Problem Solver (DI03011021) - Problem Solver

Antigravity AI

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Problem Solver

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To my students,
who constantly inspire me to find new analogies,
break down complex theories, and make learning
an engineered science.

Preface

About this Book
Welcome to **Computer Networks**! This textbook is meticulously designed to serve as a comprehensive problem solver and study companion. It provides a robust and intuitive foundation in the core principles of this subject, bridging the gap between theoretical models and practical applications. A unique feature of this book is its focus on decoding complex concepts using clear, step-by-step methodologies and real-world analogies.

Target Audience
This book is intended for students and professionals looking to master the concepts of Computer Networks. Whether you are revising for exams, seeking to solidify your fundamental understanding, or looking for practical examples to clarify abstract concepts, this book has been structured to support your learning journey. It serves as an accessible guide designed to remove the fear of complex topics, replacing it with an appreciation for the underlying mechanics.

Scope and Coverage
The coverage is divided logically to follow standard academic curriculums. Each chapter focuses on key topics, breaking them down into digestible sections:

- **Theoretical Insights** – Core concepts explained intuitively.
- **Method Blocks** – Standardized, step-by-step procedures for solving specific types of problems.
- **Worked Examples** – Fully solved problems that apply the methods in practice.

We hope this resource proves invaluable in your studies and helps you achieve excellence in **Computer Networks**. Happy learning!

Acknowledgments

Writing a comprehensive book that connects the fundamental forces of Chemistry with practical engineering applications requires more than just academic knowledge—it requires a community of support. I would like to express my sincere gratitude to everyone who contributed to the realization of this project.

Specially, I extend my heartfelt gratitude to the **Department of Technical Education (DTE), Government of Gujarat**, and **Government Polytechnic, Palanpur**, for providing an environment that fosters innovation, academic rigor, and continuous professional development.

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Finally, my deepest appreciation goes to my students. Your curiosity, challenging questions, and continuous feedback are the true driving force behind the unique analogies and streamlined structure of this book. This textbook was engineered for you.

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About the Author

Milav Jayeshkumar Dabgar is an engineering educator and R&D professional with over 9 years of experience in electronics hardware development, embedded systems, AI/ML, and full-stack software engineering. He currently serves as a **Lecturer (Class - II) & Innovation Leader** at Government Polytechnic, Palanpur, under the Education Department of the Government of Gujarat.

Milav holds a Master of Engineering (ME) in Communication Systems from L.D. College of Engineering and a Bachelor of Engineering (BE) in Electronics & Communication. He is also currently pursuing a **BS in Data Science and Applications from IIT Madras**, where he has completed both the **Diploma in Programming** and the **Diploma in Data Science** with distinction.

Most recently, he completed the **AICTE QIP PG Certificate Program in Deep Learning from SVNIT Surat** with a **perfect 10/10 CGPA**, topping his batch.

A dedicated mentor, Milav has guided numerous student teams in securing over **INR 45 lakhs** in innovation funding, including INR 25 lakhs from Shark Tank India and INR 20 lakhs through iHub Gujarat, resulting in two student patents. He is recognized as an **NPTEL Evangelist** and **NPTEL Discipline Star** in Computer Science, reflecting his commitment to continuous learning and academic excellence.

His technical expertise spans from embedded systems (8051, STM32, Arduino) to modern web technologies (Next.js, Python, PostgreSQL) and Data Science (TensorFlow, PyTorch). Through this textbook, he aims to simplify complex Engineering Chemistry concepts by integrating relatable, real-world analogies drawn from modern tech and engineering landscapes.

Milav is also an active content creator, running a popular **YouTube channel** (@milav.dabgar) where he shares technical tutorials and academic resources. He maintains a personal blog and resource hub at milav.in and can be reached for professional collaborations on LinkedIn.

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CHAPTER 1

Operational Amplifiers (Op-Amps)

1.1 Introduction to Operational Amplifiers

An operational amplifier (Op-Amp) is a direct-coupled high-gain amplifier consisting of one or more differential amplifiers followed by a level translator and an output stage. The most commonly used Op-Amp is the IC 741.

1.1.1 Basic Block Diagram and Working

An Op-Amp internally consists of:

1. **Input Stage:** A dual-input balanced-output differential amplifier. Provides most of the voltage gain and determines input resistance.
2. **Intermediate Stage:** A dual-input unbalanced-output differential amplifier. Provides additional voltage gain.
3. **Level Shifting Stage:** Brings the DC level down to zero volts with respect to ground.
4. **Output Stage:** A push-pull complementary amplifier to increase output current capabilities and lower output resistance.

1.1.2 IC 741 Description and Pin Configuration

The IC 741 is an 8-pin dual-in-line package (DIP).

- **Pin 1 & 5:** Offset null
- **Pin 2:** Inverting input ($-$)
- **Pin 3:** Non-inverting input ($+$)
- **Pin 4:** Negative power supply ($-V_{EE}$)
- **Pin 6:** Output
- **Pin 7:** Positive power supply ($+V_{CC}$)
- **Pin 8:** No connection (NC)

1.1.3 The Ideal Op-Amp and Equivalent Circuit

An ideal Op-Amp has the following characteristics:

- Infinite voltage gain ($A = \infty$)
- Infinite input impedance ($R_{in} = \infty$)
- Zero output impedance ($R_{out} = 0$)
- Infinite bandwidth

- Infinite Common Mode Rejection Ratio (CMRR)
- Infinite Slew Rate
- Zero offset voltage (output is zero when inputs are zero)

The equivalent circuit of an Op-Amp models it as a voltage-controlled voltage source. The output voltage is $V_{out} = A(V_1 - V_2)$, where V_1 is the non-inverting input voltage and V_2 is the inverting input voltage.

1.2 Op-Amp Parameters

Methodology:

Op-Amp Parameter Calculations

1. **Common Mode Rejection Ratio (CMRR):** It is the ratio of differential voltage gain (A_d) to common-mode voltage gain (A_{cm}).

$$\text{CMRR} = \frac{A_d}{A_{cm}}$$

   In decibels (dB):

$$\text{CMRR (dB)} = 20 \log_{10} \left(\frac{A_d}{A_{cm}} \right)$$

2. **Slew Rate (SR):** The maximum rate of change of output voltage with time.

$$\text{SR} = \left. \frac{dV_{out}}{dt} \right|_{\text{max}} \quad \text{(typically in } V/\mu s\text{)}$$

   For a sinusoidal output $V_{out} = V_m \sin(2\pi f t)$, the Slew Rate is:

$$\text{SR} = 2\pi f V_m$$

3. **Input Bias Current (I_B):** The average of the currents entering the two input terminals.

$$I_B = \frac{I_{B1} + I_{B2}}{2}$$

4. **Input Offset Current (I_{io}):** The difference between the two input bias currents.

$$I_{io} = |I_{B1} - I_{B2}|$$

5. **Output Offset Voltage (V_{oo}):** The voltage appearing at the output when both inputs are grounded. It depends on the input offset voltage (V_{io}) and closed-loop gain (A_{CL}).

$$V_{oo} = V_{io} \times \left(1 + \frac{R_f}{R_1} \right)$$

Worked Example:

Calculate CMRR An Op-Amp has a differential gain of 10^5 and a common-mode gain of 0.2. Calculate its CMRR in linear ratio and in decibels.
Solution:

1. Identify the given values: Differential gain $A_d = 10^5$

   Common-mode gain $A_{cm} = 0.2$

2. Calculate CMRR as a ratio:

$$\text{CMRR} = \frac{A_d}{A_{cm}} = \frac{100000}{0.2} = 500000$$

3. Calculate CMRR in decibels:

$$\text{CMRR (dB)} = 20\log_{10}(500000) \approx 20 \times 5.699 = 113.98 \text{ dB}$$

Worked Example:

Calculate Slew Rate Maximum Frequency An Op-Amp is used as an inverting amplifier with a gain of 10. The Slew Rate of the Op-Amp is 0.5 V/μs. If the input signal is 0.5 V peak, calculate the maximum frequency of the input signal for undistorted output.

Solution:

1. Determine the peak output voltage V_m:

$$V_m = \text{Gain} \times V_{in} = 10 \times 0.5 \text{ V} = 5 \text{ V}$$

2. Use the Slew Rate formula to find $f_{\max}$:

$$\text{SR} = 2\pi f_{\max} V_m$$

$$0.5 \times 10^6 \text{ V/s} = 2\pi f_{\max}(5)$$

3. Solve for $f_{\max}$:

$$f_{\max} = \frac{0.5 \times 10^6}{10\pi} = \frac{50000}{3.1416} \approx 15.9 \text{ kHz}$$

1.3 Open Loop and Closed Loop Configurations

In open-loop configuration, no feedback is applied from the output to the input. The Op-Amp acts as a high-gain comparator. In closed-loop configuration, a portion of the output is fed back to the input, drastically reducing the overall gain but improving stability, bandwidth, and linearity.

1.4 Applications of Op-Amps

1.4.1 Inverting and Non-Inverting Amplifiers

Methodology:

Amplifier Gain and Output Voltage

1. **Inverting Amplifier:** The input signal is applied to the inverting terminal through resistor R_1 and feedback is provided through R_f.

$$V_{out} = -\left(\frac{R_f}{R_1}\right) V_{in}$$

   Closed-loop gain $A_{CL} = -\frac{R_f}{R_1}$

2. **Non-Inverting Amplifier:** The input signal is applied to the non-inverting terminal. Feedback is provided to the inverting terminal.

$$V_{out} = \left(1 + \frac{R_f}{R_1}\right) V_{in}$$

   Closed-loop gain $A_{CL} = 1 + \frac{R_f}{R_1}$

Worked Example:

Design an Inverting Amplifier Design an inverting amplifier with a closed-loop gain of -15. Assume the input resistor $R_1 = 10$ kΩ. Calculate the required feedback resistor R_f.
Solution:

1. State the given parameters: $A_{CL} = -15$, $R_1 = 10$ kΩ.
2. Use the inverting amplifier gain formula:
$$A_{CL} = -\frac{R_f}{R_1}$$
3. Substitute the values:
$$-15 = -\frac{R_f}{10\text{ k}\Omega}$$
4. Solve for R_f:
$$R_f = 15 \times 10\text{ k}\Omega = 150\text{ k}\Omega$$

Worked Example:

Calculate Output for Non Inverting Amplifier A non-inverting amplifier has $R_f = 50$ kΩ and $R_1 = 10$ kΩ. If an input voltage of 1.5 V is applied, calculate the output voltage.
Solution:

1. State the given parameters: $R_f = 50$ kΩ, $R_1 = 10$ kΩ, $V_{in} = 1.5$ V.
2. Calculate the closed-loop gain:
$$A_{CL} = 1 + \frac{R_f}{R_1} = 1 + \frac{50\text{ k}\Omega}{10\text{ k}\Omega} = 1 + 5 = 6$$
3. Calculate the output voltage:
$$V_{out} = A_{CL} \times V_{in} = 6 \times 1.5\text{ V} = 9\text{ V}$$

1.4.2 Summing Amplifier

Methodology:

Summing Amplifier Analysis An inverting summing amplifier produces an output voltage that is the inverted sum of multiple input voltages, each scaled by a factor. For inputs V_1, V_2, V_3 and corresponding input resistors R_1, R_2, R_3:
$$V_{out} = -\left(\frac{R_f}{R_1}V_1 + \frac{R_f}{R_2}V_2 + \frac{R_f}{R_3}V_3\right)$$
If $R_1 = R_2 = R_3 = R$, then:
$$V_{out} = -\frac{R_f}{R}(V_1 + V_2 + V_3)$$

Worked Example:

Calculate Summing Amplifier Output An inverting summing amplifier has $R_f = 100$ kΩ, $R_1 = 20$ kΩ, $R_2 = 50$ kΩ. The input voltages are $V_1 = 2$ V and $V_2 = -1$ V. Find the output voltage V_{out}.
Solution:

1. Identify the formula for a two-input summing amplifier:

$$V_{out} = -\left(\frac{R_f}{R_1}V_1 + \frac{R_f}{R_2}V_2\right)$$

2. Substitute the given values:

$$V_{out} = -\left(\frac{100\text{ k}\Omega}{20\text{ k}\Omega}(2\text{ V}) + \frac{100\text{ k}\Omega}{50\text{ k}\Omega}(-1\text{ V})\right)$$

3. Simplify each term:

$$V_{out} = -\left(5 \times 2 + 2 \times (-1)\right)$$

$$V_{out} = -(10 - 2) = -8\text{ V}$$

1.4.3 Comparator and Schmitt Trigger

Methodology:

Schmitt Trigger Threshold Voltages A Schmitt Trigger is a comparator with positive feedback which introduces hysteresis.

