Lecture 22: Effect of Emitter Bypass Capacitor and Coupling Capacitor on...

Frequency Response of Transistor Amplifier

DI02011011: Electronics Circuit and Application (ECA)

Lecture Agenda & Learning Roadmap

- Low-Frequency Limitations and High-Pass RC Filter Action in Amplifiers
- Input Coupling Capacitor (C_{C1}) Network & Cutoff Frequency (f_{L1}) Derivation
- Output Coupling Capacitor (C_{C2}) Network & Cutoff Frequency (f_{L2}) Derivation
- Emitter Bypass Capacitor (C_E) AC Impedance & Cutoff Frequency (f_{LE}) Derivation
- Pole-Zero Analysis of the Emitter Bypass Impedance Network $Z_E(s)$
- Dominant Pole Approximation & Low-Frequency Asymptotic Bode Plotting
- Detailed Step-by-Step Numerical Calculation of f_{L1}, f_{L2}, f_{LE}, and Overall f_L
- Low-Frequency Phase Lead Response and Pulse Distortion (Sag/Tilt)
- Engineering Design Guidelines for Low-Frequency Capacitor Selection
- Design Calculation Example for Target Lower Cutoff Frequency $f_L = 50$ Hz
- Comprehensive Summary and Design Rules

Low-Frequency Signal Attenuation Mechanisms in BJT Amplifiers

- Capacitive Reactance Frequency Dependency: AC reactance $X_C(f) = \frac{1}{2\pi fC}$ approaches infinity as frequency $f \to 0$ Hz, acting as a open circuit to DC and attenuating low-frequency AC signals.
- High-Pass Filter Topology: Coupling capacitors C_{C1} and C_{C2} are in series with the signal path, forming high-pass RC filter sections with circuit resistances.
- Bypass Capacitor Degeneration: Emitter bypass capacitor C_E is in parallel with emitter resistor R_E. At low frequencies where X_{CE} is large, negative feedback across R_E reduces AC voltage gain.
- Midband Transition: As frequency increases above f_L, $X_C \to 0$, capacitors act as AC short circuits, and amplifier gain stabilizes at midband value A_M.

Input Coupling Capacitor (C_{C1}) Network & Cutoff Frequency (f_{L1})

- Equivalent Input Resistance ($R_{in,C1}$): Seen looking into the nodes connected to C_{C1}: $R_{in,C1} = R_S + R_{in,stage} = R_S + (R_1 \parallel R_2 \parallel R_{in,base})$, where $R_{in,base} = r_\pi + (1+\beta_0)R_E$ if unbypassed at DC.
- Transfer Function Contribution: $T_1(s) = \frac{V_b(s)}{V_s(s)} = \left(\frac{R_{in,stage}}{R_S+R_{in,stage}}\right)\frac{s}{s+\omega_{L1}}$, where $\omega_{L1} = \frac{1}{C_{C1}R_{in,C1}}$.
- Lower Cutoff Pole Frequency (f_{L1}): $f_{L1} = \frac{1}{2\pi C_{C1}R_{in,C1}} = \frac{1}{2\pi C_{C1}[R_S+(R_1||R_2||R_{in,base})]}$.
- Low-Frequency Roll-Off & Phase Shift: Below f_{L1}, voltage transfer drops at $+20$ dB/decade with frequency decrease, and introduces a phase lead of $+45^\circ$ at $f = f_{L1}$.

Output Coupling Capacitor (C_{C2}) Network & Cutoff Frequency (f_{L2})

- Equivalent Output Resistance ($R_{out,C2}$): Seen looking from the terminals of C_{C2}: $R_{out,C2} = R_{out,stage} + R_L = R_C + R_L$ (assuming transistor output resistance $r_o \gg R_C$).
- Output Transfer Function Expression: $T_2(s) = \frac{V_o(s)}{V_c(s)} = \left(\frac{R_L}{R_C+R_L}\right) \frac{s}{s+\omega_{L2}}$, where $\omega_{L2} = \frac{1}{C_{C2}(R_C+R_L)}$.
- Output Cutoff Pole Frequency (f_{L2}): $f_{L2} = \frac{1}{2\pi C_{C2}(R_C+R_L)}$.
- Loading Effect Impact: Larger load resistance R_L increases $R_{out,C2}$, which lowers the required capacitance C_{C2} for a given low-frequency cutoff target f_{L2}.

