Unit 3: Integration and its Applications

Applied Mathematics — 4320001 / DI02000011

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Unit 3 — Table of Contents

Syllabus Coverage

Weightage: ≈14 marks (20%) | Hours: 10–11

3.1 Integration as Anti-Differentiation

What is Integration?

Definition

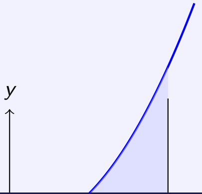
Integration is the **reverse of differentiation**.

If $\dfrac{d}{dx}[F(x)] = f(x)$, then $\displaystyle\int f(x)\,dx = F(x) + C$

where C is the **constant of integration**.

Geometric Meaning

$\displaystyle\int_a^b f(x)\,dx =$ **area** under the curve $y = f(x)$ between $x = a$ and $x = b$.



3.2 Standard Integrals

Standard Integrals Table (Part 1)

Algebraic and Exponential

$$\int x^n\,dx = \frac{x^{n+1}}{n+1} + C \quad (n \neq -1)$$

$$\int \frac{1}{x}\,dx = \ln|x| + C$$

$$\int e^x\,dx = e^x + C$$

$$\int a^x\,dx = \frac{a^x}{\ln a} + C$$

$$\int c\,dx = cx + C$$

$$\int \frac{1}{x^2}\,dx = -\frac{1}{x} + C$$

GTU MCQ Examples

- ✓ $\int \frac{1}{x^2}\,dx = -\frac{1}{x} + C$ (W'22)
- ✓ $\int 5x^4\,dx = x^5 + C$ (Su'24)
- ✓ $\int x^2\,dx = \frac{x^3}{3} + C$ (W'24)
- ✓ $\int a^x\,dx = \frac{a^x}{\log_e a} + C$ (N-W'25)

Standard Integrals Table (Part 2)

Trigonometric

$$\int \sin x\,dx = -\cos x + C \qquad \int \cos x\,dx = \sin x + C$$

$$\int \sec^2 x\,dx = \tan x + C \qquad \int \csc^2 x\,dx = -\cot x + C$$

$$\int \sec x \tan x\,dx = \sec x + C \qquad \int \csc x \cot x\,dx = -\csc x + C$$

Inverse Trig & Rational

$$\int \frac{1}{\sqrt{1-x^2}}\,dx = \sin^{-1} x + C \qquad \int \frac{1}{1+x^2}\,dx = \tan^{-1} x + C$$

$$\int \frac{1}{x^2+a^2}\,dx = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + C \qquad \int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\left(\frac{x}{a}\right) + C$$

3.3 Rules of Integration

Basic Integration Rules

Rules

1. **Constant:** $\int k\, f(x)\, dx = k \int f(x)\, dx$
2. **Sum/Difference:** $\int [f(x) \pm g(x)]\, dx = \int f\, dx \pm \int g\, dx$
3. **No quotient rule / product rule** for integration directly
   $\Rightarrow$ use substitution or integration by parts

GTU Su'25: $\int \left(\sqrt{x} + \frac{1}{\sqrt{x}}\right)^2 dx$

Expand: $\left(\sqrt{x} + \dfrac{1}{\sqrt{x}}\right)^2 = x + 2 + \dfrac{1}{x}$

$$\int \left(x + 2 + \frac{1}{x}\right) dx = \frac{x^2}{2} + 2x + \ln|x| + C$$

3.4 Integration by Substitution

Substitution Method — Key Idea

Method

Replace an inner expression by t so the integral becomes standard.

Steps:

1. Identify $t = g(x)$ so that $\frac{dt}{dx}$ appears in the integrand.
2. Substitute $dx = \dfrac{dt}{g'(x)}$.
3. Integrate in t, then substitute back.

