Pec (4331104) - Problem Solver

Antigravity AI

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Pec

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To my students,
who constantly inspire me to find new analogies,
break down complex theories, and make learning
an engineered science.

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Preface

About this Book
Welcome to **Computer Networks**! This textbook is meticulously designed to serve as a comprehensive problem solver and study companion. It provides a robust and intuitive foundation in the core principles of this subject, bridging the gap between theoretical models and practical applications. A unique feature of this book is its focus on decoding complex concepts using clear, step-by-step methodologies and real-world analogies.

Target Audience
This book is intended for students and professionals looking to master the concepts of Computer Networks. Whether you are revising for exams, seeking to solidify your fundamental understanding, or looking for practical examples to clarify abstract concepts, this book has been structured to support your learning journey. It serves as an accessible guide designed to remove the fear of complex topics, replacing it with an appreciation for the underlying mechanics.

Scope and Coverage
The coverage is divided logically to follow standard academic curriculums. Each chapter focuses on key topics, breaking them down into digestible sections:

- **Theoretical Insights** – Core concepts explained intuitively.
- **Method Blocks** – Standardized, step-by-step procedures for solving specific types of problems.
- **Worked Examples** – Fully solved problems that apply the methods in practice.

We hope this resource proves invaluable in your studies and helps you achieve excellence in **Computer Networks**. Happy learning!

Acknowledgments

Writing a comprehensive book that connects the fundamental forces of Chemistry with practical engineering applications requires more than just academic knowledge—it requires a community of support. I would like to express my sincere gratitude to everyone who contributed to the realization of this project.

Specially, I extend my heartfelt gratitude to the **Department of Technical Education (DTE), Government of Gujarat**, and **Government Polytechnic, Palanpur**, for providing an environment that fosters innovation, academic rigor, and continuous professional development.

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Finally, my deepest appreciation goes to my students. Your curiosity, challenging questions, and continuous feedback are the true driving force behind the unique analogies and streamlined structure of this book. This textbook was engineered for you.

Antigravity AI

About the Author

Milav Jayeshkumar Dabgar is an engineering educator and R&D professional with over 9 years of experience in electronics hardware development, embedded systems, AI/ML, and full-stack software engineering. He currently serves as a **Lecturer (Class - II) & Innovation Leader** at Government Polytechnic, Palanpur, under the Education Department of the Government of Gujarat.

Milav holds a Master of Engineering (ME) in Communication Systems from L.D. College of Engineering and a Bachelor of Engineering (BE) in Electronics & Communication. He is also currently pursuing a **BS in Data Science and Applications from IIT Madras**, where he has completed both the **Diploma in Programming** and the **Diploma in Data Science** with distinction.

Most recently, he completed the **AICTE QIP PG Certificate Program in Deep Learning from SVNIT Surat** with a **perfect 10/10 CGPA**, topping his batch.

A dedicated mentor, Milav has guided numerous student teams in securing over **INR 45 lakhs** in innovation funding, including INR 25 lakhs from Shark Tank India and INR 20 lakhs through iHub Gujarat, resulting in two student patents. He is recognized as an **NPTEL Evangelist** and **NPTEL Discipline Star** in Computer Science, reflecting his commitment to continuous learning and academic excellence.

His technical expertise spans from embedded systems (8051, STM32, Arduino) to modern web technologies (Next.js, Python, PostgreSQL) and Data Science (TensorFlow, PyTorch). Through this textbook, he aims to simplify complex Engineering Chemistry concepts by integrating relatable, real-world analogies drawn from modern tech and engineering landscapes.

Milav is also an active content creator, running a popular **YouTube channel** (@milav.dabgar) where he shares technical tutorials and academic resources. He maintains a personal blog and resource hub at milav.in and can be reached for professional collaborations on LinkedIn.

CHAPTER 1

Basics of Communication System

Communication systems form the backbone of modern information exchange. The fundamental parameters defining analog communication methods like Amplitude Modulation (AM) and Frequency Modulation (FM) govern system design. This chapter focuses on calculating modulation indices, bandwidth requirements, and power distributions for AM and FM systems.

1.1 Amplitude Modulation Parameters

Amplitude Modulation involves varying the amplitude of a high-frequency carrier signal in accordance with the instantaneous amplitude of a low-frequency modulating (message) signal.

Methodology:

AM Modulation Index and Bandwidth The modulation index (also called modulation depth) and frequency spectrum parameters for AM are calculated using the following formulas:

1. **Modulation Index (m):**
$$m = \frac{V_m}{V_c}$$
Where V_m is the peak amplitude of the modulating signal and V_c is the peak amplitude of the carrier signal.

2. **Modulation Index from Voltage Extremes:**
$$m = \frac{V_{max} - V_{min}}{V_{max} + V_{min}}$$
Where V_{max} and V_{min} are the maximum and minimum peak voltages of the modulated envelope.

3. **Sideband Frequencies:**
$$f_{USB} = f_c + f_m$$
$$f_{LSB} = f_c - f_m$$
Where f_c is the carrier frequency and f_m is the modulating frequency.

4. **Bandwidth (BW):**
$$BW = f_{USB} - f_{LSB} = 2f_m$$

Worked Example:

Calculating AM Modulation Index and Sidebands A carrier signal of 10 V peak and frequency 1 MHz is amplitude modulated by a sine wave of 4 V peak and frequency 10 kHz. Calculate the modulation index, upper sideband frequency, lower sideband frequency, and the bandwidth.

Solution:

1. **Identify given parameters:**
   - Carrier amplitude $V_c = 10$ V
   - Modulating signal amplitude $V_m = 4$ V
   - Carrier frequency $f_c = 1$ MHz $= 1000$ kHz
   - Modulating frequency $f_m = 10$ kHz
2. **Calculate the modulation index (m):**

$$m = \frac{V_m}{V_c} = \frac{4}{10} = 0.4$$

   Percentage modulation is 40%.

3. **Calculate sideband frequencies:**

$$f_{USB} = f_c + f_m = 1000 \text{ kHz} + 10 \text{ kHz} = 1010 \text{ kHz}$$
$$f_{LSB} = f_c - f_m = 1000 \text{ kHz} - 10 \text{ kHz} = 990 \text{ kHz}$$

4. **Calculate bandwidth (BW):**

$$BW = 2f_m = 2 \times 10 \text{ kHz} = 20 \text{ kHz}$$

Worked Example:

Modulation Index from Oscilloscope Trace An AM wave displayed on an oscilloscope has a maximum peak-to-peak voltage of 30 V and a minimum peak-to-peak voltage of 10 V. Calculate the modulation index.
Solution:

1. **Identify given parameters:**
   - Maximum peak voltage $V_{max} = \frac{30}{2} = 15$ V
   - Minimum peak voltage $V_{min} = \frac{10}{2} = 5$ V

   Note: You can also use peak-to-peak voltages directly in the formula since the factor of 2 cancels out.

2. **Calculate the modulation index (m):**

$$m = \frac{V_{max} - V_{min}}{V_{max} + V_{min}} = \frac{15 - 5}{15 + 5} = \frac{10}{20} = 0.5$$

1.2 Power and Current Relations in AM

In Amplitude Modulation, the total transmitted power is shared between the carrier and the sidebands. The carrier itself contains no information; all the information is carried by the sidebands.

Methodology:

AM Power and Current Formulas The power and antenna current in an AM system are evaluated as follows:

1. **Total Transmitted Power (P_t):**

$$P_t = P_c \left(1 + \frac{m^2}{2}\right)$$

   Where P_c is the unmodulated carrier power.

2. **Sideband Power (P_{SB}):**

$$P_{SB} = P_{USB} + P_{LSB} = P_c \left(\frac{m^2}{2} \right)$$

Since the upper and lower sidebands are symmetrical, $P_{USB} = P_{LSB} = P_c \left(\frac{m^2}{4} \right)$.

3. **Antenna Current (I_t):** Because Power $P \propto I^2$ for a given antenna resistance R,

$$I_t = I_c \sqrt{1 + \frac{m^2}{2}}$$

Where I_t is the total modulated antenna current and I_c is the unmodulated carrier current.

Worked Example:

Calculating AM Power Distribution An AM transmitter has an unmodulated carrier power of 10 kW. It is modulated by a sinusoidal audio signal with a modulation index of 0.8. Calculate the total transmitted power and the power in each sideband.

