Lecture 21: Amplifier Parameters Gain, Bandwidth and Gain Bandwidth prod...

Frequency Response of Transistor Amplifier

DI02011011: Electronics Circuit and Application (ECA)

Lecture Agenda & Learning Roadmap

- Voltage, Current, and Power Gain Definitions and Decibel Conversions
- Amplifier Frequency Response: Half-Power (-3 dB) Cutoff Frequencies f_L and f_H
- Bandwidth (BW) and Transfer Function Representation of Single-Pole Amplifiers
- Gain-Bandwidth Product (GBW) and Unity-Gain Frequency (f_T)
- Gain-Bandwidth Trade-Off via Negative Feedback Configuration
- BJT High-Frequency Figures of Merit: Cutoff Frequencies f_α, f_β, and f_T
- Physical Factors Influencing GBW: Transconductance g_m and Parasitic Capacitances
- Op-Amp Closed-Loop Bandwidth Determination and Noise Gain Considerations
- Gain-Bandwidth Shrinkage in Multistage Cascaded Amplifiers
- Step-by-Step Numerical Examples and Comprehensive Summary

Amplifier Gain Metrics: Voltage, Current, and Power Gains in Decibels

- Voltage Gain Definition: Linear ratio $A_v = \frac{V_o}{V_i}$ and decibel representation $A_v(\text{dB}) = 20\log_{10}\left|\frac{V_o}{V_i}\right|$, quantifying signal voltage amplification.
- Current Gain Definition: Linear ratio $A_i = \frac{I_o}{I_i}$ and decibel representation $A_i(\text{dB}) = 20\log_{10}\left|\frac{I_o}{I_i}\right|$, measuring current multiplication across low-impedance nodes.
- Power Gain Relationship: Overall power amplification $A_p = \frac{P_o}{P_i} = A_v \cdot A_i$. In decibels, $A_p(\text{dB}) = 10\log_{10} A_p = A_v(\text{dB}) + A_i(\text{dB})$ when input and output load resistances are equal.
- Source-to-Load System Gain (A_{vs}): Incorporates signal source internal resistance R_S and load resistance R_L: $A_{vs} = \frac{V_o}{V_s} = \left(\frac{R_{in}}{R_S+R_{in}}\right) A_v \left(\frac{R_L}{R_{out}+R_L}\right)$.

Frequency Response Characteristics & Bandwidth (BW)

- Midband Voltage Gain (A_M): The maximum, frequency-independent voltage gain in the intermediate frequency range where coupling/bypass capacitors act as AC short circuits and parasitic capacitances act as open circuits.
- Half-Power (-3 dB) Cutoff Frequencies: Lower cutoff f_L and upper cutoff f_H represent frequencies where the magnitude drops to $|A(f)| = \frac{A_M}{\sqrt{2}} \approx 0.707 A_M$.
- Power Reduction at Cutoff: At f_L and f_H, output power drops to half of midband power ($P_{out} = 0.5 P_{mid}$), corresponding to a -3.01 dB change: $20 \log_{10}(0.707) = -3.01$ dB.
- Bandwidth Definition: $BW = f_H - f_L$. For broadband amplifiers where $f_H \gg f_L$ (e.g., $f_H = 1$ MHz and $f_L = 10$ Hz), the bandwidth is approximated as $BW \approx f_H$.

Single-Pole Transfer Function Analysis & High-Frequency Roll-off

- Single-Pole High-Frequency Transfer Function: $A(s) = \frac{A_M}{1+s/\omega_H}$, where $\omega_H = 2\pi f_H$ is the high-frequency pole.
- Frequency Response Vector Expression: Substituting $s = j\omega$ yields $A(j\omega) = \frac{A_M}{1+j(\omega/\omega_H)} = \frac{A_M}{\sqrt{1+(f/f_H)^2}}\angle - \arctan(f/f_H)$.
- Asymptotic High-Frequency Roll-Off: For frequencies well above cutoff ($f \gg f_H$), the gain magnitude simplifies to $|A(f)| \approx A_M \cdot \frac{f_H}{f}$, giving a roll-off rate of -20 dB/decade (or -6 dB/octave).
- Phase Response Shift: Phase lag at midband ($f \ll f_H$) is 0°, at upper cutoff ($f = f_H$) is -45°, and approaches -90° as $f \to \infty$.

