Lecture 31: Voltage gain of feedback amplifier

Feedback in amplifiers

DI02011011: Electronics Circuit and Application (ECA)

Lecture Agenda & Systematic Outline

- System Architecture & Feedback Block Diagram Representation
- Derivation of Closed-Loop Voltage Gain Equation ($A_f = \frac{A}{1+A\beta}$)
- Asymptotic Limits & Feedback Ratio (β) Behavior at High Loop Gain
- Loop Transmission, Return Ratio, and Amount of Feedback in Decibels
- Gain Desensitivity & Fractional Variation Analysis ($\frac{dA_f}{A_f} = \frac{1}{1+A\beta}\frac{dA}{A}$)
- Voltage-Series vs. Voltage-Shunt Topologies Impact on Voltage Gain
- Two-Port h-Parameter Modeling of Loaded Feedback Networks
- Non-Inverting Op-Amp Voltage Gain Derivation ($A_{Vf} = 1 + \frac{R_2}{R_1}$)
- Inverting Op-Amp Transresistance to Voltage Gain Conversion ($A_{Vf} = -\frac{R_f}{R_1}$)
- High-Frequency Response & Gain-Bandwidth Constancy ($A_{0f} \cdot \omega_{Hf} = A_0 \cdot \omega_H$)
- Numerical Problem: Gain Variation & Desensitivity Calculation
- Circuit Analysis Example: Discrete BJT Voltage-Series Feedback Amplifier
- Summary & Engineering Synthesis

System Architecture of Feedback Amplifiers

- Basic Block Structure: Consists of an open-loop basic amplifier with gain $A = V_o / V_i$, a passive feedback network with transfer ratio $\beta = V_f / V_o$, and an input summing junction where $V_i = V_s - V_f$.
- Signal Definitions: Source signal V_s, net internal error signal $V_i = V_s - \beta V_o$, output voltage $V_o = A \cdot V_i$, and feedback signal $V_f = \beta V_o$.
- Negative Feedback Phase Condition: Requires feedback signal V_f to subtract from input V_s, establishing a 180° phase shift relative to open-loop output to stabilize total gain.
- Ideal Feedback Assumptions: Basic amplifier is unilateral (A transmits forward only), feedback network is unilateral (β transmits backward only), and β network loading is fully accounted for.

Mathematical Derivation of Closed-Loop Gain (A_f)

- Output Equation Setup: Output voltage expressed in terms of open-loop gain A and net error input V_i: $V_o = AV_i = A(V_s - V_f)$.
- Feedback Substitution: Substituting feedback relationship $V_f = \beta V_o$ yields $V_o = A(V_s - \beta V_o) = AV_s - A\beta V_o$.
- Algebraic Rearrangement: Collecting output terms on the left side: $V_o + A\beta V_o = AV_s \implies V_o(1 + A\beta) = AV_s$.
- Closed-Loop Gain Formula: Dividing by V_s gives closed-loop voltage gain $A_f = \frac{V_o}{V_s} = \frac{A}{1+A\beta}$, where term $(1 + A\beta)$ is the desensitivity factor or amount of feedback.

Asymptotic Limits & Feedback Ratio (β) Behavior

- High Loop Gain Limit ($A\beta \gg 1$): When open-loop gain product $A\beta$ is extremely large ($A\beta \to \infty$), $1 + A\beta \approx A\beta$.
- Asymptotic Closed-Loop Gain: $A_f = \frac{A}{1+A\beta} \approx \frac{A}{A\beta} = \frac{1}{\beta}$. Closed-loop gain depends exclusively on feedback factor β.
- Passive Component Precision: Since β is constructed using stable passive resistors (R_1, R_2), closed-loop gain becomes virtually immune to transistor non-linearities, temperature drift, and manufacturing tolerances.
- Feedback Ratio Bounds: Passive feedback networks constrain $0 < \beta \leq 1$; for a pure unity voltage follower, $\beta = 1$, giving $A_f \approx 1$ V/V.

Loop Gain ($A\beta$), Return Ratio, and Feedback Measurement

- Loop Gain Definition: $T = A\beta$ represents the total voltage transmission around the feedback loop when broken at a specified node.
- Return Ratio Determination: Disconnect feedback path, inject test voltage V_t at break point with source ground $V_s = 0$; returned voltage $V_r = -A\beta V_t$. Return ratio $RR = -V_r/V_t = A\beta$.
- Amount of Feedback in Decibels (N_{dB}): Defined quantitatively as $N_{dB} = 20\log_{10}|1 + A\beta|$.
- Decibel Example: A desensitivity factor $(1 + A\beta) = 100$ corresponds to $20\log_{10}(100) = 40$ dB of feedback, reducing voltage gain by 40 dB while enhancing stability equally.