1. **Upper Threshold Voltage (V_{UT}):** The voltage at which the output switches from $+V_{sat}$ to $-V_{sat}$.

$$V_{UT} = \frac{R_1}{R_1 + R_2}(+V_{sat})$$

2. **Lower Threshold Voltage (V_{LT}):** The voltage at which the output switches from $-V_{sat}$ to $+V_{sat}$.

$$V_{LT} = \frac{R_1}{R_1 + R_2}(-V_{sat})$$

3. **Hysteresis Voltage (V_H):**

$$V_H = V_{UT} - V_{LT}$$

(Here R_2 is the feedback resistor to the non-inverting terminal and R_1 is the grounded resistor).

Worked Example:

Determine Schmitt Trigger Thresholds An Op-Amp Schmitt Trigger has positive feedback resistors $R_1 = 1\text{ k}\Omega$ and $R_2 = 9\text{ k}\Omega$. The saturation voltages are ± 15 V. Calculate the upper and lower threshold voltages and the hysteresis voltage.

Solution:

1. Calculate Upper Threshold Voltage (V_{UT}):

$$V_{UT} = \frac{R_1}{R_1 + R_2}(+V_{sat}) = \frac{1}{1+9}(15) = \frac{1}{10}(15) = 1.5\text{ V}$$

2. Calculate Lower Threshold Voltage (V_{LT}):

$$V_{LT} = \frac{R_1}{R_1 + R_2}(-V_{sat}) = \frac{1}{1+9}(-15) = -1.5\text{ V}$$

3. Calculate Hysteresis Voltage (V_H):

$$V_H = V_{UT} - V_{LT} = 1.5\text{ V} - (-1.5\text{ V}) = 3.0\text{ V}$$

1.4.4 Integrator and Differentiator

Methodology:

Integrator and Differentiator Operations

1. **Integrator:** The feedback element is a capacitor C and the input element is a resistor R. The output is proportional to the time integral of the input.

$$V_{out}(t) = -\frac{1}{RC}\int_0^t V_{in}(\tau)d\tau + V_{out}(0)$$

For a constant input $V_{in} = V$, the output is a ramp: $V_{out} = -\frac{V}{RC}t$.

2. **Differentiator:** The input element is a capacitor C and the feedback element is a resistor R. The output is proportional to the time derivative of the input.

$$V_{out}(t) = -RC\frac{dV_{in}(t)}{dt}$$

Worked Example:

Calculate Integrator Output An ideal Op-Amp integrator has $R = 100$ kΩ and $C = 1\mu$F. A constant DC voltage of 2 V is applied at the input at $t = 0$. Assuming initial output voltage is zero, find the output voltage after 0.5 seconds.
Solution:

1. Calculate the time constant RC:

$$RC = 100 \times 10^3 \times 1 \times 10^{-6} = 0.1 \text{ s}$$

2. Use the integrator formula for a constant DC input:

$$V_{out}(t) = -\frac{1}{RC}\int_0^t V_{in}dt = -\frac{V_{in}}{RC}t$$

3. Substitute $V_{in} = 2$ V, $RC = 0.1$ s and $t = 0.5$ s:

$$V_{out}(0.5) = -\frac{2}{0.1} \times 0.5 = -20 \times 0.5 = -10 \text{ V}$$

Worked Example:

Calculate Differentiator Output A differentiator circuit has $R = 10$ kΩ and $C = 0.1\mu$F. The input signal is a triangular wave with a rate of change of 5000 V/s. Find the magnitude of the output voltage.
Solution:

1. Calculate the time constant RC:

$$RC = 10 \times 10^3 \times 0.1 \times 10^{-6} = 10^{-3} \text{ s}$$

2. Identify the rate of change of the input:

$$\frac{dV_{in}}{dt} = 5000 \text{ V/s}$$

3. Use the differentiator formula:

$$V_{out} = -RC\frac{dV_{in}}{dt}$$

4. Substitute the values:

$$V_{out} = -(10^{-3}) \times 5000 = -5 \text{ V}$$

The magnitude of the output voltage is 5 V.

CHAPTER 2

Introduction to Thyristors

Thyristors are a family of solid-state semiconductor devices with four layers of alternating N and P-type materials. In this chapter, we focus on the construction, symbol, working principles, and computational aspects of various thyristors like SCR, DIAC, TRIAC, GTO, IGBT, and MCT, as well as Opto-Isolators.

2.1 Silicon Controlled Rectifier (SCR)

The SCR is a unidirectional four-layer (P-N-P-N) three-junction device with three terminals: Anode (A), Cathode (K), and Gate (G).

- **Construction**: Formed by alternating P and N layers (PNPN) creating three junctions (J_1, J_2, J_3).
- **Symbol**: A diode symbol with a gate terminal extending from the cathode side.
- **Working**: Operates in three modes: Forward Blocking (off state), Forward Conduction (on state triggered by Gate), and Reverse Blocking.
- **Characteristics**: The V-I curve shows high voltage blocking until the breakover voltage is reached or a gate pulse is applied, after which it enters a low-impedance conduction state.
- **Applications**: Phase control, motor drives, AC/DC converters, and relay controls.

2.1.1 Working of SCR Using Two Transistor Analogy

The SCR can be modeled as two interconnected transistors: a PNP transistor (T_1) and an NPN transistor (T_2). The collector of T_1 feeds the base of T_2, and the collector of T_2 feeds the base of T_1.

Methodology:

SCR Anode Current Calculation via Two Transistor Analogy To calculate the anode current (I_A) of an SCR based on the two-transistor model:

1. Identify the common-base current gains of the PNP transistor (α_1) and the NPN transistor (α_2).
2. Identify the gate current injected into the base of the NPN transistor (I_G).
3. Determine the individual leakage currents of the two transistors (I_{CBO1} and I_{CBO2}) and calculate the total leakage current $I_{CO} = I_{CBO1} + I_{CBO2}$.
4. Apply the anode current formula:
$$I_A = \frac{\alpha_2 I_G + I_{CO}}{1 - (\alpha_1 + \alpha_2)}$$
5. As $(\alpha_1 + \alpha_2)$ approaches 1, the SCR turns on and the current I_A becomes very large, limited only by external load resistance.

Worked Example:

Calculate SCR Anode Current An SCR is modeled using the two-transistor analogy. The PNP transistor has a current gain $\alpha_1 = 0.42$ and leakage current $I_{CBO1} = 1.5\,\mu$A. The NPN transistor has a current gain $\alpha_2 = 0.48$ and leakage current $I_{CBO2} = 2.5\,\mu$A. A gate current of $I_G = 1.2$ mA is applied. Calculate the anode current I_A.

Step 1: Identify given parameters

- $\alpha_1 = 0.42$
- $\alpha_2 = 0.48$
- $I_G = 1.2 \text{ mA} = 1200\,\mu\text{A}$
- $I_{CBO1} = 1.5\,\mu$A
- $I_{CBO2} = 2.5\,\mu$A

Step 2: Calculate total leakage current I_{CO}

$$I_{CO} = I_{CBO1} + I_{CBO2} = 1.5 + 2.5 = 4.0\,\mu\text{A}$$

Step 3: Apply the anode current formula

$$I_A = \frac{\alpha_2 I_G + I_{CO}}{1 - (\alpha_1 + \alpha_2)}$$

$$I_A = \frac{(0.48 \times 1200) + 4.0}{1 - (0.42 + 0.48)}$$

Step 4: Compute the final value

$$I_A = \frac{576 + 4.0}{1 - 0.90} = \frac{580}{0.10} = 5800\,\mu\text{A} = 5.8 \text{ mA}$$

Conclusion: The anode current I_A is 5.8 mA.

2.2 DIAC (Diode for Alternating Current)

A DIAC is a bidirectional semiconductor switch that can be turned on in both forward and reverse polarities.

- **Construction**: Three-layer, two-terminal (MT1 and MT2) structure, practically two diodes connected in parallel but opposite directions.
- **Symbol**: Two diodes connected in anti-parallel.
- **Working**: Remains off until the applied voltage exceeds its breakover voltage (V_{BO}). Once V_{BO} is reached, it exhibits negative resistance and conducts.
- **Characteristics**: Symmetrical V-I curve in both the first and third quadrants.
- **Applications**: Used primarily as a triggering device for TRIACs in phase-control circuits like light dimmers and motor speed controllers.

Methodology:

DIAC RC Snubber and Triggering Time Calculation In a typical DIAC-TRIAC trigger circuit, an RC network determines when the DIAC breakover voltage (V_{BO}) is reached.

1. Identify the DC supply voltage (V_S), charging resistor (R), charging capacitor (C), and DIAC breakover voltage (V_{BO}).
2. Formulate the capacitor voltage equation: $V_C(t) = V_S(1 - e^{-t/RC})$.

3. Set $V_C(t) = V_{BO}$ to find the trigger time (t_{trig}).

4. Solve for t_{trig}:

$$t_{trig} = -RC \ln\left(1 - \frac{V_{BO}}{V_S}\right)$$

Worked Example:

Calculate DIAC Triggering Time A triggering circuit is powered by a DC source of $V_S = 100$ V. An RC timing circuit is used with $R = 50$ kΩ and $C = 0.1\,\mu$F. The DIAC has a breakover voltage of $V_{BO} = 30$ V. Calculate the time taken for the DIAC to trigger.

Step 1: Identify given parameters

- $V_S = 100$ V
- $R = 50 \times 10^3\,\Omega$
- $C = 0.1 \times 10^{-6}$ F
- $V_{BO} = 30$ V

Step 2: Compute the time constant τ

$$\tau = RC = (50 \times 10^3) \times (0.1 \times 10^{-6}) = 5 \times 10^{-3} \text{ s} = 5 \text{ ms}$$

Step 3: Solve for t_{trig}

$$t_{trig} = -\tau \ln\left(1 - \frac{V_{BO}}{V_S}\right)$$

$$t_{trig} = -5 \times 10^{-3} \ln\left(1 - \frac{30}{100}\right) = -5 \times 10^{-3} \ln(0.7)$$

$$t_{trig} \approx -5 \times 10^{-3} \times (-0.3567) \approx 1.78 \text{ ms}$$

Conclusion: The DIAC triggers after approximately 1.78 ms.

2.3 TRIAC (Triode for Alternating Current)

A TRIAC is essentially two SCRs connected in inverse parallel, integrated on the same silicon chip, allowing it to conduct alternating current in both directions.

- **Construction**: Five-layer device with three terminals: MT1, MT2, and Gate.
- **Symbol**: Two anti-parallel SCRs with a single common gate terminal.
- **Working**: Can be triggered into conduction by either a positive or negative gate pulse, irrespective of the MT2 polarity relative to MT1.
- **Characteristics**: Similar to an SCR but operates in both the first and third quadrants of the V-I plane.
- **Applications**: AC light dimmers, fan speed controllers, and solid-state relays.