Key Formula

$$\int f(g(x)) \cdot g'(x)\,dx = \int f(t)\,dt \quad [t = g(x)]$$

Substitution Example 1 (GTU Su'24)

Q: $\int \frac{\sin\sqrt{x}}{\sqrt{x}}\,dx$

Step 1: Let $t = \sqrt{x}$, so $\frac{dt}{dx} = \frac{1}{2\sqrt{x}}$, i.e., $dx = 2\sqrt{x}\,dt$.

Step 2: Substitute: $\int \frac{\sin t}{\sqrt{x}} \cdot 2\sqrt{x}\,dt = 2\int \sin t\,dt$

Step 3: $= -2\cos t + C = \boxed{-2\cos\sqrt{x} + C}$

Substitution Example 2 (GTU W'23)

Q: $\int \frac{(1+x)e^x}{\cos^2(xe^x)}\,dx$

Step 1: Let $t = xe^x$.

$\frac{dt}{dx} = e^x + xe^x = (1+x)e^x \Rightarrow (1+x)e^x\,dx = dt$

Step 2: $\int \frac{dt}{\cos^2 t} = \int \sec^2 t\,dt$

Step 3: $= \tan t + C = \boxed{\tan(xe^x) + C}$

Substitution Example 3 (GTU W'24, Su'25)

Q: $\int \frac{\sin x \cos x}{1 + \sin^2 x}\, dx$

Step 1: Let $t = \sin^2 x$, so $\frac{dt}{dx} = 2 \sin x \cos x$, i.e., $\sin x \cos x\, dx = \frac{dt}{2}$

Step 2: $\int \frac{1}{1+t} \cdot \frac{dt}{2} = \frac{1}{2} \int \frac{dt}{1+t}$

Step 3: $= \frac{1}{2} \ln |1 + t| + C = \boxed{\frac{1}{2} \ln(1 + \sin^2 x) + C}$

Q: $\displaystyle\int \frac{\tan x}{\sec x + \tan x}\,dx$

Rationalise: multiply by $(\sec x - \tan x)$:

$$\frac{\tan x(\sec x - \tan x)}{\sec^2 x - \tan^2 x} = \frac{\tan x(\sec x - \tan x)}{1}$$

$$= \tan x \sec x - \tan^2 x$$

$$= \tan x \sec x - (\sec^2 x - 1) = \tan x \sec x - \sec^2 x + 1$$

$$\int (\sec x \tan x - \sec^2 x + 1)\,dx = \sec x - \tan x + x + C$$

Substitution: Definite (GTU W'24)

Q: $\displaystyle\int_1^e \frac{(\log x)^2}{x}\, dx$

Let $t = \log x \Rightarrow dt = \dfrac{1}{x} dx$

Limits: when $x = 1,\ t = 0$; when $x = e,\ t = 1$

$$\int_0^1 t^2\, dt = \left[\frac{t^3}{3}\right]_0^1 = \boxed{\frac{1}{3}}$$

GTU Su'24: $\int_0^1 \frac{x^2}{1+x^6} dx$

Let $t = x^3$, $dt = 3x^2\, dx$. Limits: $0 \to 0$, $1 \to 1$.

$$\frac{1}{3}\int_0^1 \frac{dt}{1+t^2} = \frac{1}{3}\left[\tan^{-1} t\right]_0^1 = \frac{1}{3} \cdot \frac{\pi}{4} = \boxed{\frac{\pi}{12}}$$

3.5 Integration by Parts (IBP)

IBP — Formula and ILATE Rule

IBP Formula

$$\int u\,v\,dx = u\int v\,dx - \int\left(\frac{du}{dx}\cdot\int v\,dx\right)dx$$

ILATE — Choose u in this order

I	**L**	**A**	**T**	**E**
Inverse trig	Log	Algebraic	Trig	Exponential

Pick $u =$ the function that appears **earlier** in ILATE.

