Lecture 36: Use of positive feedback in oscillators

Oscillators

DI02011011: Electronics Circuit and Application (ECA)

Agenda: Positive Feedback in Oscillators

- Fundamentals of Positive vs Negative Feedback Topologies
- Derivation of Closed-Loop Gain $A_f(s) = \frac{A(s)}{1-A(s)\beta(s)}$
- Initiation of Oscillations: Thermal Noise & Transient Buildup
- Phase & Gain Conditions around the Closed Loop
- Comparative Analysis: Positive Feedback Amplifiers vs Oscillators
- Circuit Case Study: Non-Inverting Wien-Bridge Feedback Topology
- Circuit Case Study: Inverting 3-Stage RC Phase-Shift Feedback Topology
- Role of Frequency-Selective Feedback Networks (RC, LC, Crystal)
- Amplitude Stabilization: Automatic Gain Control (AGC) and Transistor Saturation
- Practical Design Challenges: Frequency Drift, Loading, and Phase Noise
- Summary & Key Engineering Takeaways

Fundamental Feedback Topologies: Positive vs Negative Feedback

- Summing Node Dynamics: In negative feedback, feedback voltage opposes input ($V_{in,eff} = V_s - V_f$); in positive feedback, feedback voltage reinforces input ($V_{in,eff} = V_s + V_f$).
- System Transfer Behavior: Positive feedback increases effective gain, narrows bandwidth, and decreases stability, creating regenerative feedback loops.
- Phase Synchronization: Positive feedback requires feedback signal $V_f = \beta V_{out}$ to be in phase (0° phase difference) with input signal V_{in} at oscillation frequency f_0.
- Energy Conversion Mechanism: Amplifier active components (BJTs, FETs, Op-Amps) draw DC energy from power supply V_{CC} and convert it to AC signal energy under control of positive feedback.

Derivation of Closed-Loop Gain under Positive Feedback

- Block Diagram Relations: Amplifier output voltage $V_{out} = A \cdot V_{in,eff}$; feedback voltage $V_f = \beta \cdot V_{out}$; summing equation $V_{in,eff} = V_s + \beta V_{out}$.
- Algebraic Substitution: $V_{out} = A(V_s + \beta V_{out}) \implies V_{out}(1 - A\beta) = AV_s$.
- Closed-Loop Gain Derivation: $A_f = \frac{V_{out}}{V_s} = \frac{A}{1-A\beta}$, where A is open-loop gain and β is feedback transfer factor.
- Condition for Oscillation without External Drive ($V_s = 0$): For finite output V_{out} with zero input $V_s = 0$, denominator $(1 - A\beta)$ must equal 0, yielding loop gain condition $A\beta = 1 + j0$.

Initiation of Oscillations: Thermal Noise & Transient Buildup

- White Noise Origin: Thermal agitation of electrons in circuit resistors produces white noise with power spectral density $v_n^2 = 4kTB\Delta f$ across all frequencies.
- Frequency Filtering: Frequency-selective feedback network $\beta(f)$ filters white noise, passing only frequency components near resonant frequency f_0.
- Exponential Buildup Dynamics: Initial loop gain $|A\beta| > 1$ places closed-loop poles in right-half s-plane ($s = \sigma \pm j\omega_0$ with $\sigma > 0$), producing exponentially growing envelope $v(t) = V_{noise} e^{\sigma t} \sin(\omega_0 t)$.
- Saturation Limiting: As amplitude grows, transistor non-linearities reduce effective large-signal gain A until steady-state loop gain satisfies $|A\beta| = 1$ with poles on $j\omega$-axis ($\sigma = 0$).

Phase Shift Distribution across Amplifier and Feedback Network

- Total Loop Phase Condition: Total phase shift around feedback loop must be $\angle A(j\omega_0) + \angle\beta(j\omega_0) = 0^\circ$ or 360° ($2\pi n$ radians).
- Inverting Amplifier Configuration: Inverting stage (CE BJT or inverting Op-Amp) introduces $\angle A = 180^\circ$; feedback network must provide $\angle\beta = 180^\circ$ at frequency f_0.
- Non-Inverting Amplifier Configuration: Non-inverting stage (CC BJT or non-inverting Op-Amp) introduces $\angle A = 0^\circ$; feedback network must provide $\angle\beta = 0^\circ$ at frequency f_0.
- Frequency Stability Factor: Phase slope $S_F = \omega_0 \frac{d\phi}{d\omega}\Big|_{\omega=\omega_0}$ determines frequency stability; higher phase slope resists frequency drift caused by component variations.

Comparison: Positive Feedback Amplifiers vs Oscillators

- Operating Loop Gain: Regenerative positive feedback amplifiers maintain $|A\beta| < 1$ (e.g. 0.95); oscillators require startup $|A\beta| > 1$ and steady-state $|A\beta| = 1$.
- Input Signal Requirement: Regenerative amplifiers boost an external input signal V_{in}; oscillators operate completely without external input ($V_{in} = 0$).
- Bandwidth vs Selectivity: Regenerative amplifiers narrow amplifier bandwidth around signal frequency; oscillators produce single pure discrete frequency f_0.
- Output Stability: Amplifiers aim for linear input-output response with low distortion; oscillators intentionally operate into non-linear gain saturation for amplitude limiting.

