Lecture 27: Concept of feedback in amplifiers: Negative and Positive

Feedback in amplifiers

DI02011011: Electronics Circuit and Application (ECA)

Lecture Agenda & Outline

- 1. Universal Feedback Architecture: Open-Loop Amplifier, Feedback Network, and Summing Junctions
- 2. Closed-Loop Gain Derivation: Mathematical Formulation of A_f = A / (1 + A * beta) and Loop Gain T
- 3. Negative Feedback Desensitization: Immunity to Parameter Variations, Temperature Drift, and Aging
- 4. Performance Modifications: Bandwidth Extension, Gain-Bandwidth Invariance, and Harmonic Distortion Reduction
- 5. Positive Feedback Mechanics: Regenerative Action, Latching, and Barkhausen Stability Criterion (—A * beta— = 1, phase = 0 deg)
- 6. Quantitative Engineering Design Examples and Comparative Performance Analysis Matrix

General Feedback System Model & Block Diagram Topology

- System Components: A feedback amplifier consists of a basic open-loop forward amplifier with transfer function A(s), a feedback network with transfer function beta(s), a sampling network, and a mixing/summing junction.
- Signal Variables: Input signal x_s (voltage or current), feedback signal x_f = beta * x_o, net input signal x_i = x_s - x_f (for negative feedback) or x_i = x_s + x_f (for positive feedback).
- Forward Path Amplifier (A): Idealized open-loop gain A = x_o / x_i. Internal parameters are sensitive to temperature, DC bias shifts, and manufacturing tolerances.
- Feedback Network (beta): Passive resistive/reactive attenuation network beta = x_f / x_o (typically beta ¡= 1). Designed with precise, temperature-stable passive components (resistors/capacitors).
- Mixer/Summing Node: Combines source signal x_s and feedback signal x_f to generate error/effective drive signal x_i.

Derivation of Closed-Loop Gain A_f & Loop Gain Dynamics

- Algebraic Derivation for Negative Feedback: Output equation x_o = A x_i. *Substituting net input x_i = x_s - x_f gives x_o = A* (x_s - beta * x_o).
- Rearranging Terms: x_o + A *beta* x_o = A *x_s =¿ x_o* (1 + A *beta) = A* x_s.
- Closed-Loop Gain Formula: A_f = x_o / x_s = A / (1 + A * beta).
- Loop Gain (T): The product T = A * beta represents the total gain around the feedback loop (from mixer through forward path and feedback network back to mixer).
- Amount of Feedback (F): Defined as F = 1 + A *beta = 1 + T. Expressed in decibels as F_dB = 20* log10(—1 + A * beta—).
- Asymptotic Gain Limit: Under heavy negative feedback (where A *beta ¿¿ 1): A_f approx A / (A* beta) = 1 / beta! Closed-loop gain becomes virtually independent of open-loop gain A.

Mechanisms of Negative Feedback: Desensitization of Gain

- Sensitivity Definition: Fractional change in closed-loop gain A_f relative to fractional change in open-loop gain A: S_A^{A_f} = (dA_f / A_f) / (dA / A).
- Derivation of Sensitivity Reduction: Differentiating A_f with respect to A gives dA_f / dA = d/dA [A / (1 + A *beta)] = 1 / (1 + A* beta)^2.
- Sensitivity Equation: dA_f / A_f = (1 / (1 + A *beta))* (dA / A). Fractional gain variation is reduced by the factor (1 + A * beta).
- Thermal & Aging Immunity: Temperature-induced variations in BJT beta_0 or MOSFET g_m (which cause 50% - 100% drift in open-loop gain A) cause negligible drift (¡ 1%) in closed-loop gain A_f.
- Nonlinear Stabilization: Linearizes overall amplifier transfer characteristics, even when transistor open-loop gain A fluctuates across large signal swings.

Bandwidth Extension via Negative Feedback

- High-Frequency Response with Feedback: Single-pole open-loop amplifier A(s) = A_0 / (1 + s / omega_H), where A_0 is open-loop midband gain and omega_H = 2 *pi* f_H is upper cutoff frequency.
- Closed-Loop Frequency Transfer Function: A_f(s) = [A_0 / (1 + s / omega_H)] / [1 + beta *A_0 / (1 + s / omega_H)] = A_0 / (1 + A_0* beta + s / omega_H).
- Dividing by (1 + A_0 *beta): A_f(s) = [A_0 / (1 + A_0* beta)] / [1 + s / (omega_H *(1 + A_0* beta))].
- Closed-Loop Cutoff Frequency: New upper 3-dB frequency omega_Hf = omega_H *(1 + A_0* beta) =¿ f_Hf = f_H *(1 + A_0* beta). Upper cutoff frequency is extended by (1 + A_0 * beta)!
- Closed-Loop Lower Cutoff Frequency: Lower 3-dB frequency is lowered: f_Lf = f_L / (1 + A_0 * beta).
- Gain-Bandwidth Product Invariance: A_f0 *f_Hf = [A_0 / (1 + A_0* beta)] *[f_H* (1 + A_0 *beta)] = A_0* f_H = Constant (GBP).

