Ecn (4331101) - Problem Solver

Antigravity AI

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To my students,
who constantly inspire me to find new analogies,
break down complex theories, and make learning
an engineered science.

Contents

Preface

About this Book
Welcome to **Computer Networks**! This textbook is meticulously designed to serve as a comprehensive problem solver and study companion. It provides a robust and intuitive foundation in the core principles of this subject, bridging the gap between theoretical models and practical applications. A unique feature of this book is its focus on decoding complex concepts using clear, step-by-step methodologies and real-world analogies.

Target Audience
This book is intended for students and professionals looking to master the concepts of Computer Networks. Whether you are revising for exams, seeking to solidify your fundamental understanding, or looking for practical examples to clarify abstract concepts, this book has been structured to support your learning journey. It serves as an accessible guide designed to remove the fear of complex topics, replacing it with an appreciation for the underlying mechanics.

Scope and Coverage
The coverage is divided logically to follow standard academic curriculums. Each chapter focuses on key topics, breaking them down into digestible sections:

- **Theoretical Insights** – Core concepts explained intuitively.
- **Method Blocks** – Standardized, step-by-step procedures for solving specific types of problems.
- **Worked Examples** – Fully solved problems that apply the methods in practice.

We hope this resource proves invaluable in your studies and helps you achieve excellence in **Computer Networks**. Happy learning!

Acknowledgments

Writing a comprehensive book that connects the fundamental forces of Chemistry with practical engineering applications requires more than just academic knowledge—it requires a community of support. I would like to express my sincere gratitude to everyone who contributed to the realization of this project.

Specially, I extend my heartfelt gratitude to the **Department of Technical Education (DTE), Government of Gujarat**, and **Government Polytechnic, Palanpur**, for providing an environment that fosters innovation, academic rigor, and continuous professional development.

I am deeply thankful to my family for their unwavering patience and support during the countless hours spent researching, formatting, and refining these chapters.

Finally, my deepest appreciation goes to my students. Your curiosity, challenging questions, and continuous feedback are the true driving force behind the unique analogies and streamlined structure of this book. This textbook was engineered for you.

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About the Author

Milav Jayeshkumar Dabgar is an engineering educator and R&D professional with over 9 years of experience in electronics hardware development, embedded systems, AI/ML, and full-stack software engineering. He currently serves as a **Lecturer (Class - II) & Innovation Leader** at Government Polytechnic, Palanpur, under the Education Department of the Government of Gujarat.

Milav holds a Master of Engineering (ME) in Communication Systems from L.D. College of Engineering and a Bachelor of Engineering (BE) in Electronics & Communication. He is also currently pursuing a **BS in Data Science and Applications from IIT Madras**, where he has completed both the **Diploma in Programming** and the **Diploma in Data Science** with distinction.

Most recently, he completed the **AICTE QIP PG Certificate Program in Deep Learning from SVNIT Surat** with a **perfect 10/10 CGPA**, topping his batch.

A dedicated mentor, Milav has guided numerous student teams in securing over **INR 45 lakhs** in innovation funding, including INR 25 lakhs from Shark Tank India and INR 20 lakhs through iHub Gujarat, resulting in two student patents. He is recognized as an **NPTEL Evangelist** and **NPTEL Discipline Star** in Computer Science, reflecting his commitment to continuous learning and academic excellence.

His technical expertise spans from embedded systems (8051, STM32, Arduino) to modern web technologies (Next.js, Python, PostgreSQL) and Data Science (TensorFlow, PyTorch). Through this textbook, he aims to simplify complex Engineering Chemistry concepts by integrating relatable, real-world analogies drawn from modern tech and engineering landscapes.

Milav is also an active content creator, running a popular **YouTube channel** (@milav.dabgar) where he shares technical tutorials and academic resources. He maintains a personal blog and resource hub at milav.in and can be reached for professional collaborations on LinkedIn.

CHAPTER 1

Network Elements and Network Topology

1.1 Classification of Network Elements

Although electrical elements are physical devices, for circuit analysis they are represented by idealized mathematical models. We classify them as follows:

- **Active and Passive Elements**: Active elements can continuously supply energy to the circuit (e.g. Voltage/Current sources). Passive elements absorb or store energy (e.g. Resistors, Capacitors, Inductors).
- **Bilateral and Unilateral Elements**: A bilateral element conducts current equally well in both directions (e.g. Resistor). A unilateral element has current-voltage characteristics that depend on the direction of current (e.g. Diode).
- **Lumped and Distributed Elements**: Lumped elements are considered point entities where physical size is neglected. Distributed elements are spread over a length (e.g. transmission lines).
- **Linear and Nonlinear Elements**: Linear elements obey Ohm's law and superposition principle exhibiting a straight-line V-I characteristic. Nonlinear elements do not.

1.2 Series and Parallel Combinations

Methodology:

Equivalent Circuit Elements To simplify circuits, identical types of passive elements connected in series or parallel can be combined into a single equivalent element.

1. **Resistors in Series**: Elements carry the same current.

$$R_{eq} = R_1 + R_2 + \cdots + R_n$$

2. **Resistors in Parallel**: Elements share the same voltage.

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_n}$$

3. **Inductors in Series and Parallel**: Similar to resistors.
   - Series: $L_{eq} = L_1 + L_2 + \cdots + L_n$
   - Parallel: $\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2} + \cdots + \frac{1}{L_n}$
4. **Capacitors in Series and Parallel**: Opposite to resistors.
   - Series: $\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \cdots + \frac{1}{C_n}$
   - Parallel: $C_{eq} = C_1 + C_2 + \cdots + C_n$

Worked Example:

Calculate Equivalent Resistance and Capacitance Find the equivalent resistance R_{eq} of a 10 Ω and a 15 Ω resistor in parallel connected in series with a 4 Ω resistor. Similarly find the equivalent capacitance C_{eq} of a 6 μF and 3 μF capacitor in series connected in parallel with a 2 μF capacitor.

Solution:

1. **Resistance Calculation**:
   - Parallel combination of 10 Ω and 15 Ω:
   $$R_p = \frac{10 \times 15}{10 + 15} = \frac{150}{25} = 6\ \Omega$$
   - Series combination with 4 Ω:
   $$R_{eq} = R_p + 4 = 6 + 4 = 10\ \Omega$$
2. **Capacitance Calculation**:
   - Series combination of 6 μF and 3 μF:
   $$C_s = \frac{6 \times 3}{6 + 3} = \frac{18}{9} = 2\ \mu\text{F}$$
   - Parallel combination with 2 μF:
   $$C_{eq} = C_s + 2 = 2 + 2 = 4\ \mu\text{F}$$

1.3 Voltage and Current Division

Methodology:

Voltage and Current Division Rules

1. **Voltage Division Rule**: For a series circuit with total voltage V and resistors $R_1, R_2, \ldots, R_n$, the voltage drop V_k across resistor R_k is:
$$V_k = V \frac{R_k}{R_{eq}} \quad \text{where} \quad R_{eq} = R_1 + R_2 + \cdots + R_n$$
2. **Current Division Rule**: For a parallel circuit with total current I and two resistors R_1 and R_2, the current through R_1 is:
$$I_1 = I \frac{R_2}{R_1 + R_2}$$
For multiple parallel branches, $I_k = I \frac{R_{eq}}{R_k}$, where $\frac{1}{R_{eq}} = \sum \frac{1}{R_j}$.

Worked Example:

Apply Voltage and Current Division A 24 V source is connected across two series resistors $R_1 = 4\ \Omega$ and $R_2 = 8\ \Omega$. Find the voltage across R_2. Then assume a total current of 12 A enters a parallel combination of $R_3 = 6\ \Omega$ and $R_4 = 3\ \Omega$. Find the current through R_3.

Solution:

1. **Voltage Division**:
$$V_2 = 24 \left(\frac{8}{4 + 8} \right) = 24 \left(\frac{8}{12} \right) = 16\ \text{V}$$

2. **Current Division**:

$$I_3 = 12\left(\frac{3}{6+3}\right) = 12\left(\frac{3}{9}\right) = 4 \text{ A}$$

1.4 Energy Sources and Transformation

Methodology:

Transformation of Energy Sources An independent practical voltage source can be converted into an equivalent practical current source and vice versa. This requires the source resistance to remain identical.

1. **Voltage to Current Source**: A voltage source V_s in series with a resistor R_s is equivalent to a current source I_s in parallel with the same resistor R_s where:

$$I_s = \frac{V_s}{R_s}$$

2. **Current to Voltage Source**: A current source I_s in parallel with a resistor R_s is equivalent to a voltage source V_s in series with R_s where:

$$V_s = I_s R_s$$

Worked Example:

Convert Energy Sources Convert a 50 V voltage source with an internal series resistance of 5 Ω into an equivalent current source.

Solution:

1. Identify the voltage source $V_s = 50$ V and series resistance $R_s = 5$ Ω.

2. Calculate the equivalent current source value:

$$I_s = \frac{V_s}{R_s} = \frac{50}{5} = 10 \text{ A}$$

3. The equivalent circuit is a 10 A current source in parallel with a 5 Ω resistor.

1.5 Network Topology Fundamentals

Network topology deals with the geometric structure of a circuit completely independent of the types of elements used.

- **Node**: A point where two or more circuit elements join.
- **Branch**: A single path between two nodes containing at least one element.
- **Loop**: A closed path in a circuit.
- **Mesh**: A loop that does not contain any other loops within it.
- **Graph**: A set of nodes connected by line segments (branches).
- **Tree**: A connected subgraph that contains all the nodes of the graph but no closed loops.
- **Twig (Tree-branch)**: A branch that belongs to a tree.
- **Link**: A branch of a graph that does not belong to the selected tree.

Number of twigs $= N - 1$. Number of links $= B - (N - 1) = B - N + 1$ where B is the total branches and N is the total nodes.

1.6 Tie-Set and Cut-Set Matrices

Methodology:

Fundamental Tie-Set Matrix Formulation A fundamental tie-set is formed by adding exactly one link to a tree creating a unique loop. The loop current (link current) directs the tie-set.

1. Draw the graph and select a tree.
2. For each link form a fundamental loop with tree branches. The direction of the loop matches the link direction.
3. Form a matrix where rows correspond to loops (links) and columns correspond to branches.
4. Matrix elements are $+1$ if the branch is in the loop and in the same direction, -1 if in the opposite direction, and 0 if not in the loop.

