Lecture 43: Crystal Oscillator Piezoelectric effect, characteristics of ...

Oscillators

DI02011011: Electronics Circuit and Application (ECA)

Lecture Agenda: Quartz Crystal Resonators

- 1. Physics of the Piezoelectric Effect & Quartz (SiO_2) Crystal Cuts
- 2. Butterworth-Van Dyke (BVD) Electrical Equivalent Circuit Model (L_m, C_m, R_m, C_p)
- 3. Series (f_s) and Parallel (f_p) Resonance Frequencies & Reactance Characteristics
- 4. Quality Factor (Q) & Extreme Frequency Stability Derivations
- 5. Mathematical Parameter Calculation Example for Quartz Resonators
- 6. Pierce Crystal Oscillator Topology: Circuit Schematic, Biasing & Load Capacitance (C_L)
- 7. Advanced Stability: TCXO, OCXO, VCXO & Drive Level Considerations

The Piezoelectric Effect in Quartz (SiO2)

- Direct Piezoelectric Effect: Mechanical stress (compression or tension) applied across specific crystallographic axes of a quartz wafer induces proportional electrical charge polarization ($Q = d \cdot F$).
- Inverse Piezoelectric Effect: An applied electric field across crystal electrodes induces proportional physical mechanical deformation/strain ($\epsilon = d \cdot E$).
- Quartz Material Properties: Single-crystal Silicon Dioxide (SiO_2) exhibits high mechanical hardness, chemical inertness, elastic stability, and an exceptionally low internal friction coefficient.
- AT-Cut Crystal Characteristics: Cut at an angle of $35^\circ 15'$ relative to the optical Z-axis. AT-cut crystals operate in thickness-shear mode and exhibit a near-zero temperature coefficient around 25°C (frequency drift $< \pm 10$ ppm over -20°C to $+70^\circ$C).

Butterworth-Van Dyke (BVD) Circuit Model

- Electrical Equivalent Circuit: Represents the mechanical quartz resonator as an electrical 4-element network comprising a series motional branch (L_m, C_m, R_m) in parallel with a static electrode capacitance (C_p or C_0).
- Motional Inductance (L_m): Represents the mechanical vibrating mass of the crystal wafer; typically extremely large (mH to hundreds of H).
- Motional Capacitance (C_m): Represents the mechanical elasticity/compliance of the crystal lattice; extremely small (fF to pF).
- Motional Resistance (R_m): Represents internal mechanical friction and acoustic losses; small ($10\ \Omega$ to $100\ \Omega$).
- Shunt Capacitance (C_p): Electrostatic capacitance formed by metal electrodes sandwiching the quartz dielectric; $C_p \gg C_m$ (typically ratio $r = C_p/C_m \approx 100 - 1000$).

Series (f_s) and Parallel (f_p) Resonance Modes

- Series Resonant Frequency (f_s): Reactance of motional branch cancels out ($X_{Lm} = X_{Cm}$): $f_s = \frac{1}{2\pi\sqrt{L_m C_m}}$. At f_s, motional impedance is purely resistive and at its minimum: $Z(f_s) \approx R_m$.
- Parallel (Anti-Resonant) Frequency (f_p): Net inductive reactance of motional branch resonates with shunt capacitance C_p: $f_p = \frac{1}{2\pi\sqrt{L_m \frac{C_m C_p}{C_m + C_p}}} = f_s\sqrt{1 + \frac{C_m}{C_p}}$. At f_p, net impedance is purely resistive and reaches maximum value.
- Narrow Frequency Bandwidth: Fractional frequency separation: $\frac{\Delta f}{f_s} = \frac{f_p - f_s}{f_s} \approx \frac{C_m}{2C_p} \approx 0.05\% - 0.5\%$.
- Inductive Reactance Region: The crystal exhibits inductive reactance ONLY in the extremely narrow frequency window between f_s and f_p.