Solution:

1. **Identify given parameters:**
   - Carrier power $P_c = 10$ kW
   - Modulation index $m = 0.8$

2. **Calculate total transmitted power (P_t):**

$$P_t = P_c \left(1 + \frac{m^2}{2} \right) = 10 \left(1 + \frac{0.8^2}{2} \right)$$

$$P_t = 10 \left(1 + \frac{0.64}{2} \right) = 10(1 + 0.32) = 13.2 \text{ kW}$$

3. **Calculate total sideband power (P_{SB}):**

$$P_{SB} = P_t - P_c = 13.2 - 10 = 3.2 \text{ kW}$$

4. **Calculate power in each sideband:**

$$P_{USB} = P_{LSB} = \frac{P_{SB}}{2} = \frac{3.2}{2} = 1.6 \text{ kW}$$

Worked Example:

Determining Modulation Index from Antenna Current The antenna current of an AM broadcast transmitter is 8 A when only the carrier is sent. When the carrier is modulated by a sine wave, the antenna current increases to 8.93 A. Determine the modulation index.

Solution:

1. **Identify given parameters:**
   - Unmodulated carrier current $I_c = 8$ A
   - Total modulated current $I_t = 8.93$ A

2. **Apply the current relation formula:**

$$I_t = I_c \sqrt{1 + \frac{m^2}{2}}$$

3. **Solve for modulation index (m):**

$$\frac{8.93}{8} = \sqrt{1 + \frac{m^2}{2}}$$

$$1.11625 = \sqrt{1 + \frac{m^2}{2}}$$

Squaring both sides:

$$(1.11625)^2 = 1 + \frac{m^2}{2}$$

$$1.246 = 1 + \frac{m^2}{2}$$

$$0.246 = \frac{m^2}{2}$$

$$m^2 = 0.492 \implies m = \sqrt{0.492} \approx 0.701$$

The modulation index is approximately 0.7 (or 70%).

1.3 Frequency Modulation Parameters

In Frequency Modulation, the frequency of the carrier is varied in accordance with the instantaneous amplitude of the modulating signal, while the carrier amplitude remains constant.

Methodology:

FM Deviation and Bandwidth The key parameters defining an FM signal's spectrum are computed as follows:

1. **Modulation Index (m_f):**

$$m_f = \frac{\Delta f}{f_m}$$

Where Δf is the maximum frequency deviation and f_m is the modulating signal frequency.

2. **Carson's Rule for Bandwidth (BW):** Unlike AM, FM generates an infinite number of sidebands. However, a significant portion of the signal power is contained within a specific bandwidth, estimated by Carson's rule:

$$BW \approx 2(\Delta f + f_m)$$

Alternatively, using the modulation index:

$$BW \approx 2 f_m (m_f + 1)$$

Worked Example:

Calculating FM Bandwidth An FM transmission has a maximum frequency deviation of 75 kHz and is modulated by an audio signal of 15 kHz. Calculate the modulation index and the approximate bandwidth required for transmission.

Solution:

1. **Identify given parameters:**
   - Frequency deviation $\Delta f = 75$ kHz
   - Modulating frequency $f_m = 15$ kHz

2. **Calculate the modulation index (m_f):**

$$m_f = \frac{\Delta f}{f_m} = \frac{75 \text{ kHz}}{15 \text{ kHz}} = 5$$

3. **Calculate the bandwidth using Carson's rule:**

$$BW = 2(\Delta f + f_m) = 2(75 \text{ kHz} + 15 \text{ kHz}) = 2(90 \text{ kHz}) = 180 \text{ kHz}$$

Worked Example:

FM Frequency Range A 100 MHz carrier is frequency modulated by a 10 kHz sine wave. If the peak frequency deviation is 50 kHz, determine the modulation index and the highest and lowest frequencies attained by the modulated signal.

Solution:

1. **Identify given parameters:**
   - Carrier frequency $f_c = 100 \text{ MHz} = 100,000 \text{ kHz}$
   - Frequency deviation $\Delta f = 50 \text{ kHz}$
   - Modulating frequency $f_m = 10 \text{ kHz}$

2. **Calculate the modulation index (m_f):**

$$m_f = \frac{\Delta f}{f_m} = \frac{50 \text{ kHz}}{10 \text{ kHz}} = 5$$

3. **Determine the frequency range:** The instantaneous frequency of the FM wave swings between $(f_c - \Delta f)$ and $(f_c + \Delta f)$.

$$f_{highest} = f_c + \Delta f = 100,000 \text{ kHz} + 50 \text{ kHz} = 100,050 \text{ kHz} = 100.05 \text{ MHz}$$
$$f_{lowest} = f_c - \Delta f = 100,000 \text{ kHz} - 50 \text{ kHz} = 99,950 \text{ kHz} = 99.95 \text{ MHz}$$

CHAPTER 2

Analog Modulation Techniques

2.1 Amplitude Modulation

Methodology:

Amplitude Modulation Basics and Modulation Index Amplitude Modulation (AM) is the process where the amplitude of the high-frequency carrier signal is varied in accordance with the instantaneous amplitude of the low-frequency modulating signal.

Steps to calculate AM parameters:

1. Identify the modulating signal $v_m(t) = V_m \sin(2\pi f_m t)$ and carrier signal $v_c(t) = V_c \sin(2\pi f_c t)$, where V_m and V_c are the peak voltages, and f_m and f_c are the frequencies.

2. Calculate the modulation index (also called modulation factor or depth of modulation) m:

$$m = \frac{V_m}{V_c}$$

The percentage of modulation is $M = m \times 100\%$.

3. Determine the standard equation of the AM wave:

$$v_{AM}(t) = V_c[1 + m\sin(2\pi f_m t)]\sin(2\pi f_c t)$$

4. If the maximum envelope voltage V_{max} and minimum envelope voltage V_{min} are given, calculate m:

$$m = \frac{V_{max} - V_{min}}{V_{max} + V_{min}}$$

5. Find the peak carrier voltage V_c and peak modulating voltage V_m from envelope voltages:

$$V_c = \frac{V_{max} + V_{min}}{2}, \quad V_m = \frac{V_{max} - V_{min}}{2}$$

Worked Example:

Calculate modulation index and peak voltages from AM envelope An AM wave displayed on an oscilloscope has a maximum peak-to-peak voltage of 50 V and a minimum peak-to-peak voltage of 10 V. Calculate the modulation index, peak carrier voltage, and peak modulating voltage.

Solution:

1. Identify V_{max} and V_{min}: Since the given values are peak-to-peak, the peak envelope voltages are:

$$V_{max} = \frac{50\text{ V}}{2} = 25\text{ V}$$

$$V_{min} = \frac{10\text{ V}}{2} = 5\text{ V}$$

2. Calculate the modulation index m:

$$m = \frac{V_{max} - V_{min}}{V_{max} + V_{min}} = \frac{25 - 5}{25 + 5} = \frac{20}{30} = 0.667$$

Percentage modulation is 66.7%.

3. Calculate the peak carrier voltage V_c:

$$V_c = \frac{V_{max} + V_{min}}{2} = \frac{25 + 5}{2} = \frac{30}{2} = 15 \text{ V}$$

4. Calculate the peak modulating voltage V_m:

$$V_m = \frac{V_{max} - V_{min}}{2} = \frac{25 - 5}{2} = \frac{20}{2} = 10 \text{ V}$$

Alternatively, verify $m = \frac{V_m}{V_c} = \frac{10}{15} = 0.667$.

Methodology:

AM Frequency Spectrum and Bandwidth An AM signal consists of the carrier frequency and two sideband frequencies.

Steps to calculate frequency spectrum and bandwidth:

1. Identify the carrier frequency f_c and modulating frequency f_m.
2. Calculate the Upper Sideband Frequency (USB):

$$f_{USB} = f_c + f_m$$

3. Calculate the Lower Sideband Frequency (LSB):

$$f_{LSB} = f_c - f_m$$

4. Calculate the Bandwidth (BW) required to transmit the AM signal:

$$BW = f_{USB} - f_{LSB} = (f_c + f_m) - (f_c - f_m) = 2f_m$$

Worked Example:

Calculate sideband frequencies and bandwidth A 1 MHz carrier is amplitude modulated by a 5 kHz audio signal. Find the sideband frequencies and the bandwidth required.