Methodology:

Fundamental Cut-Set Matrix Formulation A fundamental cut-set separates the graph into two parts and contains exactly one tree branch (twig) with the rest being links. The cut-set direction matches the twig direction.

1. Draw the graph and select a tree.
2. For each twig define a cut-set that includes that twig and necessary links to divide the graph.
3. Form a matrix where rows are cut-sets (twigs) and columns are branches.
4. Elements are $+1$ if the branch direction matches the cut-set direction, -1 if opposite, and 0 if not in the cut-set.

Worked Example:

Formulate Tie-Set and Cut-Set Matrices Consider a directed graph with 4 nodes (a, b, c, d) and 6 branches (1, 2, 3, 4, 5, 6). Let the selected tree consist of branches 1, 2, 3. The remaining branches 4, 5, 6 are links. Assuming arbitrary orientations write the steps to form tie-set and cut-set matrices.
Solution:

1. **Identify Nodes and Branches**: $N = 4$, $B = 6$. Number of twigs $= 4 - 1 = 3$. Number of links $= 6 - 4 + 1 = 3$.
2. **Tie-Set Formation**: We have 3 fundamental loops corresponding to links 4, 5, and 6.
   - Loop 1 (Link 4): Formed by link 4 and twigs forming a closed path. The loop current I_4 is the link current.
   - Loop 2 (Link 5): Formed by link 5 and twigs forming a closed path. Loop current is I_5.
   - Loop 3 (Link 6): Formed by link 6 and twigs forming a closed path. Loop current is I_6.

   The rows of the Tie-Set matrix represent these loops showing Kirchhoff's Voltage Law equations in terms of branch voltages.
3. **Cut-Set Formation**: We have 3 fundamental cut-sets corresponding to twigs 1, 2, and 3.
   - Cut-set 1 (Twig 1): Divides the graph. Cut-set orientation matches twig 1. The cut-set defines a node-pair voltage (tree-branch voltage) V_1.
   - Cut-set 2 (Twig 2): Divides the graph. Cut-set orientation matches twig 2.
   - Cut-set 3 (Twig 3): Divides the graph. Cut-set orientation matches twig 3.

The rows of the Cut-Set matrix represent these cut-sets showing Kirchhoff's Current Law equations in terms of branch currents.

1.7 Mesh and Nodal Analysis

Methodology:

Mesh Current Method The mesh current method uses Kirchhoff's Voltage Law (KVL) to find unknown currents. It is applicable to planar circuits.

1. Identify the total number of meshes in the circuit.
2. Assign a mesh current to each mesh typically in the clockwise direction (e.g. $i_1, i_2, \ldots, i_m$).
3. Apply KVL around each mesh. Express the voltage drops across elements in terms of the mesh currents. For shared branches the net current is the algebraic sum of the individual mesh currents.
4. Solve the resulting system of linear equations to find the mesh currents.

Worked Example:

Solve Circuit using Mesh Current Method A 2-mesh circuit has a 10 V source in mesh 1 and a 5 V source opposing it in mesh 2. Mesh 1 has a 2 Ω resistor, mesh 2 has a 4 Ω resistor, and they share a 3 Ω resistor. Find mesh currents i_1 and i_2.
Solution:

1. **Mesh 1 KVL Equation**: Start from the voltage source and traverse clockwise:

$$10 - 2i_1 - 3(i_1 - i_2) = 0$$

Simplify:

$$5i_1 - 3i_2 = 10 \quad \text{— (Eq. 1)}$$

2. **Mesh 2 KVL Equation**: Traverse clockwise:

$$-5 - 4i_2 - 3(i_2 - i_1) = 0$$

Simplify:

$$-3i_1 + 7i_2 = -5 \quad \text{— (Eq. 2)}$$

3. **Solve Equations**: Multiply Eq. 1 by 3 and Eq. 2 by 5:

$$15i_1 - 9i_2 = 30$$

$$-15i_1 + 35i_2 = -25$$

Add them together:

$$26i_2 = 5 \implies i_2 = \frac{5}{26} \approx 0.192 \text{ A}$$

Substitute i_2 back into Eq. 1:

$$5i_1 - 3\left(\frac{5}{26}\right) = 10 \implies 5i_1 = 10 + \frac{15}{26} = \frac{275}{26}$$

$$i_1 = \frac{55}{26} \approx 2.115 \text{ A}$$

Methodology:

Loop Current Method The loop current method generalizes the mesh method to any network including non-planar ones. It uses fundamental loops defined by a selected tree.

1. Select a tree for the network graph.
2. Assign a loop current to each fundamental loop formed by adding one link at a time. The loop current is the link current.
3. Apply KVL around each fundamental loop.
4. Express branch voltages in terms of loop currents and solve the resulting system of equations.

Worked Example:

Solve Circuit using Loop Current Method Consider a network with 3 nodes and 4 branches. The selected tree leaves two links defining two fundamental loops with currents I_{L1} and I_{L2}. The KVL equations derived from the tie-set matrix are: $4I_{L1} - 2I_{L2} = 12$ and $-2I_{L1} + 6I_{L2} = 0$. Find the loop currents.
Solution:

1. **Analyze Equations**: From the second equation:

$$6I_{L2} = 2I_{L1} \implies I_{L1} = 3I_{L2}$$

2. **Substitute and Solve**: Substitute I_{L1} into the first equation:

$$4(3I_{L2}) - 2I_{L2} = 12$$

$$12I_{L2} - 2I_{L2} = 12 \implies 10I_{L2} = 12 \implies I_{L2} = 1.2 \text{ A}$$

3. **Find Remaining Current**:

$$I_{L1} = 3(1.2) = 3.6 \text{ A}$$

Methodology:

Nodal Voltage Method Node to Datum The nodal voltage method (node-to-datum) uses Kirchhoff's Current Law (KCL) to find unknown node voltages relative to a reference node.

1. Identify all nodes in the circuit and choose one as the reference node (datum or ground) assigning it a voltage of 0 V.
2. Assign node voltages (e.g. $V_1, V_2, \ldots, V_n$) to the remaining non-reference nodes.
3. Apply KCL at each non-reference node. Express currents leaving the node in terms of node voltages and admittances (or resistances).
4. Solve the resulting system of linear equations to find the node voltages.

Worked Example:

Solve Circuit using Node-to-Datum Method A circuit has two non-reference nodes V_1 and V_2. A 5 A current source enters node 1. A 2 Ω resistor connects node 1 to ground, a 4 Ω resistor connects node 2 to ground, and a 2 Ω resistor connects node 1 to node 2. Find V_1 and V_2.
Solution:

1. **Node 1 KCL Equation**: Current entering equals current leaving.

$$5 = \frac{V_1 - 0}{2} + \frac{V_1 - V_2}{2}$$

Multiply by 2:

$$10 = V_1 + V_1 - V_2 \implies 2V_1 - V_2 = 10 \quad \text{— (Eq. 1)}$$

2. **Node 2 KCL Equation**: Current entering equals current leaving.

$$\frac{V_1 - V_2}{2} = \frac{V_2 - 0}{4}$$

Multiply by 4:

$$2(V_1 - V_2) = V_2 \implies 2V_1 - 2V_2 = V_2 \implies 2V_1 - 3V_2 = 0 \quad \text{— (Eq. 2)}$$

3. **Solve Equations**: From Eq. 2, $2V_1 = 3V_2$. Substitute into Eq. 1:

$$3V_2 - V_2 = 10 \implies 2V_2 = 10 \implies V_2 = 5 \text{ V}$$

Substitute V_2 into $2V_1 = 3(5)$:

$$2V_1 = 15 \implies V_1 = 7.5 \text{ V}$$

Methodology:

Node-Pair Voltage Method Rather than measuring all voltages against a single datum node, voltages can be assigned to independent node-pairs. This is heavily tied to the graph topology.

1. Construct a graph of the network and choose a tree.
2. The branches of the tree define a set of independent node-pair voltages (tree-branch voltages).
3. Apply KCL in the form of fundamental cut-sets for each tree branch.
4. Express branch currents in terms of the tree-branch voltages using Ohm's law.
5. Solve for the node-pair voltages.

CHAPTER 2

Two Port Networks

2.1 Network Classification and Terminology

In network analysis, electrical circuits and elements are classified based on their operating characteristics and behavioral properties.

- **Passive and Active:** Passive networks do not contain any internal energy source (e.g. Resistors, Capacitors, Inductors). Active networks contain energy sources (e.g. Voltage or Current sources) capable of supplying power indefinitely.
- **Linear and Non-linear:** Linear networks obey Ohm's Law and the principle of superposition; their V-I characteristics are straight lines passing through the origin. Non-linear networks do not follow a linear relationship.
- **Lumped and Distributed:** Lumped networks have physically separable elements. Distributed networks have resistance, inductance, and capacitance distributed along their entire length (e.g. transmission lines).
- **Unilateral and Bilateral:** Bilateral networks conduct current equally well in both directions, offering the same impedance (e.g. Resistors). Unilateral networks conduct differently depending on current polarity (e.g. Diodes).
- **Single Port and Two Port:** A single port network has only one pair of terminals for either input or output. A two port network has one pair of terminals designated for input (Port 1) and a separate pair for output (Port 2).

2.2 T and Pi Networks

Two port networks are often structurally represented using T or π (Pi) topologies.

- **Symmetrical T or π Networks:** A T-network is symmetrical if its series arms are identical ($Z_1 = Z_2$). A π-network is symmetrical if its shunt arms are identical ($Z_A = Z_B$). Their electrical characteristics remain the same when input and output ports are interchanged.
- **Asymmetrical T or π Networks:** The structural arms are unequal ($Z_1 \neq Z_2$ in a T-network, $Z_A \neq Z_B$ in a π-network).

Methodology:

T to Pi and Pi to T Conversion A T-section (Star) consists of three impedances Z_1, Z_2, and Z_3 meeting at a common node. A π-section (Delta) consists of three impedances Z_A, Z_B, and Z_C forming a closed loop between three terminals.