Methodology:

RMS Voltage Calculation for TRIAC Phase Control To compute the RMS voltage delivered to a resistive load by a TRIAC under phase angle control:

1. Identify the peak supply voltage V_m or the RMS supply voltage $V_{S,rms}$. Note that $V_m = \sqrt{2}V_{S,rms}$.

2. Identify the firing angle α in radians.

3. Use the RMS output voltage formula for symmetric bidirectional phase control:

$$V_{rms} = V_{S,rms}\sqrt{\frac{1}{\pi}\left(\pi - \alpha + \frac{\sin(2\alpha)}{2}\right)}$$

4. The power delivered to the resistive load R is $P = \frac{V_{rms}^2}{R}$.

Worked Example:

Calculate TRIAC Output Voltage and Power A TRIAC is used to control power to a $100\,\Omega$ resistive heating load from a 230 V RMS, 50 Hz supply. If the firing angle is $\alpha = 90^\circ$ ($\frac{\pi}{2}$ radians), calculate the RMS output voltage and the power delivered to the load.

Step 1: Identify given parameters

- $V_{S,rms} = 230$ V
- $R = 100\,\Omega$
- $\alpha = \frac{\pi}{2}$ radians

Step 2: Calculate RMS output voltage

$$V_{rms} = 230\sqrt{\frac{1}{\pi}\left(\pi - \frac{\pi}{2} + \frac{\sin(\pi)}{2}\right)}$$

$$V_{rms} = 230\sqrt{\frac{1}{\pi}\left(\frac{\pi}{2} + 0\right)} = 230\sqrt{\frac{1}{2}} = \frac{230}{\sqrt{2}} \approx 162.63 \text{ V}$$

Step 3: Calculate power delivered to the load

$$P = \frac{V_{rms}^2}{R} = \frac{(162.63)^2}{100} = \frac{26448.5}{100} \approx 264.49 \text{ W}$$

Conclusion: The RMS output voltage is 162.63 V and the load power is 264.49 W.

2.4 Gate Turn-Off Thyristor (GTO)

A GTO is an SCR that can be turned ON by a positive gate pulse and turned OFF by a negative gate current pulse.

- **Construction**: Modified PNPN structure with a highly interdigitated gate-cathode structure to ensure fast turn-off.
- **Symbol**: SCR symbol with a double-arrowed line on the gate terminal indicating bidirectional gate current capability.
- **Working**: A positive pulse turns it on. A large negative pulse at the gate withdraws carriers from the base region, bringing the device out of conduction.
- **Characteristics**: Similar forward characteristics to an SCR, but features a specified Turn-off Gain.
- **Applications**: High-power traction drives, induction heating, and heavy industrial inverters.

Methodology:

GTO Turn-Off Gain Calculation The Turn-off Gain (β_{off}) evaluates how effectively a negative gate current turns off the GTO.

1. Identify the continuous anode current I_A flowing through the GTO before turn-off.

2. Identify the peak negative gate current $I_{G(off)}$ required to turn off the GTO.
3. Calculate the Turn-off Gain using the formula:

$$\beta_{off} = \frac{I_A}{I_{G(off)}}$$

4. A higher Turn-off Gain implies that a smaller negative gate current is needed to interrupt a given anode current.

Worked Example:

Calculate Negative Gate Current for GTO A GTO is carrying an anode current of 800 A. The device datasheet specifies a Turn-off Gain of $\beta_{off} = 4$. Calculate the peak negative gate current required to turn off the GTO.

Step 1: Identify given parameters

- $I_A = 800$ A
- $\beta_{off} = 4$

Step 2: Apply the Turn-off Gain formula

$$\beta_{off} = \frac{I_A}{I_{G(off)}}$$

Step 3: Solve for $I_{G(off)}$

$$I_{G(off)} = \frac{I_A}{\beta_{off}} = \frac{800}{4} = 200 \text{ A}$$

Conclusion: A negative gate pulse of 200 A is required to successfully turn off the device.

2.5 Insulated Gate Bipolar Transistor (IGBT)

The IGBT combines the high input impedance of a MOSFET with the high current density and low on-state voltage drop of a BJT.

- **Construction**: Consists of a MOSFET gate driving a wide-base PNP bipolar transistor.
- **Symbol**: A BJT-like symbol with an isolated gate parallel to the base line.
- **Working**: Voltage applied to the gate induces a channel allowing base current to drive the internal PNP transistor. Removing the gate voltage turns the device off.
- **Characteristics**: Low saturation voltage and high switching speed. Exhibits both MOSFET-like input and BJT-like output characteristics.
- **Applications**: Motor drives, UPS, electric vehicles, and switched-mode power supplies.

Methodology:

Calculation of Power Dissipation in IGBT Total power loss in an IGBT includes both conduction and switching losses.

1. Determine conduction parameters: On-state saturation voltage $V_{CE(sat)}$, Collector current I_C, and Duty cycle D.
2. Calculate Conduction Loss:

$$P_{cond} = V_{CE(sat)} \times I_C \times D$$

3. Determine switching parameters: Turn-on energy E_{on}, Turn-off energy E_{off}, and Switching frequency

f_{sw}.

4. Calculate Switching Loss:
$$P_{sw} = f_{sw} \times (E_{on} + E_{off})$$
5. Calculate Total Power Dissipation:
$$P_{total} = P_{cond} + P_{sw}$$

Worked Example:

Calculate Total Power Dissipation in IGBT An IGBT operates in a converter with a switching frequency of 15 kHz and a duty cycle of 40%. The collector current is 50 A, and the saturation voltage is 2.0 V. The switching energies are $E_{on} = 4$ mJ and $E_{off} = 6$ mJ. Calculate the total power dissipated by the IGBT.

Step 1: Identify given parameters

- $f_{sw} = 15000$ Hz
- $D = 0.40$
- $I_C = 50$ A
- $V_{CE(sat)} = 2.0$ V
- $E_{on} = 4 \times 10^{-3}$ J
- $E_{off} = 6 \times 10^{-3}$ J

Step 2: Calculate Conduction Loss

$$P_{cond} = 2.0 \text{ V} \times 50 \text{ A} \times 0.40 = 40 \text{ W}$$

Step 3: Calculate Switching Loss

$$P_{sw} = 15000 \times (4 \times 10^{-3} + 6 \times 10^{-3}) = 15000 \times 10 \times 10^{-3} = 150 \text{ W}$$

Step 4: Calculate Total Power Dissipation

$$P_{total} = 40 \text{ W} + 150 \text{ W} = 190 \text{ W}$$

Conclusion: The total power dissipated by the IGBT is 190 W.

2.6 MOS-Controlled Thyristor (MCT)

An MCT is a voltage-controlled thyristor that combines the features of an SCR with integrated MOS gates for both turning ON and OFF.

- **Construction**: An inherently high-density device with multiple microcells containing N-channel MOSFETs (for turn-off) and P-channel MOSFETs (for turn-on) integrated onto a PNPN structure.
- **Symbol**: Thyristor symbol combined with a MOSFET gate.
- **Working**: A negative gate pulse (relative to anode) turns on a P-MCT, and a positive gate pulse turns it off.
- **Characteristics**: Extremely low forward voltage drop, high current density, and very fast switching capability compared to GTOs.
- **Applications**: High power aerospace switches, reactive power compensators, and induction heating.

Methodology:

MCT Switching Frequency Limits Due to internal junction capacitances, thyristors have thermal limits based on switching frequencies. To determine maximum switching limits:

1. Identify total allowable power dissipation P_{max}.
2. Identify the conduction loss P_{cond}.
3. Calculate available margin for switching losses: $P_{sw\_max} = P_{max} - P_{cond}$.
4. Using total energy per cycle $E_{cycle} = E_{on} + E_{off}$, calculate the maximum switching frequency:

$$f_{max} = \frac{P_{sw\_max}}{E_{cycle}}$$

Worked Example:

Calculate Maximum Switching Frequency for MCT An MCT is used in a high-power inverter. The total power dissipation limit of its heat sink setup is 250 W. At its operating current, the conduction loss is calculated to be 70 W. If the total energy lost per switching cycle is 12 mJ, find the maximum allowable switching frequency.

Step 1: Identify given parameters

- $P_{max} = 250$ W
- $P_{cond} = 70$ W
- $E_{cycle} = 12 \times 10^{-3}$ J

Step 2: Calculate allowed switching power loss

$$P_{sw\_max} = P_{max} - P_{cond} = 250 - 70 = 180 \text{ W}$$

Step 3: Calculate maximum switching frequency

$$f_{max} = \frac{180}{12 \times 10^{-3}} = 15000 \text{ Hz} = 15 \text{ kHz}$$

Conclusion: The MCT can be switched safely up to a maximum frequency of 15 kHz.

2.7 Opto-Isolators and Solid State Relays

An opto-isolator (or optocoupler) uses light to transfer electrical signals between two isolated circuits, offering high electrical isolation.

- **Construction**: An infrared LED on the input side and a photosensitive device on the output side (like a phototransistor, LASCR, or Opto-TRIAC), usually housed in an opaque package.
- **Working**: Current flowing through the LED emits IR light, which is detected by the photosensor, switching it into conduction.
- **Advantages**: Complete electrical isolation, immunity to electromagnetic interference (EMI), and noise suppression.
- **Types**:
  - **Opto-transistor**: Uses a phototransistor for linear or digital signal transfer.
  - **Opto-SCR (LASCR)**: Light-Activated SCR. Used to switch high DC voltages.
  - **Opto-TRIAC**: A bidirectional opto-switch used to drive larger external TRIACs in AC applications.

- **Solid State Relay (SSR) using Opto-Isolators**: An SSR uses an opto-isolator for the low-voltage control side to trigger a high-power TRIAC or back-to-back SCRs on the load side. This replaces mechanical relays, offering silent, fast, and arc-free switching.

Methodology:

Opto-Isolator LED Resistor Sizing and Output Current Calculation To safely drive an opto-isolator and determine the resulting output capability:

1. Identify the control logic voltage V_{in}, the LED forward voltage drop V_F, and the desired forward current I_F.
2. Calculate the series current-limiting resistor R_S:

$$R_S = \frac{V_{in} - V_F}{I_F}$$

3. Identify the Current Transfer Ratio (CTR) from the device datasheet (often expressed as a percentage or fraction).
4. Calculate the expected output collector current I_C:

$$I_C = I_F \times \text{CTR}$$

Worked Example:

Design Opto-Transistor Driver Circuit A microcontroller with a 5 V output logic level is used to drive an opto-transistor. The opto-LED requires a forward current of $I_F = 15$ mA and has a forward voltage drop of $V_F = 1.2$ V. The opto-isolator has a CTR of 50%. Calculate the required series resistor value and the output collector current.

Step 1: Identify the given parameters.

- $V_{in} = 5$ V
- $V_F = 1.2$ V
- $I_F = 15$ mA $= 0.015$ A
- CTR $= 50\% = 0.50$

Step 2: Calculate the series resistor R_S

$$R_S = \frac{5 - 1.2}{0.015} = \frac{3.8}{0.015} \approx 253.33\,\Omega$$

(A standard resistor value of $270\,\Omega$ or $220\,\Omega$ could be selected in practice.)

Step 3: Calculate the expected output collector current I_C

$$I_C = I_F \times \text{CTR} = 15 \text{ mA} \times 0.50 = 7.5 \text{ mA}$$

Conclusion: A series resistor of approximately 253.33 Ω is needed, and the resulting output current capability is 7.5 mA.

CHAPTER 3

Turn on and Turn off Methods of Thyristor

This chapter focuses on the computational aspects of Thyristor (SCR) turn-on (triggering) methods, turn-off (commutation) techniques, and the essential protection circuits required for reliable operation.