Nonlinear Distortion & Noise Reduction Dynamics

- Harmonic Distortion Reduction: Transistor non-linear transfer functions generate output harmonic distortion D. Applying negative feedback reduces output harmonic distortion to D_f = D / (1 + A * beta).
- Linearity Enhancement: By suppressing second and third harmonic amplitudes, negative feedback transforms non-linear amplifiers into high-fidelity linear stages.
- Signal-to-Noise Ratio (SNR) Analysis: Consider noise voltage V_n injected inside an amplifier stage: V_o = A_1 *A_2* V_s + A_2 * V_n.
- Closed-Loop Output with Noise: V_of = (A_1 *A_2* V_s) / (1 + A_1 *A_2* beta) + (A_2 *V_n) / (1 + A_1* A_2 * beta).
- Noise Limitation: Negative feedback reduces output noise generated WITHIN the feedback loop, but CANNOT improve SNR for noise present at the primary input source (V_ni)!

Mechanisms of Positive Feedback & Regenerative Action

- Positive Feedback Transfer Function: Feedback signal x_f is added in-phase at the mixer: x_i = x_s + x_f = x_s + beta * x_o.
- Closed-Loop Gain Formulation: A_f = A / (1 - A * beta).
- Regenerative Gain Enhancement: For 0 ¡ A *beta ¡ 1, the denominator (1 - A* beta) ¡ 1, causing closed-loop gain A_f to EXCEED open-loop gain A (gain enhancement).
- Instability & Saturation Region: As loop gain A * beta approaches +1, closed-loop gain A_f -¿ infinity. The system becomes unstable; any microscopic noise pulse triggers exponential growth until circuit supply rails latch.
- Applications of Positive Feedback: Controlled positive feedback is utilized in oscillators (sinusoidal/waveform generation), Schmitt triggers (bistable multivibrators with hysteresis), and regenerative receivers.

Barkhausen Criterion for Sustained Oscillations

- Oscillator Definition: An electronic circuit that converts DC power into a periodic AC output signal without requiring any external AC input (x_s = 0).
- Barkhausen Condition 1 (Loop Gain Magnitude): The magnitude of the loop transmission around the feedback loop must equal unity at the desired frequency of oscillation f_0: —T(j *omega_0)—* = —*A(j* omega_0) *beta(j* omega_0)— = 1.
- Barkhausen Condition 2 (Loop Phase Shift): The total phase shift around the feedback loop must equal zero degrees or an integer multiple of 360 degrees: angle(A *beta) = 2* n * pi (where n = 0, 1, 2...).
- Practical Oscillation Build-Up: To guarantee initial startup from thermal noise, practical designs set initial loop gain magnitude slightly higher: —A * beta— approx 1.05 - 1.1.
- Amplitude Stabilization: Non-linear amplitude limiting (such as FET AGC circuits, thermistors, or diode clamps) reduces effective gain to —A * beta— = 1 precisely at steady state to prevent signal clipping.

Frequency Response & Stability Margins (Gain & Phase Margins)

- Potential Instability in Negative Feedback Amplifiers: At high frequencies, internal amplifier poles introduce phase lag. If phase lag reaches 180 degrees while —A * beta— ¿= 1, negative feedback turns into positive feedback!
- Gain Crossover Frequency (f_c): The frequency at which loop gain magnitude equals unity (0 dB): —A(j *omega_c)* beta(j * omega_c)— = 1 (0 dB).
- Phase Crossover Frequency (f_180): The frequency at which loop phase shift equals -180 degrees: angle(A(j *omega_180)* beta(j * omega_180)) = -180 deg.
- Phase Margin (PM): PM = 180 deg + angle(A(j *omega_c)* beta(j * omega_c)). For stable operation without ringing, PM ¿= 45 deg (60 deg optimal).
- Gain Margin (GM): GM = -20 *log10(—A(j* omega_180) *beta(j* omega_180)—). Indicates how much loop gain can increase before instability occurs (typically GM ¿= 6 dB - 10 dB).