Circuit Analysis: Non-Inverting Wien-Bridge Oscillator

- Circuit Topology: Op-Amp non-inverting amplifier with gain $A_v = 1 + \frac{R_f}{R_1}$, paired with series-parallel RC Wien-Bridge feedback network.
- Feedback Transfer Factor Derivation: $\beta(s) = \frac{Z_p}{Z_s+Z_p} = \frac{sRC}{s^2R^2C^2+3sRC+1}$; at resonant frequency $\omega_0 = \frac{1}{RC}$, $\beta(j\omega_0) = \frac{1}{3}\angle 0°$.
- Gain Threshold Calculation: For loop gain $A\beta = 1$, op-amp voltage gain must be $A_v = 3$, requiring resistor ratio $\frac{R_f}{R_1} = 2$.
- Practical Startup Criterion: Design sets $\frac{R_f}{R_1} = 2.1$ ($A_v = 3.1, |A\beta| = 1.033 > 1$) for fast startup, with dynamic gain reduction to $A_v = 3.0$ at steady-state amplitude.

Role of Frequency-Selective Networks in Feedback Loops

- Function of $\beta(\omega)$ Network: Acts as narrow bandpass filter establishing phase condition $\angle\beta(j\omega_0) = -\angle A(j\omega_0)$ at single frequency f_0.
- RC Feedback Networks: Lead-lag networks (Wien Bridge) and RC ladder networks (Phase Shift) for audio frequencies (10 Hz – 100 kHz).
- LC Resonant Tanks: Inductive-capacitive networks (Hartley, Colpitts) for radio frequencies (100 kHz – 500 MHz) using energy exchange between L and C.
- Piezoelectric Crystal Networks: Quartz crystals utilizing mechanical resonance for precision clocks (32.768 kHz – 200 MHz) with ultra-high quality factor $Q > 10,000$.

Circuit Analysis: Inverting 3-Stage RC Phase-Shift Oscillator

- Circuit Configuration: Inverting op-amp or BJT CE amplifier (180° phase shift) cascaded with 3-stage high-pass RC ladder feedback network.
- Transfer Function Analysis: \{}beta(s) = \{}frac{s^3 R^3 C^3}{s^3 R^3 C^3 + 6 s^2 R^2 C^2 + 5 s R C + 1}.
- Phase Shift & Resonant Frequency: Imaginary term vanishes at $\omega_0 = \frac{1}{\sqrt{6}RC}$, where phase shift across 3 RC stages is exactly 180°.
- Feedback Attenuation & Gain Requirement: At ω_0, feedback factor $\beta(j\omega_0) = -\frac{1}{29}$; amplifier must provide gain $|A_v| \geq 29$ to satisfy $|A\beta| \geq 1$.

Amplitude Stabilization Mechanisms in Positive Feedback Systems

- Need for Non-linear Gain Limiting: Linear positive feedback loops with $|A\beta| > 1$ grow indefinitely until hard transistor saturation/clipping occurs, causing severe harmonic distortion.
- Automatic Gain Control (AGC) with JFET: Dynamic drain-source resistance $r_{ds}(V_{out})$ of JFET in feedback loop automatically reduces gain A_v as output amplitude increases.
- Thermistors / PTC Resistors: Temperature-dependent resistor in op-amp feedback loop heats up with signal current, increasing resistance and lowering loop gain to exactly 1.
- Diode Clamping Networks: Antiparallel diodes in parallel with feedback resistor R_f conduct during signal peaks, lowering effective R_f and soft-clamping oscillation amplitude.

Practical Design Considerations & Non-Idealities

- Temperature Drift & Frequency Instability: Thermal variations in R and C values cause frequency drift $\Delta f_0 = \frac{\partial f_0}{\partial R}\Delta R + \frac{\partial f_0}{\partial C}\Delta C$.
- Power Supply Rejection: Voltage fluctuations on supply rails V_{CC} alter transistor bias and junction capacitances C_π, C_μ, shifting oscillation frequency.
- Loading Effects: Connecting external load impedance R_L directly across oscillator feedback network lowers effective loop gain below 1, killing oscillations.
- Buffer Stage Requirement: High input-impedance buffer amplifiers (emitter follower or op-amp unity follower) must isolate oscillator tank from external load.

Summary: Principles of Positive Feedback in Oscillators

- Regenerative Positive Feedback: Positive feedback feeds back output signal in phase with input ($V_f = \beta V_{out}$), enabling self-sustained waveform generation from DC power.
- Closed-Loop Criterion: Closed-loop gain $A_f = \frac{A}{1-A\beta}$ yields self-sustained output at $V_s = 0$ when loop gain satisfies $A\beta = 1\angle 0°$.
- Startup & Stabilization: Initial startup uses thermal noise seed with $|A\beta| > 1$; non-linear saturation or AGC dynamically adjusts $|A\beta| = 1$ in steady state.
- Frequency & Phase Control: Frequency-selective networks (RC, LC, Crystal) dictate oscillation frequency f_0, while buffer stages prevent load pulling.