3.1 Triggering Methods of SCR

Triggering is the process of turning on an SCR from a forward-blocking state to a forward-conduction state. The most common and reliable method is gate triggering.

3.1.1 Gate Drive Circuit Design

To turn on an SCR, a positive voltage must be applied between the gate and the cathode. A series resistor is used to limit the gate current to a safe value.

Methodology:

Calculation of Gate Series Resistance To determine the required gate series resistance (R_g) for a simple DC gate drive circuit:

1. Identify the DC trigger source voltage (E_s).
2. Identify the required gate voltage (V_g) and gate current (I_g) to turn on the SCR.
3. Ensure that the maximum gate power dissipation P_{gm} is not exceeded: $P_g = V_g \cdot I_g \leq P_{gm}$.
4. Apply Kirchhoff's Voltage Law (KVL) to the gate circuit loop:

$$E_s = I_g R_g + V_g$$

5. Rearrange the formula to solve for R_g:

$$R_g = \frac{E_s - V_g}{I_g}$$

Worked Example:

Calculate Required Gate Resistance An SCR is triggered from a 15 V DC source. The minimum required gate current is 100 mA and the corresponding gate voltage is 2 V. The maximum permissible gate power dissipation is 5 W. Calculate the value of the series gate resistance required and verify if the power dissipation is within limits.

Solution:

1. **Given Data:** Source voltage $E_s = 15$ V, Gate current $I_g = 100$ mA $= 0.1$ A, Gate voltage $V_g = 2$ V, Max power $P_{gm} = 5$ W.

2. **Calculate R_g:** Using the formula for gate resistance:

$$R_g = \frac{E_s - V_g}{I_g} = \frac{15 - 2}{0.1} = \frac{13}{0.1} = 130\ \Omega$$

3. **Check Power Dissipation:** The actual gate power dissipation is:

$$P_g = V_g \cdot I_g = 2 \times 0.1 = 0.2\ \text{W}$$

Since $0.2\ \text{W} \leq 5\ \text{W}$, the power dissipation is well within the safe limits.

The required series gate resistance is 130 Ω.

3.1.2 Pulse Triggering and Power Calculation

Instead of continuous DC triggering, pulse triggering is often used to reduce power dissipation in the gate circuit.

Methodology:

Pulse Triggering Power Calculation When a train of pulses is applied to the gate:

1. Identify the pulse width (t_p) and the frequency of the pulse train (f).
2. Calculate the periodic time $T = \frac{1}{f}$.
3. The duty cycle δ of the pulse is given by $\delta = \frac{t_p}{T} = t_p \cdot f$.
4. The average gate power dissipation (P_{gav}) is calculated as:

$$P_{gav} = P_{gm} \cdot \delta = P_{gm} \cdot t_p \cdot f$$

where P_{gm} is the peak gate power during the pulse.

Worked Example:

Compute Average Gate Power A thyristor is triggered by a pulse train of 2 kHz frequency. The pulse width is 100 μs. If the peak gate power is 4 W, calculate the average gate power dissipated.
Solution:

1. **Given Data:** Frequency $f = 2000$ Hz, Pulse width $t_p = 100 \times 10^{-6}$ s, Peak power $P_{gm} = 4$ W.
2. **Calculate Duty Cycle (δ):**

$$\delta = t_p \cdot f = (100 \times 10^{-6}) \times 2000 = 0.2$$

3. **Calculate Average Gate Power (P_{gav}):**

$$P_{gav} = P_{gm} \cdot \delta = 4 \times 0.2 = 0.8\ \text{W}$$

The average gate power dissipated is 0.8 W.

3.2 Commutation Techniques of SCR

Commutation is the process of turning off a conducting SCR. In DC circuits, forced commutation techniques (using reactive components like capacitors and inductors) are required because the DC voltage does not go to zero naturally.

3.2.1 Class C Commutation (Complementary Commutation)

In Class C commutation, a capacitor is used to apply a reverse voltage across the conducting SCR when an auxiliary SCR (or the other main SCR) is triggered.

Methodology:

Class C Commutation Component Calculation To design a Class C commutation circuit:

1. Identify the DC supply voltage (V_{dc}), the load resistance (R), and the turn-off time of the SCR (t_q).
2. Calculate the load current $I_0 = \frac{V_{dc}}{R}$.
3. The circuit turn-off time t_c provided by the commutating capacitor must be strictly greater than the SCR turn-off time t_q. Typically, $t_c = (1.5 \text{ to } 2) \cdot t_q$ is chosen.
4. The expression for the circuit turn-off time in Class C commutation is:

$$t_c = R \cdot C \cdot \ln(2) \approx 0.693 \cdot R \cdot C$$

5. Rearrange to find the required commutating capacitor C:

$$C = \frac{t_c}{0.693 \cdot R}$$

Worked Example:

Find the Commutating Capacitor for Class C A Class C commutation circuit is supplied with a DC voltage of 200 V. The load resistance is 10 Ω. The thyristor has a turn-off time of 40 μs. Determine the required value of the commutating capacitor assuming a safety factor of 1.5 for the circuit turn-off time.
Solution:

1. **Given Data:** $V_{dc} = 200$ V, $R = 10\ \Omega$, $t_q = 40\ \mu\text{s} = 40 \times 10^{-6}$ s.
2. **Calculate Required Circuit Turn-off Time (t_c):**

$$t_c = 1.5 \times t_q = 1.5 \times 40\ \mu\text{s} = 60\ \mu\text{s}$$

3. **Calculate Commutating Capacitor (C):** Using the formula $t_c = 0.693 \cdot R \cdot C$:

$$C = \frac{60 \times 10^{-6}}{0.693 \times 10} = \frac{60 \times 10^{-6}}{6.93} \approx 8.66\ \mu\text{F}$$

The required value of the commutating capacitor is 8.66 μF.

3.2.2 Class B Commutation (Resonant Commutation)

In Class B commutation, an LC resonant circuit is connected in parallel with the SCR.

Methodology:

Class B Commutation LC Design

1. Identify the DC supply voltage (V_{dc}), the maximum load current (I_0), and the SCR turn-off time (t_q).
2. The peak commutation current (I_p) produced by the LC circuit must be greater than the load current. Typically, $I_p = 1.5 \cdot I_0$.

3. The peak resonant current is given by:

$$I_p = V_{dc}\sqrt{\frac{C}{L}}$$

4. The conduction time of the resonant pulse is $t_p = \pi\sqrt{LC}$.

5. To ensure the circuit provides sufficient turn-off time (t_c), simplified approximations often link the capacitor and inductor values directly to the peak current and desired t_c:

$$C = \frac{I_p \cdot t_c}{V_{dc}} \quad \text{and} \quad L = \frac{V_{dc} \cdot t_c}{I_p}$$

3.3 Thyristor Protection

Thyristors are delicate power semiconductor devices and can be easily destroyed by excessive voltage, current, or rapid changes in voltage and current.

3.3.1 Over-voltage and di/dt Protection

Rapid variations in current ($\frac{di}{dt}$) during turn-on can cause localized heating and destroy the SCR. An inductor is connected in series with the SCR to limit the rate of rise of current.

Methodology:

Series Inductor Design for Rate of Current Rise Protection To protect the SCR against high $\frac{di}{dt}$:

1. Identify the maximum operating supply voltage (V_m) and the maximum permissible rate of rise of current $\left(\frac{di}{dt}\right)_{max}$ specified in the SCR datasheet.

2. Apply the inductor voltage equation:

$$V_m = L_s \cdot \left(\frac{di}{dt}\right)_{max}$$

3. Solve for the required series inductance L_s:

$$L_s = \frac{V_m}{\left(\frac{di}{dt}\right)_{max}}$$

Worked Example:

Calculate Series Inductor for Current Rise Limitation An SCR operates from a 230 V RMS AC supply. The rated maximum $\frac{di}{dt}$ for the SCR is 50 A/μs. Calculate the minimum value of series inductance required to protect the SCR.

Solution:

1. **Given Data:** $V_{rms} = 230$ V, $\left(\frac{di}{dt}\right)_{max} = 50$ A/μs $= 50 \times 10^6$ A/s.

2. **Calculate Maximum Voltage (V_m):**

$$V_m = V_{rms} \times \sqrt{2} = 230 \times 1.414 = 325.22 \text{ V}$$

3. **Calculate Required Series Inductance (L_s):**

$$L_s = \frac{V_m}{\left(\frac{di}{dt}\right)_{max}} = \frac{325.22}{50 \times 10^6} = 6.5 \times 10^{-6} \text{ H} = 6.5\ \mu\text{H}$$

The required series inductance is 6.5 μH.

3.3.2 Snubber Circuit Design (dv/dt Protection)

A high rate of rise of voltage across the SCR ($\frac{dv}{dt}$) can cause unintended turn-on due to the internal junction capacitance. An RC Snubber circuit (a resistor and capacitor in series) is connected in parallel with the SCR to limit $\frac{dv}{dt}$.

Methodology:

RC Snubber Circuit Design To design a basic Snubber Circuit:

1. Identify the maximum voltage V_m, the maximum allowable $\left(\frac{dv}{dt}\right)_{max}$, and the maximum allowable discharge current of the capacitor I_{TD}.
2. The snubber resistor R_s limits the discharge current of the capacitor when the SCR turns on. The resistor is calculated by:
$$R_s = \frac{V_m}{I_{TD}}$$
3. The snubber capacitor C_s limits the $\frac{dv}{dt}$ when the SCR is in the blocking state. Determine C_s using the approximate boundary condition formula:
$$C_s = \frac{V_m}{R_s \cdot \left(\frac{dv}{dt}\right)_{max}}$$

Worked Example:

Calculate Snubber Circuit Components An SCR is subjected to a peak voltage of 400 V. The maximum specified $\frac{dv}{dt}$ is 100 V/μs and the maximum permissible discharge current through the SCR is 50 A. Determine the values of snubber resistor R_s and snubber capacitor C_s.

Solution:

1. **Given Data:** $V_m = 400$ V, $\left(\frac{dv}{dt}\right)_{max} = 100$ V/μs $= 100 \times 10^6$ V/s, $I_{TD} = 50$ A.
2. **Calculate Snubber Resistor (R_s):**
$$R_s = \frac{V_m}{I_{TD}} = \frac{400}{50} = 8\ \Omega$$
3. **Calculate Snubber Capacitor (C_s):** Using the standard design equation:
$$C_s = \frac{V_m}{R_s \cdot \left(\frac{dv}{dt}\right)_{max}} = \frac{400}{8 \times 100 \times 10^6}$$
$$C_s = \frac{400}{800 \times 10^6} = 0.5 \times 10^{-6}\ \text{F} = 0.5\ \mu\text{F}$$

The designed snubber components are $R_s = 8\ \Omega$ and $C_s = 0.5\ \mu$F.

3.3.3 Over-current and Gate Protection

- **Over-current Protection:** High currents can melt the semiconductor junction. Fast-acting semiconductor fuses with an I^2t rating lower than the I^2t rating of the SCR are used for protection.
- **Gate Protection:** The gate circuit is protected against over-voltage by using a Zener diode across the gate-cathode terminals. It is protected against high-frequency noise and false triggering by connecting an RC filter (a resistor and a small capacitor in parallel) across the gate and cathode.

Methodology:

Fuse Selection using I Squared t Rating

1. Identify the maximum I^2t limit of the SCR from the manufacturer's datasheet. This represents the maximum thermal energy the SCR can withstand.