1. **T to Pi (Star to Delta) Conversion:**

$$Z_A = \frac{Z_1Z_2 + Z_2Z_3 + Z_3Z_1}{Z_2}$$

$$Z_B = \frac{Z_1Z_2 + Z_2Z_3 + Z_3Z_1}{Z_3}$$

$$Z_C = \frac{Z_1Z_2 + Z_2Z_3 + Z_3Z_1}{Z_1}$$

2. **Pi to T (Delta to Star) Conversion:**

$$Z_1 = \frac{Z_B Z_C}{Z_A + Z_B + Z_C}$$

$$Z_2 = \frac{Z_C Z_A}{Z_A + Z_B + Z_C}$$

$$Z_3 = \frac{Z_A Z_B}{Z_A + Z_B + Z_C}$$

Worked Example:

Convert T Network to Pi Network Given a T-network with series arms $Z_1 = 10\,\Omega$ and $Z_2 = 20\,\Omega$ and a shunt arm $Z_3 = 40\,\Omega$. Find the equivalent π-network impedances Z_A, Z_B, and Z_C.

Step 1: Calculate the numerator product sum.

$$\text{Numerator} = Z_1Z_2 + Z_2Z_3 + Z_3Z_1$$

$$\text{Numerator} = (10)(20) + (20)(40) + (40)(10) = 200 + 800 + 400 = 1400$$

Step 2: Calculate Delta impedances.

$$Z_A = \frac{1400}{Z_2} = \frac{1400}{20} = 70\,\Omega$$

$$Z_B = \frac{1400}{Z_3} = \frac{1400}{40} = 35\,\Omega$$

$$Z_C = \frac{1400}{Z_1} = \frac{1400}{10} = 140\,\Omega$$

The equivalent π-network has impedances $Z_A = 70\,\Omega$, $Z_B = 35\,\Omega$ and $Z_C = 140\,\Omega$.

2.3 Network Impedances

Methodology:

Definitions of Network Impedances

1. **Driving Point Impedance:** The ratio of voltage to current measured at the same port (e.g. $Z_{11} = V_1/I_1$).

2. **Transfer Impedance:** The ratio of voltage at one port to the current at the other port (e.g. $Z_{21} = V_2/I_1$).

3. **Input Impedance:** The net equivalent impedance measured at the input terminals when the output is terminated by a specified load impedance.

4. **Output Impedance:** The net equivalent impedance measured at the output terminals when the input is terminated by a specified source impedance.

5. **Terminating Impedance:** The physical impedance connected across the output terminals acting as a load.

6. **Image Impedances (Z_{i1} and Z_{i2}):** Two impedances such that if port 2 is terminated in Z_{i2}, the input impedance at port 1 is exactly Z_{i1}, and if port 1 is terminated in Z_{i1}, the input impedance at port 2 is exactly Z_{i2}.

2.3.1 Characteristic Impedance of Symmetrical Networks

For perfectly symmetrical networks, the two image impedances are equal ($Z_{i1} = Z_{i2} = Z_0$). This unified value is called the **Characteristic Impedance (Z_0)**.

Methodology:

Characteristic Impedance Formulae Let a symmetrical network be defined conventionally by total series impedance Z_1 and total shunt impedance Z_2.

1. **Characteristic Impedance of Symmetrical T-Network (Z_{0T}):** Here, the two series arms are $Z_1/2$ and the single shunt arm is Z_2.

$$Z_{0T} = \sqrt{Z_{oc}Z_{sc}} = \sqrt{\frac{Z_1^2}{4} + Z_1 Z_2}$$

where Z_{oc} is open-circuit impedance and Z_{sc} is short-circuit impedance.

2. **Characteristic Impedance of Symmetrical π-Network ($Z_{0\pi}$):** Here, the single series arm is Z_1 and the two shunt arms are $2Z_2$.

$$Z_{0\pi} = \sqrt{Z_{oc}Z_{sc}} = \sqrt{\frac{Z_1 Z_2}{1 + \frac{Z_1}{4Z_2}}} = \frac{Z_1 Z_2}{Z_{0T}}$$

3. **Mathematical Relation between Z_{0T} and $Z_{0\pi}$:**

$$Z_{0T} \times Z_{0\pi} = Z_1 Z_2$$

Worked Example:

Compute Characteristic Impedances A symmetrical T-network has identical series arms of $100\,\Omega$ each and a central shunt arm of $500\,\Omega$. Calculate its characteristic impedance Z_{0T} and the characteristic impedance of its equivalent π-network $Z_{0\pi}$.
Step 1: Identify standard parameters Z_1 and Z_2. For a T-network, series arm $= Z_1/2 = 100 \implies Z_1 = 200\,\Omega$. Shunt arm $= Z_2 = 500\,\Omega$.
Step 2: Calculate Z_{0T}.

$$Z_{0T} = \sqrt{\frac{Z_1^2}{4} + Z_1 Z_2} = \sqrt{\frac{200^2}{4} + (200)(500)}$$

$$Z_{0T} = \sqrt{10000 + 100000} = \sqrt{110000} \approx 331.66\,\Omega$$

Step 3: Calculate $Z_{0\pi}$. Using the relationship $Z_{0T}Z_{0\pi} = Z_1 Z_2$:

$$Z_{0\pi} = \frac{Z_1 Z_2}{Z_{0T}} = \frac{(200)(500)}{331.66} = \frac{100000}{331.66} \approx 301.51\,\Omega$$

2.4 Two Port Network Parameters

A linear, bilateral two port network is completely characterized by four variable sets: input voltage V_1, input current I_1, output voltage V_2, and output current I_2.

2.4.1 Z Parameters (Open Circuit Impedance)

Methodology:

Z Parameters Calculation The parameters relate voltages to currents via:

$$V_1 = Z_{11}I_1 + Z_{12}I_2$$

$$V_2 = Z_{21}I_1 + Z_{22}I_2$$

1. **Open Output Port ($I_2 = 0$):** $Z_{11} = \frac{V_1}{I_1}\Big|_{I_2=0}$ (Input driving point impedance)

   $Z_{21} = \frac{V_2}{I_1}\Big|_{I_2=0}$ (Forward transfer impedance)

2. **Open Input Port ($I_1 = 0$):** $Z_{12} = \frac{V_1}{I_2}\Big|_{I_1=0}$ (Reverse transfer impedance)

   $Z_{22} = \frac{V_2}{I_2}\Big|_{I_1=0}$ (Output driving point impedance)

Conditions: Reciprocity requires $Z_{12} = Z_{21}$. Symmetry requires $Z_{11} = Z_{22}$.

Worked Example:

Find Z Parameters of a T Network Find the Z parameters for a purely resistive T-network with series arms $R_1 = 5\,\Omega$, $R_2 = 10\,\Omega$, and a shunt arm $R_3 = 20\,\Omega$.
Step 1: Open circuit the output ($I_2 = 0$). Apply a test voltage V_1 at port 1. Current flows strictly through R_1 and R_3.

$$V_1 = I_1(R_1 + R_3) \implies Z_{11} = \frac{V_1}{I_1} = 5 + 20 = 25\,\Omega$$

The voltage at port 2 is simply the drop across R_3 (since $I_2 = 0$, drop across R_2 is 0).

$$V_2 = I_1R_3 \implies Z_{21} = \frac{V_2}{I_1} = 20\,\Omega$$

Step 2: Open circuit the input ($I_1 = 0$). Apply a test voltage V_2 at port 2. Current flows strictly through R_2 and R_3.

$$V_2 = I_2(R_2 + R_3) \implies Z_{22} = \frac{V_2}{I_2} = 10 + 20 = 30\,\Omega$$

The voltage at port 1 is the drop across R_3.

$$V_1 = I_2R_3 \implies Z_{12} = \frac{V_1}{I_2} = 20\,\Omega$$

The Z-parameter matrix is:

$$\begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} = \begin{bmatrix} 25 & 20 \\ 20 & 30 \end{bmatrix}\Omega$$

2.4.2 Y Parameters (Short Circuit Admittance)

Methodology:

Y Parameters Calculation The parameters relate currents to voltages via:

$$I_1 = Y_{11}V_1 + Y_{12}V_2$$

$$I_2 = Y_{21}V_1 + Y_{22}V_2$$

1. **Short Output Port ($V_2 = 0$):** $Y_{11} = \frac{I_1}{V_1}\Big|_{V_2=0}$ (Input driving point admittance)

   $Y_{21} = \frac{I_2}{V_1}\Big|_{V_2=0}$ (Forward transfer admittance)

2. **Short Input Port ($V_1 = 0$):** $Y_{12} = \frac{I_1}{V_2}\Big|_{V_1=0}$ (Reverse transfer admittance)

   $Y_{22} = \frac{I_2}{V_2}\Big|_{V_1=0}$ (Output driving point admittance)

Conditions: Reciprocity requires $Y_{12} = Y_{21}$. Symmetry requires $Y_{11} = Y_{22}$.

2.4.3 ABCD Parameters (Transmission)

Methodology:

ABCD Parameters Calculation Also known as transmission parameters. They relate input variables to output variables:

$$V_1 = AV_2 - BI_2$$

$$I_1 = CV_2 - DI_2$$

1. **Open Output Port ($I_2 = 0$):** $A = \frac{V_1}{V_2}\Big|_{I_2=0}$ (Reverse voltage ratio)

   $C = \frac{I_1}{V_2}\Big|_{I_2=0}$ (Transfer admittance)

2. **Short Output Port ($V_2 = 0$):** $B = -\frac{V_1}{I_2}\Big|_{V_2=0}$ (Transfer impedance)

   $D = -\frac{I_1}{I_2}\Big|_{V_2=0}$ (Reverse current ratio)

Conditions: Reciprocity requires $AD - BC = 1$. Symmetry requires $A = D$.

