (0.606 \times T)$ m/s.
  - Higher temperatures increase the speed, causing slight calculation errors if not compensated.
- **Target Surface Material:**
  - Hard, flat surfaces (metal, wood, concrete) reflect sound well.
  - Soft materials (foam, cloth, wool) absorb sound, leading to weak or missed echoes.
- **Target Angle:**
  - If a flat surface is heavily angled away from the sensor, the echo deflects away from the receiver.

Advantages of Ultrasonic Sensors

- **Non-contact Measurement:** No physical wear and tear; safe for measuring hazardous liquids or delicate objects.
- **Unaffected by Color/Transparency:** Unlike optical sensors, they can easily detect clear glass, water, or completely black objects.
- **Unaffected by Lighting:** Works perfectly in complete darkness or bright sunlight.
- **Cost-Effective:** Modules like HC-SR04 are very inexpensive.
- **High Reliability:** Solid-state components with no moving parts.

Disadvantages and Limitations

- **Medium Dependent:** Cannot work in a vacuum (e.g., space applications).
- **Resolution Limits:** Slower response time compared to optical sensors due to the slow speed of sound (compared to light).
- **Blind Zone:** Cannot measure objects closer than a minimum distance (e.g., < 2 cm) due to ringing of the transducer.
- **Environmental Sensitivity:** Drastic changes in temperature or strong air currents can distort the reading.

Applications of Ultrasonic Sensors

- **Industrial & Commercial:**
  - **Liquid Level Measurement:** Monitoring fluid levels in large chemical tanks or silos.
- **Automotive:**
  - Parking assist systems (bumper sensors) and blind-spot monitoring.
- **Robotics:**
  - Obstacle avoidance and mapping in autonomous robots and drones.
- **Medical:**
  - **Ultrasonography:** Using extremely high frequencies (1-18 MHz) for internal body imaging.
- **Automation:**
  - Presence detection on conveyor belts, counting objects, and automatic door opening.

Quick Check

Question 1

Why is an ultrasonic sensor capable of detecting a clear glass bottle, while an infrared (IR) sensor might fail?

Answer: IR sensors rely on light reflection, which passes through glass. Ultrasonic relies on sound reflection, which bounces off the solid glass surface.

Question 2

If the 'Echo' pulse duration is 1000 μs, what is the approximate distance? (Assume $v = 0.034$ cm/μs)

Answer: $D = \frac{0.034 \times 1000}{2} = \frac{34}{2} = 17$ cm.

Summary

- **Ultrasonic Sensors** use sound waves above 20 kHz to measure distance.
- **Principle:** They rely on the Time of Flight (ToF) of sound echoes.
- **Formula:** Distance is half the product of the speed of sound and the total trip time.
- **HC-SR04:** Uses a 10μs Trig pulse and outputs a variable width Echo pulse.
- **Pros/Cons:** Excellent for transparent/dark targets; poor for sound-absorbing materials and vulnerable to temperature fluctuations.

Preview for Next Lecture

Lecture 44: Optical Encoders

Moving from acoustic sensors to optoelectronic sensors.

- What is an Optical Encoder?
- Absolute vs. Incremental Encoders.
- Understanding the A, B, and Z (or C) waveform outputs.
- Applications in motor speed and position control.

Agenda for Today

- What is an Optical Encoder?
- Basic components and principle
- Incremental Encoders
    - Working mechanism
    - A, B, and C (Index) waveform outputs
    - Direction and resolution
- Absolute Encoders
    - Working mechanism
    - Binary and Gray code disks
- Comparison: Absolute vs. Incremental
- Summary & Applications

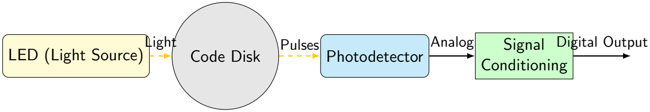
What is an Optical Encoder?

Definition and Purpose

- An optical encoder is an electromechanical device that converts angular position or motion of a shaft into analog or digital output codes.
- It provides feedback for position, velocity, and direction.

