Lecture 38: Overall gain of positive feedback amplifier

Oscillators

DI02011011: Electronics Circuit and Application (ECA)

Agenda: Overall Gain of Positive Feedback Amplifier

- Derivation of Closed-Loop Gain $A_f = \frac{A}{1-A\beta}$
- Operating Regimes based on Loop Gain Magnitude ($A\beta < 1$, $A\beta = 1$, $A\beta > 1$)
- Sensitivity Analysis of Closed-Loop Gain ($S_A^{A_f} = \frac{1}{1-A\beta}$)
- Bandwidth Narrowing & Gain-Bandwidth Product Conservation
- Non-linear Distortion Multiplier & SNR Characteristics
- Quantitative Numerical Example: Gain Spikes & Sensitivity
- Circuit Case Study: Op-Amp Non-Inverting Positive Feedback Stage
- Historical Application: Armstrong Regenerative Receiver
- Comparative Matrix: Open-Loop vs Positive vs Negative Feedback
- Engineering Guidelines to Prevent Unintended Positive Feedback Oscillations
- Summary & Key Engineering Takeaways

Derivation of Closed-Loop Gain Expression

- Summing Node Equation: Effective input voltage $V_i = V_s + V_f = V_s + \beta V_o$, where V_s is source voltage, V_f is feedback voltage, and β is feedback factor.
- Amplifier Output Equation: Output voltage $V_o = A \cdot V_i = A(V_s + \beta V_o)$, where A is open-loop voltage gain.
- Algebraic Expansion: $V_o - A\beta V_o = AV_s \implies V_o(1 - A\beta) = AV_s$.
- Closed-Loop Voltage Gain: $A_f = \frac{V_o}{V_s} = \frac{A}{1-A\beta}$, where loop gain is defined as $T = A\beta$.

Operating Regimes based on Loop Gain Magnitude

- Regime 1 ($0 < A\beta < 1$): Stable Positive Feedback Amplification. Denominator $(1 - A\beta) < 1$, yielding closed-loop gain higher than open-loop gain ($A_f > A$).
- Regime 2 ($A\beta = 1$): Oscillation Threshold. Denominator $(1 - A\beta) = 0 \implies A_f \to \infty$. Circuit produces continuous output V_o with zero input $V_s = 0$.
- Regime 3 ($A\beta > 1$): Super-Critical Instability. Output grows exponentially until active devices saturate or hit power supply rails.
- Regenerative Threshold: Keeping $A\beta$ close to 1 (e.g. 0.99) produces enormous gain ($A_f = 100A$) in a single amplifier stage.

Sensitivity Analysis of Closed-Loop Gain

- Definition of Gain Sensitivity: Sensitivity of closed-loop gain A_f with respect to open-loop gain A: $S_A^{A_f} = \frac{dA_f/A_f}{dA/A} = \frac{dA_f}{dA}\frac{A}{A_f}$.
- Mathematical Derivation: Differentiation yields $\frac{dA_f}{dA} = \frac{(1-A\beta)-A(-\beta)}{(1-A\beta)^2} = \frac{1}{(1-A\beta)^2}$.
- Sensitivity Factor Formula: $S_A^{A_f} = \left[\frac{1}{(1-A\beta)^2}\right]\left[\frac{A}{\frac{A}{1-A\beta}}\right] = \frac{1}{1-A\beta}$.
- Comparison with Negative Feedback: Negative feedback reduces gain sensitivity ($S = \frac{1}{1+A\beta} < 1$), whereas positive feedback amplifies sensitivity ($S = \frac{1}{1-A\beta} > 1$), making gain vulnerable to thermal and parameter variations.

Bandwidth Narrowing & Gain-Bandwidth Product Conservation

- Open-Loop Frequency Response: Single-pole amplifier response $A(s) = \frac{A_0}{1+s/\omega_H}$, where A_0 is DC gain and ω_H is 3dB bandwidth.
- Closed-Loop Transfer Function Derivation: $A_f(s) = \frac{\frac{A_0}{1+s/\omega_H}}{1-\beta\frac{A_0}{1+s/\omega_H}} = \frac{A_0}{1-A_0\beta+s/\omega_H} = \frac{\frac{A_0}{1-A_0\beta}}{1+s/[\omega_H(1-A_0\beta)]}$.
- Closed-Loop Bandwidth Expression: Closed-loop 3dB bandwidth is $\omega_{Hf} = \omega_H(1 - A_0\beta)$.
- Gain-Bandwidth Conservation: $A_{f0} \cdot \omega_{Hf} = \left(\frac{A_0}{1-A_0\beta}\right)(\omega_H(1 - A_0\beta)) = A_0\omega_H = GBW$ (Constant).

