Lecture 41: LC oscillators: (1) Hartley oscillator, (2) Colpitts oscilla...

Oscillators

DI02011011: Electronics Circuit and Application (ECA)

Lecture Agenda: LC Resonant Oscillators

- 1. Principles of LC Resonance & Barkhausen Criterion for Sustained Oscillation
- 2. Hartley Oscillator Topology: Circuit Schematic, Tapped Inductor & Operation
- 3. Mathematical Derivation of Hartley Oscillation Frequency (f_o) and Minimum Gain (A_v)
- 4. Colpitts Oscillator Topology: Circuit Schematic, Capacitive Divider & Operation
- 5. Mathematical Derivation of Colpitts Oscillation Frequency (f_o) and Minimum Gain (A_v)
- 6. Op-Amp vs. Transistor (CE) Implementations & Practical High-Frequency Constraints
- 7. Clapp Oscillator Enhancement & Frequency Stability Improvements

Principles of LC Resonance & Barkhausen Criterion

- Parallel LC Resonant Tank: Consists of an inductor L and capacitor C in parallel. Tank impedance is given by $Z_{tank} = \frac{j\omega L}{1-\omega^2 LC}$. At resonance $\omega_0 = 1/\sqrt{LC}$, $Z_{tank} \to \infty$ (purely resistive $R_p = Q\omega_0 L$).
- Barkhausen Criterion for Oscillation: Loop gain must satisfy $|T(j\omega_0)| = |A(j\omega_0)\beta(j\omega_0)| \geq 1$ with total phase shift $\angle T(j\omega_0) = 0^\circ$ or 360°.
- Phase Distribution: Active amplifying device (e.g., Common-Emitter BJT) provides 180° phase inversion; feedback network $\beta(j\omega)$ must provide an additional 180° phase shift at resonance ω_0.
- Energy Exchange Dynamics: Energy continuously exchanges between the magnetic field of inductor L ($E_B = \frac{1}{2}LI^2$) and electric field of capacitor C ($E_E = \frac{1}{2}CV^2$). The active amplifier replenishing I^2R tank losses to maintain constant amplitude.

Hartley Oscillator Topology & Operation

- Tank Circuit Architecture: Employs a center-tapped (split) inductor (L_1, L_2) or two series inductors connected in parallel with a single tuning capacitor C.
- Feedback Mechanism: The tap point of the inductor is connected to AC ground (or cathode/emitter reference). Inductor L_1 forms collector load; inductor L_2 connects to base/emitter feedback path.
- Autotransformer Action: Currents flowing in opposite directions relative to the grounded tap create a $180°$ phase shift between input and output voltages across the tank.
- Total Equivalent Inductance: $L_{eq} = L_1 + L_2 + 2M$, where M is mutual inductance between coils ($L_{eq} = L_1 + L_2$ if mutual coupling is negligible).
- DC Biasing & Isolation: DC collector bias supplied through RF Choke (RFC) presenting high impedance to RF signals; blocking capacitors isolate DC voltages from tank coils.

Hartley Oscillator Derivation & Mathematical Analysis

- Feedback Factor (β): Defined as ratio of feedback voltage across L_2 to output voltage across L_1: $\beta = \frac{V_{fb}}{V_{out}} = \frac{X_{L2}}{X_{L1}} = \frac{j\omega L_2 + j\omega M}{j\omega L_1 + j\omega M} = \frac{L_2+M}{L_1+M}$.
- Oscillation Frequency (f_o): Resonant frequency where net tank reactance is zero: $f_o = \frac{1}{2\pi\sqrt{L_{eq}C}} = \frac{1}{2\pi\sqrt{(L_1+L_2+2M)C}}$.
- Minimum Gain Requirement: Substituting β into Barkhausen criterion $|A_v\beta| \geq 1$ yields $|A_v| \geq \frac{1}{\beta} = \frac{L_1+M}{L_2+M}$. Without mutual coupling ($M = 0$), $|A_v| \geq \frac{L_1}{L_2}$.
- Transconductance Condition for BJT CE Stage: $g_m R'_L \geq \frac{L_1}{L_2}$, where $R'_L = R_C \parallel R_{in,stage} \parallel r_o$ is equivalent load resistance.