Gain Desensitivity & Fractional Variation Analysis

- Sensitivity Derivative: Differentiating closed-loop gain $A_f = \frac{A}{1+A\beta}$ with respect to open-loop gain A yields $\frac{dA_f}{dA} = \frac{(1+A\beta)-A\beta}{(1+A\beta)^2} = \frac{1}{(1+A\beta)^2}$.
- Fractional Gain Sensitivity Equation: Expressing relative change: $\frac{dA_f}{A_f} = \frac{dA}{A} \cdot \frac{A}{A_f} \cdot \frac{dA_f}{dA} = \frac{dA}{A} \cdot (1 + A\beta) \cdot \frac{1}{(1+A\beta)^2} = \frac{1}{1+A\beta} \frac{dA}{A}$.
- Desensitivity Factor Signification: Closed-loop gain fractional variation $\frac{dA_f}{A_f}$ is reduced by factor $(1 + A\beta)$ compared to open-loop variation $\frac{dA}{A}$.
- Sensitivity Function $S_A^{A_f}$: Defined as $S_A^{A_f} = \frac{\%\Delta A_f}{\%\Delta A} = \frac{1}{1+A\beta}$, proving that high loop gain renders the system desensitized to internal variations.

Voltage-Series vs. Voltage-Shunt Topology Impact on Gain

- Voltage-Series (Series-Shunt) Topology: Samples output voltage V_o and mixes feedback voltage V_f in series with source V_s. Ideal pure voltage amplifier with closed-loop voltage gain $A_{Vf} = \frac{A_V}{1+\beta_v A_V}$.
- Voltage-Shunt (Shunt-Shunt) Topology: Samples output voltage V_o and mixes feedback current I_f in shunt with source current I_s. Basic amplifier has transresistance gain $R_m = V_o/I_i$.
- Closed-Loop Transresistance (R_{mf}): $R_{mf} = \frac{R_m}{1+\beta_g R_m}$, where $\beta_g = I_f/V_o$ is feedback transconductance in Siemens.
- Terminal Voltage Gain Conversion: For voltage-shunt feedback with source resistance R_s, effective voltage gain $A_{Vf} = \frac{V_o}{V_s} = \frac{V_o}{I_s R_s} = \frac{R_{mf}}{R_s} = \frac{R_m/(1+\beta_g R_m)}{R_s}$.

Two-Port Parameter Modeling of Loaded Feedback Networks

- Loading Effect Realization: Practical feedback networks load both input and output ports of the basic amplifier, altering open-loop gain from A to modified open-loop gain A'.
- Two-Port h-Parameter Extraction: For voltage-series feedback, β-network is modeled as an h-parameter two-port block: $h_{11} = Z_{in,\beta}|_{V_o=0}$, $h_{22}^{-1} = Z_{out,\beta}|_{I_i=0}$, $\beta = h_{12} = \frac{V_f}{V_o}\Big|_{I_i=0}$.
- Modified Basic Amplifier Construction: Include h_{11} in series with input impedance R_{in} and h_{22}^{-1} in parallel with output load R_L to form modified amplifier A'.
- Loaded Voltage Gain Equation: Compute $A' = \frac{V_o'}{V_i'}\Big|_{loaded}$; exact closed-loop gain is then $A_{Vf} = \frac{A'}{1+h_{12}A'}$.

Voltage Gain Analysis of Non-Inverting Op-Amp Amplifier

- Circuit Topology: Input voltage V_s applied to non-inverting terminal ($V_+ = V_s$); feedback voltage divider R_1, R_2 connected between output V_o and inverting terminal ($V_- = V_f$).
- Feedback Factor Calculation: $\beta = \frac{V_f}{V_o} = \frac{R_1}{R_1+R_2}$. Op-amp open-loop voltage gain is A_{OL}.
- Exact Voltage Gain Equation: $A_{Vf} = \frac{A_{OL}}{1+A_{OL}\beta} = \frac{A_{OL}}{1+A_{OL}\frac{R_1}{R_1+R_2}}$.
- Ideal Op-Amp Simplification ($A_{OL} \to \infty$): $A_{Vf} \approx \frac{1}{\beta} = \frac{R_1+R_2}{R_1} = 1 + \frac{R_2}{R_1}$. Voltage gain is completely determined by passive resistor ratios.