Solution:

1. Identify frequencies:

$$f_c = 1 \text{ MHz} = 1000 \text{ kHz}$$
$$f_m = 5 \text{ kHz}$$

2. Calculate Upper Sideband (USB):

$$f_{USB} = f_c + f_m = 1000 \text{ kHz} + 5 \text{ kHz} = 1005 \text{ kHz}$$

3. Calculate Lower Sideband (LSB):

$$f_{LSB} = f_c - f_m = 1000 \text{ kHz} - 5 \text{ kHz} = 995 \text{ kHz}$$

4. Calculate Bandwidth (BW):

$$BW = 2f_m = 2 \times 5 \text{ kHz} = 10 \text{ kHz}$$

The sideband frequencies are 995 kHz and 1005 kHz, and the bandwidth is 10 kHz.

Methodology:

Power Calculations in Amplitude Modulation The total power of an AM wave is the sum of the carrier power and the power in the upper and lower sidebands.

Steps to calculate AM power:

1. Calculate the carrier power P_c:

$$P_c = \frac{V_{c(rms)}^2}{R} = \frac{(V_c/\sqrt{2})^2}{R} = \frac{V_c^2}{2R}$$

   where V_c is the peak carrier voltage and R is the load or antenna resistance.

2. Calculate the total power P_t:

$$P_t = P_c \left(1 + \frac{m^2}{2}\right)$$

3. Calculate the total sideband power P_{SB} (which contains the useful information):

$$P_{SB} = P_t - P_c = P_c \frac{m^2}{2}$$

4. Calculate the power in each individual sideband (P_{USB} and P_{LSB}):

$$P_{USB} = P_{LSB} = \frac{P_{SB}}{2} = P_c \frac{m^2}{4}$$

Worked Example:

Calculate AM power components An AM transmitter radiates a carrier power of 10 kW at a modulation index of 0.8. Calculate the total radiated power, the total sideband power, and the power in each sideband.

Solution:

1. Identify given values:

$$P_c = 10 \text{ kW}$$

$$m = 0.8$$

2. Calculate total power P_t:

$$P_t = P_c \left(1 + \frac{m^2}{2}\right) = 10 \left(1 + \frac{0.8^2}{2}\right) = 10 \left(1 + \frac{0.64}{2}\right) = 10(1 + 0.32) = 13.2 \text{ kW}$$

3. Calculate total sideband power P_{SB}:

$$P_{SB} = P_t - P_c = 13.2 \text{ kW} - 10 \text{ kW} = 3.2 \text{ kW}$$

   Alternatively, $P_{SB} = P_c \frac{m^2}{2} = 10 \times 0.32 = 3.2$ kW.

4. Calculate power in each sideband (P_{USB} and P_{LSB}):

$$P_{USB} = P_{LSB} = \frac{P_{SB}}{2} = \frac{3.2 \text{ kW}}{2} = 1.6 \text{ kW}$$

Methodology:

Current Calculations in Amplitude Modulation Since power is proportional to the square of the current ($P = I^2R$), the power relations can be expressed in terms of antenna currents.

Steps to calculate AM currents:

1. Relate unmodulated carrier current I_c to carrier power P_c:

$$P_c = I_c^2 R \implies I_c = \sqrt{\frac{P_c}{R}}$$

2. Relate total modulated current I_t to total power P_t:

$$P_t = I_t^2 R \implies I_t = \sqrt{\frac{P_t}{R}}$$

3. Substitute the power equation $P_t = P_c(1 + \frac{m^2}{2})$ to find the total current I_t:

$$I_t = I_c\sqrt{1 + \frac{m^2}{2}}$$

4. Calculate modulation index m from measured currents:

$$m = \sqrt{2\left(\left(\frac{I_t}{I_c}\right)^2 - 1\right)}$$

Worked Example:

Calculate modulation index from antenna currents The antenna current of an AM transmitter is 8 A when only the carrier is sent, but it increases to 8.93 A when the carrier is sinusoidally modulated. Find the percentage modulation.

Solution:

1. Identify given values:

$$I_c = 8 \text{ A}$$

$$I_t = 8.93 \text{ A}$$

2. Use the current relation formula to find m:

$$m = \sqrt{2\left(\left(\frac{I_t}{I_c}\right)^2 - 1\right)}$$

3. Substitute the values:

$$\frac{I_t}{I_c} = \frac{8.93}{8} = 1.11625$$

$$\left(\frac{I_t}{I_c}\right)^2 = (1.11625)^2 \approx 1.246$$

$$\left(\frac{I_t}{I_c}\right)^2 - 1 = 1.246 - 1 = 0.246$$

4. Solve for m:

$$m = \sqrt{2 \times 0.246} = \sqrt{0.492} \approx 0.701$$

5. Convert to percentage modulation:

$$M = m \times 100\% = 0.701 \times 100\% = 70.1\%$$

2.2 Frequency Modulation

Methodology:

Frequency Modulation Parameters In Frequency Modulation (FM), the frequency of the carrier signal is varied proportionally to the instantaneous amplitude of the modulating signal.
Steps to calculate FM parameters:

1. Identify the unmodulated carrier frequency f_c and modulating signal frequency f_m.
2. Identify the maximum frequency deviation Δf. This tells us how much the carrier frequency shifts above and below its resting value.
3. Calculate the modulation index for FM, denoted by m_f or β:
$$m_f = \frac{\Delta f}{f_m}$$
Unlike AM, the FM modulation index can be greater than 1.
4. Determine the standard time-domain equation of an FM signal:
$$v_{FM}(t) = V_c \sin(2\pi f_c t + m_f \sin(2\pi f_m t))$$
where V_c is the peak amplitude of the unmodulated carrier.

Worked Example:

Calculate FM modulation index and maximum frequency deviation An FM signal is given by the equation $v(t) = 12\sin(6 \times 10^8 t + 5\sin(1250t))$. Calculate the carrier frequency, modulating frequency, modulation index, and maximum frequency deviation.
Solution:

1. Compare the given equation with the standard FM equation:
$$v(t) = 12\sin(6 \times 10^8 t + 5\sin(1250t))$$
$$v_{FM}(t) = V_c \sin(2\pi f_c t + m_f \sin(2\pi f_m t))$$
2. Extract angular frequencies and parameters:
$$2\pi f_c = 6 \times 10^8 \implies f_c = \frac{6 \times 10^8}{2\pi} \approx 95.49 \text{ MHz}$$
$$2\pi f_m = 1250 \implies f_m = \frac{1250}{2\pi} \approx 198.9 \text{ Hz}$$
$$m_f = 5$$
3. Calculate maximum frequency deviation Δf: We know $m_f = \frac{\Delta f}{f_m}$. Rearranging this gives:
$$\Delta f = m_f \times f_m = 5 \times 198.9 \text{ Hz} = 994.5 \text{ Hz}$$

Methodology:

Bandwidth of Frequency Modulated Signal An FM signal generates theoretically infinite sidebands, but for practical purposes, the bandwidth is limited to sidebands containing significant power.
Steps to calculate FM bandwidth using Carson's Rule:

1. Identify the maximum frequency deviation Δf and the highest modulating frequency f_m.

2. Use Carson's rule to estimate the bandwidth (BW) containing 98% of the signal power:

$$BW \approx 2(\Delta f + f_m)$$

3. For Wideband FM (WBFM) where $\Delta f \gg f_m$ (typically $m_f > 10$), the bandwidth can be approximated as:

$$BW \approx 2\Delta f$$

4. For Narrowband FM (NBFM) where $\Delta f \ll f_m$ (typically $m_f < 1$), the bandwidth can be approximated as:

$$BW \approx 2f_m$$

which is identical to the bandwidth of an AM signal.

Worked Example:

Calculate bandwidth using Carson's rule In an FM system, the modulating frequency is 15 kHz and the maximum frequency deviation is 75 kHz. Calculate the bandwidth required.

Solution:

1. Identify given values:

$$f_m = 15 \text{ kHz}$$

$$\Delta f = 75 \text{ kHz}$$

2. Calculate the modulation index to check the type of FM:

$$m_f = \frac{\Delta f}{f_m} = \frac{75}{15} = 5$$

Since $m_f > 1$, this is Wideband FM.

3. Apply Carson's rule for bandwidth estimation:

$$BW = 2(\Delta f + f_m)$$

$$BW = 2(75 \text{ kHz} + 15 \text{ kHz})$$

$$BW = 2(90 \text{ kHz}) = 180 \text{ kHz}$$

The required bandwidth is 180 kHz.

