Lecture 20: Amplifier Parameters Gain, Bandwidth and Gain Bandwidth prod...

Frequency Response of Transistor Amplifier

DI02011011: Electronics Circuit and Application (ECA)

Lecture Blueprint: Amplifier Parameters & Frequency Response

- Fundamental Amplifier Gain Definitions: Voltage, Current, Transconductance, Transresistance.
- Decibel (dB) Scale Representation & Logarithmic Bode Plot Curves.
- Low-Frequency Response (f_L): Coupling (C_{C1}, C_{C2}) & Emitter Bypass (C_E) Capacitor Poles.
- High-Frequency Response (f_H): Stray Parasitic & Transistor Junction Capacitances (C_π, C_μ).
- Bandwidth (BW) & Half-Power (-3 dB) Corner Frequency Definitions.
- The Gain-Bandwidth Product (GBP) & High-Frequency Hybrid-\{}pi$ Model.
- Transition Frequency (f_T) Derivation & The Miller Effect Theorem.
- Feedback Effects on Gain-Bandwidth Trade-Offs & Worked Numerical Problems.

Fundamental Amplifier Gain Parameters

- Voltage Gain (A_v): Ratio of AC output voltage to AC input voltage: $A_v = \frac{v_{out}}{v_{in}} = \frac{v_o}{v_i}$ (Dimensionless).
- Current Gain (A_i): Ratio of AC output current to AC input current: $A_i = \frac{i_{out}}{i_{in}} = \frac{i_o}{i_i}$ (Dimensionless).
- Transconductance Gain (G_m): Ratio of AC output current to AC input voltage: $G_m = \frac{i_{out}}{v_{in}}$ (Units: Siemens, S or Ω^{-1}).
- Transresistance Gain (R_m): Ratio of AC output voltage to AC input current: $R_m = \frac{v_{out}}{i_{in}}$ (Units: Ohms, Ω).
- Power Gain (A_p): Ratio of AC output power to AC input power: $A_p = \frac{P_{out}}{P_{in}} = A_v \cdot A_i$.

Decibel (dB) Scale Representation & Logarithmic Bode Curves

- Voltage Gain in Decibels: $A_{v,\text{dB}} = 20\log_{10}|A_v| = 20\log_{10}\left|\frac{v_o}{v_i}\right|$.
- Power Gain in Decibels: $A_{p,\text{dB}} = 10\log_{10} A_p = 10\log_{10}\left(\frac{P_o}{P_i}\right)$.
- Half-Power (-3 dB) Point Definition: Frequency at which output power drops to half its midband value ($P_{out} = 0.5P_{mid}$), corresponding to voltage magnitude dropping to $\frac{1}{\sqrt{2}} \approx 0.707$ of midband voltage gain (A_{mid}).
- Decibel Loss at Cutoff: $20\log_{10}(0.7071) = -3.0103\,\text{dB} \approx -3\,\text{dB}$.
- Bode Plot Slope Rates: Single-pole RC roll-off produces a attenuation slope of $-20\,\text{dB}/\text{decade}$ ($-6\,\text{dB}/\text{octave}$).

Low-Frequency Cutoff (f_L) Dynamics: Coupling & Bypass Capacitors

- Low-Frequency Attenuation Mechanism: At low frequencies ($f \to 0$), capacitive reactances $X_C = \frac{1}{2\pi fC} \to \infty$, acting as open circuits that attenuate signals.
- Input Coupling Capacitor Pole (C_{C1}): Forms high-pass filter with total input resistance $R'_{in} = R_{sig} + (R_1 \parallel R_2 \parallel r_p)$: $f_{L1} = \frac{1}{2\pi(R_{sig}+R_{in})C_{C1}}$.
- Output Coupling Capacitor Pole (C_{C2}): Forms high-pass filter with load resistance: $f_{L2} = \frac{1}{2\pi(R_C+R_L)C_{C2}}$.
- Emitter Bypass Capacitor Pole (C_E): Bypasses emitter resistor R_E. Resistance looking into emitter is $R'_e = R_E \parallel \left[r_e + \frac{R_{TH}\parallel R_{sig}}{1+\beta}\right]$: $f_{LE} = \frac{1}{2\pi R'_e C_E}$.
- Overall Lower Cutoff Frequency: Dominant pole approximation: $f_L \approx \sqrt{f_{L1}^2 + f_{L2}^2 + f_{LE}^2}$ or $f_L \approx f_{L1} + f_{L2} + f_{LE}$.

