# Lecture 23: Frequency Response of Single Stage Amplifier

## Frequency Response of Transistor Amplifier

DI02011011: Electronics Circuit and Application (ECA)

# Lecture Agenda & Learning Roadmap

- Overview of the Three Frequency Response Regions (Low, Midband, High)
- Midband Voltage Gain Derivation using Small-Signal Hybrid-$\pi$ Model
- BJT High-Frequency Hybrid-$\pi$ Model & Junction Capacitances ($C_\pi$, $C_\mu$)
- Miller's Theorem Derivation and Equivalent Input/Output Capacitances
- High-Frequency Input and Output Cutoff Frequencies ($f_{H,in}$, $f_{H,out}$)
- Construction of Complete Bode Magnitude and Phase Response Curves
- Comprehensive Numerical Example: Full Frequency Response Calculation
- High-Frequency Performance Comparison Across Configurations (CE, CB, CC)
- MOSFET Common-Source (CS) High-Frequency Analysis
- Design Problem: Bandwidth Extension Techniques for CE Amplifiers
- Comprehensive Summary and Key Takeaways

# Overview of the Complete Frequency Response Curve

- Low-Frequency Region ($f < f_L$): Gain attenuates at $+20$ dB/decade per pole due to increasing reactance of coupling ($C_{C1}, C_{C2}$) and bypass ($C_E$) capacitors.
- Midband Frequency Region ($f_L \leq f \leq f_H$): Voltage gain magnitude is constant at maximum value $A_M$. External capacitors act as ideal AC short circuits, while internal parasitic capacitances act as open circuits.
- High-Frequency Region ($f > f_H$): Gain rolls off at $-20$ dB/decade per pole due to decreasing reactance of internal junction capacitances $C_\pi$ (base-emitter) and $C_\mu$ (base-collector).
- Bandwidth & Half-Power Points: Operating bandwidth $BW = f_H - f_L$. Gain magnitude at $f_L$ and $f_H$ is $|A(f)| = \frac{A_M}{\sqrt{2}} = A_M - 3.01$ dB.

# Midband Small-Signal Voltage Gain Derivation

- Midband Hybrid-$\pi$ Circuit: External capacitors are replaced by AC short circuits. Transistor replaced by small-signal parameters $r_\pi$ and $g_m v_b e$.
- Input Voltage Divider: Base-emitter signal voltage $V_{be} = V_s \left( \frac{R_{in}}{R_S + R_{in}} \right)$, where $R_{in} = R_1 \parallel R_2 \parallel r_\pi$.
- Output Voltage Expression: Output voltage $V_o = -g_m V_{be}(R_C \parallel R_L)$ (negative sign indicates 180° phase inversion).
- Overall Midband System Gain ($A_{v,mid}$): $A_{v,mid} = \frac{V_o}{V_s} = -\left( \frac{R_1 \| R_2 \| r_\pi}{R_S + R_1 \| R_2 \| r_\pi} \right) g_m(R_C \parallel R_L)$, where $g_m = \frac{I_C}{V_T}$ and $r_\pi = \frac{\beta_0}{g_m}$.

# High-Frequency BJT Hybrid-$\pi$ Model & Parasitic Capacitances

- Base-Emitter Junction Capacitance ($C_\pi$): Sum of forward-bias depletion capacitance $C_{jE}$ and diffusion capacitance $C_{DE} = g_m \tau_F$: $C_\pi = C_{jE} + g_m \tau_F$ (typically $20 - 100$ pF).
- Base-Collector Junction Capacitance ($C_\mu$): Reverse-bias depletion capacitance $C_{jC} = \frac{C_{j0}}{(1+V_{CB}/V_0)^m}$ (typically $1 - 5$ pF). Crosses the base and collector terminals.
- Collector-Substrate Capacitance ($C_{cs}$): Parasitic capacitance to substrate in integrated circuits (typically $1 - 3$ pF).
- Wiring & Layout Parasitics: Wiring capacitances $C_{w,in}$ and $C_{w,out}$ added in parallel with input and output nodes.

# Miller's Theorem & High-Frequency Input/Output Capacitance Derivation

- Miller's Theorem Concept: An impedance $Z$ connected between input node (1) and output node (2) with voltage gain $K = \frac{V_2}{V_1}$ can be replaced by two equivalent shunting impedances: $Z_1 = \frac{Z}{1-K}$ and $Z_2 = \frac{Z}{1-1/K}$.
- Miller Input Capacitance ($C_{M,in}$): For bridging feedback capacitance $C_\mu$ with stage inverting gain $K = A_{v,stage} = -g_m(R_C \parallel R_L)$: $C_{M,in} = C_\mu(1 - A_{v,stage}) = C_\mu(1 + |A_{v,stage}|)$.
- Miller Output Capacitance ($C_{M,out}$): $C_{M,out} = C_\mu \left(1 - \frac{1}{A_{v,stage}}\right) \approx C_\mu$ since $|A_{v,stage}| \gg 1$.
- Total High-Frequency Input Capacitance ($C'_{in}$): $C'_{in} = C_\pi + C_{M,in} + C_{w,in} = C_\pi + C_\mu(1 + |A_{v,stage}|) + C_{w,in}$.
- Impact of Miller Multiplication: Even a small $C_\mu = 4$ pF with gain $A_{v,stage} = -100$ creates $C_{M,in} = 4 \text{ pF} \times 101 = 404$ pF, completely dominating input capacitance!