Methodology:

Frequency Deviation Sensitivity The maximum frequency deviation is directly proportional to the peak amplitude of the modulating signal.

Steps to use frequency deviation sensitivity:

1. Identify the deviation sensitivity k_f, usually given in Hz/V or kHz/V.
2. Identify the peak voltage of the modulating signal V_m.
3. Calculate the maximum frequency deviation Δf:

$$\Delta f = k_f \times V_m$$

4. If V_m is changed to a new value V_{m2}, the new deviation Δf_2 is:

$$\Delta f_2 = k_f \times V_{m2}$$

Since k_f is constant, $\frac{\Delta f_1}{V_{m1}} = \frac{\Delta f_2}{V_{m2}}$.

Worked Example:

Calculate new frequency deviation for altered input voltage An FM modulator has a frequency deviation sensitivity of 5 kHz/V. If a modulating signal of 2 V peak is applied, find the maximum frequency deviation and the modulation index if the modulating frequency is 2 kHz.

Solution:

1. Identify given values:

$$k_f = 5 \text{ kHz/V}$$
$$V_m = 2 \text{ V}$$
$$f_m = 2 \text{ kHz}$$

2. Calculate the maximum frequency deviation Δf:

$$\Delta f = k_f \times V_m = 5 \text{ kHz/V} \times 2 \text{ V} = 10 \text{ kHz}$$

3. Calculate the modulation index m_f:

$$m_f = \frac{\Delta f}{f_m} = \frac{10 \text{ kHz}}{2 \text{ kHz}} = 5$$

CHAPTER 3

Analog Receivers

This chapter focuses on the computational aspects of analog receivers including both AM and FM systems. The primary mathematical concepts involve calculating intermediate frequencies, image frequencies, rejection ratios, envelope detector time constants, and phase-locked loop (PLL) sensitivities.

3.1 Receiver Characteristics and Superheterodyne Principles

Methodology:

Receiver Characteristics Calculations Radio receivers are evaluated based on computational figures of merit:

1. **Sensitivity:** The minimum input RF signal voltage required to produce a specified audio output power. Often expressed in microvolts (μV) or decibels referenced to 1 milliwatt (dBm).
2. **Selectivity:** The ability to separate the desired frequency f_s from adjacent interfering frequencies. Computed using the Quality Factor (Q) of the tuned circuit:

$$Q = \frac{f_r}{BW}$$

where f_r is the resonant frequency and BW is the bandwidth.

3. **Fidelity:** The uniform reproduction of the audio frequency spectrum. It requires the receiver bandwidth to be at least $2 \times f_{m(max)}$ for AM to accommodate the highest audio frequency.
4. **Image Frequency Rejection Ratio (IFRR):** The measure of how well the receiver suppresses the unwanted image frequency.

Methodology:

Superheterodyne Receiver Calculations A superheterodyne receiver down-converts the incoming RF signal (f_s) to a fixed Intermediate Frequency (f_{IF}) using a Local Oscillator (f_{LO}).

1. **Local Oscillator Frequency (f_{LO}):** For high-side injection (most common in standard AM receivers):

$$f_{LO} = f_s + f_{IF}$$

For low-side injection:

$$f_{LO} = f_s - f_{IF}$$

2. **Image Frequency (f_{si}):** The image frequency is an unwanted input signal that can also mix with the LO to produce the IF. For high-side injection:

$$f_{si} = f_s + 2f_{IF}$$

3. **Image Frequency Rejection Ratio (IFRR):**

$$\text{IFRR} = \sqrt{1 + Q^2\rho^2}$$

where Q is the quality factor of the tuned RF amplifier, and ρ is the fractional frequency deviation given by:

$$\rho = \left(\frac{f_{si}}{f_s}\right) - \left(\frac{f_s}{f_{si}}\right)$$

Often IFRR is expressed in decibels: IFRR (dB) $= 20\log_{10}(\text{IFRR})$.

Worked Example:

Calculate Image Frequency and IFRR An AM superheterodyne receiver has an Intermediate Frequency (IF) of 455 kHz and is tuned to a station at 1000 kHz. The RF amplifier has a Q-factor of 100. Calculate the local oscillator frequency, the image frequency, and the Image Frequency Rejection Ratio in dB. Assume high-side injection.

Step 1: Calculate the Local Oscillator Frequency (f_{LO}).

$$f_{LO} = f_s + f_{IF} = 1000 \text{ kHz} + 455 \text{ kHz} = 1455 \text{ kHz}$$

Step 2: Calculate the Image Frequency (f_{si}).

$$f_{si} = f_s + 2f_{IF} = 1000 \text{ kHz} + 2(455 \text{ kHz}) = 1910 \text{ kHz}$$

Step 3: Calculate the fractional frequency deviation (ρ).

$$\rho = \left(\frac{f_{si}}{f_s}\right) - \left(\frac{f_s}{f_{si}}\right)$$

$$\rho = \left(\frac{1910}{1000}\right) - \left(\frac{1000}{1910}\right) = 1.91 - 0.5236 = 1.3864$$

Step 4: Calculate the IFRR.

$$\text{IFRR} = \sqrt{1 + Q^2\rho^2} = \sqrt{1 + (100 \times 1.3864)^2} = \sqrt{1 + (138.64)^2} \approx 138.64$$

Step 5: Convert IFRR to Decibels.

$$\text{IFRR (dB)} = 20\log_{10}(138.64) \approx 42.84 \text{ dB}$$

3.2 Intermediate Frequency Selection and Tuning

Methodology:

Tuning Range and Capacitor Ratio The receiver is tuned to different RF stations using a variable capacitor. The required variation in capacitance depends on the frequency band to be covered.

1. The resonant frequency of the tuned circuit is given by $f = \frac{1}{2\pi\sqrt{LC}}$.
2. Let the tuning frequency range be from f_{min} to f_{max}.
3. The ratio of maximum to minimum capacitance (C_{max}/C_{min}) required to tune this range is derived directly from the frequency ratio squared:

$$\frac{C_{max}}{C_{min}} = \left(\frac{f_{max}}{f_{min}}\right)^2$$

Note: C_{max} corresponds to f_{min}, and C_{min} corresponds to f_{max}.

Worked Example:

Determine Tuning Capacitor Ratio An AM broadcast receiver must tune the frequency range from 540 kHz to 1600 kHz. Calculate the required tuning capacitor ratio. If the minimum capacitance of the variable capacitor is 20 pF, determine the required maximum capacitance.

Step 1: Calculate the frequency ratio.

$$\frac{f_{max}}{f_{min}} = \frac{1600 \text{ kHz}}{540 \text{ kHz}} \approx 2.963$$

Step 2: Calculate the required capacitance ratio.

$$\frac{C_{max}}{C_{min}} = \left(\frac{f_{max}}{f_{min}}\right)^2 = (2.963)^2 \approx 8.78$$

Step 3: Calculate the maximum capacitance (C_{max}). Given $C_{min} = 20$ pF.

$$C_{max} = 8.78 \times C_{min} = 8.78 \times 20 \text{ pF} = 175.6 \text{ pF}$$

3.3 AM Demodulation: Envelope Detector

Methodology:

Envelope Detector Design constraints An envelope detector uses a diode followed by a parallel RC filter to recover the modulating signal. Proper selection of the RC time constant ($\tau = RC$) is critical for avoiding signal distortion.

1. **Carrier Filtering:** To effectively filter out the high-frequency RF carrier, the time constant must be large compared to the carrier period:

$$RC \gg \frac{1}{f_c}$$

2. **Envelope Tracking:** To follow the highest audio frequency without sluggishness, the time constant must be small compared to the highest audio period:

$$RC \ll \frac{1}{f_m}$$

3. **Avoidance of Diagonal Clipping:** To prevent the capacitor from discharging too slowly during rapid downward swings of the AM envelope, the RC time constant must strictly satisfy:

$$RC \leq \frac{\sqrt{1-m^2}}{m\omega_m}$$

where m is the modulation index and $\omega_m = 2\pi f_m$ is the maximum angular modulating frequency.

Worked Example:

Design of RC Time Constant for Envelope Detector An envelope detector is used to demodulate an AM signal. The carrier frequency is 1 MHz, the maximum modulating frequency is 5 kHz, and the maximum modulation index is 0.8. Select a suitable value for the RC time constant and verify if $C = 0.01\,\mu$F and $R = 10$ kΩ is an appropriate combination.

