Lecture 30: Types of feedback

Feedback in amplifiers

DI02011011: Electronics Circuit and Application (ECA)

Lecture Agenda & Outline

- 1. Mathematical Proofs: Derivation of Input Impedance for Series (KVL) and Shunt (KCL) Mixing
- 2. Mathematical Proofs: Derivation of Output Impedance for Voltage and Current Sampling using Test Sources
- 3. High-Frequency Pole Movement: Root-Locus and Q-factor Dynamics in Multi-Pole Feedback Amplifiers
- 4. Stability Metrics: Gain Margin (GM) and Phase Margin (PM) Criteria on Nyquist and Bode Plots
- 5. Frequency Compensation: Dominant Pole Placement, Lag Compensation, and Miller Pole-Splitting Mechanics
- 6. Engineering Design Examples: Multi-Stage Feedback Calculations and Miller Compensation Design

Mathematical Proof of Input Impedance Transformation

- Proof 1: Series Input Mixing (Voltage Comparison):
- Input Circuit Equations: Apply Kirchhoff's Voltage Law at input loop: V_s = V_i + V_f. Substitute V_f = beta *V_o = beta* (A * V_i).
- Substituting V_f: V_s = V_i + A *beta* V_i = V_i *(1 + A* beta).
- Input Current Equation: I_i = V_i / R_in (where R_in is open-loop input resistance).
- Closed-Loop Resistance Derivation: R_inf = V_s / I_i = [V_i *(1 + A* beta)] / (V_i / R_in) = R_in *(1 + A* beta). PROVED!
- Proof 2: Shunt Input Mixing (Current Comparison):
- Apply Kirchhoff's Current Law at input node: I_s = I_i + I_f. Substitute I_f = beta *I_o = beta* (A * I_i).
- Substituting I_f: I_s = I_i + A *beta* I_i = I_i *(1 + A* beta).
- Input Voltage Equation: V_i = I_i * R_in.
- Closed-Loop Resistance Derivation: R_inf = V_i / I_s = (I_i *R_in) / [I_i* (1 + A *beta)] = R_in / (1 + A* beta). PROVED!

Mathematical Proof of Output Impedance Transformation

- Proof Method: Deactivate input signal source (set V_s = 0 or I_s = 0), apply an external test voltage V_x at output terminals, measure resulting current I_x. R_of = V_x / I_x.
- Proof 1: Voltage Output Sampling (Shunt Output):
- For set V_s = 0 and applied V_x: Feedback voltage V_f = beta *V_x. Series input gives error V_i = 0 - V_f = -beta* V_x.
- Internal Amplifier Current: Dependent source generates current g_m *V_i = -A* V_x / R_o.
- KCL at Output Node: I_x = (V_x / R_o) - (internal source current) = (V_x / R_o) - (-A *beta* V_x / R_o) = (V_x / R_o) *(1 + A* beta).
- Closed-Loop Output Resistance: R_of = V_x / I_x = R_o / (1 + A * beta). PROVED!
- Proof 2: Current Output Sampling (Series Output):
- Applying test current I_x through output loop with V_s = 0 gives V_f = beta *I_x =¿ V_i = -beta* I_x.
- KVL around Output Loop: V_x = I_x *R_o - A* V_i = I_x *R_o - A* (-beta *I_x) = I_x* R_o *(1 + A* beta).
- Closed-Loop Output Resistance: R_of = V_x / I_x = R_o *(1 + A* beta). PROVED!

Summary & Matrix of Impedance Transformation Rules

- Master Impedance Matrix:
- Topology 1 (Voltage-Series): Series Input -¿ R_inf = R_in *(1 + A_v* beta_v); Voltage Output -¿ R_of = R_o / (1 + A_v * beta_v).
- Topology 2 (Current-Series): Series Input -¿ R_inf = R_in *(1 + A_m* beta_z); Current Output -¿ R_of = R_o *(1 + A_m* beta_z).
- Topology 3 (Voltage-Shunt): Shunt Input -¿ R_inf = R_in / (1 + A_r *beta_g); Voltage Output -¿ R_of = R_o / (1 + A_r* beta_g).
- Topology 4 (Current-Shunt): Shunt Input -¿ R_inf = R_in / (1 + A_i *beta_i); Current Output -¿ R_of = R_o* (1 + A_i * beta_i).
- Signal Conditioning Matching:
- Voltage Source -¿ Requires Voltage-Series or Current-Series (High R_in).
- Current Source -¿ Requires Voltage-Shunt or Current-Shunt (Low R_in).
- Voltage Load -¿ Requires Voltage-Series or Voltage-Shunt (Low R_out).
- Current Load -¿ Requires Current-Series or Current-Shunt (High R_out).