IBP Example 1: $\int x \log x \, dx$ (GTU N-W'25)

Solution

ILATE: $u = \log x$ (L), $\quad v = x$ (A)

$$\int x \log x \, dx = \log x \cdot \frac{x^2}{2} - \int \frac{1}{x} \cdot \frac{x^2}{2} \, dx$$

$$= \frac{x^2 \log x}{2} - \frac{1}{2} \int x \, dx = \frac{x^2 \log x}{2} - \frac{x^2}{4} + C$$

$$= \boxed{\frac{x^2}{4}(2 \log x - 1) + C}$$

IBP Example 2: $\int x^2 \log x \, dx$ (GTU W'23)

Solution

ILATE: $u = \log x, \ v = x^2$

$$\int x^2 \log x \, dx = \log x \cdot \frac{x^3}{3} - \int \frac{1}{x} \cdot \frac{x^3}{3} \, dx$$

$$= \frac{x^3 \log x}{3} - \frac{1}{3} \int x^2 \, dx = \frac{x^3 \log x}{3} - \frac{x^3}{9} + C$$

$$= \boxed{\frac{x^3}{9}(3 \log x - 1) + C}$$

IBP Example 3: $\int x \sin x \, dx$ (GTU W'24)

Solution

ILATE: $u = x$ (A), $v = \sin x$ (T)

$$\int x \sin x \, dx = x(-\cos x) - \int 1 \cdot (-\cos x) \, dx$$

$$= -x \cos x + \int \cos x \, dx = \boxed{-x \cos x + \sin x + C}$$

GTU Su'25: $\int x \cos x \, dx$

$u = x$, $v = \cos x$:

$x \sin x - \int \sin x \, dx = \boxed{x \sin x + \cos x + C}$

IBP Example 4: $\int x^2 e^x \, dx$ (GTU W'22)

IBP Applied Twice

Round 1: $u = x^2$, $v = e^x$:

$$= x^2 e^x - \int 2x\, e^x \, dx$$

Round 2: For $\int 2xe^x \, dx$, $u = 2x$, $v = e^x$:

$$= 2xe^x - \int 2e^x \, dx = 2xe^x - 2e^x$$

Combine:

$$= x^2 e^x - (2xe^x - 2e^x) + C = \boxed{e^x(x^2 - 2x + 2) + C}$$

3.6 Integration by Partial Fractions

Partial Fractions — Method

When to Use

Integrand is a **rational function** $\dfrac{P(x)}{Q(x)}$ where degree of $P <$ degree of Q.

Standard Forms

$\dfrac{A}{(x-a)} + \dfrac{B}{(x-b)}$	for distinct linear factors
$\dfrac{A}{(x-a)} + \dfrac{B}{(x-a)^2}$	for repeated linear factor
$\dfrac{A}{(x-a)} + \dfrac{Bx+C}{x^2+bx+c}$	for quadratic factor

Partial Fractions Example 1 (GTU W'22)

Q: $\int \frac{x}{(x+1)(x+2)}\,dx$

Let $\frac{x}{(x+1)(x+2)} = \frac{A}{x+1} + \frac{B}{x+2}$

$x = A(x+2) + B(x+1)$

Put $x = -1$: $-1 = A(1) \Rightarrow A = -1$

Put $x = -2$: $-2 = B(-1) \Rightarrow B = 2$

$$\int \left(\frac{-1}{x+1} + \frac{2}{x+2}\right) dx = \boxed{-\ln|x+1| + 2\ln|x+2| + C}$$

Partial Fractions Example 2 (GTU W'23)

Q: $\int \frac{2x+1}{(x+1)(x-3)}\,dx$

$$\frac{2x+1}{(x+1)(x-3)} = \frac{A}{x+1} + \frac{B}{x-3}$$

$2x+1 = A(x-3) + B(x+1)$

Put $x = -1$: $-1 = A(-4) \Rightarrow A = \frac{1}{4}$

Put $x = 3$: $7 = B(4) \Rightarrow B = \frac{7}{4}$

$$\int \left(\frac{1/4}{x+1} + \frac{7/4}{x-3} \right) dx = \boxed{\frac{1}{4}\ln|x+1| + \frac{7}{4}\ln|x-3| + C}$$

