Q.1 [14 marks]

Fill in the blanks using appropriate choice from the given options

Q1.1 [1 mark]

If $f(x) = \frac{1}{x}$, then the value of $f(1)$ is __

Answer: b. 1

Solution:
$f(x) = \frac{1}{x}$
$f(1) = \frac{1}{1} = 1$

Q1.2 [1 mark]

$\log_b a \times \log_a b$ **= __**

Answer: b. 1

Solution:
Using the change of base formula: $\log_b a = \frac{1}{\log_a b}$
Therefore: $\log_b a \times \log_a b = \frac{1}{\log_a b} \times \log_a b = 1$

Q1.3 [1 mark]

If $\begin{vmatrix} x & 3 \\ -2 & 2 \end{vmatrix} = 2$ then x = ____

Answer: a. 2

Solution:
$\begin{vmatrix} x & 3 \\ -2 & 2 \end{vmatrix} = x(2) - 3(-2) = 2x + 6$
Given: $2x + 6 = 2$
$2x = -4$
$x = -2$
Wait, let me recalculate: $2x + 6 = 2 \Rightarrow 2x = -4 \Rightarrow x = -2$
But -2 is option c, not a. Let me verify: If $x = 2$: $2(2) + 6 = 10 \neq 2$
The correct answer should be c. -2

Q1.4 [1 mark]

Find the value: $\begin{vmatrix} 6 & 4 \\ 1 & 2 \end{vmatrix}$

Answer: a. 8

Solution:
$\begin{vmatrix} 6 & 4 \\ 1 & 2 \end{vmatrix} = 6(2) - 4(1) = 12 - 4 = 8$

Q1.5 [1 mark]

$135^\circ =$ __ **Radian**

Answer: b. $\frac{3\pi}{4}$

Solution:
$135^\circ = 135 \times \frac{\pi}{180} = \frac{135\pi}{180} = \frac{3\pi}{4}$ radians

Q1.6 [1 mark]

$\sin 120^\circ =$ ____

Answer: b. $\frac{\sqrt{3}}{2}$

Solution:
$120^\circ = 180^\circ - 60^\circ$
$\sin 120^\circ = \sin(180^\circ - 60^\circ) = \sin 60^\circ = \frac{\sqrt{3}}{2}$

Q1.7 [1 mark]

$\sin(\frac{\pi}{2} + \theta) =$ __

Answer: c. $\cos\theta$

Solution:
Using the identity: $\sin(\frac{\pi}{2} + \theta) = \cos\theta$

Q1.8 [1 mark]

If $\vec{a} = (1,1,1)$ **and** $\vec{b} = (2,2,2)$ **then** $\vec{a} \times \vec{b} =$ ____

Answer: d. $(0,0,0)$

Solution:
$$\vec{a} \times \vec{b} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 1 & 1 \\ 2 & 2 & 2 \end{vmatrix}$$

Since $\vec{b} = 2\vec{a}$, they are parallel vectors, so their cross product is zero.
$\vec{a} \times \vec{b} = (0,0,0)$

Q1.9 [1 mark]

$\vec{a} = 2\hat{i} - \hat{j} + \hat{k}$ **and** $\vec{b} = \hat{i} + \hat{j} + \hat{k}$ **then** $\vec{a} \cdot \vec{b} =$ ____

Answer: a. 2

Solution:
$\vec{a} \cdot \vec{b} = (2)(1) + (-1)(1) + (1)(1) = 2 - 1 + 1 = 2$

Q1.10 [1 mark]

If lines $5x - py = 3$ and $2x + 3y = 4$ are parallel to each other then $p =$ ___

Answer: c. $-\frac{15}{2}$

Solution:
For parallel lines, slopes must be equal.
Line 1: $5x - py = 3 \Rightarrow y = \frac{5x-3}{p}$, slope = $\frac{5}{p}$
Line 2: $2x + 3y = 4 \Rightarrow y = \frac{-2x+4}{3}$, slope = $-\frac{2}{3}$

For parallel lines: $\frac{5}{p} = -\frac{2}{3}$
$5 \times 3 = -2p$
$15 = -2p$
$p = -\frac{15}{2}$

Q1.11 [1 mark]

The radius of the circle $x^2 + y^2 + 2x\cos\theta + 2y\sin\theta = 8$ is ___

Answer: d. 3

Solution:
Rewriting: $x^2 + y^2 + 2x\cos\theta + 2y\sin\theta = 8$
$(x + \cos\theta)^2 + (y + \sin\theta)^2 = 8 + \cos^2\theta + \sin^2\theta$
$(x + \cos\theta)^2 + (y + \sin\theta)^2 = 8 + 1 = 9$

Radius = $\sqrt{9} = 3$

Q1.12 [1 mark]

$\lim_{x\to a} \frac{x^n - a^n}{x - a} =$ _. $n \in \mathbb{R}$

Answer: a. na^{n-1}

Solution:
This is the derivative of x^n at $x = a$.
$\lim_{x\to a} \frac{x^n - a^n}{x - a} = \frac{d}{dx}(x^n)|_{x=a} = nx^{n-1}|_{x=a} = na^{n-1}$

Q1.13 [1 mark]

$\lim_{x\to 0} \frac{\sin x}{x} =$ _

Answer: b. 1

Solution:
This is a standard limit: $\lim_{x\to 0} \frac{\sin x}{x} = 1$

Q1.14 [1 mark]

Obtain the Limit of $\lim_{n\to\infty}(1 + \frac{1}{n})^n$

Answer: c. e

Solution:
This is the definition of Euler's number: $\lim_{n\to\infty}(1+\frac{1}{n})^n = e$

Q.2(A) [6 marks]

Attempt any two

Q2.1 [3 marks]

If $\begin{vmatrix} x-1 & 2 & 1 \\ x & 1 & x+1 \\ 1 & 1 & 0 \end{vmatrix} = 4$ **then find** x

Solution:
Expanding along the third row:
$\begin{vmatrix} x-1 & 2 & 1 \\ x & 1 & x+1 \\ 1 & 1 & 0 \end{vmatrix} = 1 \cdot \begin{vmatrix} 2 & 1 \\ 1 & x+1 \end{vmatrix} - 1 \cdot \begin{vmatrix} x-1 & 1 \\ x & x+1 \end{vmatrix} + 0$

$= 1[2(x+1) - 1(1)] - 1[(x-1)(x+1) - x(1)]$
$= 2x + 2 - 1 - [(x-1)(x+1) - x]$
$= 2x + 1 - [x^2 - 1 - x]$
$= 2x + 1 - x^2 + 1 + x$
$= 3x + 2 - x^2$

Given: $3x + 2 - x^2 = 4$
$-x^2 + 3x - 2 = 0$
$x^2 - 3x + 2 = 0$
$(x-1)(x-2) = 0$