Voltage Gain Analysis of Inverting Feedback Amplifier

- Circuit Topology: Non-inverting terminal grounded ($V_+ = 0$ V); input V_s connected via R_1 to virtual ground node V_-; feedback resistor R_f connected from output V_o to V_-.
- Voltage-Shunt Analysis: Operates as voltage-shunt feedback. Transresistance gain $R_{mf} = \frac{V_o}{I_s} = -R_f$. Source current $I_s = \frac{V_s}{R_1}$.
- Closed-Loop Voltage Gain Derivation: $A_{Vf} = \frac{V_o}{V_s} = \frac{V_o}{I_s R_1} = \frac{-R_f}{R_1}$.
- Finite Op-Amp Gain Correction: Including finite A_{OL}, exact voltage gain is $A_{Vf} = \frac{-R_f/R_1}{1+\frac{1+R_f/R_1}{A_{OL}}}$, revealing gain error when A_{OL} is small.

Numerical Problem: Gain Variation & Desensitivity Calculation

- Problem Statement: An amplifier has an open-loop voltage gain $A = 10,000 \pm 20\%$. Design a voltage-series feedback network to achieve a closed-loop gain $A_{Vf} = 100$ V/V. Calculate required feedback factor β, desensitivity factor, and final percentage variation in A_{Vf}.
- Step 1 - Calculate Required Feedback Factor (β): From $A_{Vf} = \frac{A}{1+A\beta} \implies 100 = \frac{10{,}000}{1+10{,}000\beta} \implies 1 + 10,000\beta = 100 \implies \beta = \frac{99}{10{,}000} = 0.0099$ (0.99%).
- Step 2 - Determine Desensitivity Factor: $(1 + A\beta) = 1 + (10,000 \times 0.0099) = 100$ (40 dB feedback).
- Step 3 - Calculate Closed-Loop Gain Variation: $\frac{dA_{Vf}}{A_{Vf}} = \frac{1}{1+A\beta}\frac{dA}{A} = \frac{1}{100} \times (\pm 20\%) = \pm 0.20\%$. Closed-loop gain varies by only $\pm 0.20\%$, bounded between 99.8 and 100.2 V/V!

Circuit Example: Single-Stage BJT Voltage-Series Amplifier

- Circuit Parameters: Common-Emitter stage with $R_C = 4.7$ kΩ, un-bypassed emitter resistor $R_E = 470\ \Omega$, $h_{fe} = 120$, $h_{ie} = 2.4$ kΩ, source $R_S = 600\ \Omega$. Feedback sampled at collector and fed to emitter.
- Step 1 - Unloaded Open-Loop Voltage Gain (A_V): Transconductance $g_m = \frac{h_{fe}}{h_{ie}} = \frac{120}{2400\ \Omega} = 50$ mA/V. Open-loop gain $A_V = -g_m R_C = -(0.050) \times 4700 = -235$ V/V.
- Step 2 - Feedback Factor (β): Local emitter resistor provides feedback ratio $\beta = \frac{R_E}{R_C} = \frac{470}{4700} = 0.10$.
- Step 3 - Closed-Loop Gain Calculation: Desensitivity factor $(1 + |A_V|\beta) = 1 + (235 \times 0.10) = 24.5$. Closed-loop gain $A_{Vf} = \frac{-235}{24.5} = -9.59$ V/V.

Summary & Key Takeaways: Voltage Gain of Feedback Amplifiers

- Closed-Loop Voltage Gain Formula: $A_f = \frac{A}{1+A\beta}$ quantitatively describes gain reduction by desensitivity factor $(1 + A\beta)$.
- Asymptotic Precision: For high loop gain $(A\beta \gg 1)$, gain simplifies to $A_f \approx \frac{1}{\beta}$, trading raw gain magnitude for passive component precision.
- Gain Desensitivity: Relative gain variations are reduced by factor $(1 + A\beta)$ according to $\frac{dA_f}{A_f} = \frac{1}{1+A\beta}\frac{dA}{A}$.
- Topological Modeling: Voltage-series feedback directly stabilizes voltage gain A_{Vf}, whereas voltage-shunt feedback stabilizes transresistance R_{mf} converted via source resistance R_s.