Step 1: Determine the broad limits for RC based on filtering and tracking.

$$\text{Lower limit: } \frac{1}{f_c} = \frac{1}{10^6} = 1\,\mu\text{s}$$

$$\text{Upper limit: } \frac{1}{f_m} = \frac{1}{5 \times 10^3} = 200\,\mu\text{s}$$

Thus, $1\,\mu\text{s} \ll RC \ll 200\,\mu\text{s}$.
Step 2: Calculate the strict diagonal clipping limit.

$$\omega_m = 2\pi(5000) = 31415.9 \text{ rad/s}$$

$$RC \le \frac{\sqrt{1-0.8^2}}{0.8 \times 31415.9} = \frac{\sqrt{0.36}}{25132.7} = \frac{0.6}{25132.7} \approx 23.87\,\mu\text{s}$$

So, RC must be less than or equal to $23.87\,\mu$s to prevent diagonal clipping at 80% modulation.
Step 3: Evaluate the proposed RC combination.

$$RC_{proposed} = (10 \text{ k}\Omega) \times (0.01\,\mu\text{F}) = (10^4) \times (10^{-8}) = 100\,\mu\text{s}$$

Since $100\,\mu$s is significantly greater than the $23.87\,\mu$s limit, this combination will result in severe diagonal clipping. A smaller R or C must be chosen (e.g., changing R to 2 kΩ gives $RC = 20\,\mu$s, which safely satisfies the condition).

3.4 FM Receivers and Demodulators

Methodology:

FM Receiver Block Parameters A basic FM superheterodyne receiver uses a block structure similar to an AM receiver but includes a limiter and a frequency discriminator to process angular modulation.

1. **Amplitude Limiter:** Clips the IF signal to a constant amplitude to remove any amplitude variations caused by channel noise. It requires a minimum input voltage (threshold) to perform limiting.
2. **FM Demodulator (Discriminator):** Converts instantaneous frequency variations back into voltage variations. The amplitude of the output voltage is directly proportional to the instantaneous frequency deviation.
3. **Discriminator Sensitivity (K_d):** The constant of proportionality for the demodulator, expressed in Volts per Hertz (V/Hz) or Volts per kilohertz (V/kHz).

$$V_{out} = K_d \times \Delta f$$

where Δf is the instantaneous frequency deviation of the input FM signal from its resting unmodulated center frequency.

Worked Example:

Calculate FM Demodulator Output Voltage An FM receiver utilizes a discriminator with a sensitivity of 0.5 V/kHz. The incoming FM signal has a frequency deviation of ± 50 kHz caused by a 2 kHz sinusoidal modulating signal. Determine the peak-to-peak output voltage of the demodulator.
Step 1: Identify the maximum frequency deviation. The maximum frequency deviation from the center carrier frequency is given as $\Delta f = 50$ kHz.
Step 2: Calculate the peak output voltage (V_{peak}). The output voltage magnitude corresponds directly to the frequency deviation.

$$V_{peak} = K_d \times \Delta f = 0.5 \text{ V/kHz} \times 50 \text{ kHz} = 25 \text{ V}$$

Step 3: Calculate the peak-to-peak output voltage. Since the frequency deviates continuously by ± 50 kHz, the output voltage swings symmetrically by ± 25 V.

$$V_{p-p} = 2 \times V_{peak} = 2 \times 25 \text{ V} = 50 \text{ V}$$

3.5 FM Demodulator Using Phase-Locked Loop

Methodology:

PLL FM Demodulator Calculations A Phase-Locked Loop (PLL) tracks the instantaneous frequency of an incoming FM signal. The internally generated error voltage, which forces the Voltage Controlled Oscillator (VCO) to track the input frequency, serves as the demodulated audio output.

1. **VCO Operation:** The frequency of the VCO (f_{VCO}) is controlled by its input voltage (V_{out}).

$$f_{VCO} = f_0 + K_{VCO} V_{out}$$

where f_0 is the free-running frequency and K_{VCO} is the VCO sensitivity factor in Hz/V.

2. **Lock Condition:** When the PLL is continuously locked onto the incoming FM signal, $f_{VCO} = f_{in}$.

3. **Demodulated Output (V_{out}):** Since the input FM signal varies as $f_{in} = f_c + \Delta f(t)$, and setting the free-running frequency $f_0 = f_c$:

$$V_{out}(t) = \frac{f_{in}(t) - f_0}{K_{VCO}} = \frac{\Delta f(t)}{K_{VCO}}$$

Thus, the output voltage is a direct, linear reproduction of the frequency deviation $\Delta f(t)$, representing the original message signal.

Worked Example:

Calculate Output Voltage of PLL Demodulator A PLL is used as an FM demodulator. Its VCO has a free-running frequency of 10.7 MHz and a sensitivity of 20 kHz/V. The input to the PLL is an FM signal with a center frequency of 10.7 MHz and a maximum frequency deviation of 75 kHz. Determine the peak output voltage generated from the PLL demodulator.
Step 1: Identify given circuit parameters. Free-running frequency $f_0 = 10.7$ MHz
Carrier frequency $f_c = 10.7$ MHz (Note: $f_0 = f_c$)
VCO Sensitivity $K_{VCO} = 20$ kHz/V
Peak Frequency Deviation $\Delta f_{peak} = 75$ kHz
Step 2: Apply the lock condition formula to find the peak output voltage. When locked, the maximum peak deviation in frequency produces the corresponding peak output voltage.

$$V_{out(peak)} = \frac{\Delta f_{peak}}{K_{VCO}}$$

Step 3: Compute the final voltage.

$$V_{out(peak)} = \frac{75\text{ kHz}}{20\text{ kHz/V}} = 3.75\text{ V}$$

The peak output voltage corresponding to the maximum frequency deviation of 75 kHz is 3.75 V.

CHAPTER 4

Sampling Theory and Waveform Coding

4.1 Sampling Theorem

Methodology:

Finding Nyquist Rate and Nyquist Interval The sampling theorem states that a bandlimited signal with maximum frequency component f_{max} can be perfectly reconstructed from its samples if the sampling frequency f_s satisfies the condition:

1. **Nyquist Rate** (f_N): The minimum sampling rate required to avoid aliasing.

$$f_N = 2f_{max}$$

2. **Nyquist Interval** (T_N): The maximum time interval between samples.

$$T_N = \frac{1}{f_N} = \frac{1}{2f_{max}}$$

For a signal given as a sum of multiple sinusoidal components, f_{max} is the highest frequency among all components. For a product of signals, the maximum frequency is the sum of the maximum frequencies of individual signals.

Worked Example:

Nyquist Rate for a Single Sinusoid Determine the Nyquist rate and Nyquist interval for the signal $x(t) = 10\cos(2000\pi t) + 5\sin(5000\pi t)$.

Solution:

1. Extract the frequencies from the given signal. The general form is $\cos(2\pi f t)$.
2. For the first term $10\cos(2000\pi t)$:

$$2\pi f_1 = 2000\pi \implies f_1 = 1000 \text{ Hz}$$

3. For the second term $5\sin(5000\pi t)$:

$$2\pi f_2 = 5000\pi \implies f_2 = 2500 \text{ Hz}$$

4. Find the maximum frequency component f_{max}:

$$f_{max} = \max(1000, 2500) = 2500 \text{ Hz}$$

5. Calculate the Nyquist Rate f_N:

$$f_N = 2 \times f_{max} = 2 \times 2500 = 5000 \text{ Hz}$$

6. Calculate the Nyquist Interval T_N:

$$T_N = \frac{1}{f_N} = \frac{1}{5000} = 0.0002 \text{ seconds} = 0.2 \text{ ms}$$

Worked Example:

Nyquist Rate for a Product of Signals Find the Nyquist rate for the signal $x(t) = \text{sinc}(100t) \times \text{sinc}(200t)$.
Solution:

1. Recall that for a signal of the form $\text{sinc}(Kt)$, the maximum frequency is $f = \frac{K}{2}$.
2. For the first term $\text{sinc}(100t)$:
$$f_1 = \frac{100}{2} = 50 \text{ Hz}$$
3. For the second term $\text{sinc}(200t)$:
$$f_2 = \frac{200}{2} = 100 \text{ Hz}$$
4. When two signals are multiplied in the time domain, their spectra are convolved in the frequency domain. The maximum frequency of the product signal is the sum of their individual maximum frequencies:
$$f_{max} = f_1 + f_2 = 50 + 100 = 150 \text{ Hz}$$
5. Calculate the Nyquist rate f_N:
$$f_N = 2 \times f_{max} = 2 \times 150 = 300 \text{ Hz}$$

