Lecture 3.24

3.4 Signal Cycle Time Design

Review of Webster's Core Equations and Design Procedure

Before solving numerical problems, we review the exact sequence of mathematical equations used in Webster's signal design procedure.

Following a systematic step-by-step process prevents errors in volume ratio summation and green split allocation.

- Step 1: Compute Critical Volume Ratio per Phase: $y_i = \frac{q_i}{s_i}$
- Step 2: Calculate Sum of Critical Volume Ratios: $Y = \sum_{i=1}^{n} y_i$
- Step 3: Determine Total Cycle Lost Time: $L = 2n + R$ (or $\sum l_i$)
- Step 4: Compute Optimum Cycle Length: $C_o = \frac{1.5L+5}{1-Y}$
- Step 5: Calculate Effective Green: $g_i = \frac{y_i}{Y}(C_o - L)$ and Actual Green G_i

Numerical Problem 1: 2-Phase Intersection Data

Consider a 2-phase signalized cross intersection with approach traffic flows and saturation flows given below.

Phase 1 accommodates North-South traffic, while Phase 2 accommodates East-West traffic. Lost time per phase is 2.5 seconds, and total all-red clearance time is 4 seconds.

Parameter	**Phase 1 (N-S)**	**Phase 2 (E-W)**
Peak Flow (q)	800 PCU/h	600 PCU/h
Saturation Flow (s)	2400 PCU/h	2000 PCU/h
Lost Time (l)	2.5 s	2.5 s
Amber Time (A)	3.0 s	3.0 s

Target: Find Optimum Cycle Length C_o and actual green displays G_1, G_2.

Solution Step-by-Step for Problem 1

We execute the 5-step procedure to compute volume ratios, total lost time, optimum cycle length, and phase green allocations.

Detailed arithmetic calculations demonstrate how Webster's formula allocates green splits.

- Volume Ratios: $y_1 = \frac{800}{2400} = 0.333$, $y_2 = \frac{600}{2000} = 0.300$
- Sum Y: $Y = 0.333 + 0.300 = 0.633$ ($Y < 1.0$, solution is feasible!)
- Total Lost Time L: $L = (2.5 + 2.5) + 4.0 = 9.0$ seconds
- Optimum Cycle C_o: $C_o = \frac{1.5(9.0)+5}{1-0.633} = \frac{13.5+5}{0.367} = \frac{18.5}{0.367} \approx 50.4$ seconds
- Adopted Cycle Length: Round up to $C_o = 52$ seconds

Green Split Calculation for Problem 1

With adopted cycle length $C = 52$ seconds and total lost time $L = 9$ seconds, available total effective green time is $C - L = 43$ seconds.

We divide effective green between Phase 1 and Phase 2 based on their relative y_i/Y proportions.

- Effective Green Phase 1: $g_1 = \frac{0.333}{0.633} \times 43 = 22.6$ seconds
- Effective Green Phase 2: $g_2 = \frac{0.300}{0.633} \times 43 = 20.4$ seconds
- Actual Green Display G_1:
  $G_1 = g_1 + l_1 - A_1 = 22.6 + 2.5 - 3.0 = 22.1 \approx 22$ s
- Actual Green Display G_2:
  $G_2 = g_2 + l_2 - A_2 = 20.4 + 2.5 - 3.0 = 19.9 \approx 20$ s
- Verification Check: $G_1 + A_1 + G_2 + A_2 + AR = 22 + 3 + 20 + 3 + 4 = 52$ s

Numerical Problem 2: 4-Phase Complex Intersection Data

Consider a major 4-phase signalized intersection with dedicated turning phases. Approach critical lane volumes (q_i) and saturation flows (s_i) for all four phases are provided.

Total lost time per cycle is $L = 12$ seconds. Amber time is 3 seconds per phase.

- Phase 1 (North-South Straight): $q_1 = 650$ PCU/h, $s_1 = 1800$ PCU/h
- Phase 2 (North-South Right Turn): $q_2 = 300$ PCU/h, $s_2 = 1500$ PCU/h
- Phase 3 (East-West Straight): $q_3 = 550$ PCU/h, $s_3 = 1800$ PCU/h
- Phase 4 (East-West Right Turn): $q_4 = 250$ PCU/h, $s_4 = 1400$ PCU/h
- Target: Calculate Y, C_o, effective green times g_1, g_2, g_3, g_4, and verify IRC bounds

Solution Step-by-Step for Problem 2

We calculate volume ratios y_i for all four phases, sum them to check intersection saturation, and compute C_o.

This problem illustrates how multi-phase intersections require longer cycle lengths due to accumulated phase lost times.