Gain-Bandwidth Product (GBW) and Unity-Gain Frequency (f_T)

- Gain-Bandwidth Product Definition: $GBW = A_M \cdot f_H$, representing the constant product of midband voltage gain magnitude A_M and upper 3-dB bandwidth f_H for a single-pole system.
- Unity-Gain Frequency (f_T): The frequency at which the short-circuit/open-loop voltage gain drops to unity ($|A(f_T)| = 1 = 0$ dB).
- Equivalence of GBW and f_T: Setting $|A(f_T)| = \frac{A_M}{\sqrt{1+(f_T/f_H)^2}} = 1$ yields $f_T \approx A_M \cdot f_H = GBW$ when $A_M \gg 1$.
- Figure of Merit Significance: GBW remains constant across different closed-loop gain configurations, serving as an intrinsic device parameter determined by internal transistor dynamics.

Gain vs. Bandwidth Trade-off via Negative Feedback

- Closed-Loop Gain Derivation: Applying negative feedback with feedback factor β reduces low-frequency gain: $A_{cl}(s) = \frac{A(s)}{1+\beta A(s)}$.
- Closed-Loop Transfer Function Expansion: Substituting $A(s) = \frac{A_0}{1+s/\omega_H}$ yields $A_{cl}(s) = \frac{A_0/(1+A_0\beta)}{1+\frac{s}{\omega_H(1+A_0\beta)}}$.
- Closed-Loop Bandwidth Extension: The new midband gain is $A_{cl,0} = \frac{A_0}{1+A_0\beta}$ and new upper cutoff is $f_{H,cl} = f_H(1 + A_0\beta)$.
- Invariance of Gain-Bandwidth Product: Multiplying closed-loop gain by closed-loop bandwidth gives $GBW_{cl} = A_{cl,0} \cdot f_{H,cl} = \left(\frac{A_0}{1+A_0\beta}\right) \cdot f_H(1 + A_0\beta) = A_0 f_H = GBW$.

Numerical Example 1: BJT Amplifier Bandwidth and GBW Calculation

- Problem Statement: A single-stage BJT amplifier has an open-loop midband voltage gain $A_0 = 80$ dB $(10,000$ V/V$)$ and a unity-gain frequency $f_T = 50$ MHz. (1) Calculate open-loop bandwidth f_H. (2) If negative feedback reduces closed-loop gain to $A_{cl} = 40$ dB (100 V/V), find new bandwidth $f_{H,cl}$.
- Step 1 - Open-Loop Bandwidth f_H: Using $GBW = A_0 \cdot f_H = f_T$, we get $f_H = \frac{f_T}{A_0} = \frac{50\times10^6 \text{ Hz}}{10{,}000} = 5,000 \text{ Hz} = 5 \text{ kHz}$.
- Step 2 - Closed-Loop Bandwidth $f_{H,cl}$: Using constant GBW, $f_{H,cl} = \frac{GBW}{A_{cl}} = \frac{50\times10^6 \text{ Hz}}{100} = 500,000 \text{ Hz} = 500 \text{ kHz}$.
- Verification of Feedback Factor β: $1 + A_0\beta = \frac{A_0}{A_{cl}} = \frac{10000}{100} = 100 \implies \beta = \frac{99}{10000} = 0.0099$. Bandwidth expanded 100-fold from 5 kHz to 500 kHz.

Transistor High-Frequency Limits: Cutoff Frequencies f_α, f_β, and f_T

- Common-Base Short-Circuit Cutoff (f_α): Frequency where common-base short-circuit current gain α drops by 3 dB from low-frequency value α_0: $f_\alpha = \frac{1}{2\pi\tau_F}$, where τ_F is base transit time.
- Common-Emitter Short-Circuit Cutoff (f_β): Frequency where common-emitter current gain $\beta(f)$ drops by 3 dB: $f_\beta = \frac{1}{2\pi r_\pi(C_\pi + C_\mu)} = \frac{g_m}{2\pi\beta_0(C_\pi + C_\mu)}$.
- Relationship Between f_T and f_β: Short-circuit current gain relation $\beta(f) = \frac{\beta_0}{1+j(f/f_\beta)}$. Unity gain frequency occurs when $|\beta(f_T)| = 1 \implies f_T = \beta_0 f_\beta = \frac{g_m}{2\pi(C_\pi + C_\mu)}$.
- Alpha-Beta Cutoff Relation: Since $\beta_0 \gg 1$, $f_\beta = \frac{f_T}{\beta_0}$ is substantially lower than f_T and $f_\alpha \approx 1.2 f_T$.