3.7 Trigonometric Integrals

Product-to-Sum Formulae

Product-to-Sum Identities

$$\sin A \sin B = \tfrac{1}{2}[\cos(A-B) - \cos(A+B)]$$
$$\cos A \cos B = \tfrac{1}{2}[\cos(A-B) + \cos(A+B)]$$
$$\sin A \cos B = \tfrac{1}{2}[\sin(A+B) + \sin(A-B)]$$

GTU W'23, W'24: $\int \sin 5x \sin 6x \, dx$

$$\sin 5x \sin 6x = \tfrac{1}{2}[\cos(5x-6x) - \cos(5x+6x)] = \tfrac{1}{2}[\cos(-x) - \cos 11x]$$
$$= \tfrac{1}{2}\int(\cos x - \cos 11x)\,dx = \boxed{\frac{\sin x}{2} - \frac{\sin 11x}{22} + C}$$

Split Trig Fraction (GTU W'22)

Q: $\int \frac{\sin^3 x + \cos^3 x}{\sin^2 x \cos^2 x}\,dx$

Split: $\frac{\sin^3 x}{\sin^2 x \cos^2 x} + \frac{\cos^3 x}{\sin^2 x \cos^2 x} = \frac{\sin x}{\cos^2 x} + \frac{\cos x}{\sin^2 x} = \sec x \tan x + \csc x \cot x$

Wrong sign trap! $\int \csc x \cot x\,dx = -\csc x$

$$\int (\sec x \tan x + \csc x \cot x)\,dx = \sec x - \csc x + C$$

Warning

$\int \csc x \cot x\,dx = -\csc x + C$ (note the negative sign!)

Trig Integral: GTU Su'25

Q: $\int (\sqrt{x} + \frac{1}{\sqrt{x}})^2 dx$ (revisited with trig sense)

Expand: $x + 2 + \dfrac{1}{x}$

$$\int \left(x + 2 + \frac{1}{x} \right) dx = \frac{x^2}{2} + 2x + \ln|x| + C$$

GTU N-W'25: $\int \left(\frac{1}{x} + e^x \right) dx$

Direct standard forms:

$$\int \frac{1}{x}\, dx + \int e^x\, dx = \boxed{\ln|x| + e^x + C}$$

3.8 Definite Integrals

Definite Integral — Definition

Fundamental Theorem of Calculus

$$\int_a^b f(x)\,dx = \Big[F(x)\Big]_a^b = F(b) - F(a)$$

where $F'(x) = f(x)$.

GTU Examples

$$\int_0^1 e^x dx = [e^x]_0^1 = e - 1 \quad \text{(W'22, Su}$$

$$\int_1^2 x^2 dx = \left[\frac{x^3}{3}\right]_1^2 = \frac{8-1}{3} = \frac{7}{3} \quad \text{(W'23)}$$

$$\int_1^3 \frac{1}{x} dx = [\ln x]_1^3 = \ln 3 - 0 = \ln 3 \quad \text{(W'24)}$$

$$\int_2^4 (x^2 + 3x - 1) dx = \left[\frac{x^3}{3} + \frac{3x^2}{2} - x\right]_2^4 = \frac{64-8}{3} + \frac{3(16-4)}{2} - (4-2) \quad \text{(Su'25)}$$

Q: $\int_2^4 (x^2 + 3x - 1)\, dx$

$$\left[\frac{x^3}{3} + \frac{3x^2}{2} - x\right]_2^4$$

At $x = 4$: $\dfrac{64}{3} + \dfrac{48}{2} - 4 = \dfrac{64}{3} + 24 - 4 = \dfrac{64}{3} + 20$

At $x = 2$: $\dfrac{8}{3} + \dfrac{12}{2} - 2 = \dfrac{8}{3} + 6 - 2 = \dfrac{8}{3} + 4$

