

Lecture 28: Concept of feedback in amplifiers: Negative and Positive

Feedback in amplifiers

DI02011011: Electronics Circuit and Application (ECA)

Lecture Agenda & Outline

- 1. Output Signal Sampling: Voltage (Node/Shunt) vs. Current (Loop/Series) Sampling Mechanics
- 2. Input Signal Summing: Series (Voltage Comparison) vs. Shunt (Current Comparison) Mixing Mechanics
- 3. Two-Port Parameter Matrices: Selection of h , g , y , and z Parameters for Feedback Networks
- 4. Feedback Loading Analysis: Derivation of Input/Output Loading Resistances (r_{11} and r_{22})
- 5. The Systematic 5-Step Feedback Analysis Algorithm for Discrete Transistor Circuits
- 6. Step-by-Step Circuit Calculations: Voltage-Series and Current-Series Transistor Amplifier Case Studies

Output Signal Sampling: Voltage (Shunt) vs. Current (Series)

- Voltage (Shunt) Sampling Mechanism: The feedback network is connected in PARALLEL across output load terminals to sense output voltage V_o .
- Ideal Voltage Sampler Properties: To avoid loading the output, an ideal voltage sampler presents infinite input impedance looking into the feedback network ($R_{in,beta} \rightarrow \infty$).
- Effect on Output Impedance: Voltage sampling stabilizes output voltage V_o , resulting in a DRAMATIC REDUCTION of closed-loop output resistance: $R_{of} = R_o / (1 + A \cdot \beta)$.
- Current (Series) Sampling Mechanism: The feedback network is connected in SERIES with output load loop to sense output current I_o .
- Ideal Current Sampler Properties: To avoid loading the output loop, an ideal current sampler presents zero insertion impedance looking into the feedback network ($R_{in,beta} \rightarrow 0$).
- Effect on Output Impedance: Current sampling stabilizes output current I_o , resulting in a DRAMATIC INCREASE of closed-loop output resistance: $R_{of} = R_o (1 + A \cdot \beta)$.

Input Signal Summing: Series (Voltage) vs. Shunt (Current)

- Series (Voltage Comparison) Summing: The feedback signal V_f is connected in SERIES with signal source V_s at input: $V_i = V_s - V_f$.
- Ideal Series Summer Properties: Series mixing requires high input impedance for voltage sensing. Closed-loop input resistance INCREASES: $R_{in} = R_{in} (1 + A \beta)$.
- Voltage Source Matching: Ideal for signal sources with low source resistance R_s (Thevenin equivalent voltage sources).
- Shunt (Current Comparison) Summing: The feedback signal I_f is connected in PARALLEL (shunt) with signal source I_s at input node: $I_i = I_s - I_f$.
- Ideal Shunt Summer Properties: Shunt mixing draws current from input node. Closed-loop input resistance DECREASES: $R_{in} = R_{in} / (1 + A \beta)$.
- Current Source Matching: Ideal for signal sources with high source resistance R_s (Norton equivalent current sources).

The Four Basic Feedback Topologies & Ideal Amplifier Models

- 1. Voltage-Series (Series-Shunt) Topology: Samples V_o , sums V_f in series. Ideal Voltage Amplifier (V_o / V_s). High R_{inf} , Low R_{of} .
- 2. Current-Series (Series-Series) Topology: Samples I_o , sums V_f in series. Ideal Transconductance Amplifier (I_o / V_s). High R_{inf} , High R_{of} .
- 3. Voltage-Shunt (Shunt-Shunt) Topology: Samples V_o , sums I_f in shunt. Ideal Transresistance Amplifier (V_o / I_s). Low R_{inf} , Low R_{of} .
- 4. Current-Shunt (Shunt-Series) Topology: Samples I_o , sums I_f in shunt. Ideal Current Amplifier (I_o / I_s). Low R_{inf} , High R_{of} .
- System Design Selection: Choice of topology depends entirely on source impedance R_s and load impedance R_L requirements.

