Emi (4331102) - Problem Solver

Antigravity AI

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To my students,
who constantly inspire me to find new analogies,
break down complex theories, and make learning
an engineered science.

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Preface

About this Book
Welcome to **Computer Networks**! This textbook is meticulously designed to serve as a comprehensive problem solver and study companion. It provides a robust and intuitive foundation in the core principles of this subject, bridging the gap between theoretical models and practical applications. A unique feature of this book is its focus on decoding complex concepts using clear, step-by-step methodologies and real-world analogies.

Target Audience
This book is intended for students and professionals looking to master the concepts of Computer Networks. Whether you are revising for exams, seeking to solidify your fundamental understanding, or looking for practical examples to clarify abstract concepts, this book has been structured to support your learning journey. It serves as an accessible guide designed to remove the fear of complex topics, replacing it with an appreciation for the underlying mechanics.

Scope and Coverage
The coverage is divided logically to follow standard academic curriculums. Each chapter focuses on key topics, breaking them down into digestible sections:

- **Theoretical Insights** – Core concepts explained intuitively.
- **Method Blocks** – Standardized, step-by-step procedures for solving specific types of problems.
- **Worked Examples** – Fully solved problems that apply the methods in practice.

We hope this resource proves invaluable in your studies and helps you achieve excellence in **Computer Networks**. Happy learning!

Acknowledgments

Writing a comprehensive book that connects the fundamental forces of Chemistry with practical engineering applications requires more than just academic knowledge—it requires a community of support. I would like to express my sincere gratitude to everyone who contributed to the realization of this project.

Specially, I extend my heartfelt gratitude to the **Department of Technical Education (DTE), Government of Gujarat**, and **Government Polytechnic, Palanpur**, for providing an environment that fosters innovation, academic rigor, and continuous professional development.

I am deeply thankful to my family for their unwavering patience and support during the countless hours spent researching, formatting, and refining these chapters.

Finally, my deepest appreciation goes to my students. Your curiosity, challenging questions, and continuous feedback are the true driving force behind the unique analogies and streamlined structure of this book. This textbook was engineered for you.

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About the Author

Milav Jayeshkumar Dabgar is an engineering educator and R&D professional with over 9 years of experience in electronics hardware development, embedded systems, AI/ML, and full-stack software engineering. He currently serves as a **Lecturer (Class - II) & Innovation Leader** at Government Polytechnic, Palanpur, under the Education Department of the Government of Gujarat.

Milav holds a Master of Engineering (ME) in Communication Systems from L.D. College of Engineering and a Bachelor of Engineering (BE) in Electronics & Communication. He is also currently pursuing a **BS in Data Science and Applications from IIT Madras**, where he has completed both the **Diploma in Programming** and the **Diploma in Data Science** with distinction.

Most recently, he completed the **AICTE QIP PG Certificate Program in Deep Learning from SVNIT Surat** with a **perfect 10/10 CGPA**, topping his batch.

A dedicated mentor, Milav has guided numerous student teams in securing over **INR 45 lakhs** in innovation funding, including INR 25 lakhs from Shark Tank India and INR 20 lakhs through iHub Gujarat, resulting in two student patents. He is recognized as an **NPTEL Evangelist** and **NPTEL Discipline Star** in Computer Science, reflecting his commitment to continuous learning and academic excellence.

His technical expertise spans from embedded systems (8051, STM32, Arduino) to modern web technologies (Next.js, Python, PostgreSQL) and Data Science (TensorFlow, PyTorch). Through this textbook, he aims to simplify complex Engineering Chemistry concepts by integrating relatable, real-world analogies drawn from modern tech and engineering landscapes.

Milav is also an active content creator, running a popular **YouTube channel** (@milav.dabgar) where he shares technical tutorials and academic resources. He maintains a personal blog and resource hub at milav.in and can be reached for professional collaborations on LinkedIn.

CHAPTER 1

Characteristic of Measurement and Bridges

1.1 Measurement Characteristics and Errors

Methodology:

Definitions of Measurement Characteristics While understanding instrumentation, the following computational definitions apply:

1. **Instrument:** A device or mechanism used to determine the present value of the quantity under measurement.
2. **Error (e):** The algebraic difference between the measured value (A_m) and the true value (A_t). Formula: $e = A_m - A_t$.
3. **Accuracy:** The closeness with which an instrument reading approaches the true value. Relative accuracy $A = 1 - \left|\frac{A_m - A_t}{A_t}\right|$. Percentage accuracy is $A \times 100\%$.
4. **Precision:** A measure of the consistency or repeatability of measurements. It denotes the degree of closeness between individual measurements.
5. **Reproducibility:** The degree of closeness with which a given value may be repeatedly measured under different conditions or over a long period.
6. **Repeatability:** The closeness of agreement among a number of consecutive measurements of the output for the same value of input under the same operating conditions.
7. **Sensitivity (S):** The ratio of the change in output (response) to the change in input (measured quantity). Formula: $S = \frac{\Delta\text{Output}}{\Delta\text{Input}}$.
8. **Resolution:** The smallest measurable input change that produces a detectable change in the output.
9. **Linearity:** The maximum deviation of the actual calibration curve from an idealized straight line, often expressed as a percentage of full-scale deflection (FSD).
10. **Response Time:** The time elapsed between a change in the input and the corresponding change in the output to reach a specified percentage of its final steady-state value.

Methodology:

Types of Errors and Limiting Errors Errors are broadly classified into Gross Errors, Systematic Errors, and Random Errors. For computational problem-solving, we focus on **Limiting Errors** (Guarantee Errors).
1. Absolute Limiting Error (δA): The maximum allowable deviation from the nominal value specified by the manufacturer.

$$A_m = A_s \pm \delta A$$

Where A_s is the nominal or specified value.

2. Relative Limiting Error (ϵ_r): The ratio of absolute limiting error to the specified nominal value.

$$\epsilon_r = \frac{\delta A}{A_s}$$

Percentage Limiting Error $= \%\epsilon_r = \epsilon_r \times 100$.

3. Combination of Errors:

- **Addition/Subtraction ($X = x_1 \pm x_2$):** $\delta X = \delta x_1 + \delta x_2$
- **Multiplication/Division ($X = x_1 x_2$ or $X = x_1/x_2$):** $\%\epsilon_X = \%\epsilon_{x_1} + \%\epsilon_{x_2}$
- **Powers ($X = x^n$):** $\%\epsilon_X = n \times \%\epsilon_x$

Worked Example:

Calculating Limiting Errors in a Circuit **Problem:** Three resistors have the following ratings: $R_1 = 37\Omega \pm 5\%$, $R_2 = 75\,\Omega \pm 5\%$, and $R_3 = 50\,\Omega \pm 5\%$. Determine the magnitude and percentage limiting error of the equivalent resistance when they are connected in series.

Solution:

1. **Find the nominal equivalent resistance (R_s):**

$$R_s = R_1 + R_2 + R_3 = 37 + 75 + 50 = 162\,\Omega$$

2. **Calculate the absolute limiting error for each resistor:**

$$\delta R_1 = 37 \times 0.05 = 1.85\,\Omega$$

$$\delta R_2 = 75 \times 0.05 = 3.75\,\Omega$$

$$\delta R_3 = 50 \times 0.05 = 2.50\,\Omega$$

3. **Calculate the total absolute limiting error (δR_s):** For series connection (addition), absolute errors add up:

$$\delta R_s = \delta R_1 + \delta R_2 + \delta R_3 = 1.85 + 3.75 + 2.50 = 8.10\,\Omega$$

4. **Calculate the percentage limiting error of the equivalent resistance:**

$$\%\epsilon_{R_s} = \left(\frac{\delta R_s}{R_s}\right) \times 100 = \left(\frac{8.10}{162}\right) \times 100 = 5\%$$

The equivalent resistance is $162\,\Omega \pm 5\%$.

1.2 DC and AC Bridges

Methodology:

Wheatstone Bridge for Unknown Resistance The Wheatstone bridge is a DC bridge used for measuring medium resistances ($1\,\Omega$ to $100\,\text{k}\Omega$). It consists of four resistive arms: R_1 and R_2 (ratio arms), R_3 (standard variable arm), and R_x (unknown resistance).

Balance Condition Algorithm:

1. A bridge is balanced when no current flows through the galvanometer ($I_g = 0$).
2. The balance equation is given by the cross-product of the opposite arms:

$$R_1 R_x = R_2 R_3$$

3. Solve for the unknown resistance R_x:

$$R_x = \frac{R_2}{R_1} R_3$$

Worked Example:

Wheatstone Bridge Calculation **Problem:** In a Wheatstone bridge, the ratio arms R_1 and R_2 are $1000\,\Omega$ and $100\,\Omega$ respectively. The standard arm R_3 is adjusted to $452\,\Omega$ to achieve balance. Calculate the value of the unknown resistance R_x.

Solution:

1. **Identify the given parameters:** $R_1 = 1000\,\Omega$, $R_2 = 100\,\Omega$, $R_3 = 452\,\Omega$.
2. **Apply the Wheatstone bridge balance equation:**

$$R_x = \frac{R_2}{R_1} R_3$$

3. **Substitute the values:**

$$R_x = \frac{100}{1000} \times 452$$
$$R_x = 0.1 \times 452 = 45.2\,\Omega$$

The unknown resistance is $45.2\,\Omega$.

Methodology:

Maxwells Inductance Capacitance Bridge Maxwell's bridge measures unknown inductance in terms of a known standard capacitance. It is suitable for measuring medium Q coils ($1 < Q < 10$).

Bridge Configuration:

- Arm 1: Standard capacitor C_1 in parallel with variable resistor R_1.
- Arm 2: Pure resistor R_2.
- Arm 3: Pure resistor R_3.
- Arm 4: Unknown inductor L_x in series with its internal resistance R_x.

Balance Condition Algorithm:

1. Apply the general AC bridge balance equation: $Z_1 Z_4 = Z_2 Z_3$.
2. Equate the real parts to find the unknown resistance:

$$R_x = \frac{R_2 R_3}{R_1}$$

3. Equate the imaginary parts to find the unknown inductance:

$$L_x = R_2 R_3 C_1$$

4. Calculate the Quality Factor (Q) of the coil:

$$Q = \frac{\omega L_x}{R_x} = \omega C_1 R_1$$

Worked Example:

Maxwells Bridge Parameter Calculation **Problem:** A Maxwell's bridge is used to measure an unknown inductive coil. At balance, the bridge constants are: $C_1 = 0.01\,\mu\text{F}$, $R_1 = 470\,\text{k}\Omega$, $R_2 = 5.1\,\text{k}\Omega$, and $R_3 = 100\,\text{k}\Omega$. The supply frequency is $1\,\text{kHz}$. Determine the resistance and inductance of the coil, and its Q-factor.
Solution:

1. **Calculate the unknown resistance (R_x):**

$$R_x = \frac{R_2 R_3}{R_1} = \frac{5.1 \times 10^3 \times 100 \times 10^3}{470 \times 10^3} = 1085.1\,\Omega \approx 1.085\,\text{k}\Omega$$

2. **Calculate the unknown inductance (L_x):**

$$L_x = R_2 R_3 C_1 = (5.1 \times 10^3) \times (100 \times 10^3) \times (0.01 \times 10^{-6}) = 5.1\,\text{H}$$

3. **Calculate the angular frequency (ω):**

$$\omega = 2\pi f = 2\pi(1000) = 6283\,\text{rad/s}$$

4. **Calculate the Quality Factor (Q):**

$$Q = \omega C_1 R_1 = 6283 \times (0.01 \times 10^{-6}) \times (470 \times 10^3) \approx 29.53$$

Methodology:

Schering Bridge for Capacitance Measurement The Schering bridge is widely used for measuring unknown capacitance and dielectric loss of insulating materials.
Bridge Configuration:

- Arm 1: Unknown capacitor C_x in series with equivalent resistance r_x.
- Arm 2: Standard loss-free capacitor C_2.
- Arm 3: Non-inductive resistor R_3.
- Arm 4: Non-inductive resistor R_4 in parallel with capacitor C_4.