High-Frequency Cutoff (f_H) Dynamics: Parasitic Junction Capacitances

- High-Frequency Attenuation Mechanism: At high frequencies ($f \to \infty$), parasitic internal junction reactances $X_C = \frac{1}{2\pi fC} \to 0$, shorting AC signals to ground.
- Base-Emitter Diffusion & Transition Capacitance (C_π or C_{be}): Forward-biased base-emitter junction capacitance ($10\,\text{pF} - 100\,\text{pF}$).
- Base-Collector Depletion Capacitance (C_μ or C_{bc}): Reverse-biased base-collector junction capacitance ($1\,\text{pF} - 5\,\text{pF}$).
- Stray Wiring Capacitance ($C_{w,in}$, $C_{w,out}$): Parasitic PCB trace capacitances ($2\,\text{pF} - 10\,\text{pF}$).
- Overall Upper Cutoff Frequency (f_H): Set by internal low-pass RC network poles: $f_H = \frac{1}{2\pi R_{in,eq} C_{in,total}}$.

Bandwidth (BW) & Midband Operational Range

- Bandwidth Definition: The continuous range of signal frequencies over which amplifier gain remains within 3 dB of its peak midband gain A_{mid}.
- Mathematical Formula: $BW = f_H - f_L$.
- Wideband Approximation ($f_H \gg f_L$): For standard audio and video amplifiers where $f_H \geq 100 f_L$, $BW \approx f_H$.
- Midband Region Characteristics: Frequency range where external capacitors (C_{C1}, C_{C2}, C_E) act as perfect AC short circuits AND internal capacitors (C_π, C_μ) act as perfect AC open circuits, yielding constant gain A_{mid} and zero phase shift distortion.

The Gain-Bandwidth Product (GBP/f_T): Invariance Principle

- Gain-Bandwidth Product Definition: $GBP \equiv |A_{mid}| \times BW \approx |A_{mid}| \times f_H$.
- Fundamental Invariance Principle: For a single-pole amplifier system, the product of midband voltage gain and bandwidth is a CONSTANT parameter dictated by internal transistor physics.
- Gain vs. Bandwidth Trade-Off: Increasing midband gain $|A_{mid}|$ proportionally decreases bandwidth BW; trading gain extends bandwidth.
- Transition Frequency (f_T): Frequency at which short-circuit common-emitter current gain drops to unity ($|h_{fe}(f_T)| = 1$). f_T represents the absolute maximum theoretical GBP of the device.

High-Frequency Hybrid-\{}pi Transistor Model Parameters

- Hybrid-\{}pi Model Components: Replaces low-frequency *h*-parameter model for high-frequency dynamic AC circuit analysis.
- Base Spreading Resistance ($r_b b'$ or r_x): Ohmic resistance of bulk base semiconductor material between base terminal and active junction ($20\,\Omega - 100\,\Omega$).
- Internal Base-Emitter Resistance ($r_b' e$ or r_π): Dynamic resistance of forward-biased base-emitter junction: $r_\pi = \frac{\beta_0}{g_m} = \frac{\beta_0 V_T}{I_{CQ}}$.
- Transconductance (g_m): $g_m = \frac{I_{CQ}}{V_T} = \frac{I_{CQ}}{26\,\text{mV}}$ at room temperature (25°C).
- Junction Capacitances: $C_{b'e} = C_\pi$ (diffusion + depletion) and $C_{b'c} = C_\mu$ (collector overlap depletion).

Transition Frequency (f_T) Derivation & Hybrid-\{}pi Equation

- Short-Circuit Current Gain Formulation: Output short-circuited ($R_L = 0 \implies v_{ce} = 0$). Output current $i_o = -g_m v_{b'e}$.
- Input Current Equation: $i_i = v_{b'e} \cdot \left[\frac{1}{r_\pi} + j2\pi f(C_\pi + C_\mu)\right]$.
- Short-Circuit Current Gain Expression: $A_{i,sc}(f) = \frac{i_o}{i_i} = \frac{-g_m}{\frac{1}{r_\pi}+j2\pi f(C_\pi+C_\mu)} = \frac{-h_{fe0}}{1+j\frac{f}{f_\beta}}$, where $f_\beta = \frac{1}{2\pi r_\pi(C_\pi+C_\mu)}$ is the β-cutoff frequency.
- Setting $|A_{i,sc}(f_T)| = 1$: At $f = f_T \gg f_\beta$, $|A_{i,sc}| \approx \frac{g_m}{2\pi f_T(C_\pi+C_\mu)} = 1$.
- Transition Frequency Formula: $f_T = \frac{g_m}{2\pi(C_\pi+C_\mu)}$.