Primary Components

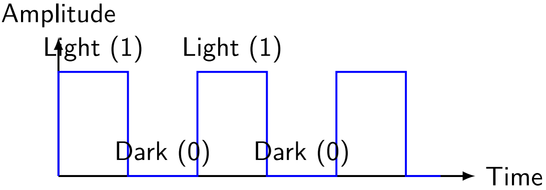
- Light Source: Usually an LED.
- Code Disk (Rotor): A disk with transparent and opaque segments.
- Photodetector Assembly: Converts light pulses into electrical signals.
- Signal Conditioning Circuit: Squares up the analog signals into digital pulses.



Basic Working Principle

How it generates a signal

- The LED emits a beam of light toward the photodetector.
- The code disk is placed between the light source and detector.
- As the shaft rotates, the opaque lines on the disk block the light, while clear windows let it pass.
- Light detected: High signal (Logic 1).
- Light blocked: Low signal (Logic 0).
- The result is a train of electrical pulses directly proportional to the rotation of the shaft.

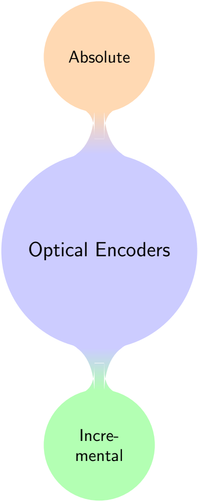


Classification of Optical Encoders

Two Main Types
Based on how they track position, optical encoders are classified into:

Incremental Encoders
- Provide relative position.
- Generate a train of pulses as the shaft rotates.
- Requires a reference point (homing) upon power-up.

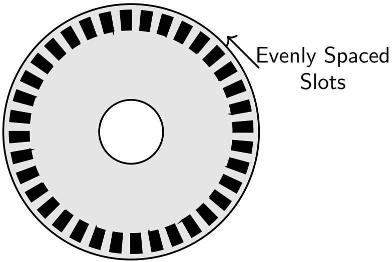
Absolute Encoders
- Provide unique absolute position.
- Output a distinct digital word for every specific angle.
- Retains position data even when power is lost.



Incremental Encoders: Overview

Characteristics

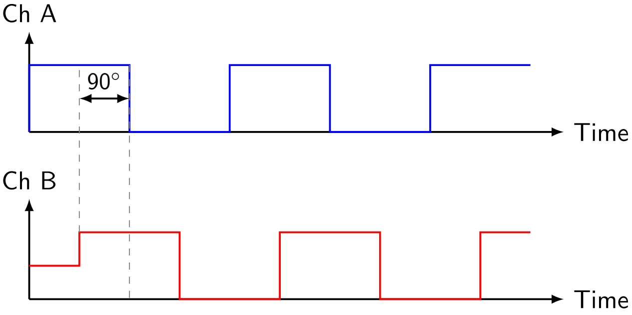
- The disk has a single track (or a few closely spaced tracks) of evenly spaced slots.
- Output is simply a series of high and low pulses.
- **Resolution**: Defined as Pulses Per Revolution (PPR). E.g., a 1000 PPR encoder gives 1000 pulses for one full 360-degree rotation.
- **Formula**: Angle per pulse $= 360^\circ/PPR$



Quadrature Outputs: A and B Channels

Determining Direction

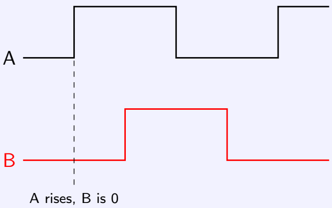
- To detect both distance and direction, incremental encoders use two signal channels: Channel A and Channel B.
- The photodetectors for A and B are positioned such that their output waveforms are 90° electrically out of phase.
- This phase shift is called **Quadrature**.
- By observing which channel leads the other, the controller can determine the direction of rotation (Clockwise vs. Counter-Clockwise).



Waveform Analysis: Direction Detection

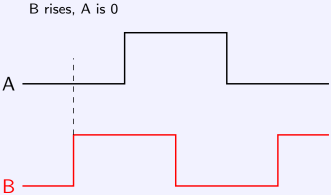
Clockwise (CW) Rotation

- Channel A leads Channel B by 90°.
- When A goes High, B is Low.



Counter-Clockwise (CCW)

- Channel B leads Channel A by 90°.
- When B goes High, A is Low.