Balance Condition Algorithm:

1. Equate real and imaginary components of the balance equation $Z_1 Z_4 = Z_2 Z_3$.
2. Calculate Unknown Capacitance:
$$C_x = C_2 \frac{R_4}{R_3}$$
3. Calculate Equivalent Series Resistance:
$$r_x = R_3 \frac{C_4}{C_2}$$
4. Calculate Dissipation Factor (Tangent of loss angle δ):

$$D = \tan\delta = \omega C_x r_x = \omega C_4 R_4$$

Worked Example:

Schering Bridge Analysis **Problem:** In a Schering bridge test at 50 Hz, balance is obtained with the following parameters: standard loss-free capacitor $C_2 = 100\,\text{pF}$, resistor $R_3 = 318\,\Omega$, resistor $R_4 = 1000\,\Omega$,

and parallel capacitor $C_4 = 0.01\,\mu\text{F}$. Calculate the unknown capacitance C_x, its series resistance r_x, and the dissipation factor D.
Solution:

1. **Calculate Unknown Capacitance (C_x):**
$$C_x = C_2 \frac{R_4}{R_3} = 100 \times 10^{-12} \times \frac{1000}{318} = 314.4\,\text{pF}$$
2. **Calculate Equivalent Series Resistance (r_x):**
$$r_x = R_3 \frac{C_4}{C_2} = 318 \times \frac{0.01 \times 10^{-6}}{100 \times 10^{-12}} = 318 \times 10^5 = 31.8\,\text{M}\Omega$$
3. **Calculate Dissipation Factor (D):**
$$\omega = 2\pi f = 2\pi(50) = 314.16\,\text{rad/s}$$
$$D = \omega C_4 R_4 = 314.16 \times (0.01 \times 10^{-6}) \times 1000 = 0.00314$$

1.3 Q Meter

Methodology:

Principle of Q Meter The Q meter is an instrument designed to measure the quality factor (Q) of electrical components, particularly coils and capacitors. Its operation is based on the characteristics of a series resonant circuit.
Core Principle and Formulas:

1. In a series RLC circuit at resonance (f_r), the inductive reactance equals the capacitive reactance ($X_L = X_C$).
2. At resonance, the voltage across the inductor (V_L) or capacitor (V_C) is Q times the applied voltage (V).
$$Q = \frac{V_L}{V} = \frac{V_C}{V}$$
3. Quality factor in terms of circuit parameters:
$$Q = \frac{\omega L}{R} = \frac{1}{\omega C R}$$
Where $\omega = 2\pi f_r$, L is inductance, C is capacitance, and R is series resistance.
4. The Q meter directly measures this voltage magnification. An oscillator provides a known injection voltage V, and an electronic voltmeter across the tuning capacitor measures V_C, which is calibrated directly in terms of Q.

Methodology:

Practical Q Meter and Distributed Capacitance A practical coil is not purely inductive; it has a self-capacitance or distributed capacitance (C_d) between its turns. This capacitance affects the effective Q and true inductance of the coil.
Algorithm to Find Distributed Capacitance (C_d):

1. Connect the coil to the Q meter. Tune the oscillator to a frequency f_1 and adjust the tuning capacitor to C_1 to obtain resonance.

2. Change the oscillator frequency to a second frequency f_2 such that $f_2 = nf_1$ (commonly $n = 2$, so $f_2 = 2f_1$).

3. Tune the capacitor again to a new value C_2 to achieve resonance at f_2.

4. Since the true inductance remains constant, use the resonant frequency formula to find C_d:

$$C_d = \frac{C_1 - n^2 C_2}{n^2 - 1}$$

If $n = 2$, the formula simplifies to:

$$C_d = \frac{C_1 - 4C_2}{3}$$

Worked Example:

Calculating Distributed Capacitance Using a Q Meter **Problem:** A coil is tested using a Q meter. At a frequency of 2 MHz, the resonance is obtained with a tuning capacitor set to 460 pF. When the frequency is increased to 4 MHz, resonance is restored by adjusting the tuning capacitor to 100 pF. Determine the distributed capacitance of the coil and the true inductance.
Solution:

1. **Identify the parameters:** $f_1 = 2\,\text{MHz}$, $C_1 = 460\,\text{pF}$ $f_2 = 4\,\text{MHz}$, $C_2 = 100\,\text{pF}$

2. **Determine the frequency ratio (n):**

$$n = \frac{f_2}{f_1} = \frac{4\,\text{MHz}}{2\,\text{MHz}} = 2$$

3. **Calculate the Distributed Capacitance (C_d):**

$$C_d = \frac{C_1 - n^2 C_2}{n^2 - 1} = \frac{460 - (2^2 \times 100)}{2^2 - 1}$$

$$C_d = \frac{460 - 400}{3} = \frac{60}{3} = 20\,\text{pF}$$

4. **Calculate the True Inductance (L):** Use the resonance condition at f_1:

$$f_1 = \frac{1}{2\pi\sqrt{L(C_1 + C_d)}} \implies L = \frac{1}{4\pi^2 f_1^2 (C_1 + C_d)}$$

Substitute $f_1 = 2 \times 10^6$ Hz and $C_1 + C_d = 480 \times 10^{-12}$ F:

$$L = \frac{1}{4\pi^2 (2 \times 10^6)^2 (480 \times 10^{-12})}$$

$$L = \frac{1}{4\pi^2 (4 \times 10^{12})(480 \times 10^{-12})} = \frac{1}{4\pi^2 \times 1920} \approx 13.19\,\mu\text{H}$$

The distributed capacitance is 20 pF and true inductance is 13.19 μH.

CHAPTER 2

Basic Electrical Parameter Measurement

This chapter focuses on the computational aspects of measuring basic electrical parameters. We explore range extension of meters, design of multi-range instruments, digital voltmeter algorithms, and error calculations in power and energy meters.

2.1 Moving Coil and Moving Iron Type Instruments

Permanent Magnet Moving Coil (PMMC) and Moving Iron (MI) instruments are fundamental analog measuring devices. Their basic measurement range can be extended using computational techniques for parallel (shunt) and series (multiplier) resistances.

Methodology:

Ammeter Range Extension To extend the range of an ammeter to measure higher currents, a low-value resistance called a shunt is connected in parallel with the meter movement.
Formulas:

1. Let I_m be the full-scale deflection current of the meter.
2. Let R_m be the internal resistance of the meter.
3. Let I be the total current to be measured.
4. The multiplying factor m is defined as $m = \frac{I}{I_m}$.
5. The required shunt resistance R_{sh} is calculated using:

$$R_{sh} = \frac{I_m R_m}{I - I_m} = \frac{R_m}{m-1}$$

Worked Example:

Calculate Shunt Resistance for Ammeter A PMMC meter has an internal resistance of 100 Ω and a full-scale deflection current of 1 mA. Calculate the required shunt resistance to extend its range to measure 1 A.
Solution:

1. Identify the given parameters: $R_m = 100\ \Omega$, $I_m = 1\text{ mA} = 0.001\text{ A}$, $I = 1\text{ A}$.
2. Calculate the multiplying factor m:

$$m = \frac{I}{I_m} = \frac{1}{0.001} = 1000$$

3. Apply the shunt resistance formula:

$$R_{sh} = \frac{R_m}{m-1} = \frac{100}{1000-1} = \frac{100}{999} \approx 0.1001\ \Omega$$

A shunt resistance of 0.1001 Ω must be connected in parallel.

Methodology:

Voltmeter Range Extension To extend the range of a voltmeter to measure higher voltages, a high-value resistance called a multiplier is connected in series with the meter movement.
Formulas:

1. Let V_m be the voltage across the meter for full-scale deflection ($V_m = I_m R_m$).
2. Let V be the total voltage to be measured.
3. The multiplying factor m is defined as $m = \frac{V}{V_m}$.
4. The required series multiplier resistance R_s is calculated using:

$$R_s = \frac{V}{I_m} - R_m = R_m(m-1)$$

Worked Example:

Calculate Multiplier Resistance for Voltmeter A basic meter movement with an internal resistance of 50 Ω and a full-scale deflection current of 2 mA is to be used as a voltmeter with a range of 0 to 10 V. Calculate the multiplier resistance.
Solution:

1. Identify the given parameters: $R_m = 50\ \Omega$, $I_m = 2$ mA $= 0.002$ A, $V = 10$ V.
2. Calculate the full-scale voltage of the meter V_m:

$$V_m = I_m R_m = 0.002 \times 50 = 0.1 \text{ V}$$

3. Calculate the multiplying factor m:

$$m = \frac{V}{V_m} = \frac{10}{0.1} = 100$$

4. Apply the series resistance formula:

$$R_s = R_m(m-1) = 50(100-1) = 50 \times 99 = 4950\ \Omega$$

A multiplier resistance of 4950 Ω must be connected in series.

2.2 DC and AC Voltmeter and Current Meter

While DC measurements rely directly on the meter movement, AC measurements typically require a rectifier circuit to convert AC to DC before measurement.

Methodology:

AC Voltmeter using Half Wave Rectifier For a half-wave rectifier coupled with a DC meter, the average (DC) voltage and the resulting AC sensitivity differ from the DC sensitivity.
Formulas:

1. DC Sensitivity $S_{dc} = \frac{1}{I_{fsd}}$ (Ω/V).
2. The DC value of a half-wave rectified sinusoidal signal is $V_{dc} = \frac{V_{peak}}{\pi} \approx 0.45 V_{rms}$.
3. AC Sensitivity for half-wave rectifier $S_{ac} = 0.45 \times S_{dc}$.
4. Total circuit resistance required $R_{total} = S_{ac} \times V_{rms}$.

5. Multiplier resistance $R_s = R_{total} - R_m - R_d$, where R_d is the forward resistance of the diode.

Worked Example:

Design of Half Wave AC Voltmeter A meter movement has $I_{fsd} = 1$ mA and $R_m = 100\ \Omega$. It is used in a half-wave rectifier AC voltmeter for a range of 10 V rms. Determine the multiplier resistance. Assume ideal diode ($R_d = 0\ \Omega$).

Solution:

1. Calculate DC sensitivity:
$$S_{dc} = \frac{1}{I_{fsd}} = \frac{1}{0.001} = 1000\ \Omega/\text{V}$$

2. Calculate AC sensitivity:
$$S_{ac} = 0.45 \times S_{dc} = 0.45 \times 1000 = 450\ \Omega/\text{V}$$

3. Calculate total required resistance for 10 V rms range:
$$R_{total} = S_{ac} \times V = 450 \times 10 = 4500\ \Omega$$

4. Calculate multiplier resistance R_s:
$$R_s = R_{total} - R_m = 4500 - 100 = 4400\ \Omega$$

Methodology:

AC Voltmeter using Full Wave Rectifier A full-wave rectifier provides a higher average voltage than a half-wave rectifier, leading to better sensitivity.

