Lecture 42: RC oscillators: (1) Phase shift oscillator, (2) Wein bridge ...

Oscillators

DI02011011: Electronics Circuit and Application (ECA)

Lecture Agenda: Audio Frequency RC Oscillators

- 1. Need for RC Oscillators & Low-Frequency Phase-Shift Principles
- 2. Op-Amp RC Phase-Shift Oscillator: Circuit Schematic & Working
- 3. Mathematical Derivation of Phase-Shift Oscillation Frequency (f_o) and Gain ($A_v \geq 29$)
- 4. BJT-Based Phase-Shift Oscillator: Transistor Loading Effects & h_{fe} Requirements
- 5. Wien-Bridge Oscillator Topology: Lead-Lag Network Resonance & Zero Phase Condition
- 6. Wien-Bridge Op-Amp Design: Derivation of $f_o = 1/(2\pi RC)$ and Gain Requirement ($A_v \geq 3$)
- 7. Automatic Gain Control (AGC) & Non-Linear Amplitude Stabilization Techniques

Low Frequency Oscillation & RC Phase-Shift Principle

- Limitations of LC Oscillators at Audio Frequencies: Resonant frequency $f_o = 1/(2\pi\sqrt{LC})$ requires impractically large inductors (H) and capacitors (μF), causing heavy core losses, bulk, and high cost.
- RC High-Pass Filter Phase Shift: Single RC section $V_{out}/V_{in} = \frac{j\omega RC}{1+j\omega RC}$ introduces a phase lead $\theta = \arctan\left(\frac{1}{\omega RC}\right)$ up to 90^circ.
- Three-Stage RC Ladder Network: An inverting amplifier provides 180° phase shift; feedback ladder must provide remaining 180°. A minimum of 3 RC stages is required (each contributing 60° phase shift at f_o).
- Transfer Function of 3-Stage High-Pass RC Ladder: $\beta(s) = \frac{s^3R^3C^3}{s^3R^3C^3+6s^2R^2C^2+5sRC+1}$.

Op-Amp RC Phase-Shift Oscillator Analysis

- Circuit Configuration: Inverting operational amplifier with gain $A_v = -R_f/R_1$ cascaded with a 3-stage high-pass RC feedback ladder (C in series, R to ground).
- Frequency of Oscillation (f_o): Substituting $s = j\omega$ into transfer function $\beta(s)$ and setting imaginary part of denominator to zero: $\text{Im}[1 - 6\omega^2R^2C^2 + j(5\omega RC - \omega^3R^3C^3)] = 0 \Rightarrow 5\omega RC = \omega^3R^3C^3 \Rightarrow \omega_0 = \frac{1}{\sqrt{6}RC} \Rightarrow f_o = \frac{1}{2\pi\sqrt{6}RC}$.
- Feedback Attenuation at ω_0: Substituting ω_0 into real part yields $\beta(j\omega_0) = \frac{1}{1-6(1/6)} = -\frac{1}{29}$. Network attenuates signal by factor of 29 with 180° phase inversion.
- Amplifier Gain Requirement: Barkhausen criterion $|A_v\beta| \geq 1 \Rightarrow |A_v| \geq 29 \Rightarrow \frac{R_f}{R} \geq 29$ (setting $R_1 = R$).

BJT-Based RC Phase-Shift Oscillator

- Transistor Loading Considerations: Input resistance of Common-Emitter BJT stage ($h_{ie} \approx \beta_0 r_e$) loads the third RC stage. To maintain equal resistance R, third resistor is chosen as $R_3' = R - h_{ie}$.
- Current Feedback Derivation: Analysis using mesh current analysis for CE transistor with collector load R_C alters frequency equation parameters.
- BJT Oscillation Frequency (f_o): $f_o = \frac{1}{2\pi RC\sqrt{6+4k}}$, where $k = R_C/R$. If $R_C = R$ ($k = 1$), $f_o = \frac{1}{2\pi\sqrt{10}RC}$.
- Minimum Transistor Current Gain (h_{fe}): $h_{fe,min} = 23 + 29\left(\frac{R}{R_C}\right) + 4\left(\frac{R_C}{R}\right)$. For $R_C = R$, minimum transistor current gain is $h_{fe,min} = 56$.