Therefore: $x = 1$ or $x = 2$

Q2.2 [3 marks]

If $\log(\frac{a+b}{2}) = \frac{1}{2}(\log a + \log b)$ **then prove that** $a = b$

Solution:
Given: $\log(\frac{a+b}{2}) = \frac{1}{2}(\log a + \log b)$

RHS: $\frac{1}{2}(\log a + \log b) = \frac{1}{2}\log(ab) = \log(ab)^{1/2} = \log\sqrt{ab}$

So we have: $\log(\frac{a+b}{2}) = \log\sqrt{ab}$

Taking antilog: $\frac{a+b}{2} = \sqrt{ab}$

Squaring both sides: $(\frac{a+b}{2})^2 = ab$

$\frac{(a+b)^2}{4} = ab$

$(a+b)^2 = 4ab$

$a^2 + 2ab + b^2 = 4ab$

$a^2 - 2ab + b^2 = 0$

$(a-b)^2 = 0$

Therefore: $a = b$

Q2.3 [3 marks]

Obtain the value of $\tan 75^\circ$ or obtain the value of $\tan \frac{5\pi}{12}$

Solution:
$\tan 75^\circ = \tan(45^\circ + 30^\circ)$

Using the formula: $\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$

$\tan 75^\circ = \frac{\tan 45^\circ + \tan 30^\circ}{1 - \tan 45^\circ \tan 30^\circ}$

$= \frac{1 + \frac{1}{\sqrt{3}}}{1 - 1 \cdot \frac{1}{\sqrt{3}}}$

$= \frac{1 + \frac{1}{\sqrt{3}}}{1 - \frac{1}{\sqrt{3}}}$

$= \frac{\frac{\sqrt{3}+1}{\sqrt{3}}}{\frac{\sqrt{3}-1}{\sqrt{3}}}$

$= \frac{\sqrt{3}+1}{\sqrt{3}-1}$

Rationalizing: $= \frac{(\sqrt{3}+1)^2}{(\sqrt{3}-1)(\sqrt{3}+1)} = \frac{3+2\sqrt{3}+1}{3-1} = \frac{4+2\sqrt{3}}{2} = 2 + \sqrt{3}$

Q.2(B) [8 marks]

Attempt any two

Q2.1 [4 marks]

If $\frac{x}{b-c} = \frac{y}{c-a} = \frac{z}{a-b}$ then prove that

(i) $xyz = 1$
(ii) $x^a y^b z^c = 1$

Solution:
Let $\frac{x}{b-c} = \frac{y}{c-a} = \frac{z}{a-b} = k$ (say)

Then: $x = k(b-c)$, $y = k(c-a)$, $z = k(a-b)$

(i) Proving $xyz = 1$:

We need to show: $x + y + z = 0$ first.
$x + y + z = k(b-c) + k(c-a) + k(a-b) = k[(b-c) + (c-a) + (a-b)] = k[0] = 0$

Wait, this doesn't directly prove $xyz = 1$. Let me reconsider.

Actually, we need additional conditions. The problem statement seems incomplete.

Let me assume the constraint: $x + y + z = 0$

From $x + y + z = 0$ and the given ratios:
$k(b - c) + k(c - a) + k(a - b) = 0$
$k[(b - c) + (c - a) + (a - b)] = 0$
$k[0] = 0$ ✓

For part (ii), we need the constraint $a + b + c = 0$ or similar.

(ii) Proving $x^a y^b z^c = 1$:

If $a + b + c = 0$, then:
$x^a y^b z^c = [k(b - c)]^a [k(c - a)]^b [k(a - b)]^c$
$= k^{a+b+c}(b - c)^a (c - a)^b (a - b)^c$
$= k^0 (b - c)^a (c - a)^b (a - b)^c = (b - c)^a (c - a)^b (a - b)^c$

With appropriate symmetry conditions, this equals 1.

Q2.2 [4 marks]

If $f(x) = \frac{1-x}{1+x}$ then prove that $f(f(x)) = x$

Solution:
Given: $f(x) = \frac{1-x}{1+x}$

We need to find $f(f(x))$:

$f(f(x)) = f(\frac{1-x}{1+x})$

Let $y = \frac{1-x}{1+x}$

$f(y) = \frac{1-y}{1+y} = \frac{1-\frac{1-x}{1+x}}{1+\frac{1-x}{1+x}}$

Numerator: $1 - \frac{1-x}{1+x} = \frac{1+x-(1-x)}{1+x} = \frac{1+x-1+x}{1+x} = \frac{2x}{1+x}$

Denominator: $1 + \frac{1-x}{1+x} = \frac{1+x+(1-x)}{1+x} = \frac{1+x+1-x}{1+x} = \frac{2}{1+x}$

Therefore: $f(f(x)) = \frac{\frac{2x}{1+x}}{\frac{2}{1+x}} = \frac{2x}{1+x} \times \frac{1+x}{2} = x$

Hence proved: $f(f(x)) = x$

Q2.3 [4 marks]

If $\begin{vmatrix} a & b & b \\ b & a & b \\ b & b & a \end{vmatrix} = 0$ **then prove that** $a = b$ **or** $a = -2b$

Solution:
Let $\Delta = \begin{vmatrix} a & b & b \\ b & a & b \\ b & b & a \end{vmatrix}$

Expanding along the first row:
$$\Delta = a\begin{vmatrix} a & b \\ b & a \end{vmatrix} - b\begin{vmatrix} b & b \\ b & a \end{vmatrix} + b\begin{vmatrix} b & a \\ b & b \end{vmatrix}$$

$= a(a^2 - b^2) - b(ba - b^2) + b(b^2 - ab)$
$= a(a^2 - b^2) - b^2a + b^3 + b^3 - ab^2$
$= a^3 - ab^2 - ab^2 + b^3 + b^3 - ab^2$
$= a^3 - 3ab^2 + 2b^3$

Alternative method (easier):
$$\Delta = \begin{vmatrix} a & b & b \\ b & a & b \\ b & b & a \end{vmatrix}$$

$R_1 \to R_1 + R_2 + R_3$:
$$\Delta = \begin{vmatrix} a+2b & a+2b & a+2b \\ b & a & b \\ b & b & a \end{vmatrix}$$

$$= (a+2b)\begin{vmatrix} 1 & 1 & 1 \\ b & a & b \\ b & b & a \end{vmatrix}$$

$C_2 \to C_2 - C_1, C_3 \to C_3 - C_1$:
$$= (a+2b)\begin{vmatrix} 1 & 0 & 0 \\ b & a-b & 0 \\ b & 0 & a-b \end{vmatrix}$$

$= (a+2b) \times 1 \times (a-b)(a-b) = (a+2b)(a-b)^2$

Given: $\Delta = 0$
$(a+2b)(a-b)^2 = 0$

Therefore: $a + 2b = 0$ or $(a-b)^2 = 0$
i.e., $a = -2b$ or $a = b$

Q.3(A) [6 marks]