Formulas:

1. The DC value of a full-wave rectified sinusoidal signal is $V_{dc} = \frac{2V_{peak}}{\pi} \approx 0.9V_{rms}$.

2. AC Sensitivity for full-wave rectifier $S_{ac} = 0.9 \times S_{dc}$.

3. Multiplier resistance $R_s = (S_{ac} \times V_{rms}) - R_m - 2R_d$ (assuming a bridge rectifier where two diodes are in conduction).

Worked Example:

Design of Full Wave AC Voltmeter Using the same meter movement ($I_{fsd} = 1$ mA, $R_m = 100\ \Omega$), design a full-wave AC voltmeter for a 10 V rms range. Assume ideal diodes.

Solution:

1. AC sensitivity for full-wave:
$$S_{ac} = 0.9 \times S_{dc} = 0.9 \times 1000 = 900\ \Omega/\text{V}$$

2. Total required resistance:
$$R_{total} = S_{ac} \times V_{rms} = 900 \times 10 = 9000\ \Omega$$

3. Multiplier resistance R_s:
$$R_s = R_{total} - R_m = 9000 - 100 = 8900\ \Omega$$

Notice the series resistance is much larger, reflecting the higher efficiency of the full-wave configuration.

2.3 Electronic Multimeter

An electronic multimeter combines multi-range voltmeters and ammeters into a single instrument. The Ayrton shunt (universal shunt) is widely used to prevent meter damage during range switching in ammeters.

Methodology:

Ayrton Universal Shunt Design The Ayrton shunt uses a tapped series combination of resistors (R_1, R_2, R_3) in parallel with the meter to provide multiple current ranges without completely opening the circuit during switching.

Formulas for a 3-range ammeter: Let the total shunt resistance be $R_{sh} = R_1 + R_2 + R_3$.

1. For the lowest current range I_1 (highest sensitivity):

$$I_m R_m = (I_1 - I_m)(R_1 + R_2 + R_3) \implies R_1 + R_2 + R_3 = \frac{I_m R_m}{I_1 - I_m}$$

2. For the intermediate current range I_2:

$$I_m(R_m + R_3) = (I_2 - I_m)(R_1 + R_2)$$

3. For the highest current range I_3:

$$I_m(R_m + R_2 + R_3) = (I_3 - I_m)R_1$$

By solving these simultaneous equations, individual resistor values R_1, R_2, and R_3 can be determined.

Worked Example:

Calculate Resistors for Ayrton Shunt Design an Ayrton shunt to provide an ammeter with ranges of 1 A, 5 A, and 10 A. The basic meter has an internal resistance $R_m = 50\ \Omega$ and $I_{fsd} = 1$ mA.

Solution: Let $I_m = 0.001$ A.

1. **For Range 1 ($I_1 = 1$ A):**

$$R_{sh} = R_1 + R_2 + R_3 = \frac{I_m R_m}{I_1 - I_m} = \frac{0.001 \times 50}{1 - 0.001} = \frac{0.05}{0.999} \approx 0.05005\ \Omega$$

2. **For Range 2 ($I_2 = 5$ A):** The meter branch has resistance $R_m + R_3$. The shunt branch has $R_1 + R_2$. The voltage drop across parallel branches is equal:

$$(I_2 - I_m)(R_1 + R_2) = I_m(R_m + R_3)$$

Substitute $R_3 = R_{sh} - (R_1 + R_2) = 0.05005 - (R_1 + R_2)$:

$$(5 - 0.001)(R_1 + R_2) = 0.001(50 + 0.05005 - (R_1 + R_2))$$

$$4.999(R_1 + R_2) = 0.05005 - 0.001(R_1 + R_2)$$

$$5(R_1 + R_2) = 0.05005 \implies R_1 + R_2 = 0.01001\ \Omega$$

Therefore, $R_3 = R_{sh} - (R_1 + R_2) = 0.05005 - 0.01001 = 0.04004\ \Omega$.

3. **For Range 3 ($I_3 = 10$ A):**

$$(I_3 - I_m)R_1 = I_m(R_m + R_2 + R_3)$$

Substitute $R_2 + R_3 = R_{sh} - R_1 = 0.05005 - R_1$:

$$(10 - 0.001)R_1 = 0.001(50 + 0.05005 - R_1)$$

$$9.999R_1 = 0.05005 - 0.001R_1 \implies 10R_1 = 0.05005 \implies R_1 = 0.005005\ \Omega$$

Therefore, $R_2 = (R_1 + R_2) - R_1 = 0.01001 - 0.005005 = 0.005005\ \Omega$.

Final values: $R_1 = 0.005005\ \Omega$, $R_2 = 0.005005\ \Omega$, $R_3 = 0.04004\ \Omega$.

2.4 Digital Voltmeter DVM Types

Digital Voltmeters (DVMs) translate analog voltage into digital readouts. Key ADC architectures used in DVMs include Integrating (Dual Slope) and Successive Approximation.

Methodology:

Dual Slope Integrating DVM The dual slope ADC integrates the unknown input voltage V_{in} for a fixed time T_1, then integrates a known negative reference voltage $-V_{ref}$ until the output returns to zero over time T_2.
Formulas:

1. Charge balance equation: $V_{in} \times T_1 = V_{ref} \times T_2$.
2. The fixed integration time T_1 corresponds to a fixed number of clock pulses N_1, so $T_1 = N_1 \times T_{clk}$.
3. The de-integration time T_2 corresponds to N_2 clock pulses, so $T_2 = N_2 \times T_{clk}$.
4. Solving for the input voltage:
$$V_{in} = V_{ref} \times \frac{T_2}{T_1} = V_{ref} \times \frac{N_2}{N_1}$$

Worked Example:

Calculate Input Voltage of Dual Slope DVM A dual slope integrating type DVM has an integrating capacitor of 0.1 μF and a resistance of 100 kΩ. The reference voltage is 2 V, and the fixed integration time is 10 ms. If the de-integration time measured by the counter is 6.5 ms, determine the input voltage.
Solution:

1. Identify parameters: $V_{ref} = 2$ V, $T_1 = 10$ ms, $T_2 = 6.5$ ms.
2. Note: The RC time constant dictates the slope of the integrator, but because it is the same for both integration and de-integration, the R and C values cancel out in the final formula.
3. Apply the voltage formula:
$$V_{in} = V_{ref} \times \frac{T_2}{T_1}$$
$$V_{in} = 2 \times \frac{6.5}{10} = 2 \times 0.65 = 1.3 \text{ V}$$

The unknown input voltage is 1.3 V.

Methodology:

Successive Approximation DVM A Successive Approximation Register (SAR) ADC determines the digital value bit by bit, from the Most Significant Bit (MSB) to the Least Significant Bit (LSB).
Computational Attributes:

1. **Resolution:** For an n-bit ADC with full-scale voltage V_{FS}, Resolution $= \frac{V_{FS}}{2^n}$.
2. **Conversion Time:** An n-bit SAR ADC requires n clock cycles to complete one conversion.
$$T_{conv} = n \times T_{clk} = \frac{n}{f_{clk}}$$
3. This conversion time is constant and independent of the magnitude of the input voltage.

Worked Example:

Determine Conversion Time and Resolution of SAR DVM An 8-bit successive approximation DVM operates with a clock frequency of 1 MHz and has a full-scale voltage range of 5 V. Calculate the resolution and the conversion time.

Solution:

1. Identify parameters: $n = 8$, $f_{clk} = 1$ MHz $= 10^6$ Hz, $V_{FS} = 5$ V.
2. Calculate Resolution:
$$\text{Resolution} = \frac{V_{FS}}{2^n} = \frac{5}{2^8} = \frac{5}{256} \approx 0.01953 \text{ V} = 19.53 \text{ mV}$$
3. Calculate Conversion Time:
$$T_{conv} = \frac{n}{f_{clk}} = \frac{8}{10^6} = 8\ \mu\text{s}$$

2.5 Clamp on Meter

A clamp-on meter (or clamp meter) allows for the measurement of high alternating currents without breaking the circuit. It acts as a current transformer.

Methodology:

Clamp on Ammeter Current Calculation The jaw of the clamp acts as the magnetic core. The current-carrying conductor acts as a single-turn primary winding, and the coil inside the meter acts as the secondary winding.

Formulas:

1. Let I_p be the primary current (current in the conductor to be measured).
2. Let I_s be the secondary current (current measured by the internal meter).
3. Let N_p be the primary turns (usually $N_p = 1$).
4. Let N_s be the secondary turns inside the clamp meter.
5. Transformer Ratio Equation: $I_p \times N_p = I_s \times N_s$.
6. Rearranging for unknown primary current:
$$I_p = \frac{I_s \times N_s}{N_p} = I_s \times N_s \quad (\text{if } N_p = 1)$$

Worked Example:

Calculate Primary Current in Clamp on Meter A clamp-on ammeter has a secondary coil with 500 turns. When clamped around a single conductor carrying an unknown AC current, the internal ammeter reads 50 mA. Calculate the actual current flowing in the conductor.

Solution:

1. Identify parameters: $N_p = 1$ (single conductor), $N_s = 500$, $I_s = 50$ mA $= 0.05$ A.
2. Apply the transformer ratio equation:
$$I_p \times 1 = I_s \times N_s$$
$$I_p = 0.05 \times 500 = 25 \text{ A}$$

The current flowing in the conductor is 25 A.

2.6 Electronic Watt Meter and Energy Meter

These meters measure active power and energy consumed over time. Computational problems typically involve calculating the percentage error in the measurement.

Methodology:

Electronic Wattmeter Power Calculation A wattmeter measures the real power in a circuit.
Formulas:

1. Real Power $P = V \times I \times \cos(\phi)$ (in Watts).
2. Percentage Error $= \left(\frac{\text{Measured Power} - \text{True Power}}{\text{True Power}}\right) \times 100\%$.

Worked Example:

Calculate Wattmeter Percentage Error A load draws 10 A from a 230 V supply at a power factor of 0.8 lagging. An electronic wattmeter connected to the circuit reads 1800 W. Calculate the percentage error of the wattmeter.
Solution:

1. Calculate True Power (P_{true}):

$$P_{true} = V \times I \times \cos(\phi) = 230 \times 10 \times 0.8 = 1840 \text{ W}$$

2. Identify Measured Power (P_{meas}): 1800 W.
3. Calculate Percentage Error:

$$\% \text{ Error} = \frac{P_{meas} - P_{true}}{P_{true}} \times 100\%$$

$$\% \text{ Error} = \frac{1800 - 1840}{1840} \times 100\% = \frac{-40}{1840} \times 100\% \approx -2.17\%$$

The wattmeter reads 2.17% lower than the actual power.