Design Example: Op-Amp Phase-Shift Oscillator

- Problem Statement: Design an Op-Amp RC phase-shift oscillator to generate an audio signal at $f_o = 1.0$ kHz using capacitors $C = 10$ nF. Calculate required resistance R and feedback resistor R_f.
- Step 1: Calculate Timing Resistance (R):
  $f_o = \frac{1}{2\pi\sqrt{6}RC} \Rightarrow R = \frac{1}{2\pi\sqrt{6}f_o C} = \frac{1}{2\pi\sqrt{6}(1000 \text{ Hz})(10\times10^{-9} \text{ F})} = \frac{1}{2\pi(2.4495)(10^{-5})} \approx 6.497 \text{ k}\Omega$ (standard 6.49 kΩ 1%).
- Step 2: Calculate Minimum Feedback Resistor (R_f):
  $|A_v| = \frac{R_f}{R} \geq 29 \Rightarrow R_f = 29 \times 6.497 \text{ k}\Omega \approx 188.41 \text{ k}\Omega$.
- Step 3: Practical Implementation Adjustment: Choose R_f as a 180 kΩ resistor in series with a 20 kΩ potentiometer (set to ~ 195 kΩ) to guarantee start-up ($|A\beta| > 1$) and trim amplitude.

Wien-Bridge Oscillator: Bridge Network & Resonance

- Bridge Topology: Incorporates a lead-lag RC bridge network comprising a series arm (R_1, C_1) and parallel arm (R_2, C_2) connected to non-inverting input of Op-Amp.
- Lead-Lag Network Transfer Function ($\beta(s)$): $\beta(s) = \frac{Z_p}{Z_s + Z_p} = \frac{\frac{R_2/(j\omega C_2)}{R_2 + 1/(j\omega C_2)}}{(R_1 + \frac{1}{j\omega C_1}) + \frac{R_2/(j\omega C_2)}{R_2 + 1/(j\omega C_2)}}$.
- Symmetrical Case ($R_1 = R_2 = R$, $C_1 = C_2 = C$): $\beta(j\omega) = \frac{j\omega RC}{1 - \omega^2 R^2 C^2 + 3j\omega RC}$.
- Zero Phase Shift Condition: Phase shift $\angle\beta(j\omega) = 0^\circ$ occurs when imaginary term $1 - \omega^2 R^2 C^2 = 0 \Rightarrow \omega_0 = \frac{1}{RC} \Rightarrow f_o = \frac{1}{2\pi RC}$.

Wien-Bridge Op-Amp Amplifier & Gain Requirements

- Attenuation at Resonant Frequency: Substituting $\omega_0 = 1/(RC)$ into transfer function: $\beta(j\omega_0) = \frac{j(1)}{0+3j(1)} = +\frac{1}{3}\angle 0^\circ$. Lead-lag network attenuates signal by factor of 3 without phase shift.
- Non-Inverting Op-Amp Amplifier: Closed-loop voltage gain equation: $A_v = 1 + \frac{R_f}{R_1}$.
- Barkhausen Criterion Compliance: Loop gain $A_v\beta = \left(1 + \frac{R_f}{R_1}\right) \times \frac{1}{3} \geq 1 \Rightarrow 1 + \frac{R_f}{R_1} \geq 3 \Rightarrow \frac{R_f}{R_1} \geq 2$.
- Exact Balance State: Maintaining $A_v = 3$ exactly $(R_f = 2R_1)$ produces sustained, stable, low-distortion sine wave oscillations.

Wien-Bridge Oscillator Component Design Example

- Problem Statement: Design a Wien-bridge oscillator for an audio signal generator operating at $f_o = 10.0$ kHz using standard capacitors $C = 1.0$ nF. Determine resistance R, feedback resistors R_1 and R_f.
- Step 1: Calculate Resistor R:
  $f_o = \frac{1}{2\pi RC} \Rightarrow R = \frac{1}{2\pi f_o C} = \frac{1}{2\pi(10{,}000\text{ Hz})(1.0\times10^{-9}\text{ F})} = \frac{1}{2\pi(10^{-5})} \approx 15.915\text{ k}\Omega$ (standard 15.8 kΩ 1%).
- Step 2: Select Ground Resistor R_1: Choose standard precision resistor $R_1 = 10.0\text{ k}\Omega$.
- Step 3: Calculate Feedback Resistor R_f: $R_f = 2 \times R_1 = 2 \times 10.0\text{ k}\Omega = 20.0\text{ k}\Omega$.
- Step 4: Real-World Adjustments: Use a 18.0 kΩ resistor in series with a 5.0 kΩ trimmer resistor to fine-tune loop gain to 3.00.