The controller uses a D-flip flop or edge-detection logic to read the state of one channel when the other channel transitions.

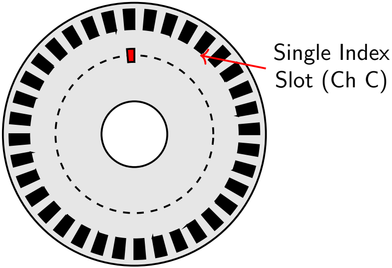
The Index Pulse: Channel C (or Z)

Homing and Reference

- Since incremental encoders lose position on power-off, a reference point is needed.
- Channel C (also known as Z or Index channel) provides **exactly one pulse per revolution**.
- It has its own separate track on the encoder disk with a single slot.

Purpose:

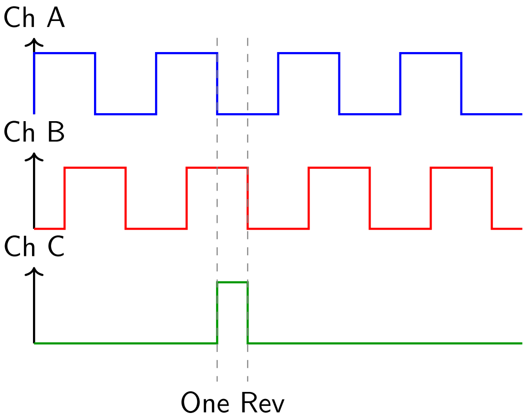
- Used for **homing** the machine to a known zero position.
- Used to **verify pulse counts** (error checking over one full revolution).



Complete A, B, C Waveform Output

The Complete Incremental Picture

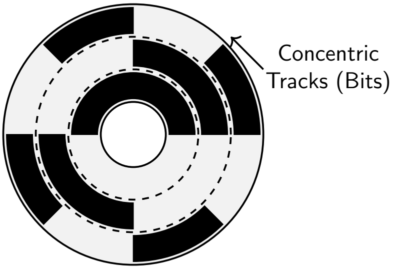
- **Channel A**: Square wave, primary counting signal.
- **Channel B**: Square wave, offset 90° from A for direction.
- **Channel C (Index)**: Single narrow pulse occurring once per 360°.
- **Decoding**: X1, X2, or X4 encoding can be used by the controller to increase resolution by counting rising and falling edges of both A and B.



Absolute Encoders: Overview

Characteristics

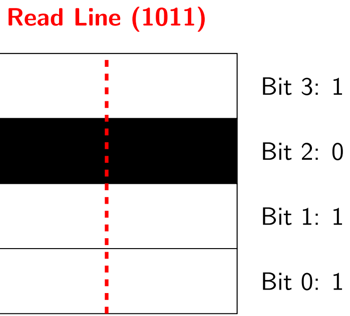
- Unlike incremental types, absolute encoders output a **unique digital code** for every discrete angle of the shaft.
- They have multiple tracks on the code disk, and a dedicated photodetector for each track.
- **Key Advantage**: Retains position information immediately upon power-up. No homing routine is required.
- **Resolution**: Defined in bits. An N-bit encoder has N tracks and can resolve 2^N unique positions per revolution.



Absolute Encoders: Working Mechanism

How it reads position

- If an encoder has 4 tracks, it is a 4-bit encoder.
- Resolution $= 2^4 = 16$ distinct positions per revolution ($360^\circ/16 = 22.5^\circ$ per position).
- A linear array of photodetectors spans the tracks radially.
- The combination of light (1) and dark (0) detected across the array forms a parallel digital word (e.g., 1011).
- Industrial absolute encoders often have 10 to 16 tracks (bits), providing extremely high resolution.



Coding Systems: Binary vs. Gray Code

The Problem with Standard Binary

- In standard binary code, multiple bits can change simultaneously (e.g., from position 3 011 to 4 100, three bits change).
- If sensors are slightly misaligned, temporary incorrect readings can occur during transitions.

The Gray Code Solution

- Absolute encoders typically use **Gray Code** formatting on the disk.
- In Gray code, **only one bit changes** between any two consecutive positions.
- This eliminates reading errors during transitions.