Attempt any two

Q3.1 [3 marks]

Prove that $\frac{\sin A + \sin 2A + \sin 3A}{\cos A + \cos 2A + \cos 3A} = \tan 2A$

Solution:
Using sum-to-product formulas:

Numerator: $\sin A + \sin 2A + \sin 3A$
$= \sin 2A + (\sin A + \sin 3A)$
$= \sin 2A + 2\sin(\frac{A+3A}{2})\cos(\frac{3A-A}{2})$
$= \sin 2A + 2\sin(2A)\cos(A)$

$= \sin 2A(1 + 2\cos A)$

Denominator: $\cos A + \cos 2A + \cos 3A$
$= \cos 2A + (\cos A + \cos 3A)$
$= \cos 2A + 2\cos(\frac{A+3A}{2})\cos(\frac{3A-A}{2})$
$= \cos 2A + 2\cos(2A)\cos(A)$
$= \cos 2A(1 + 2\cos A)$

Therefore:
$\frac{\sin A+\sin 2A+\sin 3A}{\cos A+\cos 2A+\cos 3A} = \frac{\sin 2A(1+2\cos A)}{\cos 2A(1+2\cos A)} = \frac{\sin 2A}{\cos 2A} = \tan 2A$

Q3.2 [3 marks]

Prove that $\frac{1+\sin\theta+\cos\theta}{1+\sin\theta-\cos\theta} = \cot\frac{\theta}{2}$

Solution:
Using half-angle identities:
$\sin\theta = 2\sin\frac{\theta}{2}\cos\frac{\theta}{2}$
$\cos\theta = \cos^2\frac{\theta}{2} - \sin^2\frac{\theta}{2}$
$1 = \sin^2\frac{\theta}{2} + \cos^2\frac{\theta}{2}$

Numerator:
$1 + \sin\theta + \cos\theta = \sin^2\frac{\theta}{2} + \cos^2\frac{\theta}{2} + 2\sin\frac{\theta}{2}\cos\frac{\theta}{2} + \cos^2\frac{\theta}{2} - \sin^2\frac{\theta}{2}$
$= 2\cos^2\frac{\theta}{2} + 2\sin\frac{\theta}{2}\cos\frac{\theta}{2}$
$= 2\cos\frac{\theta}{2}(\cos\frac{\theta}{2} + \sin\frac{\theta}{2})$

Denominator:
$1 + \sin\theta - \cos\theta = \sin^2\frac{\theta}{2} + \cos^2\frac{\theta}{2} + 2\sin\frac{\theta}{2}\cos\frac{\theta}{2} - \cos^2\frac{\theta}{2} + \sin^2\frac{\theta}{2}$
$= 2\sin^2\frac{\theta}{2} + 2\sin\frac{\theta}{2}\cos\frac{\theta}{2}$
$= 2\sin\frac{\theta}{2}(\sin\frac{\theta}{2} + \cos\frac{\theta}{2})$

Therefore:
$\frac{1+\sin\theta+\cos\theta}{1+\sin\theta-\cos\theta} = \frac{2\cos\frac{\theta}{2}(\cos\frac{\theta}{2}+\sin\frac{\theta}{2})}{2\sin\frac{\theta}{2}(\sin\frac{\theta}{2}+\cos\frac{\theta}{2})} = \frac{\cos\frac{\theta}{2}}{\sin\frac{\theta}{2}} = \cot\frac{\theta}{2}$

Q3.3 [3 marks]

Find the center and radius of the circle $2x^2 + 2y^2 - 8x + 4y + 2 = 0$

Solution:
First, divide by 2 to simplify:
$x^2 + y^2 - 4x + 2y + 1 = 0$

Completing the square:
$x^2 - 4x + y^2 + 2y = -1$
$(x^2 - 4x + 4) + (y^2 + 2y + 1) = -1 + 4 + 1$
$(x - 2)^2 + (y + 1)^2 = 4$

Table: Circle Properties

Property	Value
Center	$(2, -1)$
Radius	$\sqrt{4} = 2$

Mnemonic: "Complete the square to find the center's pair"

Q.3(B) [8 marks]

Attempt any two

Q3.1 [4 marks]

Plot the graph of $y = 2\sin\frac{x}{3}, 0 < x \leq 3\pi$

Solution:
For the function $y = 2\sin\frac{x}{3}$:

Table: Key Properties

Property	Value
Amplitude	2
Period	$2\pi \div \frac{1}{3} = 6\pi$
Frequency	$\frac{1}{3}$

Key Points Table:

x	$\frac{x}{3}$	$\sin\frac{x}{3}$	$y = 2\sin\frac{x}{3}$
0	0	0	0
$\frac{3\pi}{2}$	$\frac{\pi}{2}$	1	2
3π	π	0	0

```
      y
      |
  2 +       *
      |     / \
  1 +    /    \
      |  /       \
  0 +--+-----*--------> x
      0   π/2    π    3π/2   3π
   -1 +
      |
  -2 +
```

The graph shows one complete cycle from 0 to 3π with amplitude 2.

Q3.2 [4 marks]

Prove that $\tan^{-1}\frac{2}{3} + \tan^{-1}\frac{10}{11} + \tan^{-1}\frac{1}{4} = \frac{\pi}{2}$

Solution:
Let $\alpha = \tan^{-1}\frac{2}{3}, \beta = \tan^{-1}\frac{10}{11}, \gamma = \tan^{-1}\frac{1}{4}$

We need to prove: $\alpha + \beta + \gamma = \frac{\pi}{2}$

This is equivalent to proving: $\tan(\alpha + \beta + \gamma) = \infty$

Using the formula: $\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$

First, find $\tan(\alpha + \beta)$:
$\tan(\alpha + \beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta} = \frac{\frac{2}{3} + \frac{10}{11}}{1 - \frac{2}{3}\cdot\frac{10}{11}}$

$= \frac{\frac{22+30}{33}}{1 - \frac{20}{33}} = \frac{\frac{52}{33}}{\frac{13}{33}} = \frac{52}{13} = 4$

Now find $\tan(\alpha + \beta + \gamma)$:
$\tan(\alpha + \beta + \gamma) = \frac{\tan(\alpha+\beta) + \tan\gamma}{1 - \tan(\alpha+\beta)\tan\gamma}$

$= \frac{4 + \frac{1}{4}}{1 - 4\cdot\frac{1}{4}} = \frac{\frac{17}{4}}{1 - 1} = \frac{\frac{17}{4}}{0} = \infty$

Since $\tan(\alpha + \beta + \gamma) = \infty$, we have $\alpha + \beta + \gamma = \frac{\pi}{2}$

Q3.3 [4 marks]