Quantitative Analysis of Negative Feedback Amplifier

- Problem Statement: An open-loop amplifier has midband voltage gain A_0 = 10,000 (80 dB), upper 3-dB frequency f_H = 1 kHz, and second-harmonic distortion D = 10%. Calculate negative feedback properties with beta = 0.009.
- Step 1: Calculate Loop Gain & Feedback Factor: Loop gain T = A_0 *beta = 10,000* 0.009 = 90. Amount of feedback F = 1 + A_0 * beta = 1 + 90 = 91 (39.18 dB).
- Step 2: Calculate Closed-Loop Gain (A_f): A_f = A_0 / (1 + A_0 * beta) = 10,000 / 91 = 109.89 (40.82 dB). Note: 1 / beta = 1 / 0.009 = 111.11.
- Step 3: Calculate Gain Sensitivity Reduction: If open-loop gain A drops by 20% (dA/A = -0.20), closed-loop gain variation is dA_f / A_f = (-0.20) / 91 = -0.0022 = -0.22%!
- Step 4: Calculate Closed-Loop Bandwidth: f_Hf = f_H *(1 + A_0* beta) = 1 kHz * 91 = 91 kHz.
- Step 5: Calculate Reduced Distortion: D_f = D / (1 + A_0 * beta) = 10% / 91 = 0.11%.

Quantitative Oscillator Design using Barkhausen Criterion

- Problem Statement: Design an RC Phase-Shift Oscillator core using a 3-stage RC ladder network in the feedback path. Determine the required open-loop amplifier gain A and frequency of oscillation f_0.
- RC Ladder Feedback Transfer Function: beta(s) = (s *R* C)^3 / [(s *R* C)^3 + 6 *(s* R *C)^2 + 5* (s *R* C) + 1].
- Step 1: Setting Imaginary Part to Zero for Phase Shift: Setting s = j *omega and equating imaginary terms to zero gives: -omega^3* R^3 *C^3 + 5* omega *R* C = 0.
- Oscillation Frequency Derivation: omega_0^2 *R^2* C^2 = 6 =¿ omega_0 = 1 / (sqrt(6) *R* C) =¿ f_0 = 1 / (2 *pi* sqrt(6) *R* C).
- Step 2: Evaluating Feedback Factor at f_0: Substituting omega_0 into beta(j *omega_0) yields beta(j* omega_0) = -1 / 29.
- Step 3: Applying Barkhausen Gain Condition: For sustained oscillations, —A *beta— = 1 =¿ A* (-1 / 29) = -1 =¿ A = +29.
- Conclusion: The forward amplifier must provide an inverting gain of at least A = -29 (or non-inverting gain +29 with net 180 deg ladder phase shift) to sustain oscillations.

Comparative Performance Matrix: Negative vs. Positive Feedback

- Voltage Gain: Negative feedback reduces gain [A_f = A / (1 + A *beta)]; Positive feedback increases gain [A_f = A / (1 - A* beta)].
- Gain Stability: Negative feedback increases stability [dA_f/A_f = (dA/A)/(1 + A * beta)]; Positive feedback decreases stability (leads to saturation/latching).
- System Bandwidth: Negative feedback expands bandwidth [f_Hf = f_H *(1 + A* beta)]; Positive feedback narrows bandwidth.
- Nonlinear Distortion: Negative feedback suppresses distortion [D_f = D / (1 + A * beta)]; Positive feedback increases distortion.
- Terminal Noise (Internal): Negative feedback reduces internal noise; Positive feedback amplifies noise.
- Primary Applications: Negative feedback is used in audio/RF linear amplifiers and operational amplifiers; Positive feedback is used in sinusoidal oscillators, relaxation oscillators, and Schmitt triggers.

Summary & Core Takeaways of Feedback Concepts

- Closed-Loop Transfer Function: A_f = A / (1 +/- A * beta). Negative feedback uses '+', Positive feedback uses '-'.
- Desensitization & Independence: Under heavy negative feedback (A * beta ¿¿ 1), closed-loop gain depends solely on passive components: A_f approx 1 / beta.
- Performance Enhancements: Negative feedback increases upper cutoff frequency (f_Hf = f_H *(1 + A* beta)), maintains constant Gain-Bandwidth Product, and lowers distortion (D_f = D / (1 + A * beta)).
- Barkhausen Criteria: Sustained oscillation requires —A *beta— = 1 and loop phase shift angle(A* beta) = 0 deg or 360 deg.
- Stability Margin Requirements: Amplifier stability requires positive Phase Margin (PM ¿= 45 deg) and Gain Margin (GM ¿= 6 dB) to prevent unwanted oscillations.