Two-Port Network Representations of Feedback Networks

- Purpose of Two-Port Formulation: Practical feedback networks are 2-port resistive networks. Two-port analysis decouples forward transfer from feedback loading effects.
- Voltage-Series (h-parameters): Hybrid matrix where $V_1 = h_{11} I_1 + h_{12} V_2$ and $I_2 = h_{21} I_1 + h_{22} V_2$. Feedback factor $\beta = h_{12} = (V_1 / V_2) \text{ — } \{I_1=0\}$.
- Current-Series (z-parameters): Impedance matrix $V_1 = z_{11} I_1 + z_{12} I_2$ and $V_2 = z_{21} I_1 + z_{22} I_2$. Feedback factor $\beta = z_{12} = (V_1 / I_2) \text{ — } \{I_1=0\}$.
- Voltage-Shunt (y-parameters): Admittance matrix $I_1 = y_{11} V_1 + y_{12} V_2$ and $I_2 = y_{21} V_1 + y_{22} V_2$. Feedback factor $\beta = y_{12} = (I_1 / V_2) \text{ — } \{V_1=0\}$.
- Current-Shunt (g-parameters): Inverse hybrid matrix $I_1 = g_{11} V_1 + g_{12} I_2$ and $V_2 = g_{21} V_1 + g_{22} I_2$. Feedback factor $\beta = g_{12} = (I_1 / I_2) \text{ — } \{V_1=0\}$.

Feedback Network Loading Analysis: r_{11} and r_{22} Resistances

- Concept of Loaded Open-Loop Circuit: Real feedback networks load both the amplifier input and output nodes. We construct a basic amplifier WITHOUT feedback (A') by incorporating loading resistances r_{11} and r_{22} .
- Input Loading Resistance (r_{11}): Resistance looking into port 1 of feedback network with port 2 properly terminated:
- For Voltage Output (V_o sampled): Port 2 is SHORT-CIRCUITED ($V_2 = 0$). r_{11} = Input resistance looking into feedback network with output shorted.
- For Current Output (I_o sampled): Port 2 is OPEN-CIRCUITED ($I_2 = 0$). r_{11} = Input resistance looking into feedback network with output open.
- Output Loading Resistance (r_{22}): Resistance looking into port 2 of feedback network with port 1 properly terminated:
- For Series Input (V_f mixed): Port 1 is OPEN-CIRCUITED ($I_1 = 0$). r_{22} = Output resistance looking into feedback network with input open.
- For Shunt Input (I_f mixed): Port 1 is SHORT-CIRCUITED ($V_1 = 0$). r_{22} = Output resistance looking into feedback network with input shorted.

Systematic 5-Step Algorithm for Analyzing Feedback Amplifiers

- Step 1 (Identify Topology): Determine if output sampling is Voltage or Current, and if input mixing is Series or Shunt. Identify the 4 basic topologies.
- Step 2 (Determine Feedback Factor β): Find $\beta = \text{Signal_fed_back} / \text{Output_signal}$ (e.g., V_f / V_o , V_f / I_o , I_f / V_o , or I_f / I_o) with input signal set to zero.
- Step 3 (Construct Loaded Open-Loop Circuit A'): Draw basic amplifier without feedback. Include source resistance R_s , load R_L , input loading r_{i1} , and output loading r_{o2} .
- Step 4 (Calculate Open-Loop Gain A and Loop Gain T): Compute open-loop gain $A = \text{Output} / \text{Input}$ of the loaded circuit. Compute loop gain $T = A * \beta$.
- Step 5 (Compute Closed-Loop Parameters): Calculate closed-loop gain $A_f = A / (1 + A * \beta)$, closed-loop input resistance R_{in_f} , and closed-loop output resistance R_{of} .

Voltage-Series (Series-Shunt) Topology Formulation

- Signal Variables: Source voltage V_s , feedback voltage V_f , error voltage $V_i = V_s - V_f$, output voltage V_o .
- Feedback Factor: $\beta_v = (V_f / V_o) \big|_{I_1=0}$ (dimensionless ratio).
- Loading Resistances: $r_{11} = h_{11} = (V_1 / I_1) \big|_{V_2=0}$ (Port 2 shorted); $r_{22} = 1 / h_{22} = (V_2 / I_2) \big|_{I_1=0}$ (Port 1 open).
- Loaded Open-Loop Circuit (A_v): Basic amplifier voltage gain including loading: $A_v = (V_o / V_i) \big|_{\text{loaded}}$. Input resistance $R_{in} = R_s + r_{11} + r_{pi}$; Output resistance $R_o = R_C \parallel r_{22} \parallel R_L$.
- Closed-Loop Gain: $A_{vf} = V_o / V_s = A_v / (1 + A_v * \beta_v)$.
- Closed-Loop Resistances: Input resistance $R_{inf} = R_{in} (1 + A_v * \beta_v)$; Output resistance $R_{of} = R_o / (1 + A_v * \beta_v)$.