4.2 Pulse Code Modulation (PCM)

Methodology:

Calculating PCM Parameters In a Pulse Code Modulation (PCM) system, an analog signal is sampled, quantized, and encoded into a digital bitstream. The key parameters are:

1. **Number of Quantization Levels (L):** If n bits are used to encode each sample, the number of levels is:
$$L = 2^n$$
2. **Number of Bits per Sample (n):**
$$n = \log_2(L) \quad \text{(rounded up to the next integer)}$$
3. **Bit Rate (R_b):** The transmission rate in bits per second (bps), given sampling rate f_s:
$$R_b = n \times f_s$$
4. **Transmission Bandwidth (BW):** The minimum baseband bandwidth required to transmit the PCM signal:
$$BW_{min} = \frac{R_b}{2}$$

Worked Example:

Bit Rate and Bandwidth Calculation A voice signal bandlimited to 3.4 kHz is sampled at a rate 20% higher than the Nyquist rate. It is then quantized into 256 levels. Calculate the bit rate and the minimum transmission bandwidth required.
Solution:

1. Identify the maximum frequency of the signal:
$$f_{max} = 3.4 \text{ kHz} = 3400 \text{ Hz}$$

2. Calculate the Nyquist rate:

$$f_N = 2 \times f_{max} = 2 \times 3400 = 6800 \text{ Hz}$$

3. Calculate the actual sampling rate f_s which is 20% higher than f_N:

$$f_s = f_N + 0.20 f_N = 1.20 \times 6800 = 8160 \text{ Hz}$$

4. Determine the number of bits per sample n using the number of levels $L = 256$:

$$n = \log_2(256) = 8 \text{ bits/sample}$$

5. Calculate the bit rate R_b:

$$R_b = n \times f_s = 8 \times 8160 = 65280 \text{ bps} = 65.28 \text{ kbps}$$

6. Calculate the minimum transmission bandwidth:

$$BW_{min} = \frac{R_b}{2} = \frac{65280}{2} = 32640 \text{ Hz} = 32.64 \text{ kHz}$$

4.3 Quantization in PCM

Methodology:

Quantization Step Size and Error Quantization is the process of mapping continuous amplitude values of samples to discrete levels.

1. **Step Size (Δ):** Also known as the quantization interval or resolution. If the analog signal varies from V_{min} to V_{max} (total dynamic range $V_{pp} = V_{max} - V_{min}$) and uses L quantization levels:

$$\Delta = \frac{V_{max} - V_{min}}{L} = \frac{V_{pp}}{2^n}$$

2. **Maximum Quantization Error ($Q_{e,max}$):** The maximum difference between the original analog sample and the quantized value:

$$Q_{e,max} = \pm \frac{\Delta}{2}$$

3. **Quantization Noise Power (N_q):** Assuming the error is uniformly distributed over the step size:

$$N_q = \frac{\Delta^2}{12}$$

Worked Example:

Calculating Step Size and Maximum Error An analog signal with a voltage range from -5 V to +5 V is quantized using a 10-bit PCM system. Calculate the quantization step size and the maximum quantization error.

Solution:

1. Identify the given parameters:

$$V_{min} = -5 \text{ V}, \quad V_{max} = +5 \text{ V}, \quad n = 10 \text{ bits}$$

2. Calculate the peak-to-peak voltage range V_{pp}:

$$V_{pp} = V_{max} - V_{min} = 5 - (-5) = 10 \text{ V}$$

3. Calculate the number of quantization levels L:

$$L = 2^n = 2^{10} = 1024$$

4. Calculate the step size Δ:

$$\Delta = \frac{V_{pp}}{L} = \frac{10}{1024} \approx 0.00976 \text{ V} = 9.76 \text{ mV}$$

5. Calculate the maximum quantization error:

$$Q_{e,max} = \pm\frac{\Delta}{2} = \pm\frac{9.76}{2} = \pm 4.88 \text{ mV}$$

Methodology:

Signal to Quantization Noise Ratio The Signal-to-Quantization Noise Ratio (SQNR) is a measure of the quality of the quantized signal.

1. **General SQNR Formula:**

$$\text{SQNR} = \frac{\text{Signal Power}}{\text{Quantization Noise Power}} = \frac{P_s}{N_q}$$

2. **SQNR for a Sinusoidal Signal in dB:** If the input is a full-scale sinusoidal signal, the SQNR in decibels depends primarily on the number of bits n:

$$\text{SQNR}_{dB} \approx 1.76 + 6.02n$$

   This is often approximated as:

$$\text{SQNR}_{dB} \approx 1.8 + 6n$$

This formula indicates that adding one extra bit to the PCM encoder improves the SQNR by approximately 6 dB.

Worked Example:

Finding Number of Bits for a Desired SQNR A CD audio system requires a Signal-to-Quantization Noise Ratio (SQNR) of at least 90 dB for a sinusoidal test signal. Determine the minimum number of bits per sample required.

Solution:

1. State the given SQNR requirement:

$$\text{SQNR}_{dB} \geq 90 \text{ dB}$$

2. Use the approximate formula for SQNR of a sinusoidal signal:

$$1.8 + 6n \geq 90$$

3. Solve for n:

$$6n \geq 90 - 1.8$$
$$6n \geq 88.2$$
$$n \geq \frac{88.2}{6}$$
$$n \geq 14.7$$

4. Since the number of bits must be an integer, round up to the next higher integer:

$$n = 15 \text{ bits/sample}$$

Note: Standard CD audio uses 16 bits which comfortably satisfies this requirement.

4.4 Delta Modulation

Methodology:

Avoiding Slope Overload in Delta Modulation Delta Modulation (DM) uses a 1-bit quantizer to encode the difference between the current sample and the predicted previous sample. A constant step size Δ is used.

1. **Slope Overload Distortion:** Occurs when the analog input signal changes faster than the step size can track it.
2. **Condition to Avoid Slope Overload:** The maximum slope of the staircase approximated signal must be greater than or equal to the maximum slope of the input signal $m(t)$.

$$\frac{\Delta}{T_s} \geq \left|\frac{dm(t)}{dt}\right|_{max}$$

   Since $T_s = \frac{1}{f_s}$, this becomes:

$$\Delta \cdot f_s \geq \left|\frac{dm(t)}{dt}\right|_{max}$$

3. **For a Sinusoidal Signal:** If $m(t) = A_m \sin(2\pi f_m t)$, the maximum slope is $2\pi f_m A_m$. The condition becomes:

$$\Delta \cdot f_s \geq 2\pi f_m A_m$$

Worked Example:

Minimum Step Size in Delta Modulation A delta modulator is designed to operate at 3 times the Nyquist rate for a signal with a bandwidth of 3 kHz. The modulating signal is a 2 V peak sinusoid at a frequency of 2 kHz. Determine the minimum step size required to avoid slope overload.
Solution:

1. Identify the signal parameters:

$$f_{max\_bw} = 3 \text{ kHz} \quad \text{(Signal bandwidth)}$$

$$f_m = 2 \text{ kHz} = 2000 \text{ Hz} \quad \text{(Frequency of the sinusoid)}$$

$$A_m = 2 \text{ V} \quad \text{(Peak amplitude)}$$

2. Calculate the Nyquist rate based on the signal bandwidth:

$$f_N = 2 \times f_{max\_bw} = 2 \times 3000 = 6000 \text{ Hz}$$

3. Determine the actual sampling frequency f_s:

$$f_s = 3 \times f_N = 3 \times 6000 = 18000 \text{ Hz}$$

4. Apply the condition to avoid slope overload for a sinusoidal signal:

$$\Delta \cdot f_s \geq 2\pi f_m A_m$$

5. Substitute the known values to find the minimum Δ:

$$\Delta \times 18000 \geq 2\pi(2000)(2)$$

$$18000\Delta \geq 8000\pi$$
$$\Delta \geq \frac{8000\pi}{18000}$$
$$\Delta \geq \frac{4\pi}{9} \approx 1.396 \text{ V}$$

6. The minimum step size required is approximately 1.396 V.

CHAPTER 5

Line Coding and Multiplexing

Line coding transforms a sequence of digital bits into a digital signal (voltage or current levels) for transmission over a baseband channel. Multiplexing allows multiple signals to share a single communication link.