Methodology:

Energy Meter Constant and Error Analysis An electronic energy meter measures electrical energy in kilowatt-hours (kWh). The meter constant K is typically defined as impulses (or revolutions) per kWh.
Formulas:

1. True Energy Consumed E (in kWh):

$$E = P(\text{kW}) \times t(\text{hours}) = \frac{V \times I \times \cos(\phi)}{1000} \times \frac{t(\text{seconds})}{3600}$$

2. Expected number of impulses $N_{expected}$:

$$N_{expected} = K \times E$$

3. Percentage Error based on impulses:

$$\% \text{ Error} = \left(\frac{N_{actual} - N_{expected}}{N_{expected}}\right) \times 100\%$$

Worked Example:

Calculate Percentage Error of Energy Meter An electronic energy meter is rated at 230 V, 5 A and has a meter constant of 3200 impulses/kWh. In a test, a steady load of 5 A at unity power factor is connected for 10 minutes. The LED on the meter flashes 600 times. Calculate the percentage error.

Solution:

1. Calculate True Energy E consumed in 10 minutes:

$$P = V \times I \times \cos(\phi) = 230 \times 5 \times 1 = 1150 \text{ W} = 1.15 \text{ kW}$$

$$t = 10 \text{ minutes} = \frac{10}{60} \text{ hours} \approx 0.1667 \text{ hours}$$

$$E = P \times t = 1.15 \times \frac{10}{60} = \frac{11.5}{60} \approx 0.19167 \text{ kWh}$$

2. Calculate the expected number of impulses ($N_{expected}$):

$$N_{expected} = K \times E = 3200 \times \frac{11.5}{60} = \frac{36800}{60} \approx 613.33 \text{ impulses}$$

3. Calculate the Percentage Error given $N_{actual} = 600$:

$$\% \text{ Error} = \frac{N_{actual} - N_{expected}}{N_{expected}} \times 100\%$$

$$\% \text{ Error} = \frac{600 - 613.33}{613.33} \times 100\% \approx \frac{-13.33}{613.33} \times 100\% \approx -2.17\%$$

The energy meter runs slow with an error of -2.17%.

CHAPTER 3

Oscilloscopes

3.1 Measurement of Voltage and Frequency

The Cathode Ray Oscilloscope (CRO) is widely used to measure the amplitude, time period, and frequency of unknown signals.

Methodology:

Voltage Time Period and Frequency Calculations Follow these steps to calculate the electrical parameters from a CRO display:

1. **Peak-to-Peak Voltage (V_{p-p}):** Count the total number of vertical divisions occupied by the signal from its lowest peak to its highest peak. Multiply this by the vertical deflection factor (VOLTS/DIV).

$$V_{p-p} = (\text{Vertical Divisions}) \times (\text{VOLTS/DIV setting})$$

2. **Peak Voltage (V_m) and RMS Voltage (V_{rms}):**

$$V_m = \frac{V_{p-p}}{2}$$

$$V_{rms} = \frac{V_m}{\sqrt{2}} = \frac{V_{p-p}}{2\sqrt{2}}$$

3. **Time Period (T):** Count the total number of horizontal divisions occupied by one complete cycle of the waveform. Multiply this by the time base setting (TIME/DIV).

$$T = (\text{Horizontal Divisions for one cycle}) \times (\text{TIME/DIV setting})$$

4. **Frequency (f):** Calculate the reciprocal of the time period.

$$f = \frac{1}{T}$$

Worked Example:

Calculate Voltage and Frequency from CRO Screen A sinusoidal waveform is displayed on a CRO screen. It occupies 4.6 vertical divisions from peak to peak and 4 horizontal divisions for one complete cycle. The VOLTS/DIV knob is set to 2 V/div and the TIME/DIV knob is set to 5 ms/div. Calculate the peak-to-peak voltage, RMS voltage, time period, and frequency of the signal.

Solution:

1. **Given Data:**

   - Vertical divisions = 4.6 div
   - VOLTS/DIV = 2 V/div

- Horizontal divisions for one cycle = 4 div
- TIME/DIV = 5 ms/div = 5×10^{-3} s/div

2. **Calculate Peak-to-Peak Voltage (V_{p-p}):**

$$V_{p-p} = 4.6 \times 2 = 9.2 \text{ V}$$

3. **Calculate RMS Voltage (V_{rms}):**

$$V_{rms} = \frac{V_{p-p}}{2\sqrt{2}} = \frac{9.2}{2.828} = 3.25 \text{ V}$$

4. **Calculate Time Period (T):**

$$T = 4 \times 5 \times 10^{-3} = 20 \times 10^{-3} \text{ s} = 20 \text{ ms}$$

5. **Calculate Frequency (f):**

$$f = \frac{1}{T} = \frac{1}{20 \times 10^{-3}} = 50 \text{ Hz}$$

The peak-to-peak voltage is 9.2 V, RMS voltage is 3.25 V, time period is 20 ms, and frequency is 50 Hz.

Worked Example:

Determine CRO Settings for a Given Signal You need to display a 1 kHz sinusoidal signal with an RMS voltage of 5 V on a CRO screen. The screen has 8 vertical divisions and 10 horizontal divisions. Determine suitable VOLTS/DIV and TIME/DIV settings to display at least one full cycle and keep the waveform within the vertical limits.

Solution:

1. **Given Data:**
   - Frequency $f = 1000$ Hz
   - RMS Voltage $V_{rms} = 5$ V

2. **Calculate Time Period (T):**

$$T = \frac{1}{f} = \frac{1}{1000} = 1 \text{ ms}$$

   To display one full cycle across, say, 5 horizontal divisions (out of 10 available):

$$\text{TIME/DIV} = \frac{T}{5} = \frac{1 \text{ ms}}{5} = 0.2 \text{ ms/div}$$

   So, a TIME/DIV setting of 0.2 ms/div is suitable.

3. **Calculate Peak-to-Peak Voltage (V_{p-p}):**

$$V_m = V_{rms} \times \sqrt{2} = 5 \times 1.414 = 7.07 \text{ V}$$

$$V_{p-p} = 2 \times V_m = 2 \times 7.07 = 14.14 \text{ V}$$

   To fit 14.14 V within the 8 vertical divisions, the maximum divisions we can use is, say, 6 or 7.

$$\text{Required VOLTS/DIV} = \frac{14.14 \text{ V}}{6 \text{ divisions}} \approx 2.35 \text{ V/div}$$

   The nearest standard setting larger than 2.35 is usually 5 V/div. If VOLTS/DIV is 5 V/div, vertical divisions used = $14.14/5 \approx 2.8$ divisions, which fits comfortably on the screen.

Suitable settings are TIME/DIV = 0.2 ms/div and VOLTS/DIV = 5 V/div.

3.2 Measurement of Phase Angle using Lissajous Patterns

When two sinusoidal signals of the same frequency are applied to the horizontal and vertical deflection plates of a CRO in X-Y mode, a stable pattern called a Lissajous pattern is formed. The shape of the pattern depends on the phase difference between the two signals.

Methodology:

Phase Angle Calculation Follow these steps to determine the phase angle (θ) between two signals of the same frequency:

1. Measure the vertical distance from the center to the y-intercept of the ellipse. Let this be Y_1.
2. Measure the vertical distance from the center to the maximum vertical deflection of the ellipse. Let this be Y_2.
3. Calculate the sine of the phase angle:
$$\sin\theta = \frac{Y_1}{Y_2}$$
4. Calculate the phase angle θ:
$$\theta = \sin^{-1}\left(\frac{Y_1}{Y_2}\right)$$
5. **Quadrant Adjustments:**
   - If the major axis of the ellipse lies in the 1st and 3rd quadrants, the phase angle is θ or $360° - \theta$.
   - If the major axis of the ellipse lies in the 2nd and 4th quadrants, the phase angle is $180° - \theta$ or $180° + \theta$.
6. Alternatively, the horizontal measurements X_1 (x-intercept) and X_2 (maximum horizontal deflection) can be used: $\sin\theta = \frac{X_1}{X_2}$.

Worked Example:

Phase Angle from Elliptical Pattern In a phase measurement using Lissajous patterns, an ellipse is formed on the CRO screen with its major axis in the first and third quadrants. The maximum vertical deflection is 3.5 divisions, and the y-intercept is 1.8 divisions. Calculate the possible phase angles between the two applied voltages.

Solution:

1. **Given Data:**
   - Maximum vertical deflection $(Y_2) = 3.5$ div
   - y-intercept $(Y_1) = 1.8$ div
   - Ellipse orientation = 1st and 3rd quadrants
2. **Calculate Sine of Phase Angle:**
$$\sin\theta = \frac{Y_1}{Y_2} = \frac{1.8}{3.5} = 0.5143$$
3. **Calculate Primary Angle θ:**
$$\theta = \sin^{-1}(0.5143) \approx 30.95°$$
4. **Determine Possible Phase Angles:** Since the major axis is in the 1st and 3rd quadrants, the phase angle can be:
$$\phi_1 = \theta = 30.95°$$
$$\phi_2 = 360° - \theta = 360° - 30.95° = 329.05°$$

The possible phase angles are 30.95° and 329.05°.

Worked Example:

Phase Angle in Second and Fourth Quadrants A Lissajous pattern produces an ellipse in the 2nd and 4th quadrants. The distance between the two y-intercepts is 4 divisions, and the total vertical height of the ellipse is 6 divisions. Calculate the phase angle.

Solution:

1. **Given Data:**
   - Distance between y-intercepts $(2Y_1) = 4$ div $\Rightarrow Y_1 = 2$ div
   - Total vertical height $(2Y_2) = 6$ div $\Rightarrow Y_2 = 3$ div
   - Ellipse orientation = 2nd and 4th quadrants
2. **Calculate Sine of Phase Angle:**
$$\sin\theta = \frac{Y_1}{Y_2} = \frac{2}{3} = 0.6667$$
3. **Calculate Primary Angle θ:**
$$\theta = \sin^{-1}(0.6667) \approx 41.81^\circ$$
4. **Determine Possible Phase Angles:** Since the ellipse is in the 2nd and 4th quadrants, the phase angle is given by:
$$\phi_1 = 180^\circ - \theta = 180^\circ - 41.81^\circ = 138.19^\circ$$
$$\phi_2 = 180^\circ + \theta = 180^\circ + 41.81^\circ = 221.81^\circ$$

The phase angle is either 138.19° or 221.81°.

3.3 Measurement of Frequency using Lissajous Patterns

When sinusoidal signals of different frequencies are applied to the vertical and horizontal deflection plates, a complex Lissajous pattern with multiple loops is generated. The ratio of the frequencies can be determined by counting the number of tangencies (touch points) on the horizontal and vertical axes.

Methodology:

Frequency Determination from Tangencies To find an unknown frequency using a known frequency signal and a Lissajous pattern:

1. Enclose the Lissajous pattern in a bounding rectangle.
2. Count the number of times the pattern touches the horizontal boundary line (top or bottom edge). Let this be n_h (number of horizontal tangencies).
3. Count the number of times the pattern touches the vertical boundary line (left or right edge). Let this be n_v (number of vertical tangencies).
4. Apply the frequency ratio formula:
$$\frac{f_v}{f_h} = \frac{n_h}{n_v}$$
   where:
   - f_v = Frequency of the vertical signal (often the unknown frequency)
   - f_h = Frequency of the horizontal signal (usually the known reference frequency)
5. Solve for the unknown frequency.
6. **Open Loops Rule:** If the pattern has an "open end" or "free end" touching a boundary, it is counted as half a tangency (0.5).