Hartley Oscillator Component Design Example

- Problem Statement: Design a BJT Hartley oscillator to operate at $f_o = 1.0$ MHz using a tuning capacitor $C = 470$ pF. The common-emitter amplifier provides voltage gain $A_v = 15$. Determine total required inductance L_{eq} and individual inductances L_1 and L_2 (assume mutual coupling $M = 0$).
- Step 1: Compute Total Equivalent Inductance (L_{eq}): $L_{eq} = \frac{1}{4\pi^2 f_o^2 C} = \frac{1}{4\pi^2(10^6)^2(470\times10^{-12})} \approx 53.89\ \mu$H.
- Step 2: Apply Gain Criterion to determine Inductance Ratio: $|A_v| = \frac{L_1}{L_2} = 15 \Rightarrow L_1 = 15L_2$.
- Step 3: Solve System of Equations for L_1 and L_2: $L_{eq} = L_1 + L_2 = 15L_2 + L_2 = 16L_2 = 53.89\ \mu$H.
- Step 4: Calculate Component Values: $L_2 = \frac{53.89\ \mu\text{H}}{16} \approx 3.37\ \mu$H; $L_1 = 15 \times 3.37\ \mu\text{H} \approx 50.52\ \mu$H.

Colpitts Oscillator Topology & Capacitive Divider

- Tank Circuit Architecture: Consists of a single inductor L connected in parallel with a capacitive voltage divider comprising two series capacitors C_1 and C_2.
- Center Tap Grounding: Junction between C_1 and C_2 is connected to AC ground. Capacitor C_1 is connected across output (collector/drain); C_2 is connected across input (base/gate).
- Phase Inversion Mechanism: AC current through series pair (C_1, C_2) creates equal and opposite polarities across C_1 and C_2 relative to central ground node, yielding exact 180° phase shift.
- Equivalent Series Tank Capacitance (C_{eq}): $C_{eq} = \frac{C_1 C_2}{C_1 + C_2}$.
- Superior High-Frequency Performance: Parasitic input/output capacitances of transistor are absorbed directly into parallel tuning capacitors C_1 and C_2.

Colpitts Oscillator Derivation & Mathematical Analysis

- Feedback Factor (β): Defined as ratio of feedback voltage across C_2 to output voltage across C_1: $\beta = \frac{V_{fb}}{V_{out}} = \frac{X_{C2}}{X_{C1}} = \frac{1/(j\omega C_2)}{1/(j\omega C_1)} = \frac{C_1}{C_2}$.
- Oscillation Frequency (f_o): Resonant frequency of LC tank: $f_o = \frac{1}{2\pi\sqrt{LC_{eq}}} = \frac{1}{2\pi\sqrt{L\frac{C_1C_2}{C_1+C_2}}}$.
- Minimum Gain Requirement: Applying Barkhausen criterion $|A_v\beta| \geq 1$ gives $|A_v| \geq \frac{1}{\beta} = \frac{C_2}{C_1}$.
- Small-Signal BJT Transistor Condition: $h_{fe} \geq \frac{C_2}{C_1}$ or $g_m R'_L \geq \frac{C_2}{C_1}$. To ensure reliable startup, design ratio is typically set to $C_2/C_1 \approx 10$ to 20.

Colpitts Oscillator Detailed Calculation Example

- Problem Statement: A Colpitts oscillator operates at $f_o = 5.0$ MHz using an inductor $L = 10.0\ \mu$H. If the active stage requires a gain condition $A_v = C_2/C_1 = 10$, calculate C_1, C_2, and equivalent capacitance C_{eq}.
- Step 1: Calculate Required Equivalent Capacitance (C_{eq}): $C_{eq} = \frac{1}{4\pi^2 f_o^2 L} = \frac{1}{4\pi^2(5\times10^6)^2(10\times10^{-6})} = \frac{1}{4\pi^2(2.5\times10^{14})(10^{-5})} \approx 101.32$ pF.
- Step 2: Express C_{eq} in Terms of C_1 using $C_2 = 10C_1$: $C_{eq} = \frac{C_1(10C_1)}{C_1+10C_1} = \frac{10C_1^2}{11C_1} = \frac{10}{11}C_1$.
- Step 3: Solve for C_1: $C_1 = \frac{11}{10}C_{eq} = 1.1 \times 101.32$ pF ≈ 111.45 pF.
- Step 4: Solve for C_2: $C_2 = 10 \times C_1 = 10 \times 111.45$ pF ≈ 1114.5 pF (standard 1.1 nF).