5.1 Line Coding Schemes

Methodology:

Digital to Digital Conversion Rules To encode a binary sequence into a digital signal, apply the following mapping rules for each bit duration (T_b):

1. **Unipolar NRZ (Non-Return to Zero):** Bit '1' is represented by a positive voltage ($+V$). Bit '0' is represented by zero voltage (0).
2. **Polar NRZ:** Bit '1' is represented by a positive voltage ($+V$). Bit '0' is represented by a negative voltage ($-V$).
3. **Polar RZ (Return to Zero):** Uses three levels. Bit '1' transitions from $+V$ to 0 at $T_b/2$. Bit '0' transitions from $-V$ to 0 at $T_b/2$.
4. **Manchester Encoding:** Bit '1' involves a High-to-Low transition ($+V$ to $-V$) at the middle of the bit period. Bit '0' involves a Low-to-High transition ($-V$ to $+V$) at the middle.
5. **Bipolar AMI (Alternate Mark Inversion):** Bit '0' is represented by zero voltage (0). Bit '1' alternates between positive ($+V$) and negative ($-V$) voltages.

Worked Example:

Encoding a Bit Sequence Determine the voltage levels for the bit sequence **1 0 1 1 0 1** using Unipolar NRZ, Polar NRZ and Bipolar AMI line coding schemes. Assume the previous '1' in AMI was encoded as $-V$.
Solution:

1. **Analyze Unipolar NRZ:** Map $1 \to +V$ and $0 \to 0$. Sequence: $+V, 0, +V, +V, 0, +V$
2. **Analyze Polar NRZ:** Map $1 \to +V$ and $0 \to -V$. Sequence: $+V, -V, +V, +V, -V, +V$
3. **Analyze Bipolar AMI:** Map $0 \to 0$. The 1s must alternate in polarity. Since the previous 1 was $-V$, the first 1 in this sequence must be $+V$.
   - Bit 1 (1): $+V$
   - Bit 2 (0): 0
   - Bit 3 (1): alternates from previous $+V \to -V$
   - Bit 4 (1): alternates from previous $-V \to +V$
   - Bit 5 (0): 0
   - Bit 6 (1): alternates from previous $+V \to -V$

Sequence: $+V, 0, -V, +V, 0, -V$

Bit	**1**	**0**	**1**	**1**	**0**	**1**
Unipolar NRZ	$+V$	0	$+V$	$+V$	0	$+V$
Polar NRZ	$+V$	$-V$	$+V$	$+V$	$-V$	$+V$
AMI	$+V$	0	$-V$	$+V$	0	$-V$

5.2 Frequency Division Multiplexing

Methodology:

FDM Bandwidth Calculation Frequency Division Multiplexing (FDM) combines multiple signals into one medium by assigning each signal a distinct frequency band. Guard bands are used to prevent overlapping.

1. Let N be the total number of channels to be multiplexed.
2. Let B_c be the bandwidth of a single channel.
3. Let B_g be the bandwidth of a single guard band between adjacent channels.
4. The number of guard bands required is $(N-1)$.
5. The total minimum bandwidth B_T required for the multiplexed signal is calculated as:

$$B_T = N \cdot B_c + (N-1) \cdot B_g$$

Worked Example:

FDM Bandwidth with Guard Bands Five voice channels each having a bandwidth of 4 kHz are to be multiplexed together using FDM. If a guard band of 1 kHz is required between any two adjacent channels to prevent interference calculate the total bandwidth required for the multiplexed signal.
Solution:

1. **Identify given parameters:** Number of channels, $N = 5$
   Bandwidth per channel, $B_c = 4$ kHz
   Guard band bandwidth, $B_g = 1$ kHz
2. **Determine the number of guard bands:** Number of guard bands $= N - 1 = 5 - 1 = 4$
3. **Calculate the total channel bandwidth:** Total channel bandwidth $= N \times B_c = 5 \times 4$ kHz $= 20$ kHz
4. **Calculate the total guard band bandwidth:** Total guard band bandwidth $= (N-1) \times B_g = 4 \times 1$ kHz $= 4$ kHz
5. **Calculate total required bandwidth (B_T):**

$$B_T = 20 \text{ kHz} + 4 \text{ kHz} = 24 \text{ kHz}$$

The total bandwidth required is 24 kHz.

5.3 Time Division Multiplexing

Methodology:

Synchronous TDM Parameters Time Division Multiplexing (TDM) assigns individual time slots to different channels. In synchronous TDM with N input channels, each operating at a data rate of R bps:

1. **Bit duration of input channel (t_b):** $t_b = \frac{1}{R}$ seconds.
2. **Frame duration (T_f):** If multiplexing is done bit-by-bit (1 bit per channel per frame), the frame duration equals the input bit duration: $T_f = t_b = \frac{1}{R}$.
3. **Frame rate (F_s):** The number of frames transmitted per second is equal to the data rate of one input channel: $F_s = \frac{1}{T_f} = R$ frames/sec.
4. **Bits per frame (L):** If 1 bit is taken from each of the N channels, and there are k synchronization (framing) bits: $L = N \times 1 + k$ bits.
5. **Total output data rate (R_{out}):** $R_{out} = F_s \times L$ bps.

Worked Example:

Four Channel Synchronous TDM Four channels are multiplexed using Synchronous TDM. Each channel transmits data at 1 Mbps. If 1 bit is multiplexed from each channel per frame, and 1 framing bit is added to each frame for synchronization, determine: (a) Output frame rate, (b) Duration of one frame, (c) Number of bits in each frame, and (d) Total output data rate.

Solution:

1. **Identify given parameters:** Number of channels, $N = 4$
   Input data rate, $R = 1$ Mbps $= 1 \times 10^6$ bps
   Framing bits, $k = 1$
2. **Calculate output frame rate (F_s):** Since 1 bit is taken per channel per frame, the frame rate is equal to the bit rate of a single channel.
$$F_s = R = 1 \times 10^6 \text{ frames/sec} = 1 \text{ Mframes/sec}$$
3. **Calculate frame duration (T_f):** The duration of one frame is the inverse of the frame rate.
$$T_f = \frac{1}{F_s} = \frac{1}{1 \times 10^6} \text{ seconds} = 1\,\mu\text{s}$$
4. **Calculate the number of bits in each frame (L):** Each frame carries 1 bit from each of the 4 channels, plus 1 framing bit.
$$L = (N \times 1) + k = (4 \times 1) + 1 = 5 \text{ bits/frame}$$
5. **Calculate total output data rate (R_{out}):**
$$R_{out} = F_s \times L = (1 \times 10^6 \text{ frames/sec}) \times (5 \text{ bits/frame}) = 5 \text{ Mbps}$$

The output data rate is 5 Mbps.

Worked Example:

TDM with Byte Interleaving Three digital channels, each transmitting at 8 kbps, are multiplexed using byte-interleaved TDM (8 bits per channel per frame). There is no framing bit. Calculate the frame rate,

frame duration, and total output bit rate.
Solution:

1. **Identify given parameters:** Number of channels, $N = 3$
   Input data rate, $R = 8$ kbps $= 8000$ bps
   Multiplexing unit $= 8$ bits (1 byte)

2. **Calculate frame rate (F_s):** Since 8 bits are taken per channel per frame, the frame rate is the channel bit rate divided by 8.

$$F_s = \frac{R}{8} = \frac{8000}{8} = 1000 \text{ frames/sec}$$

3. **Calculate frame duration (T_f):**

$$T_f = \frac{1}{F_s} = \frac{1}{1000} \text{ seconds} = 1 \text{ ms}$$

4. **Calculate the number of bits in each frame (L):**

$$L = N \times 8 \text{ bits} = 3 \times 8 = 24 \text{ bits/frame}$$

5. **Calculate total output data rate (R_{out}):**

$$R_{out} = F_s \times L = 1000 \times 24 = 24000 \text{ bps} = 24 \text{ kbps}$$

   Note: Since there are no framing bits, R_{out} also equals $N \times R = 3 \times 8$ kbps $= 24$ kbps.