$\vec{a} = 2\hat{i} - \hat{j}$ **and** $\vec{b} = \hat{i} + 3\hat{j} - 2\hat{k}$ **then obtain** $|(\vec{a} + \vec{b}) \times (\vec{a} - \vec{b})|$

Answer:

Solution:
Given: $\vec{a} = 2\hat{i} - \hat{j}, \vec{b} = \hat{i} + 3\hat{j} - 2\hat{k}$

First, let's complete $\vec{a}$: $\vec{a} = 2\hat{i} - \hat{j} + 0\hat{k}$

$\vec{a} + \vec{b} = (2+1)\hat{i} + (-1+3)\hat{j} + (0-2)\hat{k} = 3\hat{i} + 2\hat{j} - 2\hat{k}$

$\vec{a} - \vec{b} = (2-1)\hat{i} + (-1-3)\hat{j} + (0+2)\hat{k} = \hat{i} - 4\hat{j} + 2\hat{k}$

Now, $(\vec{a} + \vec{b}) \times (\vec{a} - \vec{b})$:

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 2 & -2 \\ 1 & -4 & 2 \end{vmatrix}$$

$= \hat{i}(2 \cdot 2 - (-2)(-4)) - \hat{j}(3 \cdot 2 - (-2)(1)) + \hat{k}(3(-4) - 2(1))$

$= \hat{i}(4-8) - \hat{j}(6+2) + \hat{k}(-12-2)$

$= -4\hat{i} - 8\hat{j} - 14\hat{k}$

$|(\vec{a} + \vec{b}) \times (\vec{a} - \vec{b})| = \sqrt{(-4)^2 + (-8)^2 + (-14)^2}$

$= \sqrt{16 + 64 + 196} = \sqrt{276} = 2\sqrt{69}$

Q.4(A) [6 marks]

Attempt any two

Q4.1 [3 marks]

Find $(10\hat{i} + 2\hat{j} + 3\hat{k}) \cdot [(\hat{i} - 2\hat{j} + 2\hat{k}) \times (3\hat{i} - 2\hat{j} - 2\hat{k})]$

Solution:
Let $\vec{A} = 10\hat{i} + 2\hat{j} + 3\hat{k}$
Let $\vec{B} = \hat{i} - 2\hat{j} + 2\hat{k}$
Let $\vec{C} = 3\hat{i} - 2\hat{j} - 2\hat{k}$

We need to find $\vec{A} \cdot (\vec{B} \times \vec{C})$

This is a scalar triple product, which can be calculated as:

$$\vec{A} \cdot (\vec{B} \times \vec{C}) = \begin{vmatrix} 10 & 2 & 3 \\ 1 & -2 & 2 \\ 3 & -2 & -2 \end{vmatrix}$$

Expanding along the first row:

$$= 10 \begin{vmatrix} -2 & 2 \\ -2 & -2 \end{vmatrix} - 2 \begin{vmatrix} 1 & 2 \\ 3 & -2 \end{vmatrix} + 3 \begin{vmatrix} 1 & -2 \\ 3 & -2 \end{vmatrix}$$

$= 10[(-2)(-2) - (2)(-2)] - 2[(1)(-2) - (2)(3)] + 3[(1)(-2) - (-2)(3)]$

$= 10[4+4] - 2[-2-6] + 3[-2+6]$

$= 10(8) - 2(-8) + 3(4)$

$= 80 + 16 + 12 = 108$

Q4.2 [3 marks]

A particle under the constant forces $(1, 2, 3)$ and $(3, 1, 1)$ is displaced from point $(0, 1, -2)$ to point $(5, 1, 2)$. Calculate the total work done by the particle

Solution:
Work done = $\vec{F} \cdot \vec{d}$ where $\vec{F}$ is the resultant force and $\vec{d}$ is the displacement.

Step 1: Find resultant force
$\vec{F_1} = 1\hat{i} + 2\hat{j} + 3\hat{k}$
$\vec{F_2} = 3\hat{i} + 1\hat{j} + 1\hat{k}$
$\vec{F_{resultant}} = \vec{F_1} + \vec{F_2} = 4\hat{i} + 3\hat{j} + 4\hat{k}$

Step 2: Find displacement
Initial position: $(0, 1, -2)$
Final position: $(5, 1, 2)$
$\vec{d} = (5 - 0)\hat{i} + (1 - 1)\hat{j} + (2 - (-2))\hat{k} = 5\hat{i} + 0\hat{j} + 4\hat{k}$

Step 3: Calculate work done
$W = \vec{F_{resultant}} \cdot \vec{d} = (4\hat{i} + 3\hat{j} + 4\hat{k}) \cdot (5\hat{i} + 0\hat{j} + 4\hat{k})$
$W = 4(5) + 3(0) + 4(4) = 20 + 0 + 16 = 36$ units

Table: Work Calculation

Component	Force	Displacement	Work
x	4	5	20
y	3	0	0
z	4	4	16
Total			**36**

Q4.3 [3 marks]

$5x + 6y + 3 = 0$ and $x - 11y + 7 = 0$ are two intersecting lines find the angle between them

Answer:

Solution:
For lines $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$, the angle between them is:
$\tan\theta = \left|\frac{a_1b_2 - a_2b_1}{a_1a_2 + b_1b_2}\right|$

Line 1: $5x + 6y + 3 = 0 \rightarrow a_1 = 5, b_1 = 6$
Line 2: $x - 11y + 7 = 0 \rightarrow a_2 = 1, b_2 = -11$

$\tan\theta = \left|\frac{5(-11) - 1(6)}{5(1) + 6(-11)}\right|$

$= \left|\frac{-55-6}{5-66}\right| = \left|\frac{-61}{-61}\right| = 1$

Therefore: $\theta = \tan^{-1}(1) = 45^\circ$

Mnemonic: "Lines that intersect at forty-five, make slopes that multiply to negative one to stay alive"

Q.4(B) [8 marks]

Attempt any two

Q4.1 [4 marks]

Find the unit vector perpendicular to $\vec{a} = (1, -1, 1)$ and $\vec{b} = (2, 3, -1)$

Solution:
A vector perpendicular to both $\vec{a}$ and $\vec{b}$ is $\vec{a} \times \vec{b}$.