Frequency Response Dynamics & Pole Movement under Feedback

- Single-Pole System Root Locus: Open-loop transfer function A(s) = A_0 / (1 + s / omega_p).
- Closed-Loop Transfer Function: A_f(s) = [A_0 / (1 + s / omega_p)] / [1 + beta *A_0 / (1 + s / omega_p)] = [A_0 / (1 + A_0* beta)] / [1 + s / (omega_p *(1 + A_0* beta))].
- Pole Trajectory: The real pole moves along the negative real axis from s = -omega_p to s = -omega_pf = -omega_p *(1 + A_0* beta). System remains UNCONDITIONALLY STABLE for any amount of feedback!
- Two-Pole System Dynamics: Open-loop transfer function A(s) = A_0 / [(1 + s / omega_p1) * (1 + s / omega_p2)].
- Complex Conjugate Pole Pair: As feedback factor beta increases, the two real poles move toward each other, collide on the real axis, and break out into complex conjugate poles: s_1,2 = -sigma +/- j * omega_d.
- Peaking & Over-Shoot: Complex poles induce high-frequency gain peaking in frequency response and transient ringing in step response if quality factor Q = sqrt(1 + A_0 * beta) / (1 + omega_p1 / omega_p2) ¿ 0.707.

Multi-Pole Instability & High-Frequency Phase Shift Mechanics

- Three-Pole System Instability: Open-loop transfer function A(s) = A_0 / [(1 + s / omega_p1) *(1 + s / omega_p2)* (1 + s / omega_p3)].
- Phase Accumulation: Each pole contributes up to -90 degrees of phase shift at high frequencies. A 3-pole system accumulates up to -270 degrees of phase lag.
- Phase Crossover Frequency (omega_180): Frequency where total phase shift reaches -180 degrees: angle(A(j *omega_180)* beta) = -180 deg.
- Positive Feedback Transformation: At omega_180, e^(-j *180 deg) = -1. The denominator (1 + A* beta) becomes (1 - —A * beta—). Negative feedback transforms into POSITIVE FEEDBACK!
- Barkhausen Runaway Condition: If loop gain magnitude —A(j *omega_180)* beta— ¿= 1 (0 dB), the system undergoes self-sustained spontaneous oscillation.

Gain Margin (GM) and Phase Margin (PM) Stability Criteria

- Bode Plot Stability Evaluation: Plot 20 *log10(—A* beta—) and phase angle(A * beta) versus logarithmic frequency.
- Gain Crossover Frequency (f_c): Defined where —A(j *2* pi *f_c)* beta— = 1 (0 dB).
- Phase Crossover Frequency (f_180): Defined where angle(A(j *2* pi *f_180)* beta) = -180 deg.
- Phase Margin Formula: PM = 180 deg + angle(A(j *2* pi *f_c)* beta).
- PM Target Values: PM ¿ 0 deg for stability; PM ¿= 45 deg for acceptable transient response; PM = 60 deg for maximally flat Butterworth response (zero peaking).
- Gain Margin Formula: GM = -20 *log10(—A(j* 2 *pi* f_180) * beta—). Target: GM ¿= 6 dB - 10 dB.

Frequency Compensation: Dominant Pole & Miller Pole Splitting

- Need for Frequency Compensation: High-gain multistage op-amps have multiple poles, leading to PM ¡ 0 (unstable). Frequency compensation alters loop response to force —A * beta— ¡ 1 before phase reaches -180 degrees.
- Dominant Pole Compensation: Adding a large capacitor C_D across an internal high-resistance node pulls the first pole down to a very low frequency f_p1' ¡¡ f_p1.
- Drawback of Simple Dominant Pole: Drastically reduces open-loop bandwidth and limits amplifier Slew Rate (SR = dV/dt).
- Miller Compensation (Pole Splitting): Connecting a small compensation capacitor C_C across the high-gain inverting stage (Stage 2).
- Miller Pole-Splitting Action:
- 1. Dominant pole f_p1 is pulled LOW to f_p1' approx 1 / [2 *pi* (g_m2 *R_1* R_2 * C_C)] via Miller input capacitance multiplication.
- 2. Non-dominant pole f_p2 is pushed HIGH to f_p2' approx g_m2 / [2 *pi* (C_1 + C_2)] via negative feedback around Stage 2!
- Result: Widely separates poles, ensuring 20 dB/decade roll-off through gain crossover frequency and achieving PM approx 60 - 90 degrees!

Effect of Feedback on Noise & Internal Interference

- Noise Source Classification: Input-referred noise V_ni (from sensors/first stage) vs Internal stage noise V_n2 (from power supply ripple, second stage drivers).
- Signal-to-Noise Ratio (SNR) Equation: Output signal V_o = A_1 *A_2* V_s; Output noise from stage 2 V_on = A_2 * V_n2.
- Closed-Loop SNR Transformation: With feedback factor beta applied around overall system:
- V_of = (A_1 *A_2* V_s) / (1 + A_1 *A_2* beta), and V_onf = (A_2 *V_n2) / (1 + A_1* A_2 * beta).
- System SNR with Feedback: SNR_f = V_of / V_onf = (A_1 *A_2* V_s) / (A_2 *V_n2) = A_1* (V_s / V_n2).
- Noise Suppression Conclusion: Negative feedback dramatically suppresses internal noise V_n2 injected AFTER high-gain stage A_1. However, noise V_ni present at the primary input is amplified along with the signal!