Miller Effect Theorem: Impact of Feedback Capacitance C_μ

- Miller Effect Origin: In an inverting amplifier ($A_v = -|A_v|$), feedback capacitance C_μ connected between base (input) and collector (output) experiences an amplified AC voltage swing across its terminals.
- Input Miller Capacitance Derivation: $C_{M,in} = C_\mu\,(1 - A_v) = C_\mu\,(1 + |A_v|)$.
- Output Miller Capacitance Derivation: $C_{M,out} = C_\mu \left(1 - \frac{1}{A_v}\right) \approx C_\mu$.
- Total High-Frequency Input Capacitance: $C_{in,total} = C_\pi + C_{M,in} = C_\pi + C_\mu(1 + |A_v|)$.
- Bandwidth Impact: The massive equivalent input capacitance $C_{in,total}$ creates a low-frequency dominant pole at the input node, severely reducing high-frequency cutoff f_H.

Feedback Effects on Gain-Bandwidth Product: Desensitivity & Extension

- Closed-Loop Gain with Negative Feedback: $A_f = \frac{A_{mid}}{1+\beta_{fb}A_{mid}}$, where β_{fb} is the feedback factor.
- Closed-Loop Upper Cutoff Frequency: $f_{Hf} = f_H\left(1+\beta_{fb}A_{mid}\right)$.
- Closed-Loop Lower Cutoff Frequency: $f_{Lf} = \frac{f_L}{1+\beta_{fb}A_{mid}}$.
- Closed-Loop Bandwidth Extension: $BW_f = f_{Hf} - f_{Lf} \approx f_H\left(1+\beta_{fb}A_{mid}\right)$.
- Conservation of Gain-Bandwidth Product:
  $GBP_f = A_f \times BW_f = \left(\frac{A_{mid}}{1+\beta_{fb}A_{mid}}\right) \times [f_H(1+\beta_{fb}A_{mid})] = A_{mid} \times f_H = GBP$.

Numerical Example: High-Frequency Hybrid-\{}pi & f_T Calculation

- Given Transistor Parameters: Operating quiescent current $I_{CQ} = 2.6\,\text{mA}$, $\beta_0 = 100$, $V_T = 26\,\text{mV}$. Parasitic junction capacitances $C_\pi = 30\,\text{pF}$, $C_\mu = 2\,\text{pF}$. Inverting voltage gain $|A_v| = 50$. Source resistance $R_{sig} = 600\,\Omega$, $R_{TH} = 10\,\text{k}\Omega$.
- Step 1: Calculate Transconductance g_m: $g_m = \frac{I_{CQ}}{V_T} = \frac{2.6\,\text{mA}}{26\,\text{mV}} = 0.1\,\text{S} = 100\,\text{mS}$.
- Step 2: Calculate Input Resistance r_π: $r_\pi = \frac{\beta_0}{g_m} = \frac{100}{0.1} = 1000\,\Omega = 1.0\,\text{k}\Omega$.
- Step 3: Calculate Transition Frequency f_T:
  $f_T = \frac{g_m}{2\pi(C_\pi + C_\mu)} = \frac{0.1}{2\pi(30\text{pF}+2\text{pF})} = \frac{0.1}{2\pi \times 32 \times 10^{-12}} = 497.35\,\text{MHz}$.
- Step 4: Calculate Miller Input Capacitance: $C_{M,in} = C_\mu(1 + |A_v|) = 2\,\text{pF} \times (1 + 50) = 102\,\text{pF}$.
- Step 5: Total Input Capacitance: $C_{in,total} = C_\pi + C_{M,in} = 30\,\text{pF} + 102\,\text{pF} = 132\,\text{pF}$.

Summary & Core Amplifier Frequency Response Principles

- Frequency Response Regions: Low-frequency roll-off ($f < f_L$) set by coupling/bypass capacitors; High-frequency roll-off ($f > f_H$) set by junction capacitors (C_π, C_μ).
- Half-Power Cutoff: $-3\,\text{dB}$ point where gain drops to $0.707 A_{mid}$ and power drops to 50%.
- Miller Effect: Multiplies feedback capacitance by $(1 + |A_v|)$, creating the dominant high-frequency input pole.
- Gain-Bandwidth Invariance: $GBP = |A_{mid}| \times BW = f_T = \frac{g_m}{2\pi(C_\pi + C_\mu)}$.
- Feedback Extension: Negative feedback trades gain for bandwidth, extending $BW_f = f_H(1 + \beta_{fb} A)$ while conserving overall GBP.