Current-Series (Series-Series) Topology Formulation

- Signal Variables: Source voltage V_s , feedback voltage V_f , error voltage $V_i = V_s - V_f$, output current I_o .
- Feedback Factor: $\beta_z = (V_f / I_o) \text{ —}_{\{I_1=0\}}$ (has units of Ohms, Ohms = V / A).
- Loading Resistances: $r_{11} = z_{11} = (V_1 / I_1) \text{ —}_{\{I_2=0\}}$ (Port 2 open); $r_{22} = z_{22} = (V_2 / I_2) \text{ —}_{\{I_1=0\}}$ (Port 1 open).
- Loaded Open-Loop Transconductance Gain (A_m): $A_m = (I_o / V_i) \text{ —}_{\{\text{loaded}\}}$ (has units of Siemens, A / V).
- Loop Gain Product: $T = A_m * \beta_z$ (dimensionless, Siemens x Ohms = 1).
- Closed-Loop Parameters: Transconductance $A_{mf} = A_m / (1 + A_m \beta_z)$. Voltage gain $A_{vf} = V_o / V_s = -A_{mf} R_L$.
- Closed-Loop Resistances: Input resistance $R_{inf} = R_{in} (1 + A_m \beta_z)$; Output resistance $R_{of} = R_o (1 + A_m \beta_z)$.

Voltage-Shunt & Current-Shunt Formulations

- Voltage-Shunt Topology (Shunt-Shunt):
- Feedback factor $\beta_g = I_f / V_o$ (Siemens, A/V). Open-loop transresistance $A_r = V_o / I_i$ (Ohms, V/A). Loop gain $T = A_r * \beta_g$.
- Closed-Loop Gain: $A_{rf} = A_r / (1 + A_r * \beta_g)$. Voltage gain $A_{vf} = V_o / V_s = -A_{rf} / R_s$.
- Resistances: BOTH input and output resistances DECREASE: $R_{inf} = R_{in} / (1 + A_r * \beta_g)$, $R_{of} = R_o / (1 + A_r * \beta_g)$.
- Current-Shunt Topology (Shunt-Series):
- Feedback factor $\beta_i = I_f / I_o$ (dimensionless current ratio). Open-loop current gain $A_i = I_o / I_i$ (dimensionless).
- Closed-Loop Current Gain: $A_{if} = A_i / (1 + A_i * \beta_i)$.
- Resistances: Input resistance DECREASES, Output resistance INCREASES: $R_{inf} = R_{in} / (1 + A_i * \beta_i)$, $R_{of} = R_o (1 + A_i * \beta_i)$.

Step-by-Step Analysis: Voltage-Series BJT Feedback Stage

- Circuit Parameters: Two-stage BJT amplifier with feedback resistor $R_F = 10 \text{ k}\Omega$ from Stage 2 emitter to Stage 1 emitter. $R_{E1} = 100 \text{ }\Omega$, $R_{C1} = 4 \text{ k}\Omega$, $R_{C2} = 2 \text{ k}\Omega$, $r_{\pi1} = r_{\pi2} = 1 \text{ k}\Omega$, $h_{fe1} = h_{fe2} = 100$.
- Step 1 (Topology): Output voltage V_o is sampled at Stage 2 collector/emitter node; feedback voltage V_f across R_{E1} is in series with Stage 1 base-emitter input $\Rightarrow$ Voltage-Series Feedback.
- Step 2 (Feedback Factor β_v): $\beta_v = V_f / V_o = R_{E1} / (R_{E1} + R_F) = 100 / (100 + 10,000) = 0.0099$ approx 0.01.
- Step 3 (Loading Resistances): r_{11} (output shorted) $= R_{E1} \parallel R_F = 100 \parallel 10,000$ approx 99 Ω .
- r_{22} (input open) $= R_{E1} + R_F = 100 + 10,000 = 10.1 \text{ k}\Omega$.
- Step 4 (Open-Loop Gain A_v): Loaded first stage gain $A_{v1} = -h_{fe1} R_{C1} / (r_{\pi1} + (1+h_{fe1})r_{11}) = -100 \cdot 4k / (1k + 10199) = -36.3$.
- Loaded second stage gain $A_{v2} = -h_{fe2} (R_{C2} \parallel r_{22}) / r_{\pi2} = -100 (2k \parallel 10.1k) / 1k = -165.3$.
- Total Open-Loop Gain $A_v = A_{v1} A_{v2} = (-36.3) (-165.3) = +6000$ (75.56 dB).
- Step 5 (Closed-Loop Gain A_{vf}): $A_{vf} = A_v / (1 + A_v \beta_v) = 6000 / (1 + 6000 \cdot 0.01) = 6000 / 61 = 98.36$ (39.85 dB).