Decimal	Binary (Multiple changes)	Gray Code (1 bit changes)
0	0000	0000
1	0001	0001
2	0010 (2 bits change)	0011 (1 bit changes)
3	0011	0010
4	0100 (3 bits change)	0110 (1 bit changes)

Comparison: Absolute vs. Incremental

Feature	Incremental Encoder	Absolute Encoder
Output	Pulse train (A, B, C)	Unique digital word (e.g., Gray code)
Position on Power Loss	Lost (requires homing)	Retained
Disk Complexity	Simple (1-2 tracks)	Complex (Multiple tracks)
Cost	Generally lower	Higher
Best Used For	Speed control, relative motion	Exact positioning, safety-critical systems
Signal Processing	Counting edges over time	Reading parallel bits instantly

Applications of Optical Encoders

Where are they used?

- **Robotics**: Joint angle measurement (Absolute).
- **CNC Machining**: Precise tool positioning (Incremental/Absolute).
- **Printers and Scanners**: Paper feed and printhead positioning (Incremental).
- **Medical Equipment**: CT scanners and precision dosing pumps.
- **Automotive**: Electronic power steering systems.
- **Elevators**: Motor speed and car position tracking.

Robotics (Absolute)	CNC Machines	Printers (Incremental)

Quick Check

- **Question 1**: Which channel in an incremental encoder is used as a reference point for homing?

Quick Check

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  Answer: Channel C (or Z), as it gives one pulse per revolution.

- **Question 2**: Why do absolute encoders prefer Gray code over standard binary code?

Quick Check

- **Question 1**: Which channel in an incremental encoder is used as a reference point for homing?

  Answer: Channel C (or Z), as it gives one pulse per revolution.

- **Question 2**: Why do absolute encoders prefer Gray code over standard binary code?

  Answer: Because only one bit changes at a time, preventing multi-bit transition errors.

- **Question 3**: If an incremental encoder's Channel B leads Channel A by 90°, what does this typically signify?

Quick Check

- **Question 1**: Which channel in an incremental encoder is used as a reference point for homing?

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  Answer: Because only one bit changes at a time, preventing multi-bit transition errors.

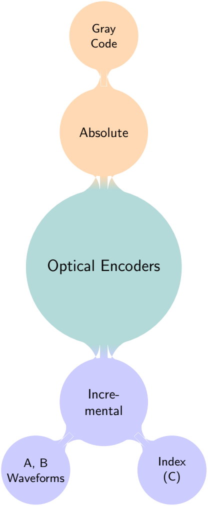
- **Question 3**: If an incremental encoder's Channel B leads Channel A by 90°, what does this typically signify?

  Answer: Rotation in the Counter-Clockwise (CCW) direction.

Summary

Key Takeaways

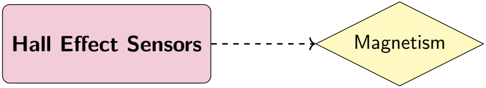
- Optical Encoders convert rotary motion into digital signals using a light source, patterned disk, and detector.
- **Incremental Encoders** output continuous pulses (A, B) to measure distance and direction, and one index pulse (C) per revolution for reference.
- **Quadrature** (A and B) allows for direction detection and resolution enhancement.
- **Absolute Encoders** output a distinct digital code (usually Gray code) for every angle, retaining position without power.



Next Lecture Preview

What's Next?

- **Lecture 45: Hall Effect Sensors and Summary of Transducers**
- Principle of the Hall Effect.
- Magnetic sensing applications.
- Wrap-up and comparative review of Unit 5 sensors.
- *Review today's notes on quadrature signals before the next class!*

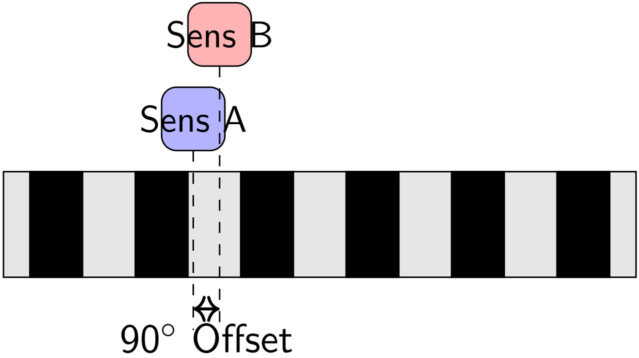


Recap: Optical Encoders

- **Incremental Encoders**
  - Generate a series of pulses as the shaft rotates.
  - Cannot determine absolute position upon power-up; requires a reference sequence.
- **Absolute Encoders**
  - Output a unique digital code for every angular position.
  - Retains position data even if power is lost.