Worked Example:

Calculate ABCD Parameters of a T Network Consider a symmetrical T-network with series impedances $Z_1 = 10\,\Omega$, $Z_2 = 10\,\Omega$ and a shunt impedance $Z_3 = 20\,\Omega$. Find its ABCD parameters.
Step 1: Open Circuit Output ($I_2 = 0$). Apply voltage V_1. Current I_1 flows through Z_1 and Z_3.

$$V_1 = I_1(Z_1 + Z_3) = I_1(10 + 20) = 30I_1$$

Voltage V_2 is the voltage across Z_3:

$$V_2 = I_1 Z_3 = 20I_1$$

Calculate A and C:

$$A = \frac{V_1}{V_2} = \frac{30I_1}{20I_1} = 1.5$$

$$C = \frac{I_1}{V_2} = \frac{I_1}{20I_1} = 0.05\,S$$

Step 2: Short Circuit Output ($V_2 = 0$). With port 2 shorted, Z_2 is perfectly in parallel with Z_3. The total equivalent input impedance is:

$$Z_{in} = Z_1 + \frac{Z_2 Z_3}{Z_2 + Z_3} = 10 + \frac{(10)(20)}{10 + 20} = 10 + \frac{200}{30} = 16.67\,\Omega$$

Thus, $V_1 = 16.67 I_1$. Using the current divider rule, the actual current leaving port 2 is positive, but ABCD convention uses I_2 entering, making leaving current $-I_2$:

$$-I_2 = I_1\left(\frac{Z_3}{Z_2 + Z_3}\right) = I_1\left(\frac{20}{30}\right) = \frac{2}{3}I_1 \implies I_1 = -1.5I_2$$

Substitute I_1 back into the voltage equation:

$$V_1 = 16.67(-1.5I_2) = -25I_2$$

Calculate B and D:

$$B = -\frac{V_1}{I_2} = -\frac{-25I_2}{I_2} = 25\,\Omega$$

$$D = -\frac{I_1}{I_2} = -\frac{-1.5I_2}{I_2} = 1.5$$

Check: Symmetry holds ($A = D = 1.5$). Reciprocity holds ($AD - BC = (1.5)(1.5) - (25)(0.05) = 2.25 - 1.25 = 1$).

2.4.4 h Parameters (Hybrid)

Methodology:

h Parameters Calculation Hybrid parameters mix voltage and current to establish relations:

$$V_1 = h_{11}I_1 + h_{12}V_2$$

$$I_2 = h_{21}I_1 + h_{22}V_2$$

1. **Short Output Port ($V_2 = 0$):** $h_{11} = \frac{V_1}{I_1}\Big|_{V_2=0}$ (Input impedance with output shorted)

   $h_{21} = \frac{I_2}{I_1}\Big|_{V_2=0}$ (Forward current gain)

2. **Open Input Port ($I_1 = 0$):** $h_{12} = \frac{V_1}{V_2}\Big|_{I_1=0}$ (Reverse voltage gain)

   $h_{22} = \frac{I_2}{V_2}\Big|_{I_1=0}$ (Output admittance with input open)

Conditions: Reciprocity requires $h_{12} = -h_{21}$. Symmetry requires $h_{11}h_{22} - h_{12}h_{21} = 1$.

Worked Example:

Calculate h Parameters Given Z Parameters Given the calculated Z parameters of a network: $Z_{11} = 40\,\Omega$, $Z_{12} = 20\,\Omega$, $Z_{21} = 20\,\Omega$, and $Z_{22} = 30\,\Omega$. Calculate the corresponding h-parameters.

Step 1: Write down the h-parameters in terms of Z-parameters.

$$h_{11} = \frac{\Delta Z}{Z_{22}}, \quad h_{12} = \frac{Z_{12}}{Z_{22}}, \quad h_{21} = -\frac{Z_{21}}{Z_{22}}, \quad h_{22} = \frac{1}{Z_{22}}$$

where the determinant is $\Delta Z = Z_{11}Z_{22} - Z_{12}Z_{21}$.

Step 2: Calculate ΔZ.

$$\Delta Z = (40)(30) - (20)(20) = 1200 - 400 = 800\,\Omega^2$$

Step 3: Compute individual h-parameters.

$$h_{11} = \frac{800}{30} = 26.67\,\Omega$$

$$h_{12} = \frac{20}{30} = 0.667$$

$$h_{21} = -\frac{20}{30} = -0.667$$

$$h_{22} = \frac{1}{30} \approx 0.0333\,S$$

The h-parameter matrix is:

$$\begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} = \begin{bmatrix} 26.67\,\Omega & 0.667 \\ -0.667 & 0.0333\,S \end{bmatrix}$$

CHAPTER 3

Network Theorems

3.1 Kirchhoff's Laws

3.1.1 Kirchhoff's Current Law (KCL)

Methodology:

Applying KCL 1. Identify the principal nodes in the circuit where three or more branches meet. 2. Assume directions for unknown currents in the branches if they are not already given. 3. Apply the rule: The algebraic sum of currents entering a node is zero ($\sum I = 0$). By convention, currents entering the node are taken as positive, and currents leaving are taken as negative.

Worked Example:

Finding Unknown Branch Currents using KCL Problem: At a principal node in a circuit, four branches meet. Currents in three branches are $I_1 = 5$ A (entering), $I_2 = 2$ A (leaving), and $I_3 = 4$ A (entering). Find the current I_4 and its direction.
Step 1: Assign polarities based on direction Entering currents are positive: $I_1 = +5$ A and $I_3 = +4$ A. Leaving currents are negative: $I_2 = -2$ A.
Step 2: Apply KCL equation $\sum I = 0$ $I_1 + I_2 + I_3 + I_4 = 0$ $(+5) + (-2) + (+4) + I_4 = 0$
Step 3: Solve for unknown current $7 + I_4 = 0$ $I_4 = -7$ A
The negative sign indicates that the assumed positive direction (entering) is incorrect. Therefore, I_4 is 7 A leaving the node.

3.1.2 Kirchhoff's Voltage Law (KVL)

Methodology:

Applying KVL 1. Identify all closed loops (or meshes) in the circuit. 2. Assign a direction of traversal (clockwise or counter-clockwise) for each loop. 3. Assign voltage polarities across resistors based on the assumed current direction (positive at entry, negative at exit). 4. Apply the rule: The algebraic sum of all voltages around any closed loop is zero ($\sum V = 0$). Note the first sign encountered while traversing the loop to write the equation consistently.

Worked Example:

Finding Voltage using KVL Problem: A series loop contains a 12 V battery and two resistors R_1 and R_2. The voltage drop across R_1 is 4 V. Find the voltage drop across R_2.
Step 1: Define the loop traversal Assume clockwise traversal. The battery voltage rises from $-$ to $+$ (so $+12$ V). The voltage drops across resistors are $-V_{R1}$ and $-V_{R2}$.
Step 2: Write KVL equation $12 - V_{R1} - V_{R2} = 0$
Step 3: Substitute known values and solve $12 - 4 - V_{R2} = 0$ $8 - V_{R2} = 0$ $V_{R2} = 8$ V

3.2 Mesh Analysis

Methodology:

Mesh Analysis Method 1. Identify the number of independent meshes (loops that do not contain any other loop). 2. Assign a mesh current to each mesh (e.g., I_1 and I_2). Usually, the clockwise direction is assumed for all meshes. 3. Apply KVL to each mesh to write equations in terms of mesh currents. For shared branches, the net current is the algebraic sum of the individual mesh currents. 4. Solve the simultaneous equations to find the unknown mesh currents.

Worked Example:

Solving a Two Mesh Circuit Problem: A circuit has two meshes. Mesh 1 contains a 10V source, a 2Ω resistor, and shares a 4Ω resistor with Mesh 2. Mesh 2 contains the shared 4Ω resistor, a 6Ω resistor, and a 5V source opposing the assumed current. Find mesh currents I_1 and I_2.
Step 1: Write KVL for Mesh 1 Assuming clockwise I_1: $10 - 2I_1 - 4(I_1 - I_2) = 0$ $10 - 6I_1 + 4I_2 = 0$ $6I_1 - 4I_2 = 10$ — (Equation 1)
Step 2: Write KVL for Mesh 2 Assuming clockwise I_2: $-4(I_2 - I_1) - 6I_2 - 5 = 0$ $-10I_2 + 4I_1 = 5$ $-4I_1 + 10I_2 = -5$ — (Equation 2)
Step 3: Solve simultaneous equations Multiply Equation 1 by 2.5: $15I_1 - 10I_2 = 25$ Add to Equation 2: $(15I_1 - 10I_2) + (-4I_1 + 10I_2) = 25 - 5$ $11I_1 = 20 \implies I_1 = 1.818$ A
Substitute I_1 into Equation 1: $6(1.818) - 4I_2 = 10$ $10.908 - 10 = 4I_2 \implies 4I_2 = 0.908 \implies I_2 = 0.227$ A

3.3 Nodal Analysis

Methodology:

Nodal Analysis Method 1. Identify all nodes in the circuit. 2. Select one node as the reference node (ground) with a potential of 0 V. 3. Assign node voltages (e.g., V_1 and V_2) to the remaining non-reference nodes with respect to ground. 4. Apply KCL at each non-reference node, expressing branch currents in terms of node voltages using Ohm's Law ($I = (V_A - V_B)/R$). Assume currents leave the node unless a current source dictates otherwise. 5. Solve the resulting simultaneous equations for the unknown node voltages.

Worked Example:

Solving a Two Node Circuit Problem: A circuit has two non-reference nodes V_1 and V_2. A 2A current source enters V_1. A 1Ω resistor connects V_1 to ground. A 2Ω resistor connects V_1 and V_2. A 3Ω resistor connects V_2 to ground. Find node voltages V_1 and V_2.
Step 1: Apply KCL at Node 1 Current entering = Currents leaving $2 = \frac{V_1}{1} + \frac{V_1 - V_2}{2}$ Multiply the entire equation by 2 to clear fractions: $4 = 2V_1 + V_1 - V_2$ $3V_1 - V_2 = 4$ — (Equation 1)
Step 2: Apply KCL at Node 2 Assume all unknown currents are leaving: $0 = \frac{V_2 - V_1}{2} + \frac{V_2}{3}$ Multiply by 6: $0 = 3(V_2 - V_1) + 2V_2$ $-3V_1 + 5V_2 = 0$ $3V_1 = 5V_2$ — (Equation 2)
Step 3: Solve equations Substitute $3V_1$ from Eq 2 into Eq 1: $5V_2 - V_2 = 4$ $4V_2 = 4 \implies V_2 = 1$ V
Find V_1: $3V_1 = 5(1) \implies V_1 = \frac{5}{3} = 1.67$ V

3.4 Principle of Duality

Methodology:

Finding the Dual of a Network 1. Place a node inside each mesh of the original circuit. Place an extra reference node outside the entire circuit. 2. Connect these nodes by drawing dashed lines through each element of the original circuit. 3. Replace each crossed element with its dual element in the new branch. Dual Pairs include: - Voltage Source ↔ Current Source - Resistance ↔ Conductance - Inductance ↔

Capacitance - Series Connection ↔ Parallel Connection - Mesh ↔ Node

Worked Example:

Drawing Dual of a Series R-L-C Circuit Problem: A circuit consists of a voltage source V in series with a resistor R, an inductor L, and a capacitor C. Describe its dual network.
Step 1: Identify original elements and topology Original elements: Series connection of Voltage Source (V), Resistance (R), Inductance (L), and Capacitance (C). Topology: 1 independent mesh circuit.
Step 2: Determine dual elements and topology Dual topology: 1 non-reference node and 1 reference node (Parallel connection). Dual elements: - Voltage source $V \to$ Current source I - Resistance $R \to$ Conductance G - Inductance $L \to$ Capacitance C_{dual} - Capacitance $C \to$ Inductance L_{dual}
Step 3: Construct dual circuit The dual circuit is a parallel G-C-L circuit driven by a current source I. All elements I, G, C_{dual}, and L_{dual} are connected in parallel between the single non-reference node and the reference node.