Emitter Bypass Capacitor (C_E) AC Impedance & Cutoff Frequency (f_{LE})

- Equivalent Resistance Seen by C_E ($R_{eq,E}$): Seen looking into the emitter terminal: $R_{eq,E} = R_E \parallel R_{emitter,in} = R_E \parallel \left[r_e + \frac{R_S \| R_1 \| R_2}{\beta_0 + 1} \right]$, where small-signal emitter resistance $r_e = \frac{V_T}{I_C} = \frac{r_\pi}{\beta_0 + 1}$.
- Emitter Pole Frequency Formula (f_{LE}): $f_{LE} = \frac{1}{2\pi C_E R_{eq,E}} = \frac{1}{2\pi C_E \left(R_E \| \left[r_e + \frac{R_S \| R_1 \| R_2}{\beta_0 + 1} \right] \right)}$.
- Small Equivalent Resistance Implication: Because r_e is small (typically $10 - 30\ \Omega$), $R_{eq,E}$ is very small ($15 - 50\ \Omega$). Thus, f_{LE} is much higher than f_{L1} and f_{L2} for equal capacitor values.
- Dominant Pole Role: To keep f_{LE} down near audio frequencies, C_E must be significantly larger ($47 - 220\ \mu$F) than C_{C1} or C_{C2} ($1 - 10\ \mu$F).

Pole-Zero Derivation of the Emitter Bypass Impedance Network $Z_E(s)$

- Impedance Expression for $R_E \parallel C_E$: $Z_E(s) = R_E \parallel \frac{1}{sC_E} = \frac{R_E}{1+sR_E C_E}$.
- Unbypassed Gain Equation: Incorporating $Z_E(s)$ into CE amplifier voltage gain: $A_v(s) = -\frac{\beta_0(R_C \| R_L)}{r_\pi+(\beta_0+1)Z_E(s)} = -g_m(R_C \parallel R_L)\frac{1+sR_E C_E}{1+sR_E C_E+g_m R_E}$.
- Zero Frequency Location (ω_z): The numerator zero occurs at $\omega_z = \frac{1}{R_E C_E} \implies f_z = \frac{1}{2\pi R_E C_E}$.
- Pole Frequency Location (ω_p): The denominator pole occurs at $\omega_p = \frac{1+g_m R_E}{R_E C_E} = \frac{1}{C_E R_{eq,E}} \implies f_{LE} = \frac{1}{2\pi C_E R_{eq,E}}$.
- Gain Transition: At frequencies below f_z, gain drops to unbypassed value $A_{v,unbypassed} = -\frac{R_C \| R_L}{R_E}$. Above f_{LE}, gain rises to full midband bypassed gain $A_M = -g_m(R_C \parallel R_L)$.

Dominant Pole Approximation & Low-Frequency Asymptotic Response

- Overall Low-Frequency Transfer Function: $A_L(s) = A_M \cdot \left(\frac{s}{s+\omega_{L1}}\right)\left(\frac{s}{s+\omega_{L2}}\right)\left(\frac{s+\omega_z}{s+\omega_{LE}}\right)$.
- Dominant Pole Condition: If one pole frequency is significantly higher than all other low-frequency poles (e.g., $f_{LE} \geq 4f_{L1}$ and $f_{LE} \geq 4f_{L2}$), f_{LE} dominates the lower 3-dB cutoff: $f_L \approx f_{LE}$.
- Non-Dominant Pole Combination Formula: When poles are closely spaced, overall lower cutoff f_L is estimated using short-circuit sum or root-square approximation: $f_L \approx \sqrt{f_{L1}^2 + f_{L2}^2 + f_{LE}^2}$ or $f_L \approx f_{L1} + f_{L2} + f_{LE}$.
- Asymptotic Bode Slope Progression: Gain magnitude slope increases from $+20$ dB/decade below f_{LE}, to $+40$ dB/decade below f_{L1}, to $+60$ dB/decade below f_{L2}.

Numerical Example 1: Determining f_{L1}, f_{L2}, f_{LE} and Overall f_L

- Circuit Parameters: $V_{CC} = 12$ V, $R_1 = 40$ kΩ, $R_2 = 10$ kΩ, $R_C = 4$ kΩ, $R_E = 2$ kΩ, $R_S = 600\ \Omega$, $R_L = 10$ kΩ, $\beta_0 = 100$, $I_C = 1$ mA. Capacitors: $C_{C1} = 10\ \mu$F, $C_{C2} = 4.7\ \mu$F, $C_E = 47\ \mu$F. Small-signal parameters: $V_T = 26$ mV $\implies r_e = 26\ \Omega$, $g_m = 38.46$ mS, $r_\pi = 2.6$ kΩ.
- Step 1 - Calculate f_{L1}: $R_B = 40\text{ k} \parallel 10\text{ k} = 8$ kΩ. $R_{in,stage} = R_B \parallel r_\pi = 8\text{ k} \parallel 2.6\text{ k} = 1.962$ kΩ. $R_{in,C1} = 600 + 1962 = 2.562$ kΩ. $f_{L1} = \frac{1}{2\pi \times 10\mu\text{F} \times 2562} = 6.21$ Hz.
- Step 2 - Calculate f_{L2}: $R_{out,C2} = R_C + R_L = 4\text{ k} + 10\text{ k} = 14$ kΩ. $f_{L2} = \frac{1}{2\pi \times 4.7\mu\text{F} \times 14000} = 2.42$ Hz.
- Step 3 - Calculate f_{LE}: $R_{base,eq} = R_S \parallel R_1 \parallel R_2 = 600 \parallel 8000 = 558\ \Omega$. $R_{emitter,in} = r_e + \frac{558}{101} = 26 + 5.52 = 31.52\ \Omega$. $R_{eq,E} = 2000 \parallel 31.52 = 31.03\ \Omega$. $f_{LE} = \frac{1}{2\pi \times 47\mu\text{F} \times 31.03} = 109.13$ Hz.
- Step 4 - Overall f_L: Dominant pole $f_{LE} = 109.13$ Hz. Root-square estimate $f_L = \sqrt{6.21^2 + 2.42^2 + 109.13^2} = 109.33$ Hz.