Op-Amp Based Hartley & Colpitts Implementations

- Inverting Op-Amp Configuration: Op-amp connected in inverting amplifier configuration provides 180° internal phase shift with voltage gain $A_v = -R_f/R_1$.
- Op-Amp Hartley Circuit: Tapped inductor tank (L_1, L_2, C) connected between output and inverting input. Closed-loop gain constraint: $\frac{R_f}{R_1} \geq \frac{L_1}{L_2}$.
- Op-Amp Colpitts Circuit: Capacitive divider tank (C_1, C_2, L) connected in feedback path. Closed-loop gain constraint: $\frac{R_f}{R_1} \geq \frac{C_2}{C_1}$.
- Frequency Limitations of Op-Amps: Limited by Gain-Bandwidth Product (GBWP) and Slew Rate $(SR = 2\pi f V_p)$, restricting Op-Amp LC oscillators to frequencies below $\sim 10 - 20$ MHz. High-frequency RF oscillators ($>$ 50 MHz) use discrete BJTs or MOSFETs.

Comparison: Hartley vs. Colpitts Oscillators

- Frequency Tuning & Adjustment: Hartley is easier to tune across a continuous frequency band using a single variable capacitor C; Colpitts requires ganged variable capacitors (C_1, C_2) or variable inductor L.
- Waveform Purity & Harmonics: Colpitts yields superior spectral purity and lower harmonic distortion because capacitors C_1 and C_2 provide low-impedance bypass paths for high-frequency harmonics.
- Parasitic Effects: Hartley suffers from parasitic inter-winding capacitance of tapped coil; Colpitts absorbs transistor parasitic input/output capacitances into C_1 and C_2.
- Frequency Range: Hartley used primarily in RF/AM broadcast bands (100 kHz – 30 MHz); Colpitts preferred in VHF/UHF radio bands (up to hundreds of MHz).

Frequency Stability & Clapp Oscillator Enhancement

- Frequency Drift Factors: Thermal expansion of inductors, temperature-coefficient of capacitors, and variations in transistor junction capacitances (C_{be}, C_{bc}) due to temperature and voltage fluctuations.
- Clapp Oscillator Architecture: A high-stability variant of Colpitts oscillator that adds a small series tuning capacitor $C_3 \ll C_1, C_2$ in series with inductor L.
- Equivalent Series Capacitance (C'_{eq}): $\frac{1}{C'_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$. Since $C_3 \ll C_1, C_2$, $C'_{eq} \approx C_3$.
- Oscillation Frequency (f_o): $f_o = \frac{1}{2\pi\sqrt{LC_3}}$. Oscillation frequency becomes virtually independent of C_1, C_2 and transistor stray capacitances, dramatically improving frequency stability.

Summary of LC Oscillators & Design Guidelines

- LC oscillators operate on Barkhausen criterion ($A\beta = 1\angle 0^\circ$) using frequency-selective LC resonant tanks for RF frequency generation.
- Hartley Oscillator uses a tapped inductor ratio (L_1/L_2) for feedback $\beta = L_2/L_1$, requiring amplifier voltage gain $|A_v| \geq L_1/L_2$.
- Colpitts Oscillator uses a split capacitor ratio (C_1/C_2) for feedback $\beta = C_1/C_2$, requiring amplifier voltage gain $|A_v| \geq C_2/C_1$.
- Colpitts provides superior harmonic suppression and high-frequency stability; Clapp topology further isolates frequency from parasitic drift using series capacitor C_3.

Formula Reference & Key Equations

- Hartley Resonant Frequency: $f_o = \frac{1}{2\pi\sqrt{(L_1+L_2+2M)C}}$, Gain Condition: $|A_v| \geq \frac{L_1+M}{L_2+M}$.
- Colpitts Resonant Frequency: $f_o = \frac{1}{2\pi\sqrt{L\frac{C_1C_2}{C_1+C_2}}}$, Gain Condition: $|A_v| \geq \frac{C_2}{C_1}$.
- Clapp Resonant Frequency: $f_o \approx \frac{1}{2\pi\sqrt{LC_3}}$ (where $C_3 \ll C_1, C_2$).
- Tank Quality Factor (Q): $Q = \frac{\omega_0 L}{R_s} = \frac{R_p}{\omega_0 L}$, where higher Q enhances frequency selectivity and phase stability.