2. Determine the short-circuit clearing time of the fuse (t_c).

3. Select a fuse such that the I^2t rating of the fuse is strictly less than the I^2t rating of the SCR.

4. Calculate the maximum fault current I_f the SCR can withstand for the duration t_c using:

$$I_f = \sqrt{\frac{(I^2t)_{\text{SCR}}}{t_c}}$$

Worked Example:

Calculate Maximum Permissible Fault Current An SCR has an I^2t rating of 45 A^2s. If a fast-acting fuse has a clearing time of 10 ms, what is the maximum fault current that the SCR can safely withstand during this clearing time?

Solution:

1. **Given Data:** $(I^2t)_{\text{SCR}} = 45\ \text{A}^2\text{s}$, clearing time $t_c = 10\ \text{ms} = 0.01\ \text{s}$.

2. **Calculate Maximum Fault Current (I_f):**

$$I_f^2 \cdot t_c = 45$$

$$I_f^2 = \frac{45}{0.01} = 4500$$

$$I_f = \sqrt{4500} \approx 67.08\ \text{A}$$

The maximum fault current the SCR can withstand without damage during the fuse clearing time is 67.08 A.

CHAPTER 4

Power Converters

Power converters form the core of industrial electronics and drives. They allow the transformation of electrical energy from one form to another, matching the characteristics of the source to the requirements of the load. This chapter addresses the computational methods for rectifiers, inverters, choppers, cycloconverters, and power supply systems.

4.1 Single Phase and Poly Phase Rectifiers

Rectifiers convert alternating current (AC) into direct current (DC). While single-phase rectifiers are common for low-power applications, poly-phase (typically three-phase) rectifiers are utilized for industrial high-power loads due to their higher efficiency, better utilization of transformers, and smoother DC output.

Methodology:

Rectifier Types and Comparison To analyze rectifiers, we define the input AC voltage as $v(t) = V_m \sin(\omega t)$. The fundamental parameters are computed as follows:

1. Single Phase Half-Wave (HW) Rectifier

- DC Output Voltage: $V_{dc} = \frac{V_m}{\pi} \approx 0.318 V_m$
- RMS Output Voltage: $V_{rms} = \frac{V_m}{2}$
- Ripple Factor: $\gamma = \sqrt{\left(\frac{V_{rms}}{V_{dc}}\right)^2 - 1} = 1.21$

2. Single Phase Full-Wave (FW) Rectifier

- DC Output Voltage: $V_{dc} = \frac{2V_m}{\pi} \approx 0.636 V_m$
- RMS Output Voltage: $V_{rms} = \frac{V_m}{\sqrt{2}}$
- Ripple Factor: $\gamma = 0.48$

3. Poly-Phase Advantages

- Increased pulse number reduces the ripple factor significantly.
- The output DC voltage is closer to the peak value of the AC input.
- Power handling capacity is substantially increased.

Worked Example:

Comparing Single Phase HW and FW Outputs A single-phase transformer delivers a secondary voltage of 230 V (RMS) at 50 Hz. Compare the average DC output voltage and output ripple voltage factor if this source is fed to a half-wave rectifier versus a full-wave bridge rectifier. Assume ideal diodes.

Step 1: Calculate the peak voltage V_m. The RMS voltage is given as 230 V.

$$V_m = \sqrt{2} \times 230 = 325.27 \text{ V}$$

Step 2: Calculate for Half-Wave Rectifier.

$$V_{dc} = \frac{V_m}{\pi} = \frac{325.27}{3.1416} = 103.54 \text{ V}$$

$$V_{rms} = \frac{V_m}{2} = \frac{325.27}{2} = 162.64 \text{ V}$$

The ripple factor is a known constant for ideal HW rectifiers: $\gamma = 1.21$.
Step 3: Calculate for Full-Wave Rectifier.

$$V_{dc} = \frac{2V_m}{\pi} = \frac{2 \times 325.27}{3.1416} = 207.07 \text{ V}$$

$$V_{rms} = \frac{V_m}{\sqrt{2}} = \frac{325.27}{1.414} = 230 \text{ V}$$

The ripple factor for ideal FW rectifiers: $\gamma = 0.48$.
Conclusion: The full-wave rectifier yields double the DC average voltage (207.07 V) compared to the half-wave rectifier (103.54 V) and significantly reduces the ripple factor from 1.21 to 0.48.

4.2 Single Phase Controlled Rectifier using SCR

A controlled rectifier uses Silicon Controlled Rectifiers (SCRs) or thyristors instead of diodes. By varying the firing angle (α), the average DC output voltage can be dynamically controlled.

Methodology:

Single Phase Controlled Rectifier Computation Let V_s be the RMS source voltage, and $V_m = \sqrt{2}V_s$ be the peak source voltage. The firing angle α is measured in radians or degrees from the zero-crossing of the positive half cycle. For a purely resistive load (R):
1. Single Phase Half-Wave Controlled Rectifier The thyristor conducts from α to π.

$$V_{dc} = \frac{1}{2\pi} \int_{\alpha}^{\pi} V_m \sin(\omega t) d(\omega t) = \frac{V_m}{2\pi}(1 + \cos \alpha)$$

$$V_{rms} = V_m \sqrt{\frac{1}{4} - \frac{\alpha}{4\pi} + \frac{\sin(2\alpha)}{8\pi}}$$

2. Single Phase Full-Wave Controlled Rectifier For a mid-point or full-bridge controlled rectifier with a resistive load, conduction occurs from α to π in each half cycle.

$$V_{dc} = \frac{1}{\pi} \int_{\alpha}^{\pi} V_m \sin(\omega t) d(\omega t) = \frac{V_m}{\pi}(1 + \cos \alpha)$$

$$V_{rms} = V_m \sqrt{\frac{1}{2} - \frac{\alpha}{2\pi} + \frac{\sin(2\alpha)}{4\pi}}$$

Worked Example:

Calculating Controlled Output Voltage for SCR Rectifiers A single-phase 230 V, 50 Hz supply is connected to a resistive load of 10 Ω via a full-wave fully controlled thyristor bridge rectifier. If the firing angle α is 60°, determine the average output voltage and the average load current.

Step 1: Identify the source parameters.

$$V_s = 230 \text{ V}$$

$$V_m = \sqrt{2} \times 230 \approx 325.27 \text{ V}$$

$$\alpha = 60° = \frac{\pi}{3} \text{ radians}$$

Step 2: Apply the full-wave controlled rectifier formula.

$$V_{dc} = \frac{V_m}{\pi}(1 + \cos\alpha)$$

$$V_{dc} = \frac{325.27}{3.1416}(1 + \cos(60°))$$

$$V_{dc} = 103.54 \times (1 + 0.5) = 103.54 \times 1.5 = 155.31 \text{ V}$$

Step 3: Calculate the average load current. By Ohm's law for average DC values with purely resistive load:

$$I_{dc} = \frac{V_{dc}}{R} = \frac{155.31}{10} = 15.53 \text{ A}$$

The average output voltage is 155.31 V and the average current is 15.53 A.

4.3 Poly Phase Rectifiers

Three-phase rectifiers are extensively used in heavy industries to drive high-power DC motors, electrochemical processes, and high-power traction systems. They can be half-wave (3-pulse) or full-wave bridge (6-pulse) configurations.

Methodology:

Three Phase Rectifier Formulations Let the 3-phase source have an RMS phase voltage of V_{ph} and an RMS line-to-line voltage of $V_L = \sqrt{3}V_{ph}$. The peak phase voltage is $V_{mp} = \sqrt{2}V_{ph}$, and the peak line voltage is $V_{mL} = \sqrt{2}V_L$.

1. Three-Phase Half-Wave (Uncontrolled) Each diode conducts for 120° ($\frac{2\pi}{3}$ radians).

$$V_{dc} = \frac{3\sqrt{3}}{2\pi}V_{mp} = 0.827V_{mp}$$

2. Three-Phase Full-Wave Bridge (Uncontrolled) The most common industrial rectifier. Diodes conduct in pairs, switching every 60° ($\frac{\pi}{3}$ radians).

$$V_{dc} = \frac{3\sqrt{3}}{\pi}V_{mp} = \frac{3}{\pi}V_{mL}$$

$$V_{dc} \approx 1.654V_{mp} \approx 0.955V_{mL}$$

Ripple factor is drastically reduced to roughly 4%.

Worked Example:

Evaluating Three Phase Rectifier Output Voltages A three-phase uncontrolled full-wave bridge rectifier is supplied from a 415 V (line-to-line RMS), 50 Hz AC source. Calculate the average DC output voltage and identify the peak inverse voltage (PIV) rating required for the diodes.

Step 1: Identify source voltages. Line-to-line RMS voltage: $V_L = 415$ V. Peak line-to-line voltage: $V_{mL} = \sqrt{2} \times 415 = 1.414 \times 415 = 586.81$ V.

Step 2: Calculate the average DC output voltage. Using the formula for a three-phase full-wave bridge:

$$V_{dc} = \frac{3}{\pi} V_{mL} = \frac{3}{3.1416} \times 586.81$$
$$V_{dc} = 0.9549 \times 586.81 = 560.34 \text{ V}$$

Step 3: Determine the PIV. In a three-phase full-wave bridge, the maximum reverse voltage across any non-conducting diode is equal to the peak line voltage.

$$\text{PIV} = V_{mL} = 586.81 \text{ V}$$

For practical purposes, a safety factor (typically 1.5 to 2) is applied, requiring a diode rated for at least 900 V – 1200 V.

4.4 Inverters

Inverters convert DC power into AC power at a desired output voltage and frequency. Depending on the connection of switching elements, they can be configured as series, parallel, or bridge inverters.

Methodology:

Inverter Output Analysis In a basic voltage source inverter (VSI), the output is a square wave or stepped wave. Let V_s be the input DC voltage.

1. Single-Phase Half-Bridge Inverter Requires a 3-wire DC source (or split capacitors).

- Peak Output Voltage: $V_p = \frac{V_s}{2}$
- Fundamental RMS Output Voltage: $V_1 = \frac{4\times(V_s/2)}{\sqrt{2}\pi} = \frac{\sqrt{2}V_s}{\pi} \approx 0.45V_s$

2. Single-Phase Full-Bridge Inverter Requires a 2-wire DC source and 4 switches.

- Peak Output Voltage: $V_p = V_s$
- Fundamental RMS Output Voltage: $V_1 = \frac{4V_s}{\sqrt{2}\pi} = \frac{2\sqrt{2}V_s}{\pi} \approx 0.9V_s$

3. Frequency Control The output frequency f is determined by the switching period T.

$$f = \frac{1}{T}$$

Worked Example:

Computing Inverter Output Power and Voltage A single-phase full-bridge inverter is connected to a 100 V DC source. It supplies power to a pure resistive load of 5 Ω. Determine the RMS value of the fundamental output voltage, the fundamental output power, and the total RMS output power.

Step 1: Calculate Fundamental RMS Voltage. Input DC voltage $V_s = 100$ V.

$$V_1 = \frac{2\sqrt{2} \times 100}{\pi} = \frac{282.84}{3.1416} = 90.03 \text{ V}$$

Step 2: Calculate Fundamental Output Power. For a resistive load, power based on the fundamental component is:

$$P_1 = \frac{V_1^2}{R} = \frac{(90.03)^2}{5} = \frac{8105.4}{5} = 1621.08 \text{ W}$$

Step 3: Calculate Total RMS Power. The output of a full bridge is a square wave alternating between $+V_s$ and $-V_s$. The total RMS voltage of a square wave is simply V_s.

$$V_{rms,total} = V_s = 100 \text{ V}$$

$$P_{total} = \frac{V_{rms,total}^2}{R} = \frac{100^2}{5} = \frac{10000}{5} = 2000 \text{ W}$$

The difference ($2000 \text{ W} - 1621.08 \text{ W} = 378.92 \text{ W}$) is the power dissipated by the higher-order harmonics.