- Volume Ratios: $y_1 = \frac{650}{1800} = 0.361$, $y_2 = \frac{300}{1500} = 0.200$, $y_3 = \frac{550}{1800} = 0.306$, $y_4 = \frac{250}{1400} = 0.179$
- Sum Y: $Y = 0.361 + 0.200 + 0.306 + 0.179 = 1.046$
- Critical Saturation Alert: $Y = 1.046 > 1.0$! The intersection is OVER-SATURATED!
- Engineering Analysis: Webster's formula yields a negative denominator $(1 - Y < 0)$. A fixed-time signal CANNOT handle these volumes!
- Corrective Action: Increase saturation flows by adding turning lanes, or restrict right turns

Modified Problem 2 with Lane Widening Solution

To fix the over-saturation in Problem 2, civil engineers widen the approach carriageways, increasing saturation flows to $s_1 = s_3 = 2200$ PCU/h and $s_2 = s_4 = 1800$ PCU/h.

We re-calculate volume ratios y_i and compute the new feasible optimum cycle length C_o.

- New Volume Ratios: $y_1 = \frac{650}{2200} = 0.295$, $y_2 = \frac{300}{1800} = 0.167$, $y_3 = \frac{550}{2200} = 0.250$, $y_4 = \frac{250}{1800} = 0.139$
- New Sum Y: $Y = 0.295 + 0.167 + 0.250 + 0.139 = 0.851$ ($Y < 1.0$, Feasible!)
- Total Lost Time L: $L = 12$ seconds
- Optimum Cycle C_o: $C_o = \frac{1.5(12)+5}{1-0.851} = \frac{18+5}{0.149} = \frac{23}{0.149} = 154.3$ seconds
- Design Decision: $C_o \approx 150$ s matches IRC upper limit for 4-phase signal

Webster's Minimum Average Delay Equation

Webster derived an equation to estimate the average delay per vehicle (d) at a signalized approach.

The equation consists of three terms: uniform delay (assuming steady arrivals), random delay (queue fluctuations), and an empirical correction factor.

- Webster's Delay Equation: $d = \frac{C(1-\lambda)^2}{2(1-\lambda x)} + \frac{x^2}{2q(1-x)} - 0.65\left(\frac{C}{q^2}\right)^{1/3} x^{(2+5\lambda)}$
- Where C: Cycle length, λ: Green ratio (g/C), q: Approach volume
- Where x: Degree of saturation ($x = \frac{q}{c} = \frac{q}{s \cdot \lambda}$)
- Simplified Formula: First term (uniform delay) accounts for 80-90% of total delay

Level of Service (LOS) Criteria for Signalized Intersections

Highway Capacity Manual (HCM) categorizes intersection performance into six Level of Service (LOS) bands (A through F) based on average control delay per vehicle.

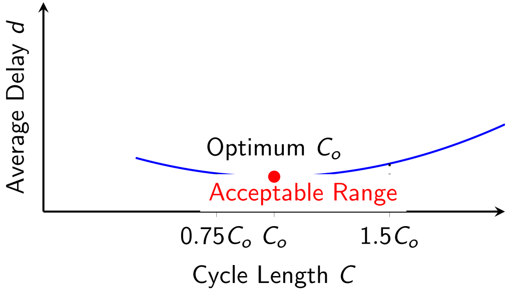
Civil engineers design signals to operate at LOS C or LOS D during peak traffic hours.

LOS	Control Delay (sec/veh)	Traffic Condition
A	≤ 10	Free flow, minimal delay
B	> 10 to 20	Good operation
C	> 20 to 35	Acceptable delay for peak hours
D	> 35 to 55	Tolerable delay, high density
E & F	> 55	Extreme delay, over-capacity gridlock

Sensitivity Analysis: Under-Design vs Over-Design of Cycle Length

Operating a signal far away from Webster's optimum cycle length C_o increases total delay sharply.

Selecting a cycle length between $0.75C_o$ and $1.5C_o$ results in minor delay penalties, providing a broad forgiving design range.



Summary of Signal Cycle Time Design Numericals

In this lecture, we solved step-by-step numerical design problems using Webster's method for 2-phase and 4-phase intersections, analyzed over-saturation conditions, and evaluated LOS delay metrics.

Mastering these numerical techniques enables students to solve GTU exam problems and design real-world intersection signal plans.

- Webster's Formula: $C_o = \frac{1.5L+5}{1-Y}$ provides optimum cycle length for minimum delay
- Volume Ratios: $y_i = q_i/s_i$; Feasibility condition $Y = \sum y_i < 1.0$
- Over-Saturation ($Y \geq 1.0$): Requires geometric lane widening or turning restrictions
- Green Split: $g_i = \frac{y_i}{Y}(C_o - L)$ distributes green proportional to demand
- LOS Thresholds: HCM grades intersection delay from LOS A ($\leq$ 10s) to LOS F ($>$ 80s)