Low-Frequency Phase Lead & Signal Distortion

- Total Low-Frequency Phase Lead Equation:
  $\theta(f) = +\arctan\left(\frac{f_{L1}}{f}\right) + \arctan\left(\frac{f_{L2}}{f}\right) + \arctan\left(\frac{f_{LE}}{f}\right) - \arctan\left(\frac{f_z}{f}\right)$.
- Midband vs Low-Frequency Phase: At midband ($f \gg f_L$), $\theta(f) \approx 0^\circ$ relative to the 180° inversion of Common-Emitter configuration.
- Pulse Response Distortion (Square Wave Sag/Tilt): Passing a pulse of width T_W through high-pass cutoff f_L causes exponential rooftop decay.
- Percentage Sag Formula: Fractional voltage droop $S = \frac{\Delta V}{V_{peak}} \approx \pi \frac{f_L}{f_{pulse}} \times 100\%$. Keeping sag under 5% requires $f_L \leq 0.016 f_{pulse}$.

Capacitor Selection Strategies for Target Lower Cutoff Frequency

- Design Budgeting Strategy: When designing for a target lower 3-dB cutoff f_L, allocate contributions across capacitors to minimize physical component sizes and costs.
- Optimal Pole Budget Rule of Thumb: Set the emitter bypass pole $f_{LE} = 0.8 f_L$, and input/output coupling poles $f_{L1} = 0.1 f_L$, $f_{L2} = 0.1 f_L$.
- Minimizing Bypass Capacitor Size: Since $C_E = \frac{1}{2\pi f_{LE} R_{eq,E}}$, allocating 80% of f_L to f_{LE} maximizes f_{LE}, reducing required physical capacitance C_E.
- Practical Considerations: Electrolytic capacitors used for C_E have wide tolerances ($\pm 20\%$) and Equivalent Series Resistance (ESR) that must be factored into high-precision designs.

Numerical Example 2: Designing Capacitors for a Specified $f_L = 50$ Hz

- Design Requirement: Size capacitors C_{C1}, C_{C2}, C_E for a CE stage with $R_{in,C1} = 3$ kΩ, $R_{out,C2} = 12$ kΩ, $R_{eq,E} = 25$ Ω to achieve a lower 3-dB cutoff $f_L = 50$ Hz.
- Step 1 - Allocate Pole Budgets: Target $f_{LE} = 0.8 \times 50$ Hz $= 40$ Hz, $f_{L1} = 0.1 \times 50$ Hz $= 5$ Hz, $f_{L2} = 0.1 \times 50$ Hz $= 5$ Hz.
- Step 2 - Calculate Input Coupling Capacitor C_{C1}:
  $C_{C1} = \frac{1}{2\pi \times 5\ \text{Hz} \times 3000\ \Omega} = 10.61\ \mu\text{F} \implies$ Select standard 15 μF.
- Step 3 - Calculate Output Coupling Capacitor C_{C2}:
  $C_{C2} = \frac{1}{2\pi \times 5\ \text{Hz} \times 12000\ \Omega} = 2.65\ \mu\text{F} \implies$ Select standard 3.3 μF.
- Step 4 - Calculate Emitter Bypass Capacitor C_E:
  $C_E = \frac{1}{2\pi \times 40\ \text{Hz} \times 25\ \Omega} = 159.15\ \mu\text{F} \implies$ Select standard 220 μF.

Summary of Bypass and Coupling Capacitor Effects on Frequency Response

- Coupling Capacitors (C_{C1}, C_{C2}): Determine high-pass cutoffs $f_{L1} = \frac{1}{2\pi C_{C1} R_{in,C1}}$ and $f_{L2} = \frac{1}{2\pi C_{C2} R_{out,C2}}$.
- Bypass Capacitor (C_E): Determines cutoff $f_{LE} = \frac{1}{2\pi C_E R_{eq,E}}$. Requires largest capacitance because $R_{eq,E} = R_E \parallel \left[r_e + \frac{R_{base}}{\beta_0 + 1} \right]$ is very small.
- Pole-Zero Behavior: Bypass network creates a zero $f_z = \frac{1}{2\pi R_E C_E}$ and pole f_{LE}. Gain transitions from unbypassed value below f_z to bypassed midband value above f_{LE}.
- Dominant Pole Approximation: Lower 3-dB cutoff $f_L \approx \sqrt{f_{L1}^2 + f_{L2}^2 + f_{LE}^2} \approx f_{LE}$ when f_{LE} is dominant.