Worked Example:

Calculate Frequency using Tangencies A standard 1 kHz signal is applied to the horizontal plates of a CRO, and an unknown signal is applied to the vertical plates. The resulting Lissajous pattern has 3 tangencies on the top horizontal edge and 2 tangencies on the right vertical edge. Calculate the frequency of the unknown signal.

Solution:

1. **Given Data:**
   - Frequency of horizontal signal $(f_h) = 1000$ Hz
   - Number of horizontal tangencies $(n_h) = 3$
   - Number of vertical tangencies $(n_v) = 2$
2. **Apply the Frequency Ratio Formula:**

$$\frac{f_v}{f_h} = \frac{n_h}{n_v}$$

3. **Calculate the Unknown Vertical Frequency (f_v):**

$$f_v = f_h \times \left(\frac{n_h}{n_v}\right)$$

$$f_v = 1000 \times \left(\frac{3}{2}\right) = 1500 \text{ Hz} = 1.5 \text{ kHz}$$

The frequency of the unknown signal is 1.5 kHz.

Worked Example:

Lissajous Pattern with Open Ends An unknown frequency f_v is connected to the vertical channel and a known frequency $f_h = 50$ Hz is connected to the horizontal channel. The resulting pattern has 5 closed loops touching the top edge and 1 closed loop plus 1 open end touching the right edge. Determine f_v.

Solution:

1. **Given Data:**
   - Horizontal frequency $(f_h) = 50$ Hz
   - Horizontal tangencies $(n_h) = 5$ (from 5 closed loops touching the top)
   - Vertical tangencies $(n_v) = 1$ (closed loop) + 0.5 (open end) = 1.5
2. **Apply the Frequency Ratio Formula:**

$$\frac{f_v}{f_h} = \frac{n_h}{n_v}$$

3. **Calculate Unknown Frequency:**

$$f_v = 50 \times \left(\frac{5}{1.5}\right) = 50 \times \frac{10}{3} = \frac{500}{3} \approx 166.67 \text{ Hz}$$

The frequency of the vertical signal is approximately 166.67 Hz.

3.4 Rise Time and Bandwidth of an Oscilloscope

The bandwidth (BW) of an oscilloscope indicates the highest frequency signal it can accurately measure. The rise time (t_r) of the oscilloscope limits its ability to display fast voltage transitions.

Methodology:

Bandwidth and Rise Time Formula The relationship between the bandwidth and the rise time of an oscilloscope (or any first-order low-pass system) is given by the empirical formula:

$$t_r \times BW = 0.35$$

where:

- t_r is the rise time in seconds.
- BW is the bandwidth in Hertz.

When measuring the rise time of a signal ($t_{r(sig)}$) using an oscilloscope with a known rise time ($t_{r(osc)}$), the displayed rise time ($t_{r(disp)}$) is given by:

$$t_{r(disp)} = \sqrt{t_{r(sig)}^2 + t_{r(osc)}^2}$$

Worked Example:

Calculate Oscilloscope Rise Time Calculate the rise time of a 20 MHz dual-trace oscilloscope.
Solution:

1. **Given Data:**
   - Bandwidth (BW) = 20 MHz = 20×10^6 Hz
2. **Apply the Rise Time Formula:**
$$t_r = \frac{0.35}{BW}$$
3. **Calculate Rise Time:**
$$t_r = \frac{0.35}{20 \times 10^6} = 0.0175 \times 10^{-6} \text{ s} = 17.5 \text{ ns}$$

The rise time of the oscilloscope is 17.5 ns.

Worked Example:

Calculate True Signal Rise Time A pulse is applied to an oscilloscope having a bandwidth of 50 MHz. The displayed rise time of the pulse on the CRO screen is 12 ns. Calculate the true rise time of the applied pulse signal.
Solution:

1. **Given Data:**
   - Oscilloscope Bandwidth (BW) = 50 MHz = 50×10^6 Hz
   - Displayed rise time ($t_{r(disp)}$) = 12 ns = 12×10^{-9} s
2. **Calculate Oscilloscope Rise Time ($t_{r(osc)}$):**
$$t_{r(osc)} = \frac{0.35}{BW} = \frac{0.35}{50 \times 10^6} = 7 \times 10^{-9} \text{ s} = 7 \text{ ns}$$
3. **Calculate True Signal Rise Time ($t_{r(sig)}$):** Using the formula $t_{r(disp)} = \sqrt{t_{r(sig)}^2 + t_{r(osc)}^2}$:
$$t_{r(sig)} = \sqrt{t_{r(disp)}^2 - t_{r(osc)}^2}$$
$$t_{r(sig)} = \sqrt{12^2 - 7^2} = \sqrt{144 - 49} = \sqrt{95} \approx 9.75 \text{ ns}$$

The true rise time of the signal is approximately 9.75 ns.

CHAPTER 4

Transducers and Sensors

4.1 Strain Gauges

Methodology:

Strain Gauge Calculations A strain gauge is a resistive transducer that converts mechanical elongation or compression into a change in electrical resistance.

1. **Strain (ϵ):** The ratio of change in length to the original length.

$$\epsilon = \frac{\Delta L}{L}$$

2. **Gauge Factor (G_f):** The ratio of fractional change in resistance to the fractional change in length (strain).

$$G_f = \frac{\Delta R/R}{\Delta L/L} = \frac{\Delta R/R}{\epsilon}$$

where:

- R = Original resistance (Ω)
- ΔR = Change in resistance (Ω)
- L = Original length (m)
- ΔL = Change in length (m)

3. **Stress (σ):** Force per unit area.

$$\sigma = \frac{F}{A}$$

4. **Young's Modulus (Y or E):** Ratio of stress to strain.

$$Y = \frac{\sigma}{\epsilon}$$

Worked Example:

Calculation of Change in Resistance A strain gauge has a nominal resistance of 120 Ω and a gauge factor of 2.0. It is bonded to a metallic member which is subjected to a strain of 10^{-4}. Calculate the change in resistance of the strain gauge.

Solution:

1. **Identify given values:**

- Original resistance, $R = 120\ \Omega$
- Gauge factor, $G_f = 2.0$
- Strain, $\epsilon = \Delta L/L = 10^{-4}$

2. **Apply the Gauge Factor formula:**
$$G_f = \frac{\Delta R/R}{\epsilon}$$
3. **Rearrange to solve for ΔR:**
$$\Delta R = G_f \times R \times \epsilon$$
4. **Substitute the values:**
$$\Delta R = 2.0 \times 120 \times 10^{-4}$$
$$\Delta R = 240 \times 10^{-4} = 0.024\ \Omega$$

The change in resistance is 0.024 Ω.

Worked Example:

Calculation of Applied Force A strain gauge with a gauge factor of 2.2 and resistance of 100 Ω is bonded to a steel bar of cross-sectional area 0.01 m^2. Young's modulus of steel is 200 GN/m^2. If the change in resistance is 0.05 Ω, calculate the applied force.

Solution:

1. **Identify given values:**
   - $G_f = 2.2$
   - $R = 100\ \Omega$
   - $\Delta R = 0.05\ \Omega$
   - $A = 0.01\ m^2$
   - $Y = 200 \times 10^9\ N/m^2$
2. **Calculate the strain (ϵ):**
$$G_f = \frac{\Delta R/R}{\epsilon} \implies \epsilon = \frac{\Delta R/R}{G_f}$$
$$\epsilon = \frac{0.05/100}{2.2} = \frac{0.0005}{2.2} = 2.27 \times 10^{-4}$$
3. **Calculate the stress (σ):**
$$Y = \frac{\sigma}{\epsilon} \implies \sigma = Y \times \epsilon$$
$$\sigma = (200 \times 10^9) \times (2.27 \times 10^{-4}) = 45.4 \times 10^6\ N/m^2$$
4. **Calculate the applied force (F):**
$$\sigma = \frac{F}{A} \implies F = \sigma \times A$$
$$F = 45.4 \times 10^6 \times 0.01 = 454 \times 10^3\ N = 454\ kN$$

The applied force is 454 kN.

4.2 Resistance Temperature Detectors (RTD)

Methodology:

RTD Resistance Temperature Relationship RTDs are metallic sensors whose resistance increases with temperature (positive temperature coefficient).

1. **Linear Approximation Formula:**

$$R_T = R_0[1 + \alpha(T - T_0)]$$

where:

- R_T = Resistance at temperature T (Ω)
- R_0 = Reference resistance at reference temperature T_0 (Ω)
- α = Temperature coefficient of resistance at T_0 ($°C^{-1}$)
- T = Operating temperature ($°C$)
- T_0 = Reference temperature (usually $0°C$ or $20°C$)

Worked Example:

Calculating Temperature from RTD Resistance A platinum RTD has a resistance of 100 Ω at $0°C$ and a temperature coefficient of 0.00385 $°C^{-1}$. The measured resistance is 138.5 Ω. Calculate the temperature.
Solution:

1. **Identify given values:**
   - $R_0 = 100\ \Omega$
   - $T_0 = 0°C$
   - $\alpha = 0.00385\ °C^{-1}$
   - $R_T = 138.5\ \Omega$
2. **State the formula:**
$$R_T = R_0[1 + \alpha(T - T_0)]$$
3. **Substitute the known values:**
$$138.5 = 100[1 + 0.00385 \times (T - 0)]$$
4. **Solve for T:**
$$\frac{138.5}{100} = 1 + 0.00385 \cdot T$$
$$1.385 - 1 = 0.00385 \cdot T$$
$$0.385 = 0.00385 \cdot T$$
$$T = \frac{0.385}{0.00385} = 100°C$$

The measured temperature is $100°C$.

4.3 Thermistors

Methodology:

Thermistor Resistance Temperature Relationship Thermistors are semiconductor devices typically with a negative temperature coefficient (NTC), meaning resistance decreases as temperature increases.

1. **Exponential Model (Steinhart-Hart simplified equation):**

$$R_T = R_0 \exp\left[\beta\left(\frac{1}{T} - \frac{1}{T_0}\right)\right]$$

where:

- R_T = Resistance at absolute temperature T (Kelvin)
- R_0 = Resistance at reference absolute temperature T_0 (Kelvin)
- β = Material constant of the thermistor (Kelvin)
- $T(K) = T(°C) + 273.15$

Worked Example:

Calculating Thermistor Resistance An NTC thermistor has a resistance of 10 $k\Omega$ at $25°C$. Its material constant β is 3950 K. Calculate its resistance at $50°C$.
Solution:

1. **Identify given values and convert temperatures to Kelvin:**
   - $R_0 = 10000\ \Omega$
   - $T_0 = 25°C = 25 + 273.15 = 298.15\ K$
   - $T = 50°C = 50 + 273.15 = 323.15\ K$
   - $\beta = 3950\ K$
2. **State the exponential model formula:**
$$R_T = R_0 \exp\left[\beta\left(\frac{1}{T} - \frac{1}{T_0}\right)\right]$$
3. **Calculate the exponent term:**
$$\frac{1}{T} - \frac{1}{T_0} = \frac{1}{323.15} - \frac{1}{298.15} = 0.0030945 - 0.0033540 = -0.0002595\ K^{-1}$$
$$\beta\left(\frac{1}{T} - \frac{1}{T_0}\right) = 3950 \times (-0.0002595) = -1.025$$
4. **Calculate R_T:**
$$R_T = 10000 \times \exp(-1.025)$$
$$R_T = 10000 \times 0.3588 = 3588\ \Omega$$

The resistance at $50°C$ is 3588 Ω.