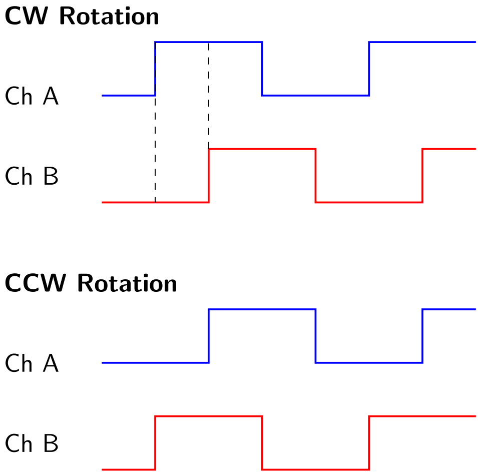
Incremental Encoders: Channel A and B Outputs

- Incremental encoders typically use two main output channels: Channel A and Channel B.
- Both channels output square waves as the optical disc's transparent and opaque lines pass the photosensors.
- The crucial feature is that these two channels are physically offset to create a 90° electrical phase shift (quadrature).



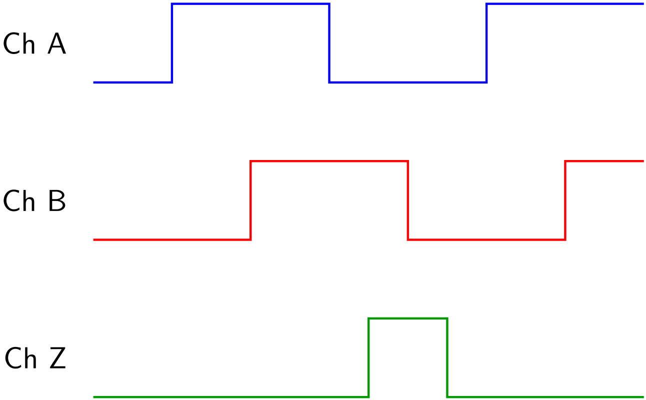
Quadrature Direction Sensing

- **Determining Direction of Rotation**
- **Clockwise (CW):** Channel A leads Channel B by 90°.
- **Counter-Clockwise (CCW):** Channel B leads Channel A by 90°.
- By checking the state of Channel B at the rising edge of Channel A, the microcontroller can instantly determine the direction.



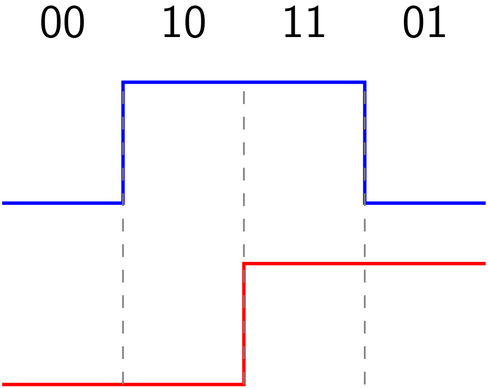
The Index Pulse: Channel Z (or C)

- A third output, often called Channel Z (or Index / Channel C), provides a single pulse per complete revolution (360°).
- **Purpose:**
  - Acts as a reference or 'home' position.
  - Allows the system to reset its counter to zero.
  - Useful for tracking the number of full rotations.



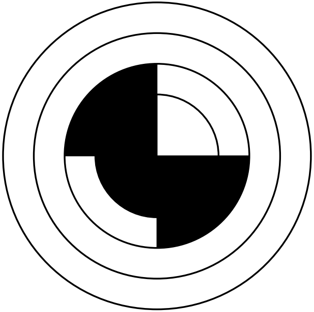
Complete A, B, Z Waveform Analysis

- **Analyzing the combined output:**
- **Resolution (PPR - Pulses Per Revolution):** Determined by the number of slits. Standard encoders can have 1000, 2048, or more PPR.
- **Quadrature Decoding (4x):** By counting every rising and falling edge of both Channel A and Channel B, we get 4 times the base resolution. A 1000 PPR encoder yields 4000 counts per revolution.