Difference: $\dfrac{64 - 8}{3} + 20 - 4 = \dfrac{56}{3} + 16 = \dfrac{56 + 48}{3} = \boxed{\dfrac{104}{3}}$

GTU MCQ on Definite Integrals

MCQ Practice

Q	**Answer**	**Paper**
$\int_{-1}^{1}(3x^2-2x+1)dx$	4	Su'24
$\int_{-\pi/2}^{\pi/2}\sin x\,dx$	0 (odd fn)	W'24
$\int_{0}^{1}e^x\,dx$	$e-1$	W'22
$\int_{1}^{2}x^2\,dx$	7/3	W'23

Odd Function Property

If $f(x)$ is odd i.e. $f(-x)=-f(x)$, then $\int_{-a}^{a}f(x)\,dx=0$

$\sin(-x)=-\sin x \Rightarrow \int_{-\pi/2}^{\pi/2}\sin x\,dx=0$

3.9 Properties of Definite Integrals

Key Properties

Important Properties

P1 $\int_a^b f(x)\,dx = \int_a^b f(t)\,dt$ (dummy var)

P2 $\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$

P3 $\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$

P4 $\int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx$

P5 $\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx$

Property P5 — GTU Star Question

Q: $\displaystyle\int_0^{\pi/2} \frac{\sin x}{\sin x + \cos x}\,dx = \frac{\pi}{4}$ (W'22, W'23, Su'24, Su'25)

Let $\displaystyle I = \int_0^{\pi/2} \frac{\sin x}{\sin x + \cos x}\,dx$

Apply P5 ($a = \pi/2$, replace $x \to \pi/2 - x$):

$$I = \int_0^{\pi/2} \frac{\cos x}{\cos x + \sin x}\,dx$$

Add: $\displaystyle 2I = \int_0^{\pi/2} \frac{\sin x + \cos x}{\sin x + \cos x}\,dx = \int_0^{\pi/2} 1\,dx = \frac{\pi}{2}$

$$\therefore I = \boxed{\frac{\pi}{4}}$$

Property P5 Variants (GTU W'23, Su'25)

Q: $\displaystyle\int_0^{\pi/2} \frac{\sqrt{\cot x}}{\sqrt{\cot x}+\sqrt{\tan x}}\,dx$

Let $I = \displaystyle\int_0^{\pi/2} \frac{\sqrt{\cot x}}{\sqrt{\cot x}+\sqrt{\tan x}}\,dx$

Apply P5 ($x \to \pi/2 - x$): $\cot x \leftrightarrow \tan x$

$$I = \int_0^{\pi/2} \frac{\sqrt{\tan x}}{\sqrt{\tan x}+\sqrt{\cot x}}\,dx$$

Add: $2I = \displaystyle\int_0^{\pi/2} 1\,dx = \frac{\pi}{2} \Rightarrow I = \boxed{\frac{\pi}{4}}$

GTU Su'24: $\int_0^{\pi/2} \frac{\sqrt{\sin x}}{\sqrt{\cos x}+\sqrt{\sin x}}dx$

Same method: apply P5, sum $= \pi/2$, answer $= \boxed{\pi/4}$

3.10 Area Under a Curve

Area Formula

Area Between Curve and X-axis

$$\text{Area} = \int_a^b |y|\,dx = \int_a^b |f(x)|\,dx$$

(Take absolute value if curve dips below X-axis)

Area of Circle $x^2 + y^2 = a^2$ (GTU W'24)

$y = \sqrt{a^2 - x^2}$ (upper half)

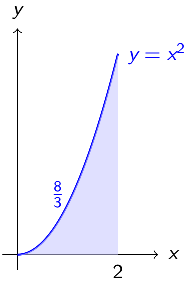
$$\text{Area} = 4\int_0^a \sqrt{a^2 - x^2}\,dx = 4 \cdot \frac{\pi a^2}{4} = \boxed{\pi a^2}$$

(Uses standard result $\int_0^a \sqrt{a^2 - x^2}\,dx = \frac{\pi a^2}{4}$)