Quality Factor (Q) & Extreme Frequency Stability

- Unmatched Quality Factor (Q): Defined as ratio of stored energy to dissipated energy per cycle: $Q = \frac{\omega_s L_m}{R_m} = \frac{1}{\omega_s C_m R_m}$.
- Numerical Comparison: Standard LC tanks achieve $Q \approx 50 - 200$; quartz crystal resonators achieve $Q = 10,000$ to $1,000,000$.
- Extreme Phase Slope ($d\phi/d\omega$): Phase slope derivative at resonance: $\frac{d\phi}{d\omega} \approx \frac{2Q}{\omega_s}$. High Q produces an extremely steep phase curve.
- Frequency Stability Impact: A massive $d\phi/d\omega$ means that any circuit noise or phase disturbance produces an infinitesimally small frequency shift Δf, yielding stability of 10^{-6} to 10^{-10} (1 ppm – 0.001 ppm).

Crystal Resonator Parameter Calculation Example

- Problem Statement: A quartz crystal has BVD equivalent parameters $L_m = 0.8$ H, $C_m = 0.012$ pF, $R_m = 50\ \Omega$, and $C_p = 4.0$ pF. Calculate: (a) Series resonant frequency f_s, (b) Parallel resonant frequency f_p, (c) Bandwidth Δf, and (d) Quality factor Q.
- Step 1: Calculate f_s:
  $f_s = \frac{1}{2\pi\sqrt{L_m C_m}} = \frac{1}{2\pi\sqrt{(0.8)(0.012\times10^{-12})}} = \frac{1}{2\pi\sqrt{9.6\times10^{-15}}} \approx 1,624,146 \text{ Hz} = 1.624146 \text{ MHz}.$
- Step 2: Calculate f_p:
  $f_p = f_s\sqrt{1+\frac{C_m}{C_p}} = 1.624146 \times \sqrt{1+\frac{0.012 \text{ pF}}{4.0 \text{ pF}}} = 1.624146 \times \sqrt{1.003} \approx 1.626584 \text{ MHz}.$
- Step 3: Compute Bandwidth (Separation Δf):
  $\Delta f = f_p - f_s = 1.626584 \text{ MHz} - 1.624146 \text{ MHz} = 2.438 \text{ kHz}.$
- Step 4: Compute Quality Factor (Q): $Q = \frac{2\pi f_s L_m}{R_m} = \frac{2\pi(1.624146\times10^{6})(0.8)}{50} \approx 163,277.$

Pierce Crystal Oscillator Topology

- Circuit Architecture: Uses a digital CMOS inverter (or BJT/Op-Amp) operating as an inverting amplifier, with the crystal connected between input and output, alongside load capacitors C_{L1}, C_{L2} to ground.
- Operation Mode: The crystal operates in its parallel inductive region between f_s and f_p, replacing the inductor in a Colpitts-like topology.
- Load Capacitance (C_L): Total capacitance presented across crystal terminals: $C_L = \frac{C_{L1}C_{L2}}{C_{L1}+C_{L2}} + C_{stray}$.
- Actual Oscillation Frequency (f_L): $f_L = f_s\sqrt{1 + \frac{C_m}{2(C_p+C_L)}}$.
- Feedback Biasing Resistor (R_f): High-value resistor ($1\ \text{M}\Omega - 10\ \text{M}\Omega$) connected across inverter to bias it in its linear linear high-gain region ($V_{in} = V_{out} = V_{DD}/2$).

Colpitts & Miller Crystal Oscillator Topologies

- Colpitts Crystal Oscillator: Replaces the LC tank inductor with a quartz crystal. Operates in series mode (f_s) when placed in the low-impedance feedback branch, or in parallel mode (f_p) when replacing the main tank coil.
- Miller Crystal Oscillator: Crystal connected between Base and Ground (or Gate and Ground); a tuned LC tank in Collector/Drain circuit is tuned slightly above f_s to present an inductive load.
- Overtone Crystal Oscillators: At frequencies above ~ 30 MHz, fundamental crystal wafers become too thin and mechanically fragile. Overtone circuits operate crystals at odd mechanical harmonics (3rd, 5th, 7th overtones).
- Tuned Selective LC Tank: An auxiliary LC tank is added to suppress fundamental oscillations and select the desired 3rd or 5th overtone mode.