3.5 Superposition Theorem

Methodology:

Superposition Theorem Method 1. Consider one independent source at a time. Turn off all other independent sources. - Replace independent voltage sources with a short circuit (0V). - Replace independent current sources with an open circuit (0A). 2. Calculate the response (voltage or current) in the desired branch due to the active source alone. 3. Repeat steps 1 and 2 for each independent source in the circuit. 4. Calculate the total response by algebraically adding the individual responses. Pay strict attention to assumed current directions and voltage polarities.

Worked Example:

Finding Current using Superposition Problem: A circuit has a 10V voltage source and a 5A current source. A 2Ω resistor is connected in series with the 10V source, and this combination is in parallel with a 4Ω resistor and the 5A current source. Find the current through the 4Ω resistor downwards.
Step 1: Response due to 10V source alone Open the 5A current source. The circuit becomes a simple series loop with the 10V source, 2Ω resistor, and 4Ω resistor. $I'_{4\Omega} = \frac{10}{2+4} = \frac{10}{6} = 1.67$ A (downwards)
Step 2: Response due to 5A source alone Short the 10V voltage source. The 5A source is now in parallel with the 2Ω and 4Ω resistors. Using the current divider rule to find current through 4Ω: $I''_{4\Omega} = 5 \times \frac{2}{2+4} = 5 \times \frac{2}{6} = 1.67$ A (downwards)
Step 3: Algebraic sum of responses Since both current components flow downwards, they add up. $I_{Total} = I'_{4\Omega} + I''_{4\Omega} = 1.67 + 1.67 = 3.34$ A (downwards)

3.6 Thevenin's Theorem

Methodology:

Thevenin Theorem Method 1. Remove the load resistor (R_L) from the circuit terminals (let's call them A and B) where the equivalent circuit is to be found. 2. Calculate the open-circuit voltage (V_{th}) across terminals A and B. This is the Thevenin voltage. 3. Turn off all independent sources (short voltage sources, open current sources) and calculate the equivalent resistance (R_{th}) looking into terminals A and B. 4. Draw the Thevenin equivalent circuit: a voltage source V_{th} in series with resistance R_{th}. 5. Reconnect R_L across A and B and find the load current: $I_L = \frac{V_{th}}{R_{th}+R_L}$.

Worked Example:

Finding Load Current using Thevenin Theorem Problem: A 12V source is in series with a 4Ω resistor. This combination is in parallel with a 6Ω resistor. A load resistor $R_L = 2\Omega$ is connected across the 6Ω resistor. Find the current through R_L.
Step 1: Remove load and find Vth Remove R_L. Terminals A and B are across the 6Ω resistor. The current in the resulting single loop is $I = \frac{12}{4+6} = 1.2$ A. V_{th} = Voltage across 6Ω resistor $= 1.2 \times 6 = 7.2$ V.
Step 2: Find Rth Short the 12V source. Looking into terminals A and B, the 4Ω and 6Ω resistors appear in parallel. $R_{th} = \frac{4\times6}{4+6} = \frac{24}{10} = 2.4\Omega$.
Step 3: Calculate load current Reconnect $R_L = 2\Omega$ to the Thevenin equivalent circuit. $I_L = \frac{V_{th}}{R_{th}+R_L} = \frac{7.2}{2.4+2} = \frac{7.2}{4.4} = 1.636$ A.

3.7 Norton's Theorem

Methodology:

Norton Theorem Method 1. Remove the load resistor (R_L) and short-circuit the load terminals A and B. 2. Calculate the short-circuit current (I_N) flowing through terminal A to B. This is the Norton current. 3. Turn off all independent sources and calculate the equivalent resistance (R_N) looking into the open terminals. Note that $R_N = R_{th}$. 4. Draw the Norton equivalent circuit: a current source I_N in parallel with resistance R_N. 5. Reconnect R_L and find the load current using the current divider rule: $I_L = I_N \times \frac{R_N}{R_N+R_L}$.

Worked Example:

Finding Load Current using Norton Theorem Problem: Using the same circuit as the Thevenin example (12V source with 4Ω series resistor, parallel with 6Ω resistor, $R_L = 2\Omega$ across 6Ω). Find current through R_L using Norton's Theorem.
Step 1: Short load terminals and find IN Remove R_L and short the terminals across the 6Ω resistor. The short circuit entirely bypasses the 6Ω resistor. The only active resistance in the circuit loop is the 4Ω resistor. $I_N = \frac{12\text{ V}}{4\Omega} = 3$ A.
Step 2: Find RN Turn off the 12V source (replace with a short circuit). Looking into the open terminals, the 4Ω and 6Ω resistors are in parallel. $R_N = \frac{4\times6}{4+6} = 2.4\Omega$.
Step 3: Calculate load current Draw Norton equivalent: 3A source in parallel with 2.4Ω resistor. Connect $R_L = 2\Omega$ in parallel across them. Using current divider rule: $I_L = I_N \times \frac{R_N}{R_N+R_L} = 3 \times \frac{2.4}{2.4+2} = 3 \times \frac{2.4}{4.4} = 1.636$ A.

3.8 Maximum Power Transfer Theorem

Methodology:

Maximum Power Transfer Method 1. Remove the variable load resistor R_L from the circuit. 2. Find the Thevenin equivalent circuit (V_{th} and R_{th}) looking into the load terminals. 3. Apply the theorem condition: For maximum power transfer to a purely resistive load, $R_L = R_{th}$. 4. Calculate the maximum power delivered to the load using the formula: $P_{max} = \frac{V_{th}^2}{4R_{th}}$.

Worked Example:

Calculating Maximum Power Transfer Problem: A circuit has a Thevenin equivalent voltage of 20V and a Thevenin resistance of 5Ω. A variable load resistor R_L is connected across the terminals. Find the value of R_L for maximum power transfer and the maximum power delivered.
Step 1: Apply condition for maximum power According to the theorem, for maximum power transfer, the load resistance must equal the Thevenin equivalent resistance. $R_L = R_{th} = 5\Omega$.

Step 2: Calculate maximum power Using the direct formula for maximum power: $P_{max} = \frac{V_{th}^2}{4R_{th}}$ $P_{max} = \frac{20^2}{4\times5} = \frac{400}{20} = 20$ W.

3.9 Reciprocity Theorem

Methodology:

Reciprocity Theorem Method 1. In a linear, bilateral, single-source network, measure the response (current I) in a specific branch due to an excitation (voltage V) in another branch. Calculate the transfer resistance $R_{transfer} = \frac{V}{I}$. 2. Swap the positions of the ideal voltage source and the ammeter (which measures the short-circuit current). 3. Recalculate or measure the new response current I' in the new branch. 4. If $I = I'$, the theorem is verified. The transfer resistance remains constant regardless of the direction of signal flow.

Worked Example:

Verifying Reciprocity Theorem Problem: A 10V source in branch A produces a current of 2A in branch B of a purely resistive linear network. If the 10V source is moved to branch B, what will be the current in branch A? Verify reciprocity.

Step 1: Analyze initial state Excitation in branch A: $V_A = 10$ V. Response in branch B: $I_B = 2$ A. Transfer resistance $R_T = \frac{V_A}{I_B} = \frac{10}{2} = 5\Omega$.

Step 2: Apply Reciprocity Theorem Move excitation to branch B: $V_B = 10$ V. According to the Reciprocity Theorem, the ratio of excitation to response must remain constant for a linear bilateral network. $R_T = \frac{V_B}{I_A} = 5\Omega$.

Step 3: Calculate new response $I_A = \frac{V_B}{R_T} = \frac{10}{5} = 2$ A. Since the response current is identical (2A) when the source and measurement points are swapped, the Reciprocity Theorem is successfully verified.

CHAPTER 4

Resonance and Coupled Circuits

4.1 Quality Factor (Q-factor) of Coil and Capacitor

Methodology:

Calculating Quality Factor The Quality Factor (Q-factor) represents the ratio of reactive power to active power in a component. It is a measure of the "goodness" of a reactive component, indicating how close it is to an ideal component.

1. Q-factor of a Coil (Inductor): A practical inductor has an internal series resistance R_L.

- Formula: $Q_L = \frac{X_L}{R_L} = \frac{2\pi f L}{R_L}$
- Where X_L is the inductive reactance, L is inductance, f is frequency, and R_L is the effective series resistance of the coil.

2. Q-factor of a Capacitor: A practical capacitor has a small internal series resistance R_C (or equivalent parallel resistance). Using the series equivalent model:

- Formula: $Q_C = \frac{X_C}{R_C} = \frac{1}{2\pi f C R_C}$
- Where X_C is the capacitive reactance, C is capacitance, and R_C is the effective series resistance of the capacitor.

Worked Example:

Coil and Capacitor Q-factor Calculation A coil has an inductance of 50 mH and a resistance of 10 Ω. A capacitor has a capacitance of 10 μF and an equivalent series resistance of 0.5 Ω. Calculate the Q-factor of both components at a frequency of 1 kHz.