# High-Frequency Cutoff Calculation ($f_{H,in}$, $f_{H,out}$ and Overall $f_H$)

- Input High-Frequency Pole ($f_{H,in}$): Equivalent resistance $R'_{in} = R_S \parallel R_1 \parallel R_2 \parallel r_\pi$. Input cutoff: $f_{H,in} = \frac{1}{2\pi R'_{in} C'_{in}} = \frac{1}{2\pi(R_S \| R_1 \| R_2 \| r_\pi)[C_\pi + C_\mu(1+|A_{v,stage}|) + C_{w,in}]}$.
- Output High-Frequency Pole ($f_{H,out}$): Equivalent resistance $R'_{out} = R_C \parallel R_L \parallel r_o \approx R_C \parallel R_L$. Output cutoff: $f_{H,out} = \frac{1}{2\pi R'_{out} C'_{out}} = \frac{1}{2\pi(R_C \| R_L)[C_\mu + C_{cs} + C_{w,out}]}$.
- Dominant High-Frequency Pole: Because $C'_{in} \gg C'_{out}$ due to Miller multiplication, $f_{H,in}$ is substantially smaller than $f_{H,out}$, making $f_{H,in}$ the dominant upper pole.
- Overall Upper 3-dB Cutoff Frequency ($f_H$): $f_H \approx f_{H,in}$ or using root-square combination: $f_H = \frac{1}{\sqrt{(1/f_{H,in})^2 + (1/f_{H,out})^2}}$.

# Bode Magnitude and Phase Plots of Single-Stage Amplifier

- Complete Transfer Function: $A(s) = A_{v,mid} \cdot \left(\frac{s}{s+\omega_L}\right) \cdot \left(\frac{1}{1+s/\omega_H}\right)$.
- Bode Magnitude Asymptotes: Flat response at $A_{v,mid}$(dB) in midband ($f_L < f < f_H$). $+20$ dB/decade roll-off for $f < f_L$, $-20$ dB/decade roll-off for $f > f_H$.
- Corner Frequencies: At $f = f_L$ and $f = f_H$, actual gain magnitude is $-3$ dB below asymptotic lines.
- Bode Phase Plot Progression: At $f \ll f_L$, phase is $+270°$ ($+90°$ lead over midband $180°$). At $f = f_L$, phase is $+225°$. At midband, phase is $180°$. At $f = f_H$, phase is $135°$ ($45°$ lag). At $f \gg f_H$, phase approaches $+90°$ ($90°$ lag).

# Comprehensive Numerical Example: Complete Frequency Response of CE Amplifier

- Given Circuit & Transistor Parameters: $V_{CC} = 15$ V, $R_1 = 33$ k$\Omega$, $R_2 = 10$ k$\Omega$, $R_C = 3.3$ k$\Omega$, $R_E = 1$ k$\Omega$, $R_S = 500$ $\Omega$, $R_L = 10$ k$\Omega$, $\beta_0 = 150$, $I_C = 1.5$ mA. Parasitics: $C_\pi = 30$ pF, $C_\mu = 4$ pF, $C_{w,in} = 5$ pF, $C_{w,out} = 3$ pF. External: $C_{C1} = 10$ $\mu$F, $C_{C2} = 10$ $\mu$F, $C_E = 100$ $\mu$F. Small-signal parameters: $V_T = 26$ mV $\implies$ $r_e = 17.33$ $\Omega$, $g_m = 57.69$ mS, $r_\pi = 2.6$ k$\Omega$.
- Step 1 - Midband Gain ($A_{v,mid}$): $R_{in} = 33\text{ k} \parallel 10\text{ k} \parallel 2.6\text{ k} = 1.954$ k$\Omega$. Loaded collector resistance $R'_L = R_C \parallel R_L = 3.3\text{ k} \parallel 10\text{ k} = 2.481$ k$\Omega$. Internal stage gain $A_{v,stage} = -g_m R'_L = -57.69\text{ mS} \times 2481 = -143.13$ V/V. Overall gain $A_{v,mid} = -\left(\frac{1954}{500+1954}\right) \times 143.13 = -113.97$ V/V (41.14 dB).
- Step 2 - Miller Input Capacitance ($C_{M,in}$): $C_{M,in} = C_\mu(1 + |A_{v,stage}|) = 4\text{ pF} \times (1 + 143.13) = 576.52$ pF. Total input capacitance $C'_{in} = C_\pi + C_{M,in} + C_{w,in} = 30 + 576.52 + 5 = 611.52$ pF.
- Step 3 - High-Frequency Input Pole ($f_{H,in}$): Equivalent resistance $R'_{in} = R_S \parallel R_{in} = 500 \parallel 1954 = 398.1$ $\Omega$. $f_{H,in} = \frac{1}{2\pi \times 398.1\ \Omega \times 611.52\text{ pF}} = 653.8$ kHz.
- Step 4 - High-Frequency Output Pole ($f_{H,out}$): $C'_{out} = C_\mu + C_{w,out} = 4 + 3 = 7$ pF. $R'_{out} = 2.481$ k$\Omega$. $f_{H,out} = \frac{1}{2\pi \times 2481 \times 7\text{ pF}} = 9.16$ MHz. Upper cutoff $f_H \approx f_{H,in} = 653.8$ kHz.