Methodology:

Multiplexer Output Efficiency The efficiency of a TDM system evaluates how much of the transmission bandwidth is utilized for actual data payload compared to the total data transmitted (which includes framing/synchronization overhead).

1. Let payload bits per frame be $D = N \times$ bits per channel.

2. Let synchronization/framing bits per frame be k.

3. Total bits per frame $L = D + k$.

4. The efficiency (η) of the multiplexer is:

$$\eta = \frac{\text{Information Bits}}{\text{Total Bits}} \times 100\% = \frac{D}{L} \times 100\%$$

Worked Example:

TDM Efficiency Calculation A synchronous TDM system multiplexes 10 channels. Each channel contributes 2 bytes per frame. If 4 framing bits are appended to each frame for synchronization, determine the efficiency of the TDM frame structure.
Solution:

1. **Calculate the number of payload (information) bits (D):** Number of channels, $N = 10$.
   Bits contributed per channel $= 2$ bytes $\times 8$ bits/byte $= 16$ bits.
   $D = 10 \times 16$ bits $= 160$ bits.

2. **Determine framing bits (k):** Framing bits per frame, $k = 4$ bits.

3. **Calculate total bits per frame (L):**

$$L = D + k = 160 + 4 = 164 \text{ bits}$$

4. **Calculate efficiency (η):**

$$\eta = \frac{D}{L} \times 100\% = \frac{160}{164} \times 100\% \approx 97.56\%$$

The multiplexing efficiency is approximately 97.56%.

Formulae

Appendix: Key Formulae

This appendix provides a quick reference to the key formulae and mathematical relationships discussed in this book. They are organized by topic for easy access.

Unit 1 & 2: Amplitude and Frequency Modulation

Amplitude Modulation (AM)

- **Modulation Index (from voltages):**

$$m = \frac{V_{max} - V_{min}}{V_{max} + V_{min}} = \frac{V_m}{V_c}$$

  where V_{max} and V_{min} are the maximum and minimum peak-to-peak voltages of the AM envelope, V_m is the modulating signal amplitude, and V_c is the carrier amplitude.

- **Carrier and Modulating Voltages:**

$$V_c = \frac{V_{max} + V_{min}}{2}, \quad V_m = \frac{V_{max} - V_{min}}{2}$$

- **Total Transmitted Power:**

$$P_t = P_c \left(1 + \frac{m^2}{2}\right)$$

  where P_c is the unmodulated carrier power.

- **Total Antenna Current:**

$$I_t = I_c \sqrt{1 + \frac{m^2}{2}}$$

  where I_c is the unmodulated carrier current.

- **Modulation Index (from currents):**

$$m = \sqrt{2\left(\left(\frac{I_t}{I_c}\right)^2 - 1\right)}$$

- **Sideband Power:**

$$P_{SB} = P_t - P_c = P_c \left(\frac{m^2}{2}\right)$$

$$P_{USB} = P_{LSB} = \frac{P_{SB}}{2} = P_c \frac{m^2}{4}$$

  where P_{USB} and P_{LSB} are the upper and lower sideband powers.

- **Sideband Frequencies:**

$$f_{USB} = f_c + f_m, \quad f_{LSB} = f_c - f_m$$

- **AM Bandwidth:**

$$BW = f_{USB} - f_{LSB} = 2f_m$$

- **AM Signal Equation:**
$$v_{AM}(t) = V_c[1 + m\sin(2\pi f_m t)]\sin(2\pi f_c t)$$
- **Diagonal Clipping Constraint (AM Envelope Detector):**
$$RC \leq \frac{\sqrt{1-m^2}}{m\omega_m}$$

Frequency Modulation (FM)

- **Modulation Index:**
$$m_f = \frac{\Delta f}{f_m}$$
where Δf is the frequency deviation and f_m is the modulating frequency.
- **Frequency Deviation:**
$$\Delta f = k_f \times V_m$$
where k_f is the frequency sensitivity of the modulator.
- **Carson's Rule for FM Bandwidth:**
$$BW \approx 2(\Delta f + f_m) = 2f_m(m_f + 1)$$
- **FM Signal Equation:**
$$v_{FM}(t) = V_c \sin(2\pi f_c t + m_f \sin(2\pi f_m t))$$
- **Frequency Limits:**
$$f_{highest} = f_c + \Delta f, \quad f_{lowest} = f_c - \Delta f$$

Unit 3: Radio Receivers and Noise

- **Noise Factor and Figure:**
$$F = \frac{SNR_{in}}{SNR_{out}}$$
$$NF = 10\log_{10}(F)\ \text{dB}$$
- **Equivalent Noise Temperature:**
$$T_e = T_0(F-1)$$
where T_0 is the reference temperature (typically 290 K).
- **Friis Formula for Cascaded Stages:**
$$F_{total} = F_1 + \frac{F_2 - 1}{G_1} + \frac{F_3 - 1}{G_1 G_2} + \dots$$
- **Local Oscillator Frequency:**
$$f_{LO} = f_s + f_{IF} \quad \text{(High-side injection)}$$
$$f_{LO} = f_s - f_{IF} \quad \text{(Low-side injection)}$$
where f_s is the signal frequency and f_{IF} is the intermediate frequency.
- **Image Frequency:**
$$f_{si} = f_s + 2f_{IF}$$

- **Image Frequency Rejection Ratio (IFRR):**

$$\text{IFRR} = \sqrt{1 + Q^2\rho^2}$$

where $Q = \frac{f_r}{BW}$ is the quality factor of the tuned circuit, and $\rho = \frac{f_{si}}{f_s} - \frac{f_s}{f_{si}}$.

- **Capacitance Tuning Ratio:**

$$\frac{C_{max}}{C_{min}} = \left(\frac{f_{max}}{f_{min}}\right)^2$$

- **PLL Output Voltage:**

$$V_{out}(t) = \frac{\Delta f(t)}{K_{VCO}} = K_d \times \Delta f$$

where K_{VCO} is the VCO sensitivity and K_d is the phase detector sensitivity.

- **VCO Output Frequency:**

$$f_{VCO} = f_0 + K_{VCO}V_{out}$$

Unit 4: Pulse Modulation and Digital Communication

- **Nyquist Rate and Interval:**

$$f_N = 2f_{max}, \quad T_N = \frac{1}{f_N} = \frac{1}{2f_{max}}$$

where f_{max} is the maximum frequency component of the signal.

- **Quantization Levels and Bits per Sample:**

$$L = 2^n, \quad n = \log_2(L)$$

where L is the number of quantization levels and n is the number of bits.

- **Quantization Step Size:**

$$\Delta = \frac{V_{max} - V_{min}}{L} = \frac{V_{pp}}{2^n}$$

- **Quantization Error and Noise Power:**

$$Q_{e,max} = \pm\frac{\Delta}{2}$$

$$N_q = \frac{\Delta^2}{12}$$

- **Signal-to-Quantization Noise Ratio (SQNR):**

$$\text{SQNR} = \frac{\text{Signal Power}}{\text{Quantization Noise Power}} = \frac{P_s}{N_q}$$

For a sinusoidal signal:

$$\text{SQNR}_{dB} \approx 1.8 + 6n \quad \text{or} \quad \text{SQNR}_{dB} \approx 1.76 + 6.02n$$

- **Bit Rate and Minimum Bandwidth:**

$$R_b = n \times f_s$$

$$BW_{min} = \frac{R_b}{2}$$

where f_s is the sampling frequency.

- **Delta Modulation (Avoiding Slope Overload):**

$$\Delta \cdot f_s \geq \left|\frac{dm(t)}{dt}\right|_{max}$$

For a sinusoidal signal $m(t) = A_m \sin(2\pi f_m t)$:

$$\Delta \cdot f_s \geq 2\pi f_m A_m$$

Unit 5: Multiplexing

- **Frame Duration:**
$$T_f = \frac{1}{F_s}$$
where F_s is the frame rate.
- **Bits per Frame (TDM):**
$$L = (N \times \text{bits per channel}) + k$$
where N is the number of channels and k is the number of synchronization/framing bits.
- **Output Bit Rate (TDM):**
$$R_{out} = F_s \times L$$
- **Framing Efficiency:**
$$\eta = \frac{\text{Information Bits}}{\text{Total Bits}} \times 100\%$$
- **Total Bandwidth (FDM):**
$$B_T = N \cdot B_c + (N-1) \cdot B_g$$
where B_c is the channel bandwidth and B_g is the guard band.