4.5 Chopper Circuits

Choppers are DC-to-DC converters. They process unregulated DC voltage to produce a regulated or variable DC output voltage. They are heavily used in electric vehicle motor control and switched-mode power supplies.

Methodology:

DC Chopper Design and Analysis Let V_s be the fixed DC input, V_0 the average DC output, T_{on} the ON-time of the switch, and T_{off} the OFF-time. Total time period $T = T_{on} + T_{off}$, and switching frequency $f = 1/T$. Duty cycle $D = T_{on}/T$.

1. Step-Down (Buck) Chopper Used when output voltage must be less than input voltage.

$$V_0 = D \cdot V_s = \frac{T_{on}}{T_{on} + T_{off}} V_s$$

2. Step-Up (Boost) Chopper Used when output voltage must be greater than input voltage.

$$V_0 = \frac{V_s}{1-D} = V_s \frac{T}{T - T_{on}} = V_s \frac{T_{on} + T_{off}}{T_{off}}$$

By adjusting the duty cycle D (where $0 \leq D < 1$), the desired voltage level is achieved.

Worked Example:

Step Up Chopper Timing Calculation A step-up chopper is used to deliver 150 V to a load from a 50 V DC source. If the OFF time of the switching element is 20 μs, compute the required ON time and the chopping frequency.

Step 1: Extract the given parameters.

$$V_s = 50 \text{ V}$$

$$V_0 = 150 \text{ V}$$

$$T_{off} = 20 \ \mu\text{s}$$

Step 2: Find the Duty Cycle (D). Using the step-up chopper equation:

$$V_0 = \frac{V_s}{1-D}$$

$$150 = \frac{50}{1-D}$$

$$1 - D = \frac{50}{150} = \frac{1}{3}$$

$$D = 1 - \frac{1}{3} = \frac{2}{3} \approx 0.667$$

Step 3: Calculate the ON time (T_{on}). We know $1 - D = \frac{T_{off}}{T_{on}+T_{off}}$. Alternatively, $V_0 = V_s \frac{T_{on}+T_{off}}{T_{off}}$:

$$150 = 50 \times \frac{T_{on} + 20}{20}$$

$$3 = \frac{T_{on} + 20}{20}$$

$$60 = T_{on} + 20 \implies T_{on} = 40\ \mu s$$

Step 4: Determine the chopping frequency (f).

$$T = T_{on} + T_{off} = 40 + 20 = 60\ \mu s$$

$$f = \frac{1}{T} = \frac{1}{60 \times 10^{-6}} = 16666.67 \text{ Hz} \approx 16.67 \text{ kHz}$$

4.6 Single Phase Cycloconverters

A cycloconverter is a direct AC-to-AC frequency converter. It converts alternating current of one frequency to alternating current of another frequency (usually lower) without using an intermediate DC link.

Methodology:

Cycloconverter Frequency Conversion For a single-phase step-down cycloconverter: **1. Output Frequency Determination** By synthesizing an output half-cycle from an integer number n of input half-cycles:

$$f_o = \frac{f_s}{m}$$

Where: f_o = Output AC frequency f_s = Source (input) AC frequency m = Number of source cycles forming one output cycle.
2. Output Voltage Characteristic The cycloconverter is effectively a phase-controlled rectifier that continuously varies its firing angle to trace out a low-frequency sine wave. The maximum possible fundamental output voltage V_{or} is smaller than the input voltage due to the commutation and synthesis process.

Worked Example:

Cycloconverter Output Frequency Determination A single-phase cycloconverter is fed by a 50 Hz supply. If the load requires an operating frequency of 10 Hz, determine the number of input cycles necessary to form one complete output cycle. If each half-cycle of the output contains p positive half-cycles and p negative half-cycles of the input, find p.
Step 1: Define frequencies.

$$f_s = 50 \text{ Hz}$$

$$f_o = 10 \text{ Hz}$$

Step 2: Find total number of input cycles per output cycle (m).

$$m = \frac{f_s}{f_o} = \frac{50}{10} = 5$$

Thus, it takes 5 complete input cycles to form 1 complete output cycle.
Step 3: Analyze the half-cycles. Total input cycles per output cycle = 5. Total input half-cycles per output cycle = $5 \times 2 = 10$. Since one output cycle consists of one positive output half-cycle and one negative output half-cycle, each output half-cycle is synthesized from $\frac{10}{2} = 5$ input half-cycles.

4.7 UPS and SMPS Systems

Uninterruptible Power Supplies (UPS) provide emergency power to a load when the input power source fails. Switched-Mode Power Supplies (SMPS) are highly efficient electronic power supplies that incorporate a switching regulator to convert electrical power efficiently.

Methodology:

UPS and SMPS System Sizing **1. UPS Battery Sizing and Backup Time** The energy available in a battery bank must satisfy the load power demand for the desired backup time.

$$\text{Total Battery Energy (Wh)} = \text{Battery Voltage (V)} \times \text{Battery Capacity (Ah)}$$

$$\text{Backup Time (Hours)} = \frac{\text{Total Battery Energy} \times \text{Inverter Efficiency}}{\text{Load Power (W)}}$$

2. SMPS Efficiency Analysis An SMPS converts power from an AC or DC source to DC loads at different voltage levels.

$$\text{Efficiency } (\eta) = \frac{P_{out}}{P_{in}} = \frac{V_{out}I_{out}}{V_{in}I_{in}}$$

Power Loss $P_{loss} = P_{in} - P_{out}$.

Worked Example:

Sizing UPS Battery Backup and SMPS Input Power An SMPS is designed to provide a regulated output of 12 V at 10 A to a server motherboard. Its rated efficiency is 80%. A UPS system uses a 24 V, 100 Ah battery bank to provide backup to this SMPS. The UPS inverter efficiency is 90%. Determine the power drawn by the SMPS from the UPS, and calculate the maximum theoretical backup time the UPS can provide to this single server setup during a blackout.

Step 1: Calculate the SMPS Output Power.

$$P_{out,SMPS} = V_{out} \times I_{out} = 12 \times 10 = 120 \text{ W}$$

Step 2: Calculate the SMPS Input Power. This input power is drawn directly from the UPS output.

$$P_{in,SMPS} = \frac{P_{out,SMPS}}{\eta_{SMPS}} = \frac{120}{0.80} = 150 \text{ W}$$

So, the load on the UPS is 150 W.

Step 3: Determine the Total Battery Energy available.

$$\text{Total Energy} = \text{Voltage} \times \text{Capacity} = 24 \text{ V} \times 100 \text{ Ah} = 2400 \text{ Wh}$$

Step 4: Calculate the Backup Time. Using the UPS efficiency to account for losses during DC-to-AC conversion:

$$\text{Backup Time} = \frac{\text{Total Energy} \times \eta_{UPS}}{\text{Load Power on UPS}}$$

$$\text{Backup Time} = \frac{2400 \times 0.90}{150} = \frac{2160}{150} = 14.4 \text{ hours}$$

The SMPS draws 150 W from the UPS, and the UPS can sustain this load for 14.4 hours.

CHAPTER 5

Industrial Electronics Applications

This chapter covers the computational methodologies and problem-solving techniques for various industrial electronics applications. The focus is exclusively on the formulas, design algorithms, and step-by-step calculations required to implement and analyze these systems.

5.1 Static Switches and Circuit Breakers

Static switches and circuit breakers utilize SCRs to control power flow and interrupt fault currents rapidly. The design requires calculating the power dissipation during the ON-state and ensuring the triggering conditions are met.

Methodology:

SCR Power Dissipation and Rating Calculation To select an appropriate SCR for static switching or circuit breaking, calculate the power dissipation and thermal requirements:

1. Identify the average load current I_{avg} and the RMS load current I_{rms} flowing through the SCR.
2. Determine the SCR on-state voltage drop V_T at the operating current.
3. Calculate the average conduction power loss:

$$P_{cond} = V_T \times I_{avg}$$

4. For more accurate dynamic modeling with on-state resistance R_T and threshold voltage V_{T0}:

$$P_{cond} = (V_{T0} \times I_{avg}) + (R_T \times I_{rms}^2)$$

5. Ensure that P_{cond} is less than the maximum allowable power dissipation of the chosen heat sink and SCR combination.

Worked Example:

Static Switch Power Dissipation An SCR is used as a static switch to control a DC load of 50 A. The SCR has a threshold voltage $V_{T0} = 1.0$ V and an on-state resistance $R_T = 0.005\,\Omega$. The current is purely DC, so $I_{avg} = I_{rms} = 50$ A. Calculate the total power dissipation in the SCR.

Step 1: Identify the given parameters.

- $I_{avg} = 50$ A
- $I_{avg} = I_{rms} = 50$ A (DC current)
- $V_{T0} = 1.0$ V
- $R_T = 0.005\,\Omega$

Step 2: Apply the power dissipation formula.

$$P_{cond} = (V_{T0} \times I_{avg}) + (R_T \times I_{rms}^2)$$

Step 3: Compute the terms. First term (Threshold loss):

$$P_1 = 1.0 \times 50 = 50 \text{ W}$$

Second term (Resistive loss):

$$P_2 = 0.005 \times (50)^2 = 0.005 \times 2500 = 12.5 \text{ W}$$

Step 4: Calculate total power.

$$P_{cond} = 50 + 12.5 = 62.5 \text{ W}$$

The total power dissipated by the static switch SCR is 62.5 W.

5.2 Single Phase AC Power Control

AC power controllers using Diac-Triac combinations regulate the RMS voltage supplied to a load by varying the firing angle α.

Methodology:

RMS Voltage and Power for Phase Control To analyze a single-phase AC power control circuit with a resistive load:

1. Identify the supply peak voltage $V_m = \sqrt{2}V_{supply(rms)}$ and the firing angle α (in radians).
2. Calculate the RMS output voltage delivered to the load:

$$V_{rms} = V_{supply(rms)}\sqrt{1 - \frac{\alpha}{\pi} + \frac{\sin(2\alpha)}{2\pi}}$$

3. Calculate the load power for a resistive load R:

$$P_{load} = \frac{V_{rms}^2}{R}$$

4. Calculate the input power factor (PF):

$$PF = \frac{V_{rms}}{V_{supply(rms)}}$$

Worked Example:

Triac Phase Control Calculations A single-phase AC power controller uses a Triac to control power to a 20 Ω resistive heating element. The supply voltage is 230 V RMS. If the Diac-Triac circuit is tuned to provide a firing angle of $\alpha = 90°$, calculate the RMS output voltage and the power delivered to the heater.

Step 1: Convert the firing angle to radians.

$$\alpha = 90° = \frac{\pi}{2} \text{ radians}$$

Step 2: Calculate the RMS output voltage. Using the formula:

$$V_{rms} = 230\sqrt{1 - \frac{\pi/2}{\pi} + \frac{\sin(2 \times 90°)}{2\pi}}$$

$$V_{rms} = 230\sqrt{1 - 0.5 + \frac{\sin(180°)}{2\pi}}$$

Since $\sin(180°) = 0$:

$$V_{rms} = 230\sqrt{0.5} = 230 \times 0.707 = 162.6 \text{ V}$$

Step 3: Calculate the power delivered to the load.

$$P_{load} = \frac{V_{rms}^2}{R} = \frac{(162.6)^2}{20} = \frac{26438.76}{20} = 1321.9 \text{ W}$$

The RMS output voltage is 162.6 V and the load power is 1321.9 W.

5.3 DC Power Control using SCR with UJT

A Unijunction Transistor (UJT) relaxation oscillator is widely used to generate triggering pulses for an SCR in DC and AC control circuits.

Methodology:

UJT Relaxation Oscillator Design To design and analyze a UJT triggering circuit:

1. Identify the UJT intrinsic standoff ratio η.
2. Calculate the peak voltage V_P required to fire the UJT:

$$V_P = \eta V_{BB} + V_D$$

(where V_{BB} is the supply voltage and $V_D \approx 0.7$ V is the diode drop).