Amplitude Stabilization & Automatic Gain Control (AGC)

- Thermal & Tolerance Instability: If $A_v < 3$, oscillations exponentially decay to zero; if $A_v > 3$, oscillations exponentially grow until Op-Amp clips against supply rails (V_{CC}, V_{EE}), introducing severe distortion.
- JFET Automatic Gain Control (AGC): Replace resistor R_1 with a JFET operating in its voltage-controlled linear ohmic region ($r_{ds} = \frac{r_o}{1 - V_{GS}/V_P}$). Output peak voltage is rectified and fed back to JFET gate (V_{GS}), dynamically adjusting $A_v = 3.00$.
- Incandescent Lamp / Thermistor Method: Lamp with positive temperature coefficient (PTC) placed as R_1; rising output amplitude heats filament, increasing R_1 and reducing gain $1 + R_f/R_1$ back to 3.
- Diode Limiting Network: Parallel back-to-back diodes across portion of R_f smoothly decrease effective feedback resistance as peak signal voltage exceeds diode threshold ($V_\gamma \approx 0.7$ V).

Comparison: Phase-Shift vs. Wien-Bridge Oscillators

- Operating Frequency Band: Phase-Shift (1 Hz – 100 kHz); Wien-Bridge (5 Hz – 1 MHz with high-speed Op-Amps).
- Frequency Tuning Capability: Phase-Shift requires simultaneous adjustment of 3 ganged components ($3R$'s or $3C$'s); Wien-Bridge requires adjusting only 2 ganged components ($2R$'s or $2C$'s), making it standard for laboratory signal generators.
- Spectral Purity & THD: Wien-Bridge offers ultra-low Total Harmonic Distortion (THD $< 0.01\%$ with AGC); Phase-Shift exhibits higher THD due to high required amplifier gain ($A_v = 29$).
- Gain Requirements: Phase-Shift ($A_v \geq 29$ inverting); Wien-Bridge ($A_v \geq 3$ non-inverting).

Quadrature & State-Variable RC Oscillator Variants

- Quadrature RC Oscillator: Utilizes two cascaded Op-Amp integrators and one inverter in a feedback loop to simultaneously generate sine (V_{sin}) and cosine (V_{cos}) outputs in phase quadrature (90° phase difference).
- Quadrature Frequency Equation: $f_o = \frac{1}{2\pi RC}$.
- State-Variable Active Filter Oscillator: Derived from state-variable active filter by removing damping resistor ($Q \to \infty$), causing second-order poles to sit precisely on $j\omega$-axis.
- Wide Sweep Capability: State-variable topologies enable wide continuous frequency tuning over a 1000:1 range without amplitude breakdown.

Summary of RC Oscillator Topologies

- RC Oscillators are designed for audio frequency generation (1 Hz – 1 MHz), overcoming the size and loss limitations of LC tanks.
- Phase-Shift Oscillator uses a 3-stage high-pass RC ladder providing 180° shift at $f_o = \frac{1}{2\pi\sqrt{6}RC}$, requiring an inverting gain $|A_v| \geq 29$.
- Wien-Bridge Oscillator uses a lead-lag RC bridge providing 0° shift at $f_o = \frac{1}{2\pi RC}$, requiring a non-inverting gain $A_v = 1 + R_f/R_1 \geq 3$.
- Automatic Gain Control (AGC) using JFETs or thermistors is critical in Wien-Bridge circuits to maintain $A_v = 3.00$ and achieve ultra-low harmonic distortion.

Formula Reference & Key Equations

- Op-Amp Phase-Shift Frequency: $f_o = \frac{1}{2\pi\sqrt{6}RC}$, Gain Condition: $|A_v| = \frac{R_f}{R} \geq 29$.
- BJT Phase-Shift Frequency ($R_C = R$): $f_o = \frac{1}{2\pi\sqrt{10}RC}$, Minimum BJT Current Gain: $h_{fe,min} = 56$.
- Wien-Bridge Frequency: $f_o = \frac{1}{2\pi RC}$, Gain Condition: $A_v = 1 + \frac{R_f}{R_1} \geq 3 \Rightarrow R_f \geq 2R_1$.
- JFET Resistance in Ohmic Region: $r_{ds} = \frac{r_{ds0}}{1 - V_{GS}/V_P}$, used for dynamic AGC loop gain stabilization.