Area Example 1 (GTU Su'24)

Q: Area bounded by $y = x^2$, X-axis, $x = 0$ to $x = 2$

$$\text{Area} = \int_0^2 x^2\,dx = \left[\frac{x^3}{3}\right]_0^2 = \frac{8}{3} - 0 = \boxed{\frac{8}{3}} \text{ sq. units}$$



Area Example 2 (GTU W'22)

Q: Area bounded by $y = 2x^2$, X-axis, $x = 1$ to $x = 3$

$$\text{Area} = \int_1^3 2x^2\,dx = 2\left[\frac{x^3}{3}\right]_1^3 = \frac{2}{3}(27-1) = \frac{2\times 26}{3} = \boxed{\frac{52}{3}} \text{ sq. units}$$

3.11 GTU PYQ Practice Session

MCQ Bank: 1-Mark Questions

All MCQs at a Glance

Integral	Answer	Paper
$\int x^{-2} dx$	$-1/x + C$	W'22
$\int (\log a) dx$	$x \log a + C$	W'22
$\int_0^1 e^x \, dx$	$e - 1$	W'22, Su'24
$\int \sin x \, dx$	$-\cos x + C$	W'23
$\int \frac{1}{x^2+4} dx$	$\frac{1}{2} \tan^{-1}(x/2) + C$	W'23
$\int_1^2 x^2 \, dx$	$7/3$	W'23
$\int 5x^4 \, dx$	$x^5 + C$	Su'24
$\int_{-1}^1 (3x^2 - 2x + 1) dx$	4	Su'24

MCQ Bank: More 1-Mark Questions

Continued

Integral	**Answer**	**Paper**
$\int x^2\,dx$	$x^3/3 + C$	W'24
$\int_{-\pi/2}^{\pi/2} \sin x\,dx$	0 (odd function)	W'24
$\int_1^3 \frac{1}{x}dx$	$\ln 3$	W'24
$\int \sec^2 x\,dx$	$\tan x + C$	Su'25
$\int a^x\,dx$	$a^x/\ln a + C$	N-W'25
$\int \frac{1}{x}dx$	$\ln\lvert x\rvert + C$	N-W'25

3-Mark PYQ: W'22 — Trig split

$$\int \frac{\sin^3 x + \cos^3 x}{\sin^2 x \cos^2 x}\, dx$$

Write as: $\displaystyle \int \sec x \tan x \, dx + \int \csc x \cot x \, dx$

$= \sec x + (-\csc x) + C = \boxed{\sec x - \csc x + C}$

3-Mark PYQ: Multiple Methods

W'23, W'24: $\int \sin 5x \sin 6x \, dx$

$= \frac{1}{2}\int[\cos(-x) - \cos 11x]dx = \frac{1}{2}\int[\cos x - \cos 11x]dx$

$= \boxed{\dfrac{\sin x}{2} - \dfrac{\sin 11x}{22} + C}$

Su'24: $\int \frac{\tan x}{\sec x + \tan x} dx$

Multiply by $(\sec x - \tan x)$: $= \int(\sec x \tan x - \sec^2 x + 1)\, dx$

$= \boxed{\sec x - \tan x + x + C}$

W'24, Su'25: $\int \frac{\sin x \cos x}{1+\sin^2 x} dx$

Let $t = \sin^2 x$: $= \dfrac{1}{2}\int \dfrac{dt}{1+t} = \boxed{\dfrac{1}{2}\ln(1+\sin^2 x) + C}$

4-Mark PYQ: IBP Practice Set

W'22: $\int x^2 e^x \, dx = e^x(x^2 - 2x + 2) + C$

IBP twice: $u = x^2$, then $u = 2x$
$= x^2 e^x - 2xe^x + 2e^x + C = e^x(x^2 - 2x + 2) + C$