Absolute Encoder Code Tracks

- Absolute encoders use multiple concentric tracks, each representing a bit in a digital word.
- **Example:** An 8-bit absolute encoder has 8 tracks and provides $2^8 = 256$ unique positions per revolution.
- At any given angle, the combination of light passing through the tracks forms a specific code (e.g., 10110010).



Concentric Code Tracks

Binary Code vs Gray Code

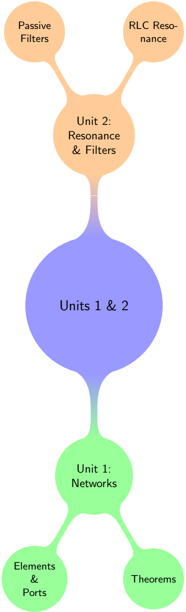
- **Standard Binary Code Issue:**
  - At boundaries (e.g., transitioning from 0111 to 1000), multiple bits change simultaneously. Slight misalignment can cause massive read errors (e.g., reading 1111 briefly).
- **Gray Code Solution:**
  - Only one bit changes at a time between successive positions.
  - Eliminates transition errors, making it the standard for absolute encoders.

Decimal	Binary Code	Gray Code
0	000	000
1	001	**001**
2	010	**011**
3	011	**010**
4	100	**110**

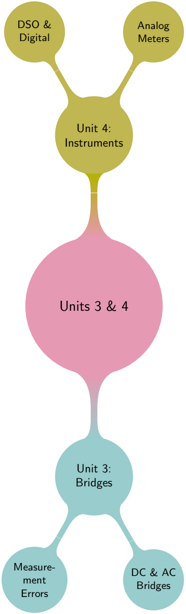
Applications of Optical Encoders

- **CNC Machines & Robotics:** Precise joint angle and tool position feedback.
- **Printers & Scanners:** Paper feed control and print-head positioning.
- **Elevators:** Tracking the exact position of the car in the shaft.
- **Servo Motors:** Integrated into the motor for closed-loop speed and position control.

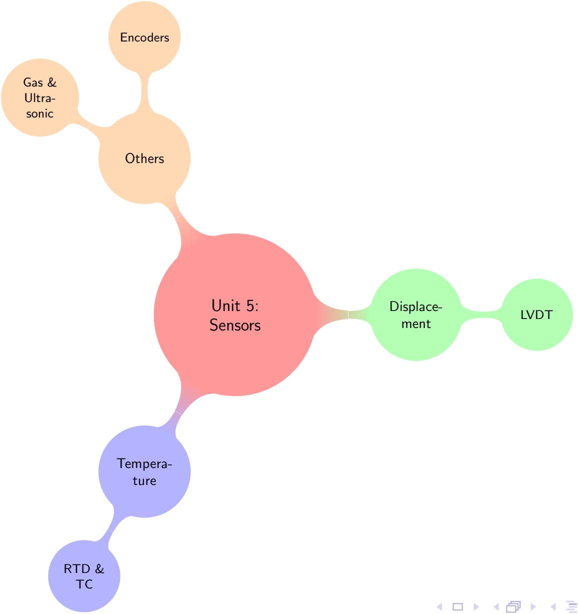
Course Summary: Unit 1 & 2



Course Summary: Unit 3 & 4



Course Summary: Unit 5



Quick Check

Self-Assessment Questions

1. What is the phase difference between Channel A and B in an incremental encoder?
2. What is the primary advantage of Gray code in absolute encoders?
3. Which network theorem replaces a complex circuit with a single voltage source and series resistance?

Final Takeaways

- **A, B, Z Waveforms:** Provide precise speed, direction (quadrature), and reference data.
- **System Integration:** Transducers and measurement instruments rely on the solid foundation of circuit theory and network analysis.
- **Real-World Engineering:** From calculating Z-parameters to reading Gray code from an encoder, electronic engineers build the bridge between theoretical mathematics and physical automation.

Best of luck with your examinations!