4.4 Capacitive Transducers

Methodology:

Capacitance Variations Capacitive transducers work on the principle of change of capacitance due to change in area, distance, or dielectric constant.

1. **Parallel Plate Capacitance Formula:**
$$C = \frac{\epsilon_0 \epsilon_r A}{d}$$
   where:
   - C = Capacitance (Farads, F)
   - ϵ_0 = Permittivity of free space ($8.854 \times 10^{-12}\ F/m$)
   - ϵ_r = Relative permittivity (dielectric constant)
   - A = Overlapping area of plates (m^2)

- d = Distance between plates (m)

2. **Sensitivity for varying distance (d):**

$$S = \frac{dC}{dd} = -\frac{\epsilon_0 \epsilon_r A}{d^2}$$

Worked Example:

Change in Distance of Capacitive Transducer A capacitive transducer uses two parallel plates of area $5 \times 10^{-4}\ m^2$ separated by air ($\epsilon_r = 1$) at a distance of $1\ mm$. If a displacement changes the distance to $0.8\ mm$, calculate the change in capacitance.

Solution:

1. **Identify given values:**
   - Area $A = 5 \times 10^{-4}\ m^2$
   - Initial distance $d_1 = 1\ mm = 10^{-3}\ m$
   - Final distance $d_2 = 0.8\ mm = 0.8 \times 10^{-3}\ m$
   - $\epsilon_0 = 8.854 \times 10^{-12}\ F/m$, $\epsilon_r = 1$

2. **Calculate initial capacitance (C_1):**

$$C_1 = \frac{\epsilon_0 \epsilon_r A}{d_1} = \frac{(8.854 \times 10^{-12}) \times 1 \times (5 \times 10^{-4})}{10^{-3}}$$

$$C_1 = 4.427 \times 10^{-12}\ F = 4.427\ pF$$

3. **Calculate final capacitance (C_2):**

$$C_2 = \frac{\epsilon_0 \epsilon_r A}{d_2} = \frac{(8.854 \times 10^{-12}) \times 1 \times (5 \times 10^{-4})}{0.8 \times 10^{-3}}$$

$$C_2 = \frac{4.427 \times 10^{-12}}{0.8} = 5.534\ pF$$

4. **Calculate change in capacitance (ΔC):**

$$\Delta C = C_2 - C_1 = 5.534\ pF - 4.427\ pF = 1.107\ pF$$

The change in capacitance is $1.107\ pF$.

4.5 Linear Variable Differential Transformer (LVDT)

Methodology:

LVDT Sensitivity and Resolution LVDT is an inductive transducer used to measure linear displacement.

1. **Sensitivity (S):** The ratio of output voltage change to the input displacement.

$$S = \frac{\Delta V_{out}}{\Delta x}$$

where ΔV_{out} is the change in RMS output voltage and Δx is the core displacement.

2. **Resolution:** The smallest measurable displacement, dependent on the voltmeter's minimum measurable voltage.

$$\text{Resolution} = \frac{\text{Minimum voltage readable}}{S}$$

Worked Example:

LVDT Resolution Calculation An LVDT has an output of 2 V rms for a displacement of 0.5 mm. The output is read on a voltmeter with a full scale of 5 V which has 100 divisions. The scale can be read to 1/5th of a division. Calculate the sensitivity and resolution of the instrument.

Solution:

1. **Calculate the Sensitivity (S):**
   - Output voltage $\Delta V_{out} = 2\ V$
   - Displacement $\Delta x = 0.5\ mm$

$$S = \frac{\Delta V_{out}}{\Delta x} = \frac{2\ V}{0.5\ mm} = 4\ V/mm$$

2. **Determine the minimum readable voltage:**
   - Full scale voltage $= 5\ V$
   - Total divisions $= 100$
   - Voltage per division $= 5\ V/100 = 0.05\ V/\text{div}$
   - Readability $= 1/5$ of a division
   - Minimum readable voltage $= \frac{1}{5} \times 0.05\ V = 0.01\ V$

3. **Calculate the Resolution:**

$$\text{Resolution} = \frac{\text{Minimum readable voltage}}{S}$$

$$\text{Resolution} = \frac{0.01\ V}{4\ V/mm} = 0.0025\ mm = 2.5\ \mu m$$

The sensitivity is 4 V/mm and the resolution is 2.5 μm.

4.6 Piezoelectric Transducers

Methodology:

Piezoelectric Calculations Piezoelectric materials generate an electrical charge when subjected to mechanical stress.

1. **Charge generated (q):**

$$q = d \times F$$

where q is charge (Coulombs), d is charge sensitivity (C/N), and F is applied force (N).

2. **Pressure or Stress (P):**
$$P = \frac{F}{A}$$
where A is the area of the crystal (m^2).

3. **Voltage generated (V):** Using the relationship between charge, capacitance, and voltage ($V = q/C$):
$$V = g \times t \times P$$
where:
   - V = Output voltage (V)
   - g = Voltage sensitivity (Vm/N)
   - t = Thickness of the crystal (m)
   - P = Applied pressure (N/m^2)

4. **Relationship between d and g:**
$$g = \frac{d}{\epsilon_0 \epsilon_r}$$

Worked Example:

Piezoelectric Output Voltage A piezoelectric crystal has a thickness of 2 mm and a voltage sensitivity of 0.05 Vm/N. It is subjected to a pressure of 1.5×10^6 N/m^2. Calculate the output voltage.
Solution:

1. **Identify given values:**
   - Thickness, $t = 2\ mm = 2 \times 10^{-3}\ m$
   - Voltage sensitivity, $g = 0.05\ Vm/N$
   - Pressure, $P = 1.5 \times 10^6\ N/m^2$
2. **Apply the voltage formula:**
$$V = g \times t \times P$$
3. **Substitute the values:**
$$V = 0.05 \times (2 \times 10^{-3}) \times (1.5 \times 10^6)$$
4. **Calculate the result:**
$$V = 0.05 \times 3000 = 150\ V$$

The output voltage is 150 V.

Worked Example:

Charge and Voltage of Quartz Crystal A quartz piezoelectric crystal having dimensions of 10 $mm \times 10$ $mm \times$ 2 mm is subjected to a force of 5 N. The charge sensitivity is 2×10^{-12} C/N and its relative permittivity is 4.5. Calculate the generated charge and the output voltage.
Solution:

1. **Identify given values:**
   - Area, $A = 10\ mm \times 10\ mm = 100 \times 10^{-6}\ m^2 = 10^{-4}\ m^2$
   - Thickness, $t = 2\ mm = 2 \times 10^{-3}\ m$
   - Force, $F = 5\ N$
   - Charge sensitivity, $d = 2 \times 10^{-12}\ C/N$

- Relative permittivity, $\epsilon_r = 4.5$
- $\epsilon_0 = 8.854 \times 10^{-12}\ F/m$

2. **Calculate the generated charge (q):**

$$q = d \times F$$

$$q = (2 \times 10^{-12}) \times 5 = 10 \times 10^{-12}\ C = 10\ pC$$

3. **Calculate the capacitance of the crystal (C):**

$$C = \frac{\epsilon_0 \epsilon_r A}{t}$$

$$C = \frac{(8.854 \times 10^{-12}) \times 4.5 \times 10^{-4}}{2 \times 10^{-3}}$$

$$C = \frac{39.843 \times 10^{-16}}{2 \times 10^{-3}} = 19.92 \times 10^{-13}\ F = 1.992\ pF$$

4. **Calculate the output voltage (V):**

$$V = \frac{q}{C}$$

$$V = \frac{10 \times 10^{-12}}{1.992 \times 10^{-12}} = \frac{10}{1.992} = 5.02\ V$$

The generated charge is 10 pC and output voltage is 5.02 V.

4.7 Hall Effect Transducers

Methodology:

Hall Effect Voltage When a current-carrying conductor is placed in a transverse magnetic field, an electromotive force (EMF) is produced across its edges perpendicular to both the current and the magnetic field.

1. **Hall Voltage (V_H):**

$$V_H = K_H \frac{B \cdot I}{t}$$

where:

- V_H = Hall voltage (V)
- K_H = Hall coefficient ($V \cdot m/(A \cdot T)$ or m^3/C)
- B = Magnetic flux density (Tesla, T or Wb/m^2)
- I = Current passing through the conductor (A)
- t = Thickness of the conductor (m)

2. **Hall Coefficient in terms of charge density:**

$$K_H = \frac{1}{n \cdot e}$$

where n is charge carrier density (m^{-3}) and e is the charge of an electron ($1.6 \times 10^{-19}\ C$).

Worked Example:

Calculating Hall Voltage A Hall effect element of thickness 2 mm has a Hall coefficient of $10^{-3}\ m^3/C$. It is placed in a magnetic field of 0.5 T. If a current of 4 mA flows through it, calculate the generated Hall voltage.

Solution:

1. **Identify given values:**
   - Thickness, $t = 2\ mm = 2 \times 10^{-3}\ m$
   - Hall coefficient, $K_H = 10^{-3}\ m^3/C$
   - Magnetic field, $B = 0.5\ T$
   - Current, $I = 4\ mA = 4 \times 10^{-3}\ A$
2. **Apply the Hall voltage formula:**
$$V_H = K_H \frac{B \cdot I}{t}$$
3. **Substitute the values:**
$$V_H = 10^{-3} \times \frac{0.5 \times (4 \times 10^{-3})}{2 \times 10^{-3}}$$
4. **Simplify and solve:**
$$V_H = 10^{-3} \times \frac{2 \times 10^{-3}}{2 \times 10^{-3}} = 10^{-3} \times 1 = 10^{-3}\ V = 1\ mV$$

The generated Hall voltage is 1 mV.

CHAPTER 5

Test And Measuring Instruments

5.1 Frequency Counters and Digital Measurements

Frequency counters are vital test instruments for measuring the frequency, period, and time interval of electronic signals. The measurement relies on counting the number of cycles within a specific time gate or counting the number of time base clock pulses during one or more signal periods.

Methodology:

Frequency and Period Measurement **1. Frequency Measurement:** The unknown frequency f_x is measured by counting the number of cycles N over a known gating time interval T_g.

$$f_x = \frac{N}{T_g}$$

If the gating time is generated by a time base clock of frequency f_{tb}, then $T_g = M \times T_{tb} = \frac{M}{f_{tb}}$ where M is a dividing factor.
2. Period Measurement: For low-frequency signals, period measurement is more accurate. The period of the unknown signal T_x is measured by counting the number of time base clock pulses N that occur during one period (or multiple periods M) of the unknown signal.

$$T_x = \frac{N \times T_{tb}}{M} = \frac{N}{M \times f_{tb}}$$

where T_{tb} is the period of the time base oscillator.
3. Measurement Error: The total percentage error in a frequency counter is typically the sum of the ± 1 count error and the time base error:

$$\%\text{Error} = \left(\frac{1}{N} + \text{Time Base Error}\right) \times 100\%$$

where N is the number of counts registered.