3. Calculate the time period T of the oscillator using the charging resistor R and capacitor C:

$$T = RC \ln\left(\frac{1}{1-\eta}\right)$$

4. Calculate the frequency of the triggering pulses:

$$f = \frac{1}{T}$$

5. Ensure R is within the valid range for oscillation:

$$\frac{V_{BB} - V_P}{I_P} > R > \frac{V_{BB} - V_V}{I_V}$$

(where I_P, V_V, and I_V are UJT parameters).

Worked Example:

UJT Triggering Frequency Calculation A UJT relaxation oscillator operates from a 12 V DC supply. The UJT has an intrinsic standoff ratio $\eta = 0.63$. The timing components are $R = 50$ kΩ and $C = 0.1\,\mu$F. Calculate the peak triggering voltage V_P and the frequency of oscillation. Assume $V_D = 0.7$ V.

Step 1: Calculate the peak voltage V_P.

$$V_P = \eta V_{BB} + V_D$$

$$V_P = (0.63 \times 12) + 0.7 = 7.56 + 0.7 = 8.26 \text{ V}$$

Step 2: Calculate the time period T.

$$T = RC \ln \left(\frac{1}{1 - \eta} \right)$$

$$T = (50 \times 10^3) \times (0.1 \times 10^{-6}) \times \ln \left(\frac{1}{1 - 0.63} \right)$$

$$T = 0.005 \times \ln \left(\frac{1}{0.37} \right) = 0.005 \times \ln(2.70)$$

Since $\ln(2.70) \approx 0.994$:

$$T \approx 0.005 \times 0.994 = 0.00497 \text{ s} \approx 4.97 \text{ ms}$$

Step 3: Calculate the frequency f.

$$f = \frac{1}{T} = \frac{1}{0.00497} \approx 201.2 \text{ Hz}$$

The peak voltage is 8.26 V and the oscillation frequency is 201.2 Hz.

5.4 Photoelectric Relays and Switches

Photoelectric switches use light-sensitive components like LDRs, LASCRs, or photodiodes to trigger a relay or an electronic switch based on ambient light intensity.

Methodology:

Voltage Divider Triggering Calculation To determine the threshold of a photoelectric switch:

1. Model the circuit as a voltage divider comprising the light sensor (e.g., LDR resistance R_{LDR}) and a fixed/variable resistor R.
2. For a dark-activated switch (LDR placed near the ground):

$$V_{out} = V_{CC} \times \frac{R_{LDR}}{R_{LDR} + R}$$

3. For a light-activated switch (LDR placed near V_{CC}):

$$V_{out} = V_{CC} \times \frac{R}{R_{LDR} + R}$$

4. Set $V_{out} \geq V_{trigger}$ (e.g., 0.7 V for a BJT base or SCR gate) to find the threshold R_{LDR} value.
5. Use the sensor's datasheet to correlate R_{LDR} to the physical lux level.

Worked Example:

Dark Activated Relay Design A dark-activated relay circuit uses a 9 V supply and an NPN transistor that requires $V_{base} \geq 0.7$ V to turn on the relay. The LDR is connected between the transistor base and ground, while a resistor R is connected between the base and 9 V. The LDR has a resistance of 10 kΩ at the desired dark threshold. Calculate the maximum value of R required to ensure the relay turns on.

Step 1: Set up the voltage divider equation. Since the LDR is tied to ground, the base voltage is:

$$V_{base} = 9 \times \frac{R_{LDR}}{R_{LDR} + R}$$

Step 2: Substitute the threshold conditions. We want the relay to turn on ($V_{base} = 0.7$ V) when $R_{LDR} = 10$ kΩ:

$$0.7 = 9 \times \frac{10 \text{ k}\Omega}{10 \text{ k}\Omega + R}$$

Step 3: Solve for R.

$$0.7 \times (10 \text{ k}\Omega + R) = 90 \text{ k}\Omega$$

$$7 \text{ k}\Omega + 0.7R = 90 \text{ k}\Omega$$

$$0.7R = 83 \text{ k}\Omega$$

$$R = \frac{83 \text{ k}\Omega}{0.7} = 118.57 \text{ k}\Omega$$

A resistor of 118.57 kΩ (or a standard value of 120 kΩ) should be used to trigger the circuit at the required light level.

5.5 Solar PV Based Power Generation

Calculating the requirements for a solar photovoltaic (PV) array involves sizing strings of modules to meet system voltage and load current constraints.

Methodology:

Solar Array Sizing Procedure To size a Solar PV array for a specific application:

1. Determine the nominal system voltage V_{sys} and the maximum load current I_{load}.
2. Obtain the single module specifications at Maximum Power Point (MPP): V_{mpp} and I_{mpp}.
3. Calculate the number of modules required in series (N_s) to meet the system voltage:

$$N_s = \left\lceil \frac{V_{sys}}{V_{mpp}} \right\rceil$$

4. Calculate the number of parallel strings (N_p) required to meet the load current:

$$N_p = \left\lceil \frac{I_{load}}{I_{mpp}} \right\rceil$$

5. Determine the total number of modules $N_{total} = N_s \times N_p$.
6. Calculate the total array power: $P_{array} = N_{total} \times V_{mpp} \times I_{mpp}$.

Worked Example:

Sizing a Solar PV Array An industrial water pump requires 48 V DC and draws a continuous current of 25 A. The available solar panels are rated at $V_{mpp} = 18$ V and $I_{mpp} = 5.5$ A. Calculate the number of series panels, parallel strings, and total panels required. Also, calculate the total array power capacity.

Step 1: Calculate series modules N_s.

$$N_s = \frac{V_{sys}}{V_{mpp}} = \frac{48}{18} = 2.67$$

Since we cannot use a fraction of a panel, round up to the next whole number: $N_s = 3$.

Step 2: Calculate parallel strings N_p.

$$N_p = \frac{I_{load}}{I_{mpp}} = \frac{25}{5.5} = 4.54$$

Round up to ensure sufficient current: $N_p = 5$.
Step 3: Calculate total modules.

$$N_{total} = N_s \times N_p = 3 \times 5 = 15 \text{ panels}$$

Step 4: Calculate total array power capacity.

$$P_{panel} = 18 \text{ V} \times 5.5 \text{ A} = 99 \text{ W}$$

$$P_{array} = N_{total} \times P_{panel} = 15 \times 99 = 1485 \text{ W} = 1.485 \text{ kW}$$

The array requires 15 panels (5 parallel strings of 3 panels in series), providing a total capacity of 1.485 kW.

5.6 RF Heating

Radio Frequency (RF) heating encompasses Induction heating (for conductive materials) and Dielectric heating (for insulating materials).

Methodology:

Induction Heating Skin Depth Estimation In induction heating, the heating effect is restricted to the surface due to the skin effect. The depth of penetration (skin depth) δ must be calculated to evaluate heating efficiency:

1. Identify the frequency of the AC field f (in Hz).
2. Identify the material's resistivity ρ (in Ω·m) and relative permeability μ_r.
3. Calculate the absolute permeability $\mu = \mu_r \mu_0$, where $\mu_0 = 4\pi \times 10^{-7}$ H/m.
4. Calculate the skin depth:

$$\delta = \sqrt{\frac{\rho}{\pi f \mu}}$$

Worked Example:

Skin Depth Calculation for Induction Heating Calculate the skin depth of induction heating for a steel workpiece at a frequency of 10 kHz. The resistivity of the steel is $\rho = 2 \times 10^{-7}\,\Omega \cdot \text{m}$ and its relative permeability is $\mu_r = 100$.
Step 1: Identify given parameters.

- $f = 10 \text{ kHz} = 10^4 \text{ Hz}$
- $\rho = 2 \times 10^{-7}\,\Omega \cdot \text{m}$
- $\mu_r = 100$
- $\mu_0 = 4\pi \times 10^{-7}$ H/m

Step 2: Calculate absolute permeability.

$$\mu = 100 \times (4\pi \times 10^{-7}) = 4\pi \times 10^{-5} \text{ H/m}$$

Step 3: Apply the skin depth formula.

$$\delta = \sqrt{\frac{2 \times 10^{-7}}{\pi \times 10^4 \times (4\pi \times 10^{-5})}}$$

$$\delta = \sqrt{\frac{2 \times 10^{-7}}{4\pi^2 \times 10^{-1}}} = \sqrt{\frac{2 \times 10^{-7}}{3.947}} = \sqrt{5.06 \times 10^{-8}}$$

$$\delta \approx 2.25 \times 10^{-4} \text{ m} = 0.225 \text{ mm}$$

The high-frequency induced current will concentrate in the outer 0.225 mm of the steel workpiece.

Methodology:

Dielectric Heating Power Calculation Dielectric heating applies to non-conductive materials placed between two electrodes. The power dissipated as heat is proportional to the material's dielectric properties:

1. Identify the applied RMS voltage V, frequency f, and the loss tangent of the material $\tan\delta$.
2. Calculate the capacitance C of the material block:

$$C = \frac{\epsilon_0 \epsilon_r A}{d}$$

(where A is area, d is thickness, ϵ_r is relative permittivity, and $\epsilon_0 = 8.854 \times 10^{-12}$ F/m).

3. Calculate the power dissipated in the dielectric:

$$P = V^2 (2\pi f) C \tan\delta$$

Worked Example:

Dielectric Heating of Wood A block of wood of area 0.5 m^2 and thickness 0.05 m is heated using dielectric heating. The applied voltage is 5 kV RMS at a frequency of 20 MHz. The wood has a relative permittivity $\epsilon_r = 4.0$ and a loss tangent $\tan\delta = 0.05$. Calculate the power absorbed by the wood.

Step 1: Calculate the capacitance of the wooden block.

$$C = \frac{8.854 \times 10^{-12} \times 4.0 \times 0.5}{0.05} = \frac{1.77 \times 10^{-11}}{0.05} = 354 \times 10^{-12} \text{ F} = 354 \text{ pF}$$

Step 2: Calculate the angular frequency ω.

$$\omega = 2\pi f = 2 \times 3.1416 \times 20 \times 10^6 \approx 1.256 \times 10^8 \text{ rad/s}$$

Step 3: Calculate the dissipated power.

$$P = (5000)^2 \times (1.256 \times 10^8) \times (354 \times 10^{-12}) \times 0.05$$

$$P = 25 \times 10^6 \times 1.256 \times 10^8 \times 354 \times 10^{-12} \times 0.05$$

$$P = 25 \times 1.256 \times 354 \times 0.05 \times 10^2$$

$$P = 111156 \text{ W} = 111.1 \text{ kW}$$

The wood absorbs 111.1 kW of RF power as heat.

5.7 Stepper Motors

Stepper motors translate digital pulse inputs into discrete mechanical movements. The computational focus lies in establishing the relationship between the applied pulses, step angle, and rotational speed.

Methodology:

Stepper Motor Step Angle and Speed To analyze the kinematics of a stepper motor:

1. Identify the number of stator phases m and the number of rotor teeth N_r.

2. Calculate the step angle α (the angular rotation per input pulse):

$$\alpha = \frac{360°}{m \times N_r}$$

3. Determine the total number of steps required for one full revolution (S_{rev}):

$$S_{rev} = \frac{360°}{\alpha} = m \times N_r$$

4. Given an input pulse frequency f (pulses per second or Hz), calculate the rotational speed N in Revolutions Per Minute (RPM):

$$N = \frac{\alpha \times f}{360°} \times 60 = \frac{\alpha \times f}{6}$$

Worked Example:

Calculating Stepper Motor Speed A 3-phase stepper motor has 20 teeth on its rotor. The motor is driven by a controller outputting pulses at a frequency of 400 Hz. Calculate the step angle, the number of steps per revolution, and the motor's speed in RPM.