Pierce Oscillator Load Capacitance Design Example

- Problem Statement: A 12.0 MHz Pierce crystal specifies a manufacturer load capacitance $C_L = 18.0$ pF. Estimated PCB trace stray capacitance is $C_{stray} = 3.0$ pF. Calculate required matching external capacitors C_{L1} and C_{L2} (assume $C_{L1} = C_{L2}$).
- Step 1: Set Up Load Capacitance Equation: $C_L = \frac{C_{L1}C_{L2}}{C_{L1}+C_{L2}} + C_{stray} \Rightarrow 18.0 = \frac{C_{L1}^2}{2C_{L1}} + 3.0 = \frac{C_{L1}}{2} + 3.0$.
- Step 2: Solve for C_{L1}: $\frac{C_{L1}}{2} = 18.0 - 3.0 = 15.0$ pF $\Rightarrow C_{L1} = 30.0$ pF.
- Step 3: Set C_{L2}: $C_{L2} = C_{L1} = 30.0$ pF (standard 30 pF or 33 pF 5% NPO ceramic capacitors).
- Step 4: Power Drive Level Verification: Check drive power $P_d = I_{rms}^2 R_m < 1.0$ mW to prevent physical crystal damage or thermal frequency pulling.

Temperature-Compensated & Oven-Controlled Oscillators

- TCXO (Temperature Compensated Crystal Oscillator): Uses a varactor diode in series with the crystal alongside a thermistor temperature-sensing network to continuously adjust load capacitance C_L and cancel thermal drift to ± 0.5 ppm.
- OCXO (Oven Controlled Crystal Oscillator): Encloses quartz crystal and heating element inside a thermally insulated oven maintained at a constant zero-temp-coefficient turnover temperature ($\sim 75°C - 85°C$); yields ± 0.001 ppm (1 ppb) stability.
- VCXO (Voltage Controlled Crystal Oscillator): Pulls crystal frequency over a narrow range (± 100 ppm) by varying DC control voltage on a series varactor diode; essential for Phase-Locked Loops (PLLs).
- Stability Hierarchy: Standard Crystal ($10 - 50$ ppm) $<$ TCXO ($0.5 - 2$ ppm) $<$ OCXO ($0.001 - 0.01$ ppm).

Crystal Drive Level & Parasitic Spurious Modes

- Drive Level Limitations: Power dissipated in motional resistor R_m ($P_d = I_{rms}^2 R_m$) must be kept below maximum rating ($10\ \mu\text{W} - 500\ \mu\text{W}$). Overdriving causes non-linear frequency frequency shifts, micro-cracking, and accelerated aging.
- Drive Level Dependency (DLD): Abrupt changes in motional resistance R_m and resonant frequency under low drive levels caused by surface micro-contaminants or crystal lattice imperfections.
- Spurious Anharmonic Resonances: Unwanted secondary mechanical vibration modes occurring near desired operating frequency; suppressed through proper crystal blank geometry and dampening.
- Aging Effects: Long-term frequency drift ($1 - 5$ ppm/year) caused by mass transfer (outgassing) onto electrodes or stress relaxation in crystal mounting structures.

Summary of Crystal Oscillators

- Quartz crystal resonators leverage direct and inverse piezoelectricity to achieve unmatched frequency stability ($10^{-6} - 10^{-10}$).
- Butterworth-Van Dyke (BVD) model consists of series motional branch (L_m, C_m, R_m) in parallel with electrostatic electrode capacitance C_p.
- Exhibits dual resonances: Series resonance $f_s = \frac{1}{2\pi\sqrt{L_m C_m}}$ (minimum impedance) and Parallel anti-resonance $f_p = f_s\sqrt{1 + C_m/C_p}$ (maximum impedance).
- Pierce oscillator circuit is the industry-standard microcontroller clock topology, operating crystal in parallel inductive mode between f_s and f_p with load capacitance C_L.

Formula Reference & Key Equations

- Series Resonant Frequency: $f_s = \frac{1}{2\pi\sqrt{L_m C_m}}$.
- Parallel Resonant Frequency: $f_p = f_s\sqrt{1 + \frac{C_m}{C_p}}$.
- Quality Factor (Q): $Q = \frac{\omega_s L_m}{R_m} = \frac{1}{\omega_s C_m R_m}$.
- Pierce Net Load Capacitance: $C_L = \frac{C_{L1} C_{L2}}{C_{L1}+C_{L2}} + C_{stray}$.
- Frequency Pulling Formula: $f_L = f_s\sqrt{1 + \frac{C_m}{2(C_p+C_L)}}$.