Step 1: Calculate the Q-factor of the coil. Given parameters: Frequency $f = 1000$ Hz, $L = 50 \times 10^{-3}$ H, $R_L = 10\ \Omega$.

$$X_L = 2\pi f L = 2\pi(1000)(50 \times 10^{-3}) = 314.16\ \Omega$$

$$Q_L = \frac{X_L}{R_L} = \frac{314.16}{10} = 31.42$$

Step 2: Calculate the Q-factor of the capacitor. Given parameters: Frequency $f = 1000$ Hz, $C = 10 \times 10^{-6}$ F, $R_C = 0.5\ \Omega$.

$$X_C = \frac{1}{2\pi f C} = \frac{1}{2\pi(1000)(10 \times 10^{-6})} = 15.92\ \Omega$$

$$Q_C = \frac{X_C}{R_C} = \frac{15.92}{0.5} = 31.84$$

4.2 Series Resonant Circuit

Methodology:

Analysis of Series Resonant Circuit In a series RLC circuit, resonance occurs when the inductive reactance exactly equals the capacitive reactance ($X_L = X_C$).

1. Resonance Frequency (f_r): The frequency at which the reactances cancel each other out.

$$f_r = \frac{1}{2\pi\sqrt{LC}} \text{ Hz}$$

2. Impedance at Resonance (Z_r): Since $X_L = X_C$, the net reactance is zero. The total impedance becomes purely resistive and reaches its minimum value.

$$Z_r = R\ \Omega$$

3. Current at Resonance (I_r): Because the circuit impedance is at its minimum, the current flowing through the circuit reaches its maximum value.

$$I_r = \frac{V}{R} \text{ A}$$

4. Quality Factor and Selectivity (Q_0): The Q-factor of the series circuit (also known as selectivity) represents the voltage magnification across the inductor or capacitor at resonance.

$$Q_0 = \frac{1}{R}\sqrt{\frac{L}{C}} = \frac{2\pi f_r L}{R}$$

5. Bandwidth (BW): The difference between the upper (f_2) and lower (f_1) half-power frequencies. A higher selectivity (high Q_0) results in a narrower bandwidth.

$$BW = f_2 - f_1 = \frac{R}{2\pi L} \text{ Hz}$$

Alternatively:

$$BW = \frac{f_r}{Q_0}$$

Worked Example:

Solving a Series Resonant Circuit A series circuit consists of a resistance of 5 Ω, an inductance of 20 mH, and a capacitance of 0.05 μF. It is connected to a 100 V variable frequency AC supply. Find (1) the resonance frequency (2) impedance and current at resonance (3) Q-factor and (4) bandwidth.

Step 1: Calculate the resonance frequency (f_r). Given $R = 5\ \Omega$, $L = 20 \times 10^{-3}$ H, $C = 0.05 \times 10^{-6}$ F.

$$f_r = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{(20 \times 10^{-3})(0.05 \times 10^{-6})}} = \frac{1}{2\pi\sqrt{1 \times 10^{-9}}}$$

$$f_r \approx 5032.9 \text{ Hz or } 5.03 \text{ kHz}$$

Step 2: Calculate impedance and current at resonance. At resonance, the impedance is purely resistive and is equal to R.

$$Z_r = R = 5\ \Omega$$

$$I_r = \frac{V}{Z_r} = \frac{100}{5} = 20 \text{ A}$$

Step 3: Calculate the Q-factor (Q_0).

$$Q_0 = \frac{1}{R}\sqrt{\frac{L}{C}} = \frac{1}{5}\sqrt{\frac{20 \times 10^{-3}}{0.05 \times 10^{-6}}} = \frac{1}{5}\sqrt{400000} = \frac{632.45}{5} = 126.49$$

Step 4: Calculate the bandwidth (BW). Using the relationship between resonance frequency and Q-factor:

$$BW = \frac{f_r}{Q_0} = \frac{5032.9}{126.49} \approx 39.79 \text{ Hz}$$

(Check with the alternative formula: $BW = \frac{R}{2\pi L} = \frac{5}{2\pi(0.02)} = 39.79$ Hz).

4.3 Parallel Resonant Circuit

Methodology:

Analysis of Practical Parallel Resonant Circuit A practical parallel resonant circuit (often called a tank circuit) consists of a coil (possessing both inductance L and internal resistance R) connected in parallel with a capacitor (C).

1. Resonance Frequency (f_r): Resonance occurs when the reactive component of the total line current is zero, causing the circuit to operate at unity power factor.

$$f_r = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{L^2}} \text{ Hz}$$

If the coil resistance is very small such that R^2/L^2 is negligible compared to $1/LC$, this simplifies to the ideal case: $f_r \approx \frac{1}{2\pi\sqrt{LC}}$.

2. Dynamic Impedance (Z_{dyn}): At resonance, the equivalent impedance of the parallel circuit is purely resistive and reaches its maximum value. It is known as dynamic impedance.

$$Z_{dyn} = \frac{L}{CR}\ \Omega$$

3. Line Current at Resonance (I_r): Because the dynamic impedance is maximum at resonance, the total line current drawn from the supply is at its minimum.

$$I_r = \frac{V}{Z_{dyn}} \text{ A}$$

4. Quality Factor (Q_0) and Bandwidth (BW): The selectivity of the tank circuit is also defined by its Q-factor.

$$Q_0 = \frac{2\pi f_r L}{R}$$

$$BW = \frac{f_r}{Q_0} \text{ Hz}$$

Worked Example:

Solving a Parallel Tank Circuit A coil of inductance 10 mH and resistance 2 Ω is connected in parallel with a 2 μF capacitor. The circuit is fed from a 20 V AC supply. Find (1) the resonance frequency (2) dynamic impedance (3) line current at resonance and (4) Quality factor.

Step 1: Calculate the resonance frequency (f_r). Given $L = 10 \times 10^{-3}$ H, $R = 2\ \Omega$, $C = 2 \times 10^{-6}$ F. First, evaluate the terms inside the square root:

$$\frac{1}{LC} = \frac{1}{(10 \times 10^{-3})(2 \times 10^{-6})} = \frac{1}{2 \times 10^{-8}} = 50 \times 10^6$$

$$\frac{R^2}{L^2} = \frac{2^2}{(10 \times 10^{-3})^2} = \frac{4}{100 \times 10^{-6}} = 40000$$

$$f_r = \frac{1}{2\pi}\sqrt{50000000 - 40000} = \frac{1}{2\pi}\sqrt{49960000} \approx \frac{7068.2}{2\pi} = 1124.9 \text{ Hz}$$

Step 2: Calculate dynamic impedance (Z_{dyn}).

$$Z_{dyn} = \frac{L}{CR} = \frac{10 \times 10^{-3}}{(2 \times 10^{-6})(2)} = \frac{10 \times 10^{-3}}{4 \times 10^{-6}} = 2500\ \Omega$$

Step 3: Calculate the line current at resonance (I_r).

$$I_r = \frac{V}{Z_{dyn}} = \frac{20}{2500} = 0.008 \text{ A or } 8 \text{ mA}$$

Step 4: Calculate the Quality factor (Q_0).

$$Q_0 = \frac{2\pi f_r L}{R} = \frac{2\pi(1124.9)(10 \times 10^{-3})}{2} = \frac{70.68}{2} = 35.34$$

4.4 Coupled Circuits

Methodology:

Mutual Inductance and Coefficient of Coupling When two coils are placed in close proximity, a time-varying current in one coil produces a changing magnetic flux that links with the second coil, inducing an electromotive force (EMF) in it. This action is known as mutual induction.

1. Mutual Inductance (M): The property of two coils by which a change in current in one induces an EMF in the other. It is measured in Henrys (H).

2. Coefficient of Coupling (k): A dimensionless factor representing the fraction of total magnetic flux from one coil that successfully links with the other coil. Its value always lies in the range $0 \leq k \leq 1$.

$$k = \frac{M}{\sqrt{L_1 L_2}} \implies M = k\sqrt{L_1 L_2}$$

Where L_1 and L_2 are the self-inductances of the two respective coils.

3. Equivalent Inductance in Series Connections: When two coupled coils are wired in series, their mutual inductance either increases or decreases the total equivalent inductance depending on the magnetic polarity (often determined by the dot convention).

- **Series Aiding:** The magnetic fluxes produced by both coils are in the same direction and reinforce each other.

$$L_{eq(aid)} = L_1 + L_2 + 2M$$

- **Series Opposing:** The magnetic fluxes produced by the coils oppose each other.

$$L_{eq(opp)} = L_1 + L_2 - 2M$$

Worked Example:

Calculating Mutual Inductance and Equivalent Inductance Two coils with self-inductances of 40 mH and 90 mH are magnetically coupled. The coefficient of coupling is 0.8. Calculate (1) the mutual inductance between them and (2) the equivalent inductance when they are connected in series aiding and series opposing.

Step 1: Calculate the mutual inductance (M). Given $L_1 = 40$ mH, $L_2 = 90$ mH, $k = 0.8$.

$$M = k\sqrt{L_1 L_2} = 0.8 \times \sqrt{40 \times 90} = 0.8 \times \sqrt{3600}$$

$$M = 0.8 \times 60 = 48 \text{ mH}$$

Step 2: Calculate equivalent inductance in series aiding.

$$L_{eq(aid)} = L_1 + L_2 + 2M$$

$$L_{eq(aid)} = 40 + 90 + 2(48) = 130 + 96 = 226 \text{ mH}$$

Step 3: Calculate equivalent inductance in series opposing.

$$L_{eq(opp)} = L_1 + L_2 - 2M$$

$$L_{eq(opp)} = 40 + 90 - 2(48) = 130 - 96 = 34 \text{ mH}$$

Worked Example:

Determining Coupling Coefficient from Series Tests Two identical coils are connected in series. When connected in a series aiding configuration, the total measured inductance is 160 mH. When the connections of one coil are reversed (series opposing), the total inductance drops to 40 mH. Calculate the self-inductance of each coil, the mutual inductance, and the coefficient of coupling.