Step 1: Calculate the step angle.

$$\alpha = \frac{360°}{m \times N_r} = \frac{360°}{3 \times 20} = \frac{360°}{60} = 6° \text{ per step}$$

Step 2: Calculate the number of steps per revolution.

$$S_{rev} = \frac{360°}{6°} = 60 \text{ steps/rev}$$

Step 3: Calculate the motor speed in RPM. Using the formula derived:

$$N = \frac{\alpha \times f}{6} = \frac{6 \times 400}{6} = 400 \text{ RPM}$$

Alternatively:

$$400 \text{ pulses/second} \div 60 \text{ steps/rev} = 6.667 \text{ rev/second}$$

$$6.667 \times 60 = 400 \text{ RPM}$$

The motor step angle is 6°, requiring 60 steps for a full revolution, and it rotates at 400 RPM.

5.8 Programmable Logic Controllers

Programmable Logic Controllers (PLCs) cyclically execute control programs. Analyzing the PLC involves calculating its scan time, which dictates the latency and responsiveness of the control system.

Methodology:

PLC Scan Time Estimation The total scan time of a PLC is the sum of times taken to read inputs, execute the program, and update outputs. To calculate this:

1. Identify the time taken to read all inputs (T_{in}) and update all outputs (T_{out}).

2. Determine the number of basic logical instructions (N_{logic}) and complex instructions ($N_{complex}$) in the ladder program.

3. Obtain the execution time per basic instruction (t_{logic}) and per complex instruction ($t_{complex}$) from the PLC datasheet.

4. Calculate the program execution time:

$$T_{prog} = (N_{logic} \times t_{logic}) + (N_{complex} \times t_{complex})$$

5. Add the system overhead time (T_{sys}) associated with self-diagnostics and communications.

6. Calculate the total scan time:

$$T_{scan} = T_{in} + T_{prog} + T_{out} + T_{sys}$$

Worked Example:

Estimating PLC Response Latency A PLC controls an industrial packaging machine. The ladder logic program consists of 1500 basic Boolean instructions and 50 complex mathematical instructions. The PLC specifications are as follows:

Step 1: Identify given parameters.

- Basic instructions: 1500
- Complex instructions: 50
- Basic instruction execution time $= 2\,\mu$s
- Complex instruction execution time $= 20\,\mu$s
- Overhead time $T_{OH} = 1.5$ ms

Step 2: Calculate execution time of user program (T_{prog}). $T_{prog} =$ (No. of basic instructions $\times$ time) + (No. of complex instructions $\times$ time)

$$T_{prog} = (1500 \times 2\,\mu\text{s}) + (50 \times 20\,\mu\text{s})$$

$$T_{prog} = 3000\,\mu\text{s} + 1000\,\mu\text{s} = 4000\,\mu\text{s} = 4.0 \text{ ms}$$

Step 2: Add all the scan cycle components.

$$T_{scan} = T_{in} + T_{prog} + T_{out} + T_{sys}$$

$$T_{scan} = 1.5 \text{ ms} + 4.0 \text{ ms} + 1.5 \text{ ms} + 1.0 \text{ ms}$$

$$T_{scan} = 8.0 \text{ ms}$$

The total scan time for the PLC to complete one cycle is 8.0 ms. This represents the minimum guaranteed latency for an input change to affect an output.

Appendix: Key Formulae

This appendix provides a quick reference to the key formulae and mathematical relationships discussed in this book.

Unit 1: Operational Amplifiers

- **Inverting Amplifier Closed-Loop Gain:**

$$A_{CL} = -\frac{R_f}{R_1} \quad \text{and} \quad V_{out} = -\left(\frac{R_f}{R_1}\right) V_{in}$$

- **Non-Inverting Amplifier Closed-Loop Gain:**

$$A_{CL} = 1 + \frac{R_f}{R_1} \quad \text{and} \quad V_{out} = \left(1 + \frac{R_f}{R_1}\right) V_{in}$$

- **Inverting Summing Amplifier Output Voltage:**

$$V_{out} = -\left(\frac{R_f}{R_1}V_1 + \frac{R_f}{R_2}V_2 + \frac{R_f}{R_3}V_3\right)$$

  If $R_1 = R_2 = R_3 = R$:

$$V_{out} = -\frac{R_f}{R}(V_1 + V_2 + V_3)$$

- **Output Offset Voltage:**

$$V_{oo} = V_{io} \times \left(1 + \frac{R_f}{R_1}\right)$$

- **Input Bias and Offset Current:**

$$I_B = \frac{I_{B1} + I_{B2}}{2}, \quad I_{io} = |I_{B1} - I_{B2}|$$

- **Schmitt Trigger Threshold Voltages:**

$$V_{UT} = \frac{R_1}{R_1 + R_2}(+V_{sat}), \quad V_{LT} = \frac{R_1}{R_1 + R_2}(-V_{sat})$$

$$V_H = V_{UT} - V_{LT} \quad \text{(Hysteresis Voltage)}$$

- **Integrator Output Voltage:**

$$V_{out}(t) = -\frac{1}{RC}\int_0^t V_{in}(\tau)d\tau + V_{out}(0)$$

- **Differentiator Output Voltage:**

$$V_{out}(t) = -RC\frac{dV_{in}(t)}{dt}$$

- **Common Mode Rejection Ratio (CMRR):**

$$\text{CMRR} = \frac{A_d}{A_{cm}}, \quad \text{CMRR (dB)} = 20\log_{10}\left(\frac{A_d}{A_{cm}}\right)$$

- **Slew Rate (SR):**

$$\text{SR} = \left.\frac{dV_{out}}{dt}\right|_{\max} \quad \text{and for sinusoidal input,} \quad \text{SR} = 2\pi f_{\max} V_m$$

Unit 2: Power Semiconductor Devices

- **Thyristor Anode Current (Two-Transistor Model):**

$$I_A = \frac{\alpha_2 I_G + I_{CO}}{1 - (\alpha_1 + \alpha_2)}$$

where $I_{CO} = I_{CBO1} + I_{CBO2}$.

- **Gate Turn-Off Thyristor (GTO) Turn-off Current Gain:**

$$\beta_{off} = \frac{I_A}{I_{G(off)}} \implies I_{G(off)} = \frac{I_A}{\beta_{off}}$$

- **Power Losses in Switches:**

$$P_{cond} = V_{CE(sat)} \times I_C \times D \quad \text{(Conduction Loss)}$$
$$P_{sw} = f_{sw} \times (E_{on} + E_{off}) \quad \text{(Switching Loss)}$$
$$P_{total} = P_{cond} + P_{sw}$$
$$f_{max} = \frac{P_{sw\_max}}{E_{cycle}} \quad \text{(Maximum Switching Frequency)}$$

- **UJT Relaxation Oscillator Triggering Time:**

$$t_{trig} = -RC \ln\left(1 - \frac{V_{BO}}{V_S}\right)$$

- **Optocoupler Relations:**

$$R_S = \frac{V_{in} - V_F}{I_F}, \quad I_C = I_F \times \text{CTR}$$

- **AC Voltage Controller (Phase Control) RMS Voltage:**

$$V_{rms} = V_{S,rms}\sqrt{\frac{1}{\pi}\left(\pi - \alpha + \frac{\sin(2\alpha)}{2}\right)}$$

Unit 3: Protection and Commutation Circuits

- **Snubber Circuit Design (dv/dt and di/dt Protection):**

$$L_s = \frac{V_m}{\left(\frac{di}{dt}\right)_{max}} \quad \text{(Series Inductor)}$$
$$R_s = \frac{V_m}{I_{TD}} \quad \text{(Snubber Resistor)}$$
$$C_s = \frac{V_m}{R_s \cdot \left(\frac{dv}{dt}\right)_{max}} \quad \text{(Snubber Capacitor)}$$

- **Fuse Selection (Thermal Energy):**

$$I_f = \sqrt{\frac{(I^2 t)_{\text{SCR}}}{t_c}}$$

- **Gate Drive Characteristics:**

$$E_s = I_g R_g + V_g \implies R_g = \frac{E_s - V_g}{I_g}$$
$$P_g = V_g \cdot I_g, \quad P_{gav} = P_{gm} \cdot \delta = P_{gm} \cdot t_p \cdot f$$

- **Commutation Circuit Components:**

$$C = \frac{I_p \cdot t_c}{V_{dc}}, \quad L = \frac{V_{dc} \cdot t_c}{I_p}, \quad I_p = V_{dc}\sqrt{\frac{C}{L}}$$
$$t_c = R \cdot C \cdot \ln(2) \approx 0.693 \cdot R \cdot C$$

Unit 4: Rectifiers and Converters

- **General Voltage Relationships:**
$$V_m = \sqrt{2}V_s, \quad V_p = V_s$$
- **Half-Wave Rectifier:**
$$V_{rms} = \frac{V_m}{2}$$
- **Full-Wave Rectifier Average DC Voltage:**
$$V_{dc} = \frac{2V_m}{\pi} \approx 0.636V_m$$
- **Fundamental RMS Voltage Component (Square Wave Inverter):**
$$V_1 = \frac{4V_s}{\sqrt{2}\pi} = \frac{2\sqrt{2}V_s}{\pi} \approx 0.9V_s$$

Unit 5: Applications of Power Electronics

- **Solar Photovoltaic Systems:**
$$P_{array} = N_{total} \times P_{panel}, \quad N_{total} = N_s \times N_p$$
$$N_s = \left\lceil \frac{V_{sys}}{V_{mpp}} \right\rceil, \quad N_p = \left\lceil \frac{I_{load}}{I_{mpp}} \right\rceil$$
- **Induction and Dielectric Heating:**
$$\delta = \sqrt{\frac{\rho}{\pi f \mu}} \quad \text{(Skin Depth)}$$
$$P = V^2(2\pi f)C \tan\delta \quad \text{(Dielectric Heating Power)}$$
$$C = \frac{\epsilon_0 \epsilon_r A}{d} \quad \text{(Capacitance)}$$
- **Programmable Logic Controllers (PLCs):**
$$T_{scan} = T_{in} + T_{prog} + T_{out} + T_{sys} \quad \text{(Total Scan Time)}$$
$$T_{prog} = (N_{logic} \times t_{logic}) + (N_{complex} \times t_{complex}) \quad \text{(Program Execution Time)}$$
- **Stepper Motors:**
$$\alpha = \frac{360°}{m \times N_r} \quad \text{(Step Angle)}$$
$$S_{rev} = \frac{360°}{\alpha} = m \times N_r \quad \text{(Steps per Revolution)}$$
$$N = \frac{\alpha \times f}{360°} \times 60 = \frac{\alpha \times f}{6} \quad \text{(Speed in RPM)}$$
- **UJT Relaxation Oscillator:**
$$V_P = \eta V_{BB} + V_D \quad \text{(Peak Voltage)}$$
$$\frac{V_{BB} - V_P}{I_P} > R > \frac{V_{BB} - V_V}{I_V} \quad \text{(Resistance Range)}$$
$$T = RC \ln\left(\frac{1}{1-\eta}\right) \quad \text{(Time Period)}$$

- **LDR Voltage Divider Circuit:**

$$V_{out} = V_{CC} \times \frac{R}{R_{LDR} + R} \quad \text{or} \quad V_{out} = V_{CC} \times \frac{R_{LDR}}{R_{LDR} + R}$$

- **Power Factor and Conduction Losses:**

$$PF = \frac{V_{rms}}{V_{supply(rms)}} \quad \text{(Power Factor)}$$

$$P_{cond} = V_T \times I_{avg} \quad \text{or} \quad P_{cond} = (V_{T0} \times I_{avg}) + (R_T \times I_{rms}^2)$$