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 1 \\ 2 & 3 & -1 \end{vmatrix}$$

$$= \hat{i}[(-1)(-1) - (1)(3)] - \hat{j}[(1)(-1) - (1)(2)] + \hat{k}[(1)(3) - (-1)(2)]$$

$$= \hat{i}[1 - 3] - \hat{j}[-1 - 2] + \hat{k}[3 + 2]$$

$$= -2\hat{i} + 3\hat{j} + 5\hat{k}$$

Magnitude: $|\vec{a} \times \vec{b}| = \sqrt{(-2)^2 + 3^2 + 5^2} = \sqrt{4 + 9 + 25} = \sqrt{38}$

Unit vector: $\hat{n} = \frac{\vec{a} \times \vec{b}}{|\vec{a} \times \vec{b}|} = \frac{-2\hat{i} + 3\hat{j} + 5\hat{k}}{\sqrt{38}}$

$$\hat{n} = \frac{-2}{\sqrt{38}}\hat{i} + \frac{3}{\sqrt{38}}\hat{j} + \frac{5}{\sqrt{38}}\hat{k}$$

Q4.2 [4 marks]

Prove that angle between vectors $3\hat{i} + \hat{j} + 2\hat{k}$ and $2\hat{i} - 2\hat{j} + 4\hat{k}$ is $\sin^{-1} \frac{2}{\sqrt{7}}$

Solution:
Let $\vec{A} = 3\hat{i} + \hat{j} + 2\hat{k}$ and $\vec{B} = 2\hat{i} - 2\hat{j} + 4\hat{k}$

Step 1: Calculate dot product
$\vec{A} \cdot \vec{B} = 3(2) + 1(-2) + 2(4) = 6 - 2 + 8 = 12$

Step 2: Calculate magnitudes
$|\vec{A}| = \sqrt{3^2 + 1^2 + 2^2} = \sqrt{9 + 1 + 4} = \sqrt{14}$
$|\vec{B}| = \sqrt{2^2 + (-2)^2 + 4^2} = \sqrt{4 + 4 + 16} = \sqrt{24} = 2\sqrt{6}$

Step 3: Find cosine of angle
$\cos\theta = \frac{\vec{A} \cdot \vec{B}}{|\vec{A}||\vec{B}|} = \frac{12}{\sqrt{14} \cdot 2\sqrt{6}} = \frac{12}{2\sqrt{84}} = \frac{6}{2\sqrt{21}} = \frac{3}{\sqrt{21}}$

Step 4: Find sine of angle
$\sin^2\theta = 1 - \cos^2\theta = 1 - \frac{9}{21} = \frac{12}{21} = \frac{4}{7}$

$\sin\theta = \frac{2}{\sqrt{7}}$

Therefore: $\theta = \sin^{-1} \frac{2}{\sqrt{7}}$

Q4.3 [4 marks]

Find the Limit of $\lim_{x \to -1} \frac{2x^3+5x^2+4x+1}{3x^3+5x^2+x-1}$

Solution:
First, let's check if direct substitution works:
At $x = -1$:
Numerator: $2(-1)^3 + 5(-1)^2 + 4(-1) + 1 = -2 + 5 - 4 + 1 = 0$
Denominator: $3(-1)^3 + 5(-1)^2 + (-1) - 1 = -3 + 5 - 1 - 1 = 0$

Since we get $\frac{0}{0}$ form, we need to factor both polynomials.

Factoring the numerator: $2x^3 + 5x^2 + 4x + 1$
Since $x = -1$ is a root, $(x + 1)$ is a factor.
Using polynomial division: $2x^3 + 5x^2 + 4x + 1 = (x + 1)(2x^2 + 3x + 1)$
Further factoring: $2x^2 + 3x + 1 = (2x + 1)(x + 1)$
So: $2x^3 + 5x^2 + 4x + 1 = (x + 1)^2(2x + 1)$

Factoring the denominator: $3x^3 + 5x^2 + x - 1$
Since $x = -1$ is a root, $(x + 1)$ is a factor.
Using polynomial division: $3x^3 + 5x^2 + x - 1 = (x + 1)(3x^2 + 2x - 1)$
Further factoring: $3x^2 + 2x - 1 = (3x - 1)(x + 1)$
So: $3x^3 + 5x^2 + x - 1 = (x + 1)^2(3x - 1)$

Therefore:
$\lim_{x \to -1} \frac{2x^3+5x^2+4x+1}{3x^3+5x^2+x-1} = \lim_{x \to -1} \frac{(x+1)^2(2x+1)}{(x+1)^2(3x-1)}$
$= \lim_{x \to -1} \frac{2x+1}{3x-1} = \frac{2(-1)+1}{3(-1)-1} = \frac{-1}{-4} = \frac{1}{4}$

Q.5(A) [6 marks]

Attempt any two

Q5.1 [3 marks]

Find the Limit of $\lim_{x \to 1} \frac{\sqrt{x+7}-\sqrt{3x+5}}{\sqrt{3x+5}-\sqrt{5x+3}}$

Solution:
At $x = 1$:
Numerator: $\sqrt{1+7} - \sqrt{3+5} = \sqrt{8} - \sqrt{8} = 0$
Denominator: $\sqrt{3+5} - \sqrt{5+3} = \sqrt{8} - \sqrt{8} = 0$

We have $\frac{0}{0}$ form. We'll rationalize both numerator and denominator.

Rationalizing the numerator:
$\sqrt{x+7} - \sqrt{3x+5} = \frac{(\sqrt{x+7}-\sqrt{3x+5})(\sqrt{x+7}+\sqrt{3x+5})}{\sqrt{x+7}+\sqrt{3x+5}}$
$= \frac{(x+7)-(3x+5)}{\sqrt{x+7}+\sqrt{3x+5}} = \frac{x+7-3x-5}{\sqrt{x+7}+\sqrt{3x+5}} = \frac{-2x+2}{\sqrt{x+7}+\sqrt{3x+5}}$

Rationalizing the denominator:
$\sqrt{3x+5}-\sqrt{5x+3}=\frac{(\sqrt{3x+5}-\sqrt{5x+3})(\sqrt{3x+5}+\sqrt{5x+3})}{\sqrt{3x+5}+\sqrt{5x+3}}$

$=\frac{(3x+5)-(5x+3)}{\sqrt{3x+5}+\sqrt{5x+3}}=\frac{3x+5-5x-3}{\sqrt{3x+5}+\sqrt{5x+3}}=\frac{-2x+2}{\sqrt{3x+5}+\sqrt{5x+3}}$

Therefore:
$\lim_{x\to1}\frac{\sqrt{x+7}-\sqrt{3x+5}}{\sqrt{3x+5}-\sqrt{5x+3}}=\lim_{x\to1}\frac{\frac{-2x+2}{\sqrt{x+7}+\sqrt{3x+5}}}{\frac{-2x+2}{\sqrt{3x+5}+\sqrt{5x+3}}}$

$=\lim_{x\to1}\frac{-2x+2}{\sqrt{x+7}+\sqrt{3x+5}}\times\frac{\sqrt{3x+5}+\sqrt{5x+3}}{-2x+2}$

$=\lim_{x\to1}\frac{\sqrt{3x+5}+\sqrt{5x+3}}{\sqrt{x+7}+\sqrt{3x+5}}$

Substituting $x=1$:
$=\frac{\sqrt{8}+\sqrt{8}}{\sqrt{8}+\sqrt{8}}=\frac{2\sqrt{8}}{2\sqrt{8}}=1$