Non-linear Distortion Multiplier & SNR Characteristics

- Harmonic Distortion Multiplier: Non-linear distortion generated in amplifier stage is multiplied by feedback factor: $D_f = \frac{D}{1-A\beta}$.
- Reduced Dynamic Range: Near $A\beta \approx 1$, small non-linearities in g_m create severe waveform clipping and intermodulation distortion.
- Noise Amplification: Internal amplifier thermal noise v_n is amplified by closed-loop gain A_f, elevating total output noise power spectral density.
- Selectivity Benefit: Despite distortion trade-offs, positive feedback provides narrow bandpass filtering around resonant frequency f_0, improving frequency selectivity.

Quantitative Numerical Example: Gain Spikes & Sensitivity

- Problem Statement: Consider an amplifier with open-loop gain $A = 100$. Calculate overall closed-loop gain A_f and gain sensitivity S for feedback factors $\beta = 0.005, 0.008, 0.009, 0.0099$.
- Case 1 ($\beta = 0.005$): $A\beta = 0.5 \implies A_f = \frac{100}{1-0.5} = 200$, Sensitivity $S = \frac{1}{0.5} = 2$.
- Case 2 ($\beta = 0.008$): $A\beta = 0.8 \implies A_f = \frac{100}{1-0.8} = 500$, Sensitivity $S = \frac{1}{0.2} = 5$.
- Case 3 ($\beta = 0.0099$): $A\beta = 0.99 \implies A_f = \frac{100}{1-0.99} = 10,000$, Sensitivity $S = \frac{1}{0.01} = 100$. (A 1% change in A causes a 100% change in A_f!).

Circuit Case Study: Op-Amp Positive Feedback Stage

- Circuit Setup: Op-amp connected with feedback resistor R_2 between output and non-inverting terminal $(+)$, and resistor R_1 from $(+)$ terminal to ground.
- Feedback Factor Derivation: Voltage division at $(+)$ terminal gives $\beta = \frac{R_1}{R_1+R_2}$.
- Closed-Loop Gain Formulation: $A_f = \frac{A_v}{1-A_v \frac{R_1}{R_1+R_2}}$, where A_v is op-amp open-loop gain.
- Hysteresis Thresholds in Schmitt Trigger: When $A_v \to \infty$, loop gain $A_v\beta \gg 1$ creates bi-stable switching with upper threshold $V_{UT} = +V_{sat} \frac{R_1}{R_1+R_2}$ and lower threshold $V_{LT} = -V_{sat} \frac{R_1}{R_1+R_2}$.

Historical Application: Armstrong Regenerative Receiver

- Historical Significance: Invented by Edwin Howard Armstrong in 1912; revolutionized radio communications.
- Operating Principle: Controlled positive feedback (regeneration) applied via a tickler coil back to grid circuit, maintaining loop gain $A\beta \approx 0.99$.
- Performance Breakthrough: Single vacuum tube achieved $1000\times$ voltage amplification and razor-sharp selectivity, receiving distant RF signals.
- Oscillation Threshold Risk: If operator adjusted tickler coil past $A\beta = 1$, receiver burst into self-oscillation, radiating interference to nearby radios.

Comparative Matrix: Open-Loop vs Positive vs Negative Feedback

- Gain Parameter: Open-Loop = A; Negative Feedback = $A_f = \frac{A}{1+A\beta} < A$; Positive Feedback = $A_f = \frac{A}{1-A\beta} > A$.
- Bandwidth Parameter: Open-Loop = BW; Negative Feedback = $BW(1 + A\beta)$ (Wider); Positive Feedback = $BW(1 - A\beta)$ (Narrower).
- Gain Stability / Sensitivity: Open-Loop = High; Negative Feedback = Extremely Low ($S = \frac{1}{1+A\beta}$); Positive Feedback = Extremely High ($S = \frac{1}{1-A\beta}$).
- Primary Applications: Open-Loop = Comparators; Negative Feedback = Linear Amplifiers; Positive Feedback = Oscillators, Schmitt Triggers, Regenerative Receivers.

Engineering Guidelines to Prevent Unintended Oscillations

- Parasitic Feedback Isolation: Minimize parasitic capacitance C_{gd} or C_μ between amplifier output and input traces on PCB.
- Power Supply Decoupling: Install ceramic (0.1 μF) and electrolytic (10 μF) decoupling capacitors directly at amplifier supply pins to prevent supply line feedback.
- Ground Plane Design: Utilize continuous low-impedance ground plane to prevent common ground impedance coupling.
- Shielding & Component Placement: Separate input and output stage components physically to prevent radiative electromagnetic positive feedback.

Summary: Key Takeaways on Overall Gain

- Closed-Loop Gain Formulation: Positive feedback increases overall gain to $A_f = \frac{A}{1 - A\beta}$.
- Operating Regimes: $0 < A\beta < 1$ yields stable gain enhancement; $A\beta = 1$ triggers self-sustained oscillation ($A_f \to \infty$).
- Gain-Bandwidth Trade-off: Gain increases by $\frac{1}{1 - A\beta}$ while 3dB bandwidth narrows by $(1 - A\beta)$, keeping GBW constant.
- Design Caution: High sensitivity $S = \frac{1}{1 - A\beta}$ and elevated distortion require strict control of feedback factor β and parasitic coupling.