Worked Example:

Calculating Frequency and Counter Error A frequency counter with a time base frequency of 1 MHz and a time base error of 1×10^{-6} (or 1 ppm) is used to measure an unknown signal. The gating time is set to 1 s. The counter registers a count of 25000. Determine: 1. The frequency of the unknown signal. 2. The maximum percentage error in the measurement.
Solution:
Step 1: Calculate the unknown frequency Given: Number of counts $N = 25000$ Gating time $T_g = 1$ s

$$f_x = \frac{N}{T_g} = \frac{25000}{1} = 25 \text{ kHz}$$

Step 2: Calculate the measurement error The ±1 count error fraction is:

$$\text{Count Error} = \frac{1}{N} = \frac{1}{25000} = 4 \times 10^{-5}$$

The time base error is given as 1×10^{-6}. Total error fraction:

$$\text{Total Error} = 4 \times 10^{-5} + 1 \times 10^{-6} = 4.1 \times 10^{-5}$$

Percentage error:

$$\%\text{Error} = (4.1 \times 10^{-5}) \times 100\% = 0.0041\%$$

The maximum percentage error is ±0.0041%.

Worked Example:

Period Measurement for Low Frequency A 50 Hz sine wave is to be measured using a frequency counter in period mode. The time base clock is 10 MHz. 1. How many counts will be registered for a single period measurement? 2. What is the count if the period is averaged over 100 cycles?

Solution:

Step 1: Calculate counts for a single period Frequency $f_x = 50$ Hz. Period $T_x = \frac{1}{f_x} = \frac{1}{50} = 0.02$ s. Time base frequency $f_{tb} = 10$ MHz $= 10^7$ Hz. Time base period $T_{tb} = \frac{1}{10^7} = 0.1\mu$s. Number of counts $N = \frac{T_x}{T_{tb}} = \frac{0.02}{10^{-7}} = 200,000$ counts.

Step 2: Calculate counts for 100 cycles averaging Total time for 100 cycles $T_{total} = 100 \times 0.02$ s $= 2$ s. Number of counts $N = \frac{T_{total}}{T_{tb}} = \frac{2}{10^{-7}} = 20,000,000$ counts.

5.2 Harmonic Distortion Analyzers

A distortion analyzer measures the extent to which a signal contains harmonic components. Harmonic distortion occurs when a system introduces integer multiples of the fundamental frequency into the output.

Methodology:

Total Harmonic Distortion THD Let a signal have a fundamental RMS voltage V_1 and harmonic RMS voltages $V_2, V_3, V_4, \ldots$

1. Individual Harmonic Distortion: The distortion factor for the n-th harmonic is:

$$D_n = \frac{V_n}{V_1}$$

2. Total Harmonic Distortion THD: The total harmonic distortion is the ratio of the RMS voltage of all harmonics combined to the RMS voltage of the fundamental:

$$\text{THD} = \frac{\sqrt{V_2^2 + V_3^2 + V_4^2 + \ldots}}{V_1}$$

It is typically expressed as a percentage:

$$\%\text{THD} = \text{THD} \times 100\%$$

3. THD based on Total RMS Voltage: If the total RMS voltage of the signal V_{total} is known along with the fundamental V_1, THD can be calculated as:

$$\text{THD} = \sqrt{\frac{V_{total}^2 - V_1^2}{V_1^2}}$$

Worked Example:

Calculating THD from Harmonic Amplitudes An audio signal is analyzed using a spectrum analyzer. The following RMS voltage values are obtained for the fundamental and its harmonics: Fundamental (f_1): 10 V Second harmonic ($2f_1$): 1.2 V Third harmonic ($3f_1$): 0.5 V Fourth harmonic ($4f_1$): 0.2 V
Calculate the individual harmonic distortions and the Total Harmonic Distortion (THD).
Solution:
Step 1: Calculate Individual Harmonic Distortions

$$D_2 = \frac{V_2}{V_1} = \frac{1.2}{10} = 0.12 \text{ (or 12\%)}$$

$$D_3 = \frac{V_3}{V_1} = \frac{0.5}{10} = 0.05 \text{ (or 5\%)}$$

$$D_4 = \frac{V_4}{V_1} = \frac{0.2}{10} = 0.02 \text{ (or 2\%)}$$

Step 2: Calculate Total Harmonic Distortion

$$\text{THD} = \frac{\sqrt{V_2^2 + V_3^2 + V_4^2}}{V_1}$$

$$\text{THD} = \frac{\sqrt{1.2^2 + 0.5^2 + 0.2^2}}{10}$$

$$\text{THD} = \frac{\sqrt{1.44 + 0.25 + 0.04}}{10} = \frac{\sqrt{1.73}}{10}$$

$$\text{THD} = \frac{1.315}{10} = 0.1315$$

Expressed as a percentage:

$$\%\text{THD} = 13.15\%$$

Worked Example:

Determining Fundamental Voltage from THD A harmonic distortion analyzer measures a total signal RMS voltage of 12 V and a THD of 5%. Assuming the fundamental definition of THD is used (and distortion is small enough that $V_{total} \approx V_1$), calculate the exact fundamental RMS voltage and the RMS voltage of the combined harmonics.
Solution:
Step 1: Set up the equations Total RMS voltage:

$$V_{total} = \sqrt{V_1^2 + V_h^2} = 12 \text{ V}$$

where V_h is the combined harmonic RMS voltage ($V_h = \sqrt{V_2^2 + V_3^2 + \ldots}$). THD definition:

$$\text{THD} = \frac{V_h}{V_1} = 0.05 \implies V_h = 0.05V_1$$

Step 2: Solve for V_1 and V_h Substitute V_h into the total voltage equation:

$$\sqrt{V_1^2 + (0.05V_1)^2} = 12$$

$$\sqrt{V_1^2 + 0.0025V_1^2} = 12$$

$$\sqrt{1.0025V_1^2} = 12$$

$$1.00125V_1 = 12$$

$$V_1 = \frac{12}{1.00125} \approx 11.985 \text{ V}$$

Now find V_h:

$$V_h = 0.05 \times 11.985 = 0.599 \text{ V}$$

5.3 Spectrum Analyzers

A spectrum analyzer is used to display the amplitude of signals as a function of frequency. For swept-tuned spectrum analyzers, parameters like resolution bandwidth (RBW), span, and sweep time are critical for accurate measurements without signal distortion.

Methodology:

Spectrum Analyzer Settings **1. Sweep Time T_s:** Sweep time is the duration taken by the analyzer to sweep across the specified frequency span. To maintain amplitude calibration and avoid ringing in the IF filter, the sweep time must be sufficiently slow. The minimum sweep time is given by:

$$T_s \geq \frac{k \times \text{Span}}{\text{RBW} \times \text{VBW}}$$

Often if the Video Bandwidth (VBW) is much larger than RBW, the formula simplifies to:

$$T_s \geq \frac{k \times \text{Span}}{\text{RBW}^2}$$

where k is a constant that depends on the filter shape (typically between 2 and 3 for Gaussian filters).
2. Frequency Resolution: The ability of a spectrum analyzer to separate two closely spaced continuous-wave (CW) signals is primarily determined by its Resolution Bandwidth (RBW). For two signals of equal amplitude to be resolved, their frequency separation Δf must be greater than or equal to the RBW:

$$\Delta f \geq \text{RBW}$$

Worked Example:

Sweep Time Calculation A spectrum analyzer is configured with a frequency span of 1 MHz and a resolution bandwidth (RBW) of 1 kHz. Assuming a filter constant $k = 2$ and a large VBW, determine the minimum allowable sweep time.
Solution:
Step 1: Identify given parameters Span $= 1$ MHz $= 10^6$ Hz RBW $= 1$ kHz $= 10^3$ Hz Filter constant $k = 2$
Step 2: Apply the minimum sweep time formula

$$T_{s(\min)} = \frac{k \times \text{Span}}{\text{RBW}^2}$$

$$T_{s(\min)} = \frac{2 \times 10^6}{(10^3)^2}$$

$$T_{s(\min)} = \frac{2 \times 10^6}{10^6} = 2 \text{ seconds}$$

The sweep time must be set to at least 2 seconds to avoid amplitude errors.

Worked Example:

Adjusting RBW for Faster Sweep For the spectrum analyzer in the previous example (Span $= 1$ MHz), the user wants to reduce the minimum sweep time to 20 ms. What is the minimum required Resolution Bandwidth (RBW)? Assume $k = 2$.
Solution:
Step 1: Identify parameters Span $= 10^6$ Hz $T_s = 20$ ms $= 0.02$ s $k = 2$
Step 2: Rearrange the formula to solve for RBW

$$T_s = \frac{k \times \text{Span}}{\text{RBW}^2} \implies \text{RBW}^2 = \frac{k \times \text{Span}}{T_s}$$

Step 3: Calculate RBW

$$\text{RBW}^2 = \frac{2 \times 10^6}{0.02} = \frac{2 \times 10^6}{2 \times 10^{-2}} = 10^8$$

$$\text{RBW} = \sqrt{10^8} = 10,000 \text{ Hz} = 10 \text{ kHz}$$

The RBW must be increased to at least 10 kHz to achieve a sweep time of 20 ms without measurement errors.

5.4 Function Generators

Function generators produce standard waveforms such as sine, square, and triangular waves over a wide frequency range. A common analog architecture involves a constant current source charging and discharging a capacitor (integrator) to produce a triangular wave, which then drives a Schmitt trigger to output a square wave.

Methodology:

Triangular and Square Wave Frequency Consider an integrator-based function generator where a capacitor C is charged and discharged by a constant current I.

1. Capacitor Voltage Rate of Change:

$$\frac{dV_c}{dt} = \frac{I}{C}$$

2. Frequency Calculation: If the Schmitt trigger has upper and lower threshold voltages V_{UT} and V_{LT}, the peak-to-peak amplitude of the triangular wave is $\Delta V = V_{UT} - V_{LT}$. The time to charge (or discharge) the capacitor by ΔV is:

$$T_{half} = \frac{C \cdot \Delta V}{I}$$

The total period T for a symmetrical waveform is $2 \times T_{half}$:

$$T = \frac{2 \cdot C \cdot \Delta V}{I}$$

The frequency f of the output waveforms (both triangular and square) is:

$$f = \frac{1}{T} = \frac{I}{2 \cdot C \cdot \Delta V}$$

Worked Example:

Function Generator Frequency A function generator circuit uses a 10 nF capacitor. The constant charging and discharging current is set to 2 mA. The voltage thresholds for the Schmitt trigger are +5 V and −5 V. Calculate the peak-to-peak voltage of the triangular wave and the frequency of the generated signals.

Solution:

Step 1: Calculate Peak-to-Peak Voltage

$$V_{UT} = 5 \text{ V}, \quad V_{LT} = -5 \text{ V}$$

$$\Delta V = V_{UT} - V_{LT} = 5 - (-5) = 10 \text{ V}$$

Step 2: Calculate Output Frequency Given: Current $I = 2 \text{ mA} = 2 \times 10^{-3}$ A Capacitance $C = 10 \text{ nF} = 10 \times 10^{-9}$ F Peak-to-Peak Voltage $\Delta V = 10$ V

Apply the frequency formula:

$$f = \frac{I}{2 \cdot C \cdot \Delta V}$$

$$f = \frac{2 \times 10^{-3}}{2 \times (10 \times 10^{-9}) \times 10}$$

$$f = \frac{2 \times 10^{-3}}{200 \times 10^{-9}} = \frac{2 \times 10^{-3}}{2 \times 10^{-7}}$$

$$f = 10^4 \text{ Hz} = 10 \text{ kHz}$$

The frequency of the generated square and triangular waves is 10 kHz.