Q5.2 [3 marks]

Find the Limit of $\lim_{x\to0}\frac{\cos(ax)-\cos(bx)}{x^2}$

Solution:
Using the identity: $\cos A-\cos B=-2\sin(\frac{A+B}{2})\sin(\frac{A-B}{2})$

$\cos(ax)-\cos(bx)=-2\sin(\frac{ax+bx}{2})\sin(\frac{ax-bx}{2})$

$=-2\sin(\frac{(a+b)x}{2})\sin(\frac{(a-b)x}{2})$

Therefore:
$\lim_{x\to0}\frac{\cos(ax)-\cos(bx)}{x^2}=\lim_{x\to0}\frac{-2\sin(\frac{(a+b)x}{2})\sin(\frac{(a-b)x}{2})}{x^2}$

$=-2\lim_{x\to0}\frac{\sin(\frac{(a+b)x}{2})}{x}\times\frac{\sin(\frac{(a-b)x}{2})}{x}$

$=-2\lim_{x\to0}\frac{\sin(\frac{(a+b)x}{2})}{\frac{(a+b)x}{2}}\times\frac{(a+b)}{2}\times\frac{\sin(\frac{(a-b)x}{2})}{\frac{(a-b)x}{2}}\times\frac{(a-b)}{2}$

Using $\lim_{u\to0}\frac{\sin u}{u}=1$:

$=-2\times1\times\frac{(a+b)}{2}\times1\times\frac{(a-b)}{2}=-2\times\frac{(a+b)(a-b)}{4}=-\frac{(a^2-b^2)}{2}=\frac{b^2-a^2}{2}$

Q5.3 [3 marks]

Find the Limit of $\lim_{x\to3}\frac{x^3-27}{\sqrt[3]{x}-\sqrt[3]{3}}$

Solution:
Let $u=\sqrt[3]{x}$, then $x=u^3$ and as $x\to3$, $u\to\sqrt[3]{3}$

$\lim_{x\to3}\frac{x^3-27}{\sqrt[3]{x}-\sqrt[3]{3}}=\lim_{u\to\sqrt[3]{3}}\frac{(u^3)^3-27}{u-\sqrt[3]{3}}=\lim_{u\to\sqrt[3]{3}}\frac{u^9-27}{u-\sqrt[3]{3}}$

Since $27=(\sqrt[3]{3})^9$, we have:
$\lim_{u\to\sqrt[3]{3}}\frac{u^9-(\sqrt[3]{3})^9}{u-\sqrt[3]{3}}$

This is of the form $\frac{f(a)-f(b)}{a-b}$ where $f(u)=u^9$, which gives us $f'(\sqrt[3]{3})$.

$f'(u) = 9u^8$
$f'(\sqrt[3]{3}) = 9(\sqrt[3]{3})^8 = 9 \times 3^{8/3} = 9 \times 3^{8/3} = 9 \times (3^2)^{4/3} = 9 \times 9^{4/3} = 9 \times 9 \times 9^{1/3} = 81 \times \sqrt[3]{9}$

Alternative approach using direct factorization:
$x^3 - 27 = x^3 - 3^3 = (x-3)(x^2 + 3x + 9)$

Let $y = \sqrt[3]{x}$, then $x = y^3$:
$\sqrt[3]{x} - \sqrt[3]{3} = y - \sqrt[3]{3}$

Using the identity $a^3 - b^3 = (a-b)(a^2 + ab + b^2)$:
$x - 3 = y^3 - (\sqrt[3]{3})^3 = (y - \sqrt[3]{3})(y^2 + y\sqrt[3]{3} + (\sqrt[3]{3})^2)$

Therefore:
$\lim_{x\to 3} \frac{x^3-27}{\sqrt[3]{x}-\sqrt[3]{3}} = \lim_{x\to 3} \frac{(x-3)(x^2+3x+9)}{\sqrt[3]{x}-\sqrt[3]{3}}$

$= \lim_{x\to 3} \frac{(y-\sqrt[3]{3})(y^2+y\sqrt[3]{3}+(\sqrt[3]{3})^2)(x^2+3x+9)}{y-\sqrt[3]{3}}$

$= \lim_{x\to 3}(y^2 + y\sqrt[3]{3} + (\sqrt[3]{3})^2)(x^2 + 3x + 9)$

At $x = 3$, $y = \sqrt[3]{3}$:
$= ((\sqrt[3]{3})^2 + \sqrt[3]{3} \cdot \sqrt[3]{3} + (\sqrt[3]{3})^2)(3^2 + 3 \cdot 3 + 9)$
$= (3^{2/3} + 3^{2/3} + 3^{2/3})(9 + 9 + 9)$
$= 3 \cdot 3^{2/3} \cdot 27 = 81 \cdot 3^{2/3} = 81\sqrt[3]{9}$

Q.5(B) [8 marks]

Attempt any two

Q5.1 [4 marks]

Find the equation of lines passing through point $A(3\sqrt{3}, 4)$ and making angle $\frac{\pi}{6}$ with line $\sqrt{3}x - 3y + 5 = 0$

Solution:
Given line: $\sqrt{3}x - 3y + 5 = 0$
Rewriting in slope form: $3y = \sqrt{3}x + 5$, so slope $m_1 = \frac{\sqrt{3}}{3} = \frac{1}{\sqrt{3}}$

Let the slope of required lines be m_2.

The angle between two lines with slopes m_1 and m_2 is given by:
$\tan\theta = \left|\frac{m_2-m_1}{1+m_1m_2}\right|$

Given $\theta = \frac{\pi}{6}$, so $\tan\frac{\pi}{6} = \frac{1}{\sqrt{3}}$

$\frac{1}{\sqrt{3}} = \left|\frac{m_2-\frac{1}{\sqrt{3}}}{1+\frac{m_2}{\sqrt{3}}}\right|$

This gives us two cases:

Case 1: $\frac{1}{\sqrt{3}} = \frac{m_2-\frac{1}{\sqrt{3}}}{1+\frac{m_2}{\sqrt{3}}}$

$\frac{1}{\sqrt{3}}(1 + \frac{m_2}{\sqrt{3}}) = m_2 - \frac{1}{\sqrt{3}}$

$\frac{1}{\sqrt{3}} + \frac{m_2}{3} = m_2 - \frac{1}{\sqrt{3}}$

$\frac{2}{\sqrt{3}} = m_2 - \frac{m_2}{3} = \frac{2m_2}{3}$

$m_2 = \frac{2}{\sqrt{3}} \times \frac{3}{2} = \frac{3}{\sqrt{3}} = \sqrt{3}$

Case 2: $\frac{1}{\sqrt{3}} = -\frac{m_2 - \frac{1}{\sqrt{3}}}{1 + \frac{m_2}{\sqrt{3}}}$

Following similar steps: $m_2 = 0$

Equations of the lines:
Using point-slope form with point $(3\sqrt{3}, 4)$:

Line 1 (slope = $\sqrt{3}$): $y - 4 = \sqrt{3}(x - 3\sqrt{3})$
$y - 4 = \sqrt{3}x - 9$
$y = \sqrt{3}x - 5$
or $\sqrt{3}x - y - 5 = 0$

Line 2 (slope = 0): $y - 4 = 0(x - 3\sqrt{3})$
$y = 4$

Q5.2 [4 marks]

Find the equation of circle passing through origin and point $(1, 2)$ and whose center lies on the X-axis

Solution:
Let the center of the circle be $(h, 0)$ since it lies on the X-axis.
Let the radius be r.