Step-by-Step Analysis: Current-Series Transconductance Stage

- Circuit Parameters: Single-stage BJT Common-Emitter amplifier with unbypassed emitter resistor $R_E = 500 \text{ Ohm}$. Transistor parameters: $g_m = 40 \text{ mS}$, $r_{\pi} = 2.5 \text{ kOhm}$, $R_C = 5 \text{ kOhm}$, $R_s = 1 \text{ kOhm}$.
- Step 1 (Topology): Output current I_c is sampled through emitter loop; feedback voltage $V_f = I_e * R_E$ is in series with base input $\Rightarrow$ Current-Series Feedback.
- Step 2 (Feedback Impedance β_z): $V_f = I_o * R_E \Rightarrow \beta_z = V_f / I_o = R_E = 500 \text{ Ohm}$.
- Step 3 (Loading Resistances): Output open ($I_o = 0$) $\Rightarrow r_{11} = R_E = 500 \text{ Ohm}$; Input open ($I_i = 0$) $\Rightarrow r_{22} = R_E = 500 \text{ Ohm}$.
- Step 4 (Open-Loop Transconductance A_m): Loaded open-loop transconductance (without feedback loop): $A_m = I_o / V_i = g_m = 40 \text{ mS}$.
- Loop Gain T : $T = A_m \beta_z = 40 \text{ mS} \cdot 500 \text{ Ohm} = 20$.
- Step 5 (Closed-Loop Parameters): Closed-loop transconductance $A_{mf} = A_m / (1 + T) = 40 \text{ mS} / (1 + 20) = 1.905 \text{ mS}$.
- Closed-Loop Voltage Gain: $A_{vf} = -A_{mf} R_C = -1.905 \text{ mS} \cdot 5 \text{ kOhm} = -9.525$. (Note: $-R_C / R_E = -5000 / 500 = -10!$).
- Input Resistance Transformation: $R_{inf} = (R_s + r_{\pi}) (1 + T) = (1k + 2.5k) (1 + 20) = 3.5 \text{ kOhm} \cdot 21 = 73.5 \text{ kOhm}$!

Comprehensive Summary & Two-Port Matrix Overview

- Feedback Topologies Summary:
- 1. Voltage-Series: $\beta_v = V_f / V_o$, $R_{in} = R_{in} (1 + A_v \beta_v)$, $R_{of} = R_o / (1 + A_v \beta_v)$ [Voltage Amp].
- 2. Current-Series: $\beta_z = V_f / I_o$, $R_{in} = R_{in} (1 + A_m \beta_z)$, $R_{of} = R_o (1 + A_m \beta_z)$ [Transconductance Amp].
- 3. Voltage-Shunt: $\beta_g = I_f / V_o$, $R_{in} = R_{in} / (1 + A_r \beta_g)$, $R_{of} = R_o / (1 + A_r \beta_g)$ [Transresistance Amp].
- 4. Current-Shunt: $\beta_i = I_f / I_o$, $R_{in} = R_{in} / (1 + A_i \beta_i)$, $R_{of} = R_o (1 + A_i \beta_i)$ [Current Amp].
- Two-Port Rule: Input series mixing INCREASES R_{in} ; Input shunt mixing DECREASES R_{in} . Voltage output sampling DECREASES R_{out} ; Current output sampling INCREASES R_{out} .
- Methodology: Always execute the systematic 5-step algorithm incorporating r_{11} and r_{22} loading resistances.