W'24: $\int x \sin x \, dx = -x \cos x + \sin x + C$

$u = x,\ v = \sin x$: $x(-\cos x) - \int(-\cos x)dx = -x\cos x + \sin x + C$

Su'25: $\int x \cos x \, dx = x \sin x + \cos x + C$

$u = x,\ v = \cos x$: $x \sin x - \int \sin x \, dx = x \sin x + \cos x + C$

4-Mark PYQ: Partial Fractions Set

W'22: $\int \frac{x}{(x+1)(x+2)}\,dx$

PF: $\dfrac{-1}{x+1} + \dfrac{2}{x+2}$
$\Rightarrow -\ln|x+1| + 2\ln|x+2| + C$

W'23: $\int \frac{2x+1}{(x+1)(x-3)}\,dx$

$A = \frac{1}{4}$, $B = \frac{7}{4}$
$\Rightarrow \dfrac{1}{4}\ln|x+1| + \dfrac{7}{4}\ln|x-3| + C$

4-Mark PYQ: Property + Area Set

W'22, W'23, Su'24, Su'25: $\int_0^{\pi/2} \frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}} dx$

Apply P5: add both forms $\rightarrow 2I = \pi/2 \Rightarrow I = \pi/4$

Su'24: Area $y = x^2$, $x = 0$ to 2

$\int_0^2 x^2\, dx = 8/3$ sq. units

W'22: Area $y = 2x^2$, $x = 1$ to 3

$2\int_1^3 x^2\, dx = \frac{2}{3}(27 - 1) = 52/3$ sq. units

W'24: Area of circle $x^2 + y^2 = a^2$

$4\int_0^a \sqrt{a^2 - x^2}\, dx = \pi a^2$ sq. units

Unit 3 Summary

Unit 3 — Key Takeaways

Methods Decision Tree

Direct	use standard integral table
Substitution	inner function + its derivative present
IBP	product of two different-type functions (ILATE)
Partial Fractions	rational function, degree(num) < degree(den)
Product-to-Sum	$\sin mx \cdot \sin nx$, $\cos mx \cdot \cos nx$, etc.

Definite Integral Tricks

✓ Odd function on $[-a, a] \Rightarrow 0$

✓ $\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx$ (use when sin/cos swap helps)

✓ Add I and its P5-version: $2I = \int_0^a 1\,dx = a$

Common Errors to Avoid

$\int \csc x \cot x\,dx = -\csc x + C$ (sign!)

$\int a^x\,dx = a^x/\ln a + C$ (not a^x/a)

Don't forget $+C$ in indefinite integrals

Unit 3 — GTU Weightage Summary

Marks Distribution

Topic	**Marks**	**Frequency**
MCQ (standard integrals)	1 each	2 per paper
Substitution	3	1–2 per paper
IBP	3–4	1 per paper
Partial Fractions	4	1 per paper
Definite Integral properties	3–4	1 per paper
Area under curve	4	1 per paper

Total: $\approx$ 14–16 marks out of 80

Exam-ready with all PYQ types covered!

IBP: $\int \log x\, dx$ and $\int e^x(\sin x + \cos x)\, dx$

GTU W'23, Su'24 — $\int \log x\, dx$

Let $u = \log x,\ dv = dx \Rightarrow du = \frac{1}{x}dx,\ v = x$
$\int \log x\, dx = x \log x - \int x \cdot \frac{1}{x}\, dx = x \log x - x + C$

GTU W'24 — $\int e^x(\sin x + \cos x)\, dx$

Use $\int e^x[f(x) + f'(x)]\, dx = e^x f(x) + C$
Here $f(x) = \sin x,\ f'(x) = \cos x$ ✓
$\therefore \int e^x(\sin x + \cos x)\, dx = e^x \sin x + C$

Substitution: $\int \frac{\cos x}{\sin^2 x}\,dx$ and $\int \frac{x}{1+x^4}\,dx$

GTU W'22 — $\int \frac{\cos x}{\sin^2 x}\,dx$

Let $t = \sin x,\ dt = \cos x\,dx$
$\int \frac{1}{t^2}\,dt = -\frac{1}{t} + C = -\csc x + C$