The general equation of circle with center (h, k) and radius r is:
$(x - h)^2 + (y - k)^2 = r^2$

Since center is $(h, 0)$: $(x - h)^2 + y^2 = r^2$

Condition 1: Circle passes through origin $(0, 0)$
$(0 - h)^2 + 0^2 = r^2$
$h^2 = r^2$ ... (1)

Condition 2: Circle passes through $(1, 2)$
$(1 - h)^2 + 2^2 = r^2$
$(1 - h)^2 + 4 = r^2$ ... (2)

From equations (1) and (2):
$h^2 = (1 - h)^2 + 4$
$h^2 = 1 - 2h + h^2 + 4$
$0 = 5 - 2h$
$h = \frac{5}{2}$

From equation (1): $r^2 = h^2 = (\frac{5}{2})^2 = \frac{25}{4}$

Table: Circle Properties

Property	Value
Center	$(\frac{5}{2}, 0)$
Radius	$\frac{5}{2}$

Equation of circle:
$(x - \frac{5}{2})^2 + y^2 = \frac{25}{4}$

Expanding: $x^2 - 5x + \frac{25}{4} + y^2 = \frac{25}{4}$
$x^2 + y^2 - 5x = 0$

Q5.3 [4 marks]

Find the equation of lines passing through point $A(-8, -10)$ and product of its intercepts on both axis is -40

Solution:
Let the equation of line be $\frac{x}{a} + \frac{y}{b} = 1$ where a and b are x-intercept and y-intercept respectively.

Given conditions:

1. Line passes through $(-8, -10)$: $\frac{-8}{a} + \frac{-10}{b} = 1$ ... (1)
2. Product of intercepts: $ab = -40$ ... (2)

From equation (2): $b = \frac{-40}{a}$

Substituting in equation (1):
$\frac{-8}{a} + \frac{-10}{\frac{-40}{a}} = 1$

$\frac{-8}{a} + \frac{-10a}{-40} = 1$

$\frac{-8}{a} + \frac{a}{4} = 1$

Multiplying by $4a$:
$-32 + a^2 = 4a$
$a^2 - 4a - 32 = 0$
$(a - 8)(a + 4) = 0$

So $a = 8$ or $a = -4$

Case 1: $a = 8$
$b = \frac{-40}{8} = -5$
Equation: $\frac{x}{8} + \frac{y}{-5} = 1$
$\frac{x}{8} - \frac{y}{5} = 1$
$5x - 8y = 40$

Case 2: $a = -4$
$b = \frac{-40}{-4} = 10$
Equation: $\frac{x}{-4} + \frac{y}{10} = 1$
$\frac{-x}{4} + \frac{y}{10} = 1$
$-10x + 4y = 40$
$10x - 4y + 40 = 0$
$5x - 2y + 20 = 0$

The two equations are:

1. $5x - 8y - 40 = 0$
2. $5x - 2y + 20 = 0$

Mathematics Formula Cheat Sheet

Determinants

- **2×2 Matrix**: $\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$
- **3×3 Matrix**: Expand along any row or column

Logarithms

- $\log_a b \times \log_b a = 1$
- $\log(xy) = \log x + \log y$
- $\log(\frac{x}{y}) = \log x - \log y$
- $\log(x^n) = n \log x$

Trigonometry

- **Basic Values**:
  - $\sin 30^\circ = \frac{1}{2}$, $\cos 30^\circ = \frac{\sqrt{3}}{2}$, $\tan 30^\circ = \frac{1}{\sqrt{3}}$
  - $\sin 60^\circ = \frac{\sqrt{3}}{2}$, $\cos 60^\circ = \frac{1}{2}$, $\tan 60^\circ = \sqrt{3}$
  - $\sin 45^\circ = \cos 45^\circ = \frac{1}{\sqrt{2}}$, $\tan 45^\circ = 1$
- **Compound Angles**:
  - $\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$
  - $\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$
  - $\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$
- **Multiple Angles**:
  - $\sin 2A = 2 \sin A \cos A$
  - $\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$

- $\tan 2A = \frac{2\tan A}{1-\tan^2 A}$

- **Half Angles**:
  - $\sin\frac{A}{2} = \pm\sqrt{\frac{1-\cos A}{2}}$
  - $\cos\frac{A}{2} = \pm\sqrt{\frac{1+\cos A}{2}}$
  - $\tan\frac{A}{2} = \frac{1-\cos A}{\sin A} = \frac{\sin A}{1+\cos A}$
- **Sum-to-Product**:
  - $\sin A + \sin B = 2\sin(\frac{A+B}{2})\cos(\frac{A-B}{2})$
  - $\sin A - \sin B = 2\cos(\frac{A+B}{2})\sin(\frac{A-B}{2})$
  - $\cos A + \cos B = 2\cos(\frac{A+B}{2})\cos(\frac{A-B}{2})$
  - $\cos A - \cos B = -2\sin(\frac{A+B}{2})\sin(\frac{A-B}{2})$
- **Allied Angles**:
  - $\sin(90^\circ - \theta) = \cos\theta$
  - $\cos(90^\circ - \theta) = \sin\theta$
  - $\sin(90^\circ + \theta) = \cos\theta$
  - $\cos(90^\circ + \theta) = -\sin\theta$
  - $\sin(180^\circ - \theta) = \sin\theta$
  - $\cos(180^\circ - \theta) = -\cos\theta$

Vectors

- **Dot Product**: $\vec{a}\cdot\vec{b} = |\vec{a}||\vec{b}|\cos\theta = a_1b_1 + a_2b_2 + a_3b_3$
- **Cross Product**: $\vec{a}\times\vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$
- **Magnitude**: $|\vec{a}| = \sqrt{a_1^2 + a_2^2 + a_3^2}$
- **Unit Vector**: $\hat{a} = \frac{\vec{a}}{|\vec{a}|}$
- **Angle between vectors**: $\cos\theta = \frac{\vec{a}\cdot\vec{b}}{|\vec{a}||\vec{b}|}$
- **Scalar Triple Product**: $\vec{a}\cdot(\vec{b}\times\vec{c}) = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$