Step 1: Set up the simultaneous equations. Let the self-inductances be equal: $L_1 = L_2 = L$. For series aiding:

$$L + L + 2M = 160 \implies 2L + 2M = 160 \implies L + M = 80 \quad \text{— (Eq 1)}$$

For series opposing:

$$L + L - 2M = 40 \implies 2L - 2M = 40 \implies L - M = 20 \quad \text{— (Eq 2)}$$

Step 2: Solve for L and M. Add Eq 1 and Eq 2 together:

$$(L + M) + (L - M) = 80 + 20 \implies 2L = 100 \implies L = 50 \text{ mH}$$

Therefore, the self-inductance of each coil is $L_1 = 50$ mH and $L_2 = 50$ mH. Substitute the value of L back into Eq 1:

$$50 + M = 80 \implies M = 30 \text{ mH}$$

Step 3: Calculate the coefficient of coupling (k).

$$k = \frac{M}{\sqrt{L_1 L_2}} = \frac{30}{\sqrt{50 \times 50}} = \frac{30}{50} = 0.6$$

CHAPTER 5

Attenuators and Filters

5.1 Attenuation Units and Relations

In communication networks, a signal experiences a decrease in strength or magnitude as it travels from the source to the load. This phenomenon is known as attenuation. Attenuation is primarily measured in two logarithmic units: the **Decibel (dB)** and the **Neper (Np)**.

Methodology:

Decibel and Neper Relation To quantify attenuation, we compare the input (source) parameters with the output (load) parameters.

1. The Decibel (dB) The decibel is defined based on the common logarithm (base 10) of power, voltage, or current ratios.

- **Power Ratio:** $D = 10\log_{10}\left(\frac{P_{in}}{P_{out}}\right)$ dB
- **Voltage Ratio:** $D = 20\log_{10}\left(\frac{V_{in}}{V_{out}}\right)$ dB (assuming equal impedances)
- **Current Ratio:** $D = 20\log_{10}\left(\frac{I_{in}}{I_{out}}\right)$ dB (assuming equal impedances)

2. The Neper (Np) The Neper is defined using the natural logarithm (base e) of the voltage or current ratios.

- **Voltage/Current Ratio:** $N = \ln\left(\frac{V_{in}}{V_{out}}\right) = \ln\left(\frac{I_{in}}{I_{out}}\right)$ Np

3. Conversion Between Units By comparing the base 10 and base e formulations, we can derive the direct relationship between Nepers and Decibels:

- 1 Neper = 8.686 Decibels
- 1 Decibel = 0.1151 Nepers

Worked Example:

Convert Decibels to Nepers An attenuator reduces the power of an input signal by 15 dB. Calculate the corresponding attenuation in Nepers.

Step 1: Identify the given attenuation in dB.

$$D = 15 \text{ dB}$$

Step 2: Use the conversion factor from dB to Nepers. We know that 1 dB = 0.1151 Np.

Step 3: Calculate the value.

$$N = 15 \times 0.1151 \text{ Np} = 1.7265 \text{ Np}$$

Answer: The attenuation is 1.7265 Np.

5.2 Attenuator Design

An attenuator is a resistive network designed to reduce the amplitude or power of a signal by a specific amount without introducing phase shift or distortion, while simultaneously providing impedance matching between the source and the load. The characteristic impedance is denoted by R_0.

In symmetrical attenuator design, we define the ratio of input current to output current as N (not to be confused with Nepers):

$$N = \frac{I_{in}}{I_{out}} = \frac{V_{in}}{V_{out}} = 10^{\frac{\text{Attenuation (in dB)}}{20}}$$

Methodology:

Symmetrical T Attenuator Design A symmetrical T attenuator consists of two series arms and one shunt arm.

- Let the total series resistance be R_1, so each of the two series arms is denoted as $\frac{R_1}{2}$.
- Let the shunt arm resistance be R_2.
- R_0 is the required characteristic impedance.
- N is the voltage/current ratio derived from the required dB attenuation.

Design Equations:

1. **Series Arm Resistors:** Each series resistor has a value of:

$$\frac{R_1}{2} = R_0 \left(\frac{N-1}{N+1}\right)$$

2. **Shunt Arm Resistor:** The shunt resistor has a value of:

$$R_2 = R_0 \left(\frac{2N}{N^2-1}\right)$$

Worked Example:

Design of Symmetrical T Attenuator Design a symmetrical T attenuator to provide an attenuation of 20 dB and to work into a characteristic impedance of $600\,\Omega$.

Step 1: Calculate the ratio N.

$$\text{Attenuation} = 20 \text{ dB}$$

$$N = 10^{\frac{20}{20}} = 10^1 = 10$$

Step 2: Calculate the value for the series arms.

$$\frac{R_1}{2} = R_0 \left(\frac{N-1}{N+1}\right)$$

$$\frac{R_1}{2} = 600 \left(\frac{10-1}{10+1}\right) = 600 \left(\frac{9}{11}\right) \approx 490.91\,\Omega$$

Each of the two series resistors is $490.91\,\Omega$.

Step 3: Calculate the value for the shunt arm.

$$R_2 = R_0 \left(\frac{2N}{N^2-1}\right)$$

$$R_2 = 600 \left(\frac{2(10)}{10^2-1}\right) = 600 \left(\frac{20}{99}\right) \approx 121.21\,\Omega$$

Answer: The T attenuator requires two series arm resistors of $490.91\,\Omega$ each, and one shunt arm resistor of $121.21\,\Omega$.

Methodology:

Symmetrical Pi Attenuator Design A symmetrical π (Pi) attenuator consists of one series arm and two shunt arms.

- Let the series arm resistance be R_1.
- Let the equivalent shunt arm resistance be $2R_2$, meaning each of the two shunt arms has a resistance of R_2. (Some texts use alternative naming; here we strictly define the series physical resistor as R_1 and the two parallel physical resistors as R_2).
- R_0 is the characteristic impedance.
- $N = 10^{\frac{\text{dB}}{20}}$ is the current ratio.

Design Equations:

1. **Series Arm Resistor:** The single series resistor has a value of:

$$R_1 = R_0 \left(\frac{N^2 - 1}{2N} \right)$$

2. **Shunt Arm Resistors:** Each of the two shunt resistors has a value of:

$$R_2 = R_0 \left(\frac{N + 1}{N - 1} \right)$$

Worked Example:

Design of Symmetrical Pi Attenuator Design a symmetrical π attenuator with an attenuation of 40 dB and characteristic impedance of $50\,\Omega$.

Step 1: Calculate the ratio N.

$$\text{Attenuation} = 40 \text{ dB}$$

$$N = 10^{\frac{40}{20}} = 10^2 = 100$$

Step 2: Calculate the value for the series arm.

$$R_1 = R_0 \left(\frac{N^2 - 1}{2N} \right)$$

$$R_1 = 50 \left(\frac{100^2 - 1}{2(100)} \right) = 50 \left(\frac{10000 - 1}{200} \right) = 50 \left(\frac{9999}{200} \right) = 2499.75\,\Omega$$

The series resistor is $2499.75\,\Omega$.

Step 3: Calculate the value for the shunt arms.

$$R_2 = R_0 \left(\frac{N + 1}{N - 1} \right)$$

$$R_2 = 50 \left(\frac{100 + 1}{100 - 1} \right) = 50 \left(\frac{101}{99} \right) \approx 51.01\,\Omega$$

Answer: The π attenuator requires one series arm resistor of $2499.75\,\Omega$, and two shunt arm resistors of $51.01\,\Omega$ each.

Methodology:

Symmetrical Lattice Attenuator Design A symmetrical lattice attenuator features a bridge-like structure with two identical series arms and two identical diagonal (cross) arms.

- Let the two series arms each have resistance R_1.

- Let the two diagonal arms each have resistance R_2.

Design Equations:

1. **Series Arms:**
$$R_1 = R_0 \left(\frac{N-1}{N+1} \right)$$
2. **Diagonal Arms:**
$$R_2 = R_0 \left(\frac{N+1}{N-1} \right)$$

Notice that for a symmetrical lattice attenuator, $R_1 \times R_2 = R_0^2$.

Worked Example:

Design of Symmetrical Lattice Attenuator Design a symmetrical lattice attenuator for 10 dB attenuation and $600\,\Omega$ characteristic impedance.

Step 1: Calculate the ratio N.
$$N = 10^{\frac{10}{20}} = 10^{0.5} \approx 3.162$$

Step 2: Calculate the series arms (R_1).
$$R_1 = 600 \left(\frac{3.162-1}{3.162+1} \right) = 600 \left(\frac{2.162}{4.162} \right) \approx 311.68\,\Omega$$

Step 3: Calculate the diagonal arms (R_2).
$$R_2 = 600 \left(\frac{3.162+1}{3.162-1} \right) = 600 \left(\frac{4.162}{2.162} \right) \approx 1155.04\,\Omega$$

Answer: The lattice attenuator requires two series arm resistors of $311.68\,\Omega$ and two diagonal arm resistors of $1155.04\,\Omega$.

5.3 Classification of Passive Filters

A filter is a reactive network that freely passes signals of desired frequencies while severely attenuating all other frequencies. Based on the frequency bands they pass or attenuate, passive filters are classified into four main types:

1. **Low Pass Filter (LPF):** Passes all frequencies from zero to a specific cutoff frequency (f_c) and attenuates all frequencies above f_c.
2. **High Pass Filter (HPF):** Attenuates all frequencies below a specific cutoff frequency (f_c) and passes all frequencies above it.
3. **Band Pass Filter (BPF):** Passes a specific band of frequencies between a lower cutoff frequency (f_1) and an upper cutoff frequency (f_2), and attenuates frequencies outside this band.
4. **Band Stop Filter (BSF):** Attenuates a specific band of frequencies between a lower cutoff (f_1) and an upper cutoff (f_2), and passes all other frequencies.

5.4 Constant K Type Filters

A constant-K filter (or image parameter filter) is constructed using series and shunt impedances such that their product is a constant ($Z_1 Z_2 = R_0^2 = K^2$), independent of frequency.

Methodology:

Constant K Low Pass Filter Design For a low pass filter, the series arm contains inductance (L) and the shunt arm contains capacitance (C). Given the cutoff frequency f_c (in Hz) and the nominal characteristic impedance R_0 (in Ω):

Base Component Values:

1. Inductor:
$$L = \frac{R_0}{\pi f_c}$$
2. Capacitor:
$$C = \frac{1}{\pi R_0 f_c}$$

Network Formations:

- **T-Section:** Two series inductors of value $\frac{L}{2}$ and one shunt capacitor of value C.
- **π-Section:** One series inductor of value L and two shunt capacitors of value $\frac{C}{2}$.