5.5 Digital IC Testers

Digital IC testers are instruments used to verify the logical functionality of digital integrated circuits. They apply a sequence of input stimuli (test vectors) to the IC's pins and compare the resulting output patterns against the expected truth table.

Methodology:

Test Vector Calculation **1. Exhaustive Testing:** For a combinational logic circuit with N input pins, exhaustive testing requires applying every possible combination of inputs. The total number of test vectors required is:

$$\text{Total Vectors} = 2^N$$

2. Sequential Testing: For sequential logic (containing M memory elements or flip-flops), exhaustive testing would theoretically require testing every possible state and input combination:

$$\text{Total Vectors} = 2^{N+M}$$

However, sequential ICs often use algorithmic test patterns rather than exhaustive testing due to the exponentially high number of required vectors.
3. Test Time: If each test vector takes time T_v to apply and evaluate, the total time for exhaustive testing is:

$$\text{Total Time} = 2^N \times T_v$$

Worked Example:

Exhaustive Test Time for Combinational Logic A Digital IC Tester is evaluating a 16-input combinational logic ALU. The tester can apply a test vector and evaluate the output in 50 ns. Determine the total number of test vectors required for exhaustive testing and the total time required to run the test.
Solution:
Step 1: Calculate total number of test vectors Number of inputs $N = 16$. Total test vectors $= 2^{16} = 65,536$.
Step 2: Calculate total test time Time per vector $T_v = 50$ ns $= 50 \times 10^{-9}$ s. Total time $= 65,536 \times 50 \times 10^{-9}$ s. Total time $= 3,276,800 \times 10^{-9}$ s ≈ 3.28 ms.

Formulae

Appendix: Key Formulae

This appendix provides a quick reference to the key formulae and mathematical relationships discussed in this book, categorized by unit.

Unit 1: Measurement Errors and AC Bridges

Errors and Accuracy:

- **Absolute Error (e):** $e = A_m - A_t$
- **Relative Error (ϵ_r):** $\epsilon_r = \frac{e}{A_t} = \frac{A_m - A_t}{A_t}$
- **Percentage Error:** $\%\epsilon_r = \epsilon_r \times 100\%$
- **Relative Accuracy (A):** $A = 1 - |\epsilon_r|$
- **Percentage Accuracy:** $\%A = A \times 100\%$

DC Bridges (Wheatstone and Kelvin):

- **Wheatstone Bridge Balance Condition:** $R_1 R_4 = R_2 R_3 \implies R_4 = \frac{R_2 R_3}{R_1}$
- **Kelvin Double Bridge Unknown Resistance:** $R = \frac{P}{Q} S + \frac{qr}{p+q+r} \left(\frac{P}{Q} - \frac{p}{q} \right)$
- If $\frac{P}{Q} = \frac{p}{q}$, then $R = \frac{P}{Q} S$

AC Bridges General Balance Conditions:

- **Magnitude Condition:** $|Z_1||Z_4| = |Z_2||Z_3|$
- **Phase Angle Condition:** $\angle\theta_1 + \angle\theta_4 = \angle\theta_2 + \angle\theta_3$

Specific AC Bridges:

- **Maxwell's Inductance-Capacitance Bridge:**

$$L_x = R_2 R_3 C_1, \quad R_x = \frac{R_2 R_3}{R_1}, \quad Q = \omega C_1 R_1$$

- **Hay's Bridge:**

$$L_x = \frac{R_2 R_3 C_1}{1 + (1/Q)^2}, \quad R_x = \frac{R_2 R_3}{R_1} \left(\frac{1}{1 + Q^2} \right), \quad Q = \frac{\omega L_x}{R_x} = \frac{1}{\omega C_1 R_1}$$

- **Schering Bridge:**

$$C_x = C_2 \frac{R_4}{R_3}, \quad r_x = R_3 \frac{C_4}{C_2}, \quad D = \omega C_x r_x = \omega C_4 R_4$$

Q-Meter:

- **Quality Factor (Q):** $Q = \frac{\omega L}{R} = \frac{1}{\omega C R} = \frac{V_C}{V}$
- **Resonant Frequency:** $f = \frac{1}{2\pi\sqrt{LC}}$
- **Distributed Capacitance (C_d):** $C_d = \frac{C_1 - n^2 C_2}{n^2 - 1}$, where $n = \frac{f_2}{f_1}$

Unit 2: Measuring Instruments

Ammeters and Voltmeters:

- **Shunt Resistance for Ammeter:** $R_{sh} = \frac{I_m R_m}{I - I_m} = \frac{R_m}{m-1}$, where $m = \frac{I}{I_m}$
- **Ayrton Shunt (Universal Shunt):** $(I_x - I_m)R_{sh} = I_m(R_m + R_{series})$ depending on tap position.
- **Series Multiplier for Voltmeter:** $R_s = \frac{V}{I_m} - R_m = R_m(m-1)$, where $m = \frac{V}{V_m}$
- **DC Voltmeter Sensitivity:** $S_{dc} = \frac{1}{I_{fsd}}$ (Ω/V)
- **AC Voltmeter Sensitivity (Full Wave):** $S_{ac} = 0.9 \times S_{dc}$
- **AC Voltmeter Sensitivity (Half Wave):** $S_{ac} = 0.45 \times S_{dc}$
- **Total Voltmeter Resistance:** $R_{total} = S \times V_{range} = R_s + R_m$

Energy Meter and Power:

- **Power in AC Circuit:** $P = V \times I \times \cos(\phi)$
- **Energy (E):** $E = P \times t$
- **Expected Revolutions ($N_{expected}$):** $N_{expected} = K \times E$
- **Percentage Error:** $\%\ \text{Error} = \frac{N_{actual} - N_{expected}}{N_{expected}} \times 100\%$

Analog to Digital Converters (ADC):

- **Resolution:** $\text{Resolution} = \frac{V_{FS}}{2^n}$ (for an n-bit ADC)
- **Conversion Time (Successive Approximation):** $T_{conv} = \frac{n}{f_{clk}}$
- **Dual Slope ADC Input Voltage:** $V_{in} = V_{ref} \times \frac{T_2}{T_1} = V_{ref} \times \frac{N_2}{N_1}$

Instrument Transformers:

- **Current Transformer Ratio:** $I_p \times N_p = I_s \times N_s$

Unit 3: Oscilloscopes and Waveform Analysis

Voltage and Time Measurements:

- **Peak-to-Peak Voltage:** $V_{p-p} = (\text{Vertical Divisions}) \times (\text{VOLTS/DIV})$
- **Peak Voltage:** $V_m = \frac{V_{p-p}}{2}$
- **RMS Voltage (Sine Wave):** $V_{rms} = \frac{V_m}{\sqrt{2}} = \frac{V_{p-p}}{2\sqrt{2}}$
- **Time Period (T):** $T = (\text{Horizontal Divisions for one cycle}) \times (\text{TIME/DIV})$
- **Frequency (f):** $f = \frac{1}{T}$

Lissajous Figures:

- **Frequency Ratio:** $\frac{f_v}{f_h} = \frac{n_h}{n_v}$
- **Phase Angle (θ):** $\theta = \sin^{-1}\left(\frac{Y_1}{Y_2}\right)$, where Y_1 is the y-intercept and Y_2 is the maximum vertical deflection.

Oscilloscope Specifications:

- **Rise Time and Bandwidth:** $t_r \times BW = 0.35$ or $t_r = \frac{0.35}{BW}$
- **Displayed Rise Time:** $t_{r(disp)} = \sqrt{t_{r(sig)}^2 + t_{r(osc)}^2}$

Unit 4: Transducers and Sensors

Strain Gauges and Stress:

- **Stress (σ):** $\sigma = \frac{F}{A}$
- **Strain (ϵ):** $\epsilon = \frac{\Delta L}{L}$
- **Young's Modulus (Y):** $Y = \frac{\sigma}{\epsilon}$
- **Gauge Factor (G_f):** $G_f = \frac{\Delta R/R}{\epsilon} = \frac{\Delta R/R}{\Delta L/L}$
- **Change in Resistance:** $\Delta R = G_f \times R \times \epsilon$

Capacitive Transducers:

- **Capacitance (C):** $C = \frac{\epsilon_0 \epsilon_r A}{d}$
- **Sensitivity (S):** $S = \frac{dC}{dd} = -\frac{\epsilon_0 \epsilon_r A}{d^2}$

Temperature Transducers:

- **RTD Resistance:** $R_T = R_0[1 + \alpha(T - T_0)]$
- **Thermistor Resistance:** $R_T = R_0 \exp\left[\beta\left(\frac{1}{T} - \frac{1}{T_0}\right)\right]$

Piezoelectric Transducers:

- **Charge Generated (q):** $q = d \times F$
- **Voltage Generated (V):** $V = \frac{q}{C} = g \times t \times P$
- **Voltage Sensitivity (g):** $g = \frac{E}{P} = \frac{d}{\epsilon_0 \epsilon_r}$

Hall Effect Transducers:

- **Hall Voltage (V_H):** $V_H = K_H \frac{B \cdot I}{t}$
- **Hall Coefficient (K_H):** $K_H = \frac{1}{n \cdot e}$

General Sensor Characteristics:

- **Sensitivity (S):** $S = \frac{\Delta \text{Output}}{\Delta \text{Input}}$
- **Resolution:** $\text{Resolution} = \frac{\text{Minimum readable output}}{S}$

Unit 5: Electronic Instruments and Signal Analysis

Digital Frequency Counters:

- **Frequency Measurement:** $f_x = \frac{N}{T_g}$, where N is the count and T_g is the gate time.
- **Count Error:** $\text{Count Error} = \frac{1}{N}$
- **Percentage Error:** $\%\text{Error} = \left(\frac{1}{N} + \text{Time Base Error}\right) \times 100\%$
- **Time Interval Measurement:** $T_x = \frac{N}{M \times f_{tb}}$

Function Generators:

- **Frequency (f):** $f = \frac{I}{2 \cdot C \cdot \Delta V}$
- **Hysteresis Voltage (ΔV):** $\Delta V = V_{UT} - V_{LT}$

- **Rate of Voltage Change:** $\frac{dV_c}{dt} = \frac{I}{C}$

Signal Analyzers:

- **Total Harmonic Distortion (THD):**

$$\text{THD} = \frac{\sqrt{V_2^2 + V_3^2 + V_4^2 + \dots}}{V_1} = \frac{V_h}{V_1}$$

$$\text{THD} = \sqrt{\frac{V_{total}^2 - V_1^2}{V_1^2}}$$

- **Percentage THD:** $\%\text{THD} = \text{THD} \times 100\%$
- **Harmonic Distortion (D_n):** $D_n = \frac{V_n}{V_1}$
- **Total RMS Voltage:** $V_{total} = \sqrt{V_1^2 + V_h^2}$
- **Spectrum Analyzer Minimum Sweep Time:**

$$T_{s(\min)} = \frac{k \times \text{Span}}{\text{RBW}^2} \quad \text{or} \quad T_{s(\min)} = \frac{k \times \text{Span}}{\text{RBW} \times \text{VBW}}$$