Coordinate Geometry

Straight Lines

- **Slope**: $m = \frac{y_2-y_1}{x_2-x_1}$
- **Point-Slope Form**: $y - y_1 = m(x - x_1)$

- **Two-Point Form**: $\frac{y-y_1}{y_2-y_1} = \frac{x-x_1}{x_2-x_1}$
- **Slope-Intercept Form**: $y = mx + c$
- **Intercept Form**: $\frac{x}{a} + \frac{y}{b} = 1$
- **General Form**: $Ax + By + C = 0$

Parallel and Perpendicular Lines

- **Parallel Lines**: $m_1 = m_2$
- **Perpendicular Lines**: $m_1 \times m_2 = -1$
- **Angle between lines**: $\tan\theta = \left|\frac{m_1-m_2}{1+m_1m_2}\right|$

Circle

- **Standard Form**: $(x-h)^2 + (y-k)^2 = r^2$
- **General Form**: $x^2 + y^2 + 2gx + 2fy + c = 0$
- **Center**: $(-g, -f)$
- **Radius**: $\sqrt{g^2 + f^2 - c}$

Limits

- **Standard Limits**:
  - $\lim_{x\to 0} \frac{\sin x}{x} = 1$
  - $\lim_{x\to 0} \frac{\tan x}{x} = 1$
  - $\lim_{x\to 0} \frac{1-\cos x}{x^2} = \frac{1}{2}$
  - $\lim_{n\to\infty}(1 + \frac{1}{n})^n = e$
  - $\lim_{x\to 0}(1 + x)^{1/x} = e$
- **L'Hôpital's Rule**: If $\lim_{x\to a} \frac{f(x)}{g(x)}$ gives $\frac{0}{0}$ or $\frac{\infty}{\infty}$, then: $\lim_{x\to a} \frac{f(x)}{g(x)} = \lim_{x\to a} \frac{f'(x)}{g'(x)}$
- **Algebraic Limits**: For polynomial $\frac{P(x)}{Q(x)}$:
  - If $P(a) \neq 0$ and $Q(a) \neq 0$: Direct substitution
  - If $P(a) = Q(a) = 0$: Factor and cancel common factors
  - For $\frac{\infty}{\infty}$: Divide by highest power

Functions

- **Even Function**: $f(-x) = f(x)$
- **Odd Function**: $f(-x) = -f(x)$
- **Composite Function**: $(f \circ g)(x) = f(g(x))$
- **Inverse Function**: If $y = f(x)$, then $x = f^{-1}(y)$

Useful Algebraic Identities

- $(a+b)^2 = a^2 + 2ab + b^2$
- $(a-b)^2 = a^2 - 2ab + b^2$
- $(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$
- $(a-b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$
- $a^3 + b^3 = (a+b)(a^2 - ab + b^2)$
- $a^3 - b^3 = (a-b)(a^2 + ab + b^2)$
- $a^4 - b^4 = (a^2 + b^2)(a+b)(a-b)$

Conversion Formulas

- **Degrees to Radians**: $\text{Radians} = \text{Degrees} \times \frac{\pi}{180}$
- **Radians to Degrees**: $\text{Degrees} = \text{Radians} \times \frac{180}{\pi}$

Important Angles in Radians

Degrees	Radians
30°	$\frac{\pi}{6}$
45°	$\frac{\pi}{4}$
60°	$\frac{\pi}{3}$
90°	$\frac{\pi}{2}$
120°	$\frac{2\pi}{3}$
135°	$\frac{3\pi}{4}$
150°	$\frac{5\pi}{6}$
180°	π

Differentiation (Basic)

- $\frac{d}{dx}(x^n) = nx^{n-1}$
- $\frac{d}{dx}(\sin x) = \cos x$
- $\frac{d}{dx}(\cos x) = -\sin x$
- $\frac{d}{dx}(\tan x) = \sec^2 x$
- $\frac{d}{dx}(e^x) = e^x$
- $\frac{d}{dx}(\ln x) = \frac{1}{x}$

Problem-Solving Tips

For Determinants

1. Always expand along the row/column with most zeros
2. Factor out common terms first
3. Use row/column operations to create zeros

For Limits

1. Try direct substitution first
2. If you get $\frac{0}{0}$, factor and cancel
3. For square roots, rationalize numerator/denominator
4. Use standard limit formulas

For Trigonometry

1. Convert everything to same angle measure (degrees or radians)
2. Use compound angle formulas for complex expressions
3. Check if angles are special angles (30°, 45°, 60°, etc.)

For Vectors

1. Write vectors in component form: $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$
2. For cross product, use determinant method
3. For dot product, multiply corresponding components and add

For Circle Problems

1. Complete the square to find center and radius
2. Use distance formula: $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
3. Remember: All points on circle are equidistant from center

For Line Problems

1. Find slope first: $m = \frac{y_2 - y_1}{x_2 - x_1}$
2. Use point-slope form: $y - y_1 = m(x - x_1)$
3. For parallel lines: same slope
4. For perpendicular lines: product of slopes = -1

Memory Tips

- **SOHCAHTOA**: Sin = Opposite/Hypotenuse, Cos = Adjacent/Hypotenuse, Tan = Opposite/Adjacent
- **CAST Rule**: In quadrants I, II, III, IV - Cosine, All, Sine, Tangent are positive respectively
- **30-60-90 Triangle**: Sides in ratio $1 : \sqrt{3} : 2$
- **45-45-90 Triangle**: Sides in ratio $1 : 1 : \sqrt{2}$

Common Mistakes to Avoid

1. **Sign errors** in trigonometric identities
2. **Forgetting to rationalize** when dealing with surds in limits
3. **Not checking domain** for inverse trigonometric functions
4. **Mixing up cross product and dot product** formulas
5. **Forgetting to complete the square** properly in circle equations
6. **Not factoring completely** in limit problems

Quick Reference Values

- $\sqrt{2} \approx 1.414$
- $\sqrt{3} \approx 1.732$
- $\pi \approx 3.14159$
- $e \approx 2.718$

Final Tips for Exam Success

Time Management

- Spend 2-3 minutes on each fill-in-the-blank question
- Allocate 8-10 minutes per 3-mark question
- Allow 12-15 minutes per 4-mark question
- Reserve 20-25 minutes per 7-8 mark question

Question Selection Strategy

- Read all options before selecting questions
- Choose questions you're most confident about
- Start with easier questions to build confidence

Presentation Tips

- Show all working steps clearly
- Draw diagrams where applicable
- Use proper mathematical notation
- Box your final answers

Common Topics That Appear Frequently

1. **Trigonometric identities and compound angles**

2. **Limits involving rationalization**
3. **Vector operations (dot and cross products)**
4. **Circle and line equations**
5. **Determinant calculations**

Best of luck with your exams! 🎯

Remember: Practice makes perfect. Work through similar problems multiple times to build speed and accuracy.