GTU Su'25 — $\int \frac{x}{1+x^4}\,dx$

Let $t = x^2,\ dt = 2x\,dx \Rightarrow x\,dx = \frac{dt}{2}$
$\frac{1}{2}\int \frac{dt}{1+t^2} = \frac{1}{2}\tan^{-1}(t) + C = \frac{1}{2}\tan^{-1}(x^2) + C$

Definite: $\int_0^{\pi} x \sin x \, dx$ and $\int_0^1 \frac{x}{(1+x^2)^2} \, dx$

GTU W'23 — IBP definite

$\int_0^{\pi} x \sin x \, dx$. Let $u = x$, $dv = \sin x \, dx$
$= [-x \cos x]_0^{\pi} + \int_0^{\pi} \cos x \, dx = \pi + [\sin x]_0^{\pi} = \pi$

GTU Su'24 — Sub. in definite

$\int_0^1 \frac{x}{(1+x^2)^2} \, dx$. Let $t = 1 + x^2$, $dt = 2x \, dx$
Limits: $x = 0 \to t = 1$; $x = 1 \to t = 2$
$\frac{1}{2} \int_1^2 t^{-2} \, dt = \frac{1}{2} \left[-\frac{1}{t} \right]_1^2 = \frac{1}{2} \left(-\frac{1}{2} + 1 \right) = \frac{1}{4}$

Standard Integral Identities — Must Know

Special Forms

$\int \frac{dx}{x^2+a^2}$	$\frac{1}{a}\tan^{-1}\frac{x}{a} + C$
$\int \frac{dx}{\sqrt{a^2-x^2}}$	$\sin^{-1}\frac{x}{a} + C$
$\int \frac{dx}{x^2-a^2}$	$\frac{1}{2a}\ln\left\lvert\frac{x-a}{x+a}\right\rvert + C$
$\int \sqrt{a^2-x^2}\,dx$	$\frac{x}{2}\sqrt{a^2-x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a} + C$
$\int e^{ax}\sin bx\,dx$	$\frac{e^{ax}(a\sin bx - b\cos bx)}{a^2+b^2} + C$
$\int e^{ax}\cos bx\,dx$	$\frac{e^{ax}(a\cos bx + b\sin bx)}{a^2+b^2} + C$

Reduction: $\int \frac{2x+3}{x^2+3x-4}\,dx$ (PF type, GTU W'24)

Spot the numerator = derivative of denominator

$\frac{d}{dx}(x^2+3x-4) = 2x+3$ — numerator is exactly the derivative!

$\therefore \int \frac{2x+3}{x^2+3x-4}\,dx = \ln|x^2+3x-4| + C$

Generalise: $\int \frac{f'(x)}{f(x)}\,dx = \ln|f(x)| + C$

This is the fastest integration technique for rational functions where numerator = derivative of denominator.

GTU often tests this as a 3-mark standalone or MCQ.

Mixed PYQ Set — 3 and 4 mark (All Years)

3-Mark Practice

1. $\int \frac{\sin x}{1+\cos^2 x}\,dx$ [W'22]
2. $\int x^2 \sin x\,dx$ [Su'23]
3. $\int \frac{\log x}{x}\,dx$ [W'23]
4. $\int_0^{\pi/2} \sin^3 x \cos^2 x\,dx$ [N-W'25]

4-Mark Practice

1. $\int \frac{x^2+1}{(x+2)(x-3)}\,dx$ [partial fractions]
2. $\int_0^2 \frac{x^2}{\sqrt{4-x^2}}\,dx$ [W'24 area-type]
3. Area bounded by $y = x^3$, x-axis, $x = 0$ to $x = 2$ [Su'25]
4. $\int_0^{\pi} \frac{x \sin x}{1+\cos^2 x}\,dx = \pi^2/4$ [property P5]

Unit 3 Complete!

Next: Unit 4 — Differential Equations