Worked Example:

Design of Constant K Low Pass Filter Design a constant-K low pass T-section and π-section filter having a cutoff frequency of 2 kHz and a design impedance of $600\,\Omega$.

Step 1: Identify given parameters. $f_c = 2000$ Hz, $R_0 = 600\,\Omega$.

Step 2: Calculate base components L and C.

$$L = \frac{600}{\pi \times 2000} = \frac{600}{6283.18} \approx 0.0955 \text{ H} = 95.5 \text{ mH}$$

$$C = \frac{1}{\pi \times 600 \times 2000} = \frac{1}{3769911.18} \approx 0.265 \times 10^{-6} \text{ F} = 0.265\,\mu\text{F}$$

Step 3: Define T-section values.

- Series inductors: $\frac{L}{2} = \frac{95.5}{2} = 47.75$ mH (each)
- Shunt capacitor: $C = 0.265\,\mu$F

Step 4: Define π-section values.

- Series inductor: $L = 95.5$ mH
- Shunt capacitors: $\frac{C}{2} = \frac{0.265}{2} = 0.1325\,\mu$F (each)

Methodology:

Constant K High Pass Filter Design For a high pass filter, the series arm contains capacitance (C) and the shunt arm contains inductance (L). Given the cutoff frequency f_c (in Hz) and the nominal characteristic impedance R_0 (in Ω):

Base Component Values:

1. Inductor:
$$L = \frac{R_0}{4\pi f_c}$$
2. Capacitor:
$$C = \frac{1}{4\pi R_0 f_c}$$

Network Formations:

- **T-Section:** Two series capacitors of value $2C$ and one shunt inductor of value L.
- **π-Section:** One series capacitor of value C and two shunt inductors of value $2L$.

Worked Example:

Design of Constant K High Pass Filter Design a constant-K high pass T-section filter having a cutoff frequency of 4 kHz and a design impedance of $500\,\Omega$.

Step 1: Identify given parameters. $f_c = 4000$ Hz, $R_0 = 500\,\Omega$.

Step 2: Calculate base components L and C.

$$L = \frac{500}{4\pi \times 4000} = \frac{500}{50265.48} \approx 0.00995 \text{ H} = 9.95 \text{ mH}$$

$$C = \frac{1}{4\pi \times 500 \times 4000} = \frac{1}{25132741} \approx 0.0398 \times 10^{-6} \text{ F} = 0.0398\,\mu\text{F}$$

Step 3: Define T-section values.

- Series capacitors: $2C = 2 \times 0.0398 = 0.0796\,\mu$F (each)
- Shunt inductor: $L = 9.95$ mH

Answer: The required T-section has two series capacitors of $0.0796\,\mu$F and one shunt inductor of 9.95 mH.

5.5 Comparison of Band Pass and Band Stop Filters

Understanding the opposing functionalities of Band Pass and Band Stop filters is crucial for frequency selection applications in network circuits.

Parameter	**Band Pass Filter (BPF)**	**Band Stop Filter (BSF)**
Function	Passes a specific range of frequencies and blocks all others.	Blocks a specific range of frequencies and passes all others.
Pass Band	$f_1 < f < f_2$	$f < f_1$ and $f > f_2$
Stop Band	$f < f_1$ and $f > f_2$	$f_1 < f < f_2$
Attenuation Curve	Minimum attenuation between lower and upper cutoff frequencies.	Maximum attenuation between lower and upper cutoff frequencies.
Series Arm Components	Formed by a series combination of an inductor (L) and a capacitor (C).	Formed by a parallel combination of an inductor (L) and a capacitor (C).
Shunt Arm Components	Formed by a parallel combination of an inductor (L) and a capacitor (C).	Formed by a series combination of an inductor (L) and a capacitor (C).

Formulae

This appendix provides a quick reference to the key formulae, mathematical relationships, and their physical explanations as discussed throughout this book.

Unit 1: DC Circuits and Network Theorems

- **Equivalent Resistance (Series)**
  When resistors are connected end-to-end, the total resistance is the algebraic sum of the individual resistances.
  $$R_{eq} = R_1 + R_2 + \cdots + R_n$$
- **Equivalent Resistance (Parallel)**
  For two resistors connected across the same two nodes, the equivalent resistance is the product over the sum.
  $$R_{parallel} = \frac{R_1 \times R_2}{R_1 + R_2}$$
- **Kirchhoff's Current Law (KCL)**
  The algebraic sum of currents entering and leaving a junction is zero.
  $$\sum I_{in} = \sum I_{out}$$
- **Ohm's Law**
  The current through a conductor is directly proportional to the voltage across it.
  $$V = I \times R \quad \text{or} \quad I = \frac{V}{R}$$
- **Delta to Star Transformation**
  Converts a Delta network to an equivalent Star network. The resistance of any arm of the star network is the product of the two adjacent delta arms divided by the sum of all three delta arms.
  $$R_1 = \frac{R_B \times R_C}{R_A + R_B + R_C}$$
- **Current Division Rule**
  Used to find the current flowing through one of two parallel branches.
  $$I_1 = I_{total} \times \frac{R_2}{R_1 + R_2}$$
- **Thevenin's Theorem**
  A linear active network can be replaced by a single voltage source (V_{th}) in series with a resistance (R_{th}). The load current is:
  $$I_L = \frac{V_{th}}{R_{th} + R_L}$$
- **Maximum Power Transfer Theorem**
  Maximum power is transferred to a load when the load resistance equals the Thevenin equivalent resistance ($R_L = R_{th}$).
  $$P_{max} = \frac{V_{th}^2}{4R_{th}}$$

- **Norton's Theorem**
  Any linear active network can be replaced by a single current source (I_N) in parallel with a resistance (R_N).
$$R_N = R_{th} \quad \text{and} \quad I_N = \frac{V_{th}}{R_{th}}$$

Unit 4: Resonance and Coupled Circuits

- **Inductive and Capacitive Reactance**
  The opposition to AC current by an inductor and a capacitor, respectively.
$$X_L = 2\pi f L$$
$$X_C = \frac{1}{2\pi f C}$$

- **Resonant Frequency (Series RLC)**
  The frequency at which inductive and capacitive reactances are equal, making the circuit purely resistive.
$$f_r = \frac{1}{2\pi\sqrt{LC}}$$

- **Resonant Frequency (Parallel RLC with coil resistance)**
  The frequency at which a parallel LC circuit (where the inductor has some resistance R) resonates.
$$f_r = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$$

- **Quality Factor (Q)**
  A measure of the sharpness of resonance or the energy storage efficiency of the circuit.
$$Q_0 = \frac{1}{R}\sqrt{\frac{L}{C}} = \frac{2\pi f_r L}{R}$$

- **Quality Factor in terms of Reactance**
  Alternatively, Q can be expressed as the ratio of reactance to resistance.
$$Q_C = \frac{X_C}{R_C}, \quad Q_L = \frac{X_L}{R_L}$$

- **Bandwidth**
  The range of frequencies over which the power is at least half of its maximum value at resonance.
$$BW = f_2 - f_1 = \frac{f_r}{Q_0} = \frac{R}{2\pi L}$$

- **Dynamic Impedance (Parallel Resonance)**
  The purely resistive impedance offered by a parallel resonant circuit at resonance.
$$Z_{dyn} = \frac{L}{CR}$$

- **Resonant Current**
  The current flowing through the circuit at resonance.
$$I_r = \frac{V}{Z_{dyn}} \quad \text{(Parallel)} \quad \text{or} \quad I_r = \frac{V}{R} \quad \text{(Series)}$$

- **Equivalent Inductance (Coupled Coils)**
  The total inductance of two mutually coupled coils connected in series aiding or opposing configurations.
  $$L_{eq(aid)} = L_1 + L_2 + 2M$$
  $$L_{eq(opp)} = L_1 + L_2 - 2M$$
- **Mutual Inductance and Coefficient of Coupling**
  The mutual inductance M depends on the self-inductances and the degree of magnetic coupling (k) between them.
  $$M = k\sqrt{L_1 L_2} \implies k = \frac{M}{\sqrt{L_1 L_2}}$$

Unit 5: Filters and Attenuators

- **Characteristic Impedance**
  The constant impedance of a symmetrical network when terminated by the same impedance.
  $$Z_1 Z_2 = R_0^2 = K^2$$
- **Attenuation (Decibels)**
  The logarithmic ratio measuring the power or voltage loss in a network.
  $$D = 10\log_{10}\left(\frac{P_{in}}{P_{out}}\right) \text{ dB} = 20\log_{10}\left(\frac{V_{in}}{V_{out}}\right) \text{ dB}$$
- **Conversion between Nepers and Decibels**
  Relationship between the two standard logarithmic units of attenuation.
  $$1 \text{ dB} = 0.1151 \text{ Np} \quad \text{and} \quad 1 \text{ Np} = 8.686 \text{ dB}$$

Unit 6: Transient Analysis and Laplace Transforms

- **General Transient Equation for DC Excitation**
  Describes the response $x(t)$ of a first-order system to a step input over time.
  $$x(t) = x(\infty) + [x(0) - x(\infty)]e^{-t/\tau}$$
- **Time Constant**
  The time required for the system's response to reach approximately 63.2% of its final value.
  $$\tau = RC \quad \text{(for RC circuits)}$$
  $$\tau = \frac{L}{R} \quad \text{(for RL circuits)}$$
- **Initial Conditions**
  The voltage across a capacitor and the current through an inductor cannot change instantaneously.
  $$v_C(0^+) = v_C(0^-)$$
  $$i_L(0^+) = i_L(0^-)$$
- **Voltage-Current Relationships (Inductor and Capacitor)**
  The fundamental differential equations governing inductors and capacitors.
  $$v_L(t) = L\frac{di(t)}{dt}$$
  $$i_C(t) = C\frac{dv_C(t)}{dt}$$
- **Transfer Function**
  The ratio of the Laplace transform of the output to the Laplace transform of the input, assuming zero initial conditions.
  $$H(s) = \frac{V_o(s)}{V_i(s)} = K\frac{(s - z_1)(s - z_2)\ldots(s - z_m)}{(s - p_1)(s - p_2)\ldots(s - p_n)}$$