Physical Parameters Governing GBW: g_m, C_π, and C_μ

- Transconductance Dependence: $g_m = \frac{I_C}{V_T}$, where I_C is DC collector current and $V_T \approx 26$ mV at 300 K. Higher bias current I_C elevates g_m and increases f_T.
- Diffusion Capacitance C_{DE}: Proportional to collector current: $C_{DE} = g_m \tau_F = \left(\frac{I_C}{V_T}\right) \tau_F$. Dominates base-emitter junction capacitance $C_\pi = C_{jE} + C_{DE}$ at higher bias currents.
- Depletion Capacitances C_{jE}, C_μ: Voltage-dependent junction capacitances: $C_j(V_R) = \frac{C_{j0}}{(1+V_R/V_0)^m}$. Collector-base capacitance C_μ is minimized by higher reverse-bias collector-base voltage V_{CB}.
- Optimum Bias Current Region: At very high I_C, high-level injection and Kirk effect increase base transit time τ_F, causing f_T and GBW to roll off after reaching a peak value.

Numerical Example 2: Operational Amplifier Closed-Loop Bandwidth

- Problem Statement: An operational amplifier has a specified Gain-Bandwidth Product $GBW = 12$ MHz. It is configured as a non-inverting voltage amplifier with feedback resistors $R_F = 18$ kΩ and $R_1 = 2$ kΩ. Determine: (1) Closed-loop midband voltage gain A_{cl}. (2) Closed-loop 3-dB bandwidth $f_{H,cl}$. (3) Gain at an operating signal frequency of $f = 2.4$ MHz.
- Step 1 - Closed-Loop Gain A_{cl}: $A_{cl} = 1 + \frac{R_F}{R_1} = 1 + \frac{18\text{ k}\Omega}{2\text{ k}\Omega} = 1 + 9 = 10$ V/V (20 dB).
- Step 2 - Closed-Loop Bandwidth $f_{H,cl}$: $f_{H,cl} = \frac{GBW}{A_{cl}} = \frac{12\times 10^6\text{ Hz}}{10} = 1.2$ MHz.
- Step 3 - Gain Magnitude at $f = 2.4$ MHz:
$|A(f)| = \frac{A_{cl}}{\sqrt{1+(f/f_{H,cl})^2}} = \frac{10}{\sqrt{1+(2.4/1.2)^2}} = \frac{10}{\sqrt{1+4}} = \frac{10}{\sqrt{5}} \approx 4.47$ V/V (13.01 dB).

Gain-Bandwidth Considerations in Multistage Amplifiers

- Multistage Bandwidth Shrinkage Effect: Cascading N identical non-interacting single-pole stages increases overall low-frequency gain to $A_{M,sys} = (A_M)^N$, but shrinks upper 3-dB bandwidth.
- Upper Cutoff Formula for N Cascaded Stages: $f_{H,sys} = f_H \cdot \sqrt{2^{1/N} - 1}$, where f_H is the upper cutoff of an individual stage.
- Bandwidth Reduction Factors: For $N = 2$, $f_{H,sys} = 0.643 f_H$. For $N = 3$, $f_{H,sys} = 0.510 f_H$. For $N = 4$, $f_{H,sys} = 0.435 f_H$.
- Overall GBW Non-Constancy: Because overall gain grows exponentially $(A_M)^N$ while bandwidth shrinks moderately, the total multi-stage GBW is not equal to single-stage GBW, requiring careful stage-by-stage optimization.

Summary & Key Engineering Formulas for Amplifier Parameters

- Gain Relationships: Voltage Gain $A_v(\text{dB}) = 20\log_{10}|A_v|$, Power Gain $A_p(\text{dB}) = 10\log_{10} A_p = A_v(\text{dB}) + A_i(\text{dB})$.
- Bandwidth & Cutoff Limits: Half-power (-3 dB) frequencies f_L, f_H; Bandwidth $BW = f_H - f_L \approx f_H$ for broadband systems.
- Gain-Bandwidth Invariance: $GBW = A_M \cdot f_H = f_T$. Negative feedback scales gain down by $(1 + A_0\beta)$ and expands upper bandwidth by $(1 + A_0\beta)$, keeping GBW constant.
- Transistor Frequency Parameters: Unity-gain frequency $f_T = \beta_0 f_\beta = \frac{g_m}{2\pi(C_\pi + C_\mu)}$, driven by bias current I_C and internal capacitances.