# High-Frequency Response Comparison: CE, CB, and CC Configurations

- Common-Emitter (CE): Provides high voltage and current gain, but suffers from strong Miller effect $C_{M,in} = C_\mu(1 + |A_v|)$, resulting in moderate upper cutoff frequency $f_H$.
- Common-Base (CB): Low input impedance ($R_{in} \approx r_e$), unit current gain, but ZERO Miller effect because collector is AC grounded relative to input. Achieves extremely high upper cutoff $f_H \approx f_T$, making it ideal for RF amplifiers.
- Common-Collector (CC / Emitter Follower): Voltage gain $A_v \approx 1 \implies C_{M,in} = C_\mu(1 - 1) = 0$. Low Miller multiplication, high input impedance, wide bandwidth, ideal as high-speed output buffers.
- Summary Comparison: CB configuration provides the widest bandwidth for voltage amplification due to elimination of Miller effect.

# MOSFET Common-Source (CS) High-Frequency Response

- MOSFET High-Frequency Parasitics: Gate-source capacitance $C_{gs}$, gate-drain capacitance $C_{gd}$, and drain-source capacitance $C_{ds}$.
- Midband Gain: $A_{v,mid} = -g_m(R_D \parallel R_L)$, where $g_m = 2\sqrt{K_n I_D} = \sqrt{2\mu_n C_{ox}\frac{W}{L} I_D}$.
- Miller Gate-Drain Capacitance: Gate-drain capacitance $C_{gd}$ bridges input and output: $C_{M,in} = C_{gd}(1 + g_m(R_D \parallel R_L))$.
- CS High-Frequency Pole ($f_{H,in}$): $f_{H,in} = \frac{1}{2\pi(R_S \| R_G)[C_{gs}+C_{gd}(1+g_m(R_D \| R_L))]}$. Very high input resistance $R_G$ makes CS amplifiers sensitive to signal source resistance $R_S$.

# Design Problem: Bandwidth Extension of a CE Stage

- Problem Challenge: An engineer needs to extend the upper 3-dB bandwidth $f_H$ of a Common-Emitter stage from 500 kHz to 2 MHz. Evaluate design options.
- Option 1 - Emitter Degeneration ($R_e$): Adding an unbypassed emitter resistor $R_e$ reduces stage gain to $A'_{v,stage} \approx -\frac{R_C \| R_L}{r_e + R_e}$. Lower gain reduces $C_{M,in} = C_\mu(1 + |A'_{v,stage}|)$, extending upper cutoff $f_H$ proportionally.
- Option 2 - Source Impedance Reduction ($R_S$): Lowering signal source resistance $R_S$ using an upstream Emitter Follower buffer reduces $R'_{in} = R_S \parallel R_{in}$, boosting $f_{H,in} = \frac{1}{2\pi R'_{in} C'_{in}}$.
- Option 3 - Device Selection: Replacing BJT with a high-$f_T$ microwave transistor having smaller $C_\mu$ (0.5 pF vs 4 pF).
- Quantitative Trade-off: Adding $R_e$ to trade gain for bandwidth preserves the constant Gain-Bandwidth Product (GBW).

# Summary of Single-Stage Amplifier Frequency Characteristics

- Complete Transfer Function: $A(s) = A_{v,mid}\left(\frac{s}{s+\omega_L}\right)\left(\frac{1}{1+s/\omega_H}\right)$.
- Midband Gain: $A_{v,mid} = -\left(\frac{R_{in}}{R_S+R_{in}}\right) g_m(R_C \parallel R_L)$.
- Low-Frequency Dominant Pole: Set by external capacitors $C_{C1}, C_{C2}, C_E$, dominated by $C_E$ due to small $R_{eq,E}$.
- High-Frequency Dominant Pole: Set by internal parasitics $C_\pi, C_\mu$, dominated by Miller input capacitance $C_{M,in} = C_\mu(1 + |A_{v,stage}|)$.
- Configuration Comparison: Common-Base provides superior high-frequency performance by eliminating Miller effect.