Design Example 1: Multi-Stage Feedback Impedance Analysis

- Problem Statement: A two-stage BJT Series-Shunt (Voltage-Series) feedback amplifier has open-loop voltage gain A_v = 2,000 (66 dB), open-loop input resistance R_in = 4 k-Ohm, open-loop output resistance R_o = 8 k-Ohm. The feedback factor is beta_v = 0.025.
- Step 1: Calculate Loop Gain T: T = A_v *beta_v = 2,000* 0.025 = 50 (33.98 dB).
- Step 2: Calculate Amount of Feedback F: F = 1 + T = 1 + 50 = 51 (34.15 dB).
- Step 3: Calculate Closed-Loop Gain A_vf: A_vf = A_v / (1 + T) = 2,000 / 51 = 39.215 (31.87 dB). (Note: Asymptotic 1 / beta_v = 1 / 0.025 = 40!).
- Step 4: Calculate Closed-Loop Input Resistance R_inf: Series input mixing boosts input resistance: R_inf = R_in *(1 + T) = 4 k-Ohm* 51 = 204 k-Ohm!
- Step 5: Calculate Closed-Loop Output Resistance R_of: Voltage output sampling lowers output resistance: R_of = R_o / (1 + T) = 8,000 Ohm / 51 = 156.86 Ohms!
- Conclusion: Feedback boosted input resistance from 4 k-Ohm to 204 k-Ohm, and dropped output resistance from 8 k-Ohm to 156.86 Ohms.

Design Example 2: Miller Compensation & Phase Margin Calculation

- Problem Statement: An uncompensated op-amp has open-loop gain A_0 = 10^5, primary pole f_p1 = 100 Hz, secondary pole f_p2 = 1 MHz, and tertiary pole f_p3 = 10 MHz. Calculate Phase Margin with beta = 0.1, and design Miller capacitor C_C for PM = 60 deg.
- Step 1: Uncompensated Phase Margin at Gain Crossover: Loop gain T(0) = 10^5 *0.1 = 10^4 (80 dB). Uncompensated crossover frequency f_c approx 100 Hz* 10^4 = 1 MHz.
- At f_c = 1 MHz: Phase lag angle(T) = -arctan(1M / 100) - arctan(1M / 1M) - arctan(1M / 10M) = -90 deg - 45 deg - 5.7 deg = -140.7 deg.
- Uncompensated Phase Margin: PM = 180 deg - 140.7 deg = 39.3 deg (Inadequate! Target PM ¿= 60 deg).
- Step 2: Miller Pole-Splitting Design for PM = 60 deg: For PM = 60 deg, new gain crossover f_c' must equal non-dominant pole f_p2' = 1 MHz.
- Required New Dominant Pole f_p1': f_c' = f_p1' *T(0) =¿ 1 MHz = f_p1'* 10^4 =¿ f_p1' = 100 Hz / 10 = 10 Hz.
- Step 3: Calculating Miller Capacitor C_C: Given stage 1 output resistance R_1 = 1 M-Ohm and stage 2 transconductance g_m2 = 5 mS:
- f_p1' = 1 / [2 *pi* g_m2 *R_1* R_2 *C_C] =¿ C_C = 1 / [2* pi *5mS* 1M-Ohm *10k-Ohm* 10Hz] = 3.18 pF.

Multi-Variable Design Trade-offs: Gain vs Bandwidth vs Stability

- 1. Voltage Gain vs Bandwidth Trade-Off: Increasing feedback factor beta lowers closed-loop voltage gain A_f, but expands system bandwidth f_Hf = f_H *(1 + A_0* beta) proportionally.
- 2. Gain Margin / Phase Margin vs Loop Gain Trade-Off: High loop gain T = A * beta provides superior gain desensitization and distortion reduction, but pushes multi-pole systems toward phase crossover, degrading Phase Margin.
- 3. Input/Output Impedance Matching: Series input boosts R_in (ideal for voltage sensing); Shunt input lowers R_in (ideal for current sensing). Voltage sampling lowers R_out; Current sampling boosts R_out.
- 4. Power & Area Overhead: Miller compensation requires silicon area for capacitor C_C and increases high-frequency power dissipation during large-signal slewing.
- Engineering Synthesis: Active circuit design demands balancing gain requirements, load impedance matching, noise figure, and Phase Margin ¿= 60 degrees.

Unit 4 Comprehensive Summary & Master Takeaways

- Closed-Loop Fundamental Formula: A_f = A / (1 + A *beta). Under heavy feedback (A* beta ¿¿ 1), A_f approx 1 / beta.
- Universal Impedance Modification: All input and output impedances are transformed by factor (1 + Loop Gain) = (1 + A * beta).
- Series Input -¿ R_inf = R_in *(1 + A* beta); Shunt Input -¿ R_inf = R_in / (1 + A * beta).
- Voltage Output -¿ R_of = R_o / (1 + A *beta); Current Output -¿ R_of = R_o* (1 + A * beta).
- Stability Criterion: Stable amplifier operation requires Phase Margin PM ¿= 45 deg (60 deg optimal) and Gain Margin GM ¿= 6 dB.
- Miller Compensation: Uses pole splitting via capacitor C_C to pull primary pole down and push